ON S – Normal Space and Some Functions

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1-Abstract and introduction

The notion of S-normal Space was introdneed by S.N.Maheshwari and R. Prasad (3) by replacing open set in the definition of normal space by semi-open set fhriefty x open sets due to levine (1).

The Purpose of this work is to find more characterization of S-normal Space by using New class of sets called Generalized Semi Open Set (brielly gs-open set) Where we shall introduce new definition equivalent to the definition of S-normal space by using (gs-open set) instead of (S-open set) also we introduce some preseration theorems conceming S-normal space by using New class of funetions as pre gs-closed and pre gs-contious funetion.

المستخلص – المقدمة

إن مفهوم الفضاء المئوي –S قدم لأول مرة من قبل العالمان S.N.Maheshwari و R.Prasad (3) وذلك بأبدال المجموعة المفتوحة في تعريف الفضاء المئوي بالمجموعة المفتوحة –S والتي قدمت لأول مرة من قبل العالم Norman Levine (1) . أن الغرض الرئيسي من هذا العمل أيجاد خصائص أخرى للفضاء المئوي – S بأستخدام مفهوم المجموعة الجديدة (المفتوحة –gs) حيث ستقوم بتقديم تعريفاً جديداً يكافىء تعريف الفضاء المئوي –S بأستخدام المجموعة المفتوحة –gs بدل المجموعة المفتوحة وكذلك ستقوم بتقديم بعض مبرهنات المحافظة التي تخص الفضاء المئوي – S بأستخدام مفاهيم جديدة من الدوال كمفهوم الدالة المغلقة –gs أولياً والدالة المستمرة –gs أولياً .

2-Preliminaries

In this work we denoted for topaces (X.T) and (Y.T) by X and Y simply . Also we denoted for the elosure . interior and complement of A by (A.A.A) respectively .

2-1 Definition (1) :

A subset A of space X is said to be S-open set if there exist an open set O in X such that O $\subseteq$ A $\subseteq$ O .
Not that every open set is S-open and the converse need not be true – see (1) .

2-2 Definition (5) :

A subset A of space X is said to be Semi-closed set (briefly s-closed set) If there exist a closed set F in X such that : F $\subseteq$ A $\subseteq$ F .

2-3 Remarks :

1- Every closed and the converse need not be true see (8) .
2- A subset A of space X is s-closed set If and only If A is s-open .

2-4 Definition (5) :

I.et A be a subset of space X then we say that for the intersection of all s-closed sets which contain A semi closure of A and denoted by (A) .

2-5 Definition (3) :

Iet A be a subset of space X the semi – interior of A is the Iargest s-open set contained in A and denoted by (A) .

2-6 Definition (6) :

A subset A of space X is said to Semi –generalized closed set (briefly sg-closed set) If A $\subseteq$ U whenever A $\subseteq$ U and U is s-open set in X . and we say that a subset B of a space X is sg –open set If B is sg-closed set .

2-7 Definition (7) :

A subset A of space X is said to be generalized Semi-closed (briefly gs-closed set)

If A c U whenever A c U and U is open set in X .
And we say that a sub set B of space X is gs-open set If B is gs-closed set .Clearly that every s-closed set is sg-closed and every sg-closed set is gs –closed set s-closed ⟶ sg-closed ⟶ gs-closed .

2-8 Remark :

None of the implication in the above diagram is reversible .See (8) .

3. S-normal space

3-1 Definition (3) :

space X is said to be S-normal space if for every disjoint closed sets A and B in X there exist disjoint s-open sets U and V in X such that : A c U and B c V .

3-2 Remark :

It is obvious that every normal space is S-normal and the converse needs not be true . see (8) .
Now we introduce the concept of SO-space which gives us an important condition in the next theorem .

3-3 Definition (8) :

A space X is said to be So-space If every s-open set in X is open

3-4 Theorems :

I et X be S-normal space ,if X is SO-space then X be normal space . Proof :- see (8) .

3-5 Lemma :

A subset A of a space X is gs-open if and only if F c A whenever F c A and F is closed set in X .
proof :- The first side :
Let A be gs-open set in X .
And Iet F c A , where F closed set in X To prove F c A .

since F c A then A c F .
And since A is gs-closed set and F is open .
Then A c F .
But A - (A) . by (7) .
Then (A) c F .
Hence F c A .
The second Side :
Let c A whenever F c A where F is closed .
To prove A is gs-open set In other words we will prove that A is gs-closed .
Let A c U where U is open in X .
Then U c A where U is closed set .
Then by hypothesis U c A . _
Then (A) c U . but (A) - (A) .
Then A c U .
And thus A is gs-closed set . Consequently A is gs-open set .
The next theorem gives defintion equivalent to the definition of S-normal space :-

3-6 Theorem :-

for a space X then the following statements are equivalent :-

1- X is S-normal .

2- For every disjoint closed sets A and B in X there exist disjoint gs-open sets U and V in X such that A c U and B c V .

Proof :-

1) $\longrightarrow$ 2) This is obvious since every open set is gs-open .

2) $\longrightarrow$ 1)

let A and B be any disjoint closed sets in X by (2) , there sxist disjoint gs-open sets U and V in X such that A c U and B c V .
let U1-U and V1 – V .
Then U1 and V1 are disjoint s-open set in X .
And by Lemma (3-5) A c U1 and B c V1 .
And hence a space X is S-normal .

3-7 Theorem :

let X be S-normal space and let Y be closed and open (clopen) sub set in X then the sub space Y is S-normal .

proof :- let A and B be any disjoint closed sets in Y .
Then A and B are disjoint closed sets in X .
Since X is S-normal .
Then there exist disjoint s-open sets U and V in X .
Such that A c U and B c V .
Now let U1 – U ^ Y and V1 – V ^ Y .
Then U1 and V1 are disjoint s-open sets in Y (Proposition 1-1-12 (8))
Such that A c U1 and B c V1 .
And hcnce the subspace Y is S-normal .

4. Some function and preservation theorems

In this section we shall introduce some functions which it will use later .

4-1 Defiinition (2) :

Let X and Y be topological space a function f : X $\longrightarrow$ Y is said to be Pre s-closed function if for cach s-closed set B in X then f (B) s-closed set in Y

4-2 Definition :

let X and Y be topological space function f : X $\longrightarrow$ Y is said to be pre gs-closed if for each s-closed set B of X then f (B) is gs-closed set in Y .

4-3 Proposition :

Every pre S-closed function is pre gs-closed .
Proof : this is obvious since every s-closed set is gs-closed set .

4-4 Remark :

The converse of previous proposition need not be true as the following example:

4-5 example :

let X – (a,b,c) , T – (x,O, :a: :b: :a,b:) be topology on X and let T = (X,O, :a: :a,b:) another topology on X

let f : (X,T') $\longrightarrow$ (X,T") defined as : f (x) = x
Then f is pre gs-closed not pre s-closed function since there exist s-closed set (a,c) of (X,T') such that f (a,c) = (a,c) is not s-closed set in (X,T")

4-6 Definition (8) :

let X and Y be topological space a function f : X $\longrightarrow$ Y is said to be S" continuous if for each s-open set (s-closed set) B of Y then f (B) is s-open (s-closed) set in X .

4-7 Definition (8) :

let X and Y be topological space a function f : X $\longrightarrow$ Y is said to be gs-conunuous If for each s-closed set (s-closed set) B of Y then f ' (B) is gs-closed (gs-open) set in X .

4-8 Proposition :

Every S " –continuous function is pre gs-continuous .
Proof : This is obvious since every s-closed set is gs-closed set .

4-9 Remark :

The converse of previous proposition need not be true. See (8) .
Now we shall introduce some preservation Theorems Concerning S-normal space .

4-10 Lemma :

subjection f : X $\longrightarrow$ Y is pre gs-closed If and only If for each $\subseteq$ Y and each s-open U of X such that f (S) $\subseteq$ U , there exist gs-open set V of Y such that S $\subseteq$ V and f (V) $\subseteq$ U .

4-11 Theorem :

Let f : X $\longrightarrow$ Y subjection . continuous pre gs-closed If X is S-normal then Y be S-normal .
Proof : let A and B any disjoint closed set of Y
Since f is continuous
Then f1 (A) and f4 (B) are disjoint closed set in X
Since X be S-normal

Then there exist disjoint s-open sets U and V in X
Such that f1 (A) c U and f4 (B) c V .
Now by Lemma (4-10)
There exist tow gs-open set G and II in Y Such that A c G and B c H
And f4 (G) c U and f4 (H) c V
Clearly G ^ H - O
Since f4 (G) ^ f4 (II) c U ^ V – O
Then f4 (G) ^ f (H) – O
Then f4 (G ^ H) – O
And since f is subjection
Then G ^ H – O
And by theorem (3 – 6) a space Y is S-normal

4-12 Corollary :

If f : X $\longrightarrow$ Y be surbection pre s-closed , continuous and X be S-normal space then Y be S-normal space .

4-13 Theorem :

let f : X $\longrightarrow$ Y be injection closed pre gs-continuous . If Y is S-normal then X be S-normal .

poof : let A and B be any disjoint closed sets in X
Since is closed and injection
Then f (A) and f (B) are disjoint closed sets in Y
Sins Y be S-normal space
Then there exist disjoint s-open sets U and V in Y
Such that f (A) c U and f (B) c V
Since f is pre gs-continuous
Then f (U) and f (V) are disjoint gs-open sets in X
Such that A c f4 (U) and B c f4 (V)
And By Theorem (3-6)
A space X be S-normal space

4-14 Corollary :

If f : X $\longrightarrow$ Y be injection , S-continuous ,closed and Y be S-normal Then X be S-normal space .

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