Dynamic Analysis of Gough-Stewart Platform Manipulator

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Abstract

A novel derivation to evaluate all the controlled forces which cause by the motors and effected along the prismatic joints on the legs of the Gough-Stewart platform manipulator based on the virtual work method is proposed in this paper. In this paper the manipulator can be considered as a multibody mechanism with rigid elements. It can be assumed that the manipulator motion was known. The aim of the dynamic analysis in this paper is to evaluate all the controlled forces which necessary to implement the manipulator programming motion.

Keywords: Gough-Stewart, Robotics, Dynamic Analysis and Manipulator

الحسابات الديناميكية للمناول نوع كوف – ستيوارت

الخلاصة

تم في هذا البحث استخدام طريقة جديدة واشتقاق جديد لحساب كل القوى المسيطر عليها التي تولدها محركات الروبوت في المفاصل الطولية في ارجل الروبوت نوع ستيوارت. لقد تم اعتماد طريقة احتساب الجهد الافتراضي بعد ان تم اعتبار الروبوت الية ميكانيكية متكونة من عدة اجسام صلبة, وان حركة الروبوت معلومة ما بين نقطتين معلومتين.

لقد تم اعتبار الارجل بدون اية انحناءات جانبية ناتجة من جراء تاثير القوى المختلفة المؤثرة عليها. ان الهدف الرئيسي من هذا البحث هو ايجاد طريقة علمية لاحتساب جميع القوى المسيطر عليها في محركات الروبوت والتي تعتبر ضرورية لانجاز الحركة المبرمجة للروبوت بين نقطتين معلومتين.

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1. Introduction.

In general, Gough-Stewart platform manipulator is a six degree of freedom with two main bodies [3]. The fixed body is called the base, while another body is regarded as movable and is called the moving plate. These two bodies are connected together by six extensible legs. In this paper we assumed that every leg of the legs of the manipulator is consists of two parts connected together with a prismatic kinematic joint (p). The prismatic joints are affected under the controlled forces which cause by the motors. All the legs are connected with the base by spherical kinematic joints (s) in the points A_i , while they are connected with the moving plate by spherical joints with fingers in the points B_i, as shown in Figure1.

Force and moment analysis for any robotic system are useful to choose the suitable motors for implementation the programming motion [5]. Several methods are proposed to derive dynamic equations of the Gough-Stewart manipulator.

The motivation of this paper is to derive a mathematical formulation for evaluation all the controlled forces in the prismatic kinematic joints of the Gough-Stewart platform manipulator.

2. Position equation

In order to describe the motion of the moving plate of the manipulator relative to the base, we assumed that the moving plate attached to the base as shown in Figure 1, so the position vector of the point A_i in the Global coordinate system can be written as:

$$r_{Ai} = \left(x_{Ai}, y_{Ai}, z_{Ai}\right)^T, i = 1,2,..,n;$$

and the position vector of the point B_i in the local coordinates system can be written as:

$$r_{Bi} = \left(x_{Bi}, y_{Bi}, z_{Bi}\right)^T, i = 1,2,...,n.$$

The rotation of transformation between the local (moving plate) and the global coordinate systems can be represented by Euler's rotation matrix with y as pitch angle, q as yaw angle and j as a roll angle [1]. Thus $\mathbf{R_{01}}$ is a matrix of coordinate transformation from the local to the global coordinates can be written as:

$$\mathbf{R}_{01} = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

where

$$A_{11} = \cos(y)\cos(j) - \sin(y)\cos(q)\sin(j),$$

$$A_{12} = \sin(y)\cos(j) + \cos(y)\cos(q)\sin(j),$$

$$A_{13} = \sin(q)\sin(j),$$

$$A_{21} = -\cos(y)\sin(j) - \sin(y)\cos(q)\cos(j),$$

$$A_{22} = -\sin(y)\sin(j) + \cos(y)\cos(q)\cos(j),$$

$$A_{23} = \sin(q)\cos(j),\quad A_{31} = \sin(y)\sin(q),$$

$$A_{32} = -\cos(y)\sin(q), \text{and } A_{33} = \cos(q).$$

Generally, kinematic relations of manipulator are expressed by the loop equations. In Figure 1 one of the loops are represented, and its equation can be written as a leg vector:

$$q_i = l_i = r_0 + R_{01} r_{Bi} - r_{Ai};$$

$$i = 1,2,\dots,n$$

where r_0 the position vector of the moving plate centre in the Global coordinate system,
Thus:

$$q_i^{\,2} = r_0^{\,T} r_0 + r_{Bi}^{\,T} r_{Bi} + r_{Ai}^{\,T} r_{Ai} + 2 r_0^{\,T} R_{01} r_{Bi} - 2 r_{Ai}^{\,T} R_{01} r_{Bi} - 2 r_0^{\,T} r_{Ai};$$
$$i = 1,2,\dots,n,$$

$$\Psi_{\Pi i}(q_i, x, y, z, y, q, j) = r_0^{\,T} r_0 + r_{Bi}^{\,T} r_{Bi} + r_{Ai}^{\,T} r_{Ai} + 2 r_0^{\,T} R_{01} r_{Bi} - 2 r_{Ai}^{\,T} R_{01} r_{Bi} - 2 r_0^{\,T} r_{Ai} - q_i^{\,2} = 0;$$
$$i = 1,..6.n$$

It is assumed that, the geometrical parameters of the moving plate centre can be represented as:

$$r = (x \quad y \quad z \quad y \quad q \quad j)^T,$$

and the legs vectors as:

$$q = (q_1 \quad q_2 \quad . \quad . \quad . \quad q_6)^T.$$

Thus the position equation will be written as:

$$\Psi_\Pi(q, r) = [\Psi_{\Pi 1}(q_1, x, .., j) \quad \Psi_{\Pi 2}(q_2, x, .., j) \quad \mathbf{K} \quad \Psi_{\Pi 6}(q_6, x, .., j)]^T$$

$$\Psi_\Pi(q, r) = 0 \qquad (1)$$

3. Forces and moments effected on the moving plate of the manipulator.

3.1. Gravity force.

It can be considered that $\overline{G}_1$ is the gravity force vector of the moving plate in the Global coordinate system, so its projector on the Global coordinate is $G_1^{(0)} = (0 \quad 0 \quad -m_1 g)^T$ and his projector on the local coordinate system is $G_1^{(1)} = R_{10} G_1^{(0)}$.
There is a moment that caused by the gravity force, its vector relative to the beginning of the local coordinate system can be given as:

$$\overline{M}_{01}\{G_1\} = \bar{r}_{C1} \times \overline{G}_1 \qquad (2)$$

3.2. Working force.

It can be assumed that the moving plate affected under a working force with a vector $\overline{P}_H$. This force will be caused a moment with a vector relative to the beginning of the local coordinate system $\overline{M}_{01}\{\overline{P}_H\}$.

3.3. Controlled forces(motors forces).

Every leg of the legs of the manipulator affected under a controlled force which effected along the prismatic joint (along the leg). This force causes by the motor Q_i

4. Inertia forces and moments evaluation.

The formulation of the inertia force of the moving plate of the manipulator can be written as:

$$\Phi_1 = -m_1 W_{C1} \qquad (3)$$

where W_{C1} is the vector projector of the acceleration of the moving plate centre on the local coordinate system [7]. Inertia moment of the moving plate relative to the beginning of the local coordinate system can be obtained as:

$$\bar{L}_{01} = -\bar{\bar{J}}_{C1} \cdot \bar{e}_1 - \bar{w}_1 \times \bar{\bar{J}}_{C1} \cdot \bar{w}_1 + \bar{r}_{C1} \times \bar{\Phi}_1 \tag{4}$$

In the above equation $\bar{\bar{J}}_{C1}$ - inertia tenser of the moving plate relative to its mass centre; $\bar{w}_1$ - moving plate angular velocity; $\bar{e}_1$ - moving plate angular acceleration and $\bar{r}_{C1}$ - radius vector of the moving plate centre. Thus the projector of the inertia moment on the local coordinate system can be written as:

$$L_{01} = -J_{C1} \cdot e_1 - \tilde{w}_1 J_{C1} w_1 + \tilde{r}_{C1} \times \Phi_1 . \tag{5}$$

where

$$\tilde{r}_{C1} = \begin{pmatrix} 0 & -z_{C1} & y_{C1} \\ z_{C1} & 0 & -x_{C1} \\ -y_{C1} & x_{C1} & 0 \end{pmatrix},$$

$$\tilde{w}_1 = \begin{pmatrix} 0 & -w_Z & w_y \\ w_z & 0 & w_x \\ -w_y & w_x & 0 \end{pmatrix}.$$

Similarly the inertia forces and moments for the legs of the manipulator can be written as:

$$\Phi_S = -m_S W_{CS},$$

and,

$$\bar{L}_{0S} = -\bar{\bar{J}}_{CS} \cdot \bar{e}_S - \bar{w}_S \times \bar{\bar{J}}_{CS} \cdot \bar{w}_S + \bar{r}_{CS} \times \bar{\Phi}_S$$

But we assumed that the mass, inertia forces and moments of the legs is very little, so we will neglect them in the evaluations of the present work.

5. Virtual work evaluation.

The moving plate of the manipulator affected under the total force which consists of its weight, working force and inertia force. The total force vector is

$$\bar{P}_1 = \bar{G}_1 + \bar{P}_H + \bar{\Phi}_1 . \tag{6}$$

The total force causes a total moment relative to beginning of the local coordinate system as follow:

$$\bar{M}_{01} = \bar{M}_{01}\{\bar{G}_1\} + \bar{M}_{01}\{\bar{P}_H\} + \bar{L}_{01} \tag{7}$$

Thus the summation of the virtual work of the moving plate by any little transformation and orientation of the moving plate can be obtained as:

$$dA_1 = \bar{P}_1 \cdot d\bar{r}_{C1} + \bar{M}_{01} \cdot d\bar{g}_1 \tag{8}$$

In the above equation $d\bar{r}_{c1}$ - a moving plate centre little translation, $d\bar{g}_1$ - a moving plate little rotations and it can be obtained as follow:

From Figure 2 it can be assumed that the moving plate angular orientation relative to Global coordinate by using Euler's angles dy_1, dq_1, dj_1 . The angular orientation can be written as:

$$d\bar{g}_1 = d\overline{y}_1 + d\overline{q}_1 + d\overline{j}_1 \\ = \bar{k}_0 d\overline{y}_1 + \bar{n}dq_1 + \bar{k}_1 dj_1 , \qquad (9)$$

Equation (9) can be written in a following form:

$$dg_1 = \Gamma_1 d\Delta_1 \qquad (10)$$

In the above equation $d\Delta_1 = \begin{pmatrix} dy_1 \\ dq_1 \\ dj_1 \end{pmatrix}$,

$$\Gamma_1 = \begin{pmatrix} \sin j_1 \sin q_1 & \cos j_1 & 0 \\ \sin q_1 \cos j_1 & -\sin j_1 & 0 \\ \cos q_1 & 0 & 1 \end{pmatrix}.$$

Thus:

$$d\bar{g}_1 = \bar{i}(dq_1 \cos j_1 + dy_1 \sin j_1 \sin q_1) \\ + \bar{j}_1(-dq_1 \sin j_1 + dy_1 \sin q_1 \cos j_1) \\ + \bar{k}_1(dy_1 \cos q_1 + dj_1)$$

Now, the virtual work can be derived in a new relation as follow:

$$dA_1 = P_1^T dr_{C1} + M_{01}^T \Gamma_1 d\Delta_1 \qquad (11)$$

If we assume that all the forces and moments effected on the moving plate can be written as:

$$F_1 = (P_{x1} \quad P_{y1} \quad P_{z1} \quad M_{01x} \quad M_{01y} \quad M_{01z})^T,$$

$$K_1 = \begin{pmatrix} E_3 & 0 \\ 0 & \Gamma_1 \end{pmatrix}$$

and the geometrical parameters of the moving plate centre can be represented as:

$$r = (x_{c1} \quad y_{c1} \quad z_{c1} \quad y_1 \quad q_1 \quad j_1)^T$$

the virtual work will be written as:

$$dA_1 = F_1^T K_1 dr \qquad (12)$$

6. The controlled forces evaluation.

In the static equilibrium condition the virtual work represented in equation (12) equal to virtual work which causes by the controlled forces which can given in the following form [9] :

$$dA_1 = Q^T dq \qquad (13)$$

Thus:

$$Q = \left(\frac{dr}{dq}\right)^T K_1^T F_1 \qquad (14)$$

From equation (1) we can derive the following relation:

$$\frac{\partial \Psi_\Pi}{\partial q} dq + \frac{\partial \Psi_\Pi}{\partial r} dr = 0 ,$$

and $\dfrac{dr}{dq} = -\left(\dfrac{\partial \Psi_\Pi}{\partial r}\right)^{-1} \left(\dfrac{\partial \Psi_\Pi}{\partial q}\right).$

Where

$$\frac{\partial \Psi_\Pi}{\partial q} = \begin{pmatrix} \dfrac{\partial \Psi_{\Pi 1}}{\partial q_1} & \dots & \dfrac{\partial \Psi_{\Pi 1}}{\partial q_6} \\ \dots & \dots & \dots \\ \dfrac{\partial \Psi_{\Pi 6}}{\partial q_1} & \dots & \dfrac{\partial \Psi_{\Pi 6}}{\partial q_6} \end{pmatrix}, \\ = diag\{2q_i\}$$

and,

$$\frac{\partial \Psi_{\Pi}}{\partial r} = \begin{pmatrix} \frac{\partial \Psi_{\Pi 1}}{\partial x_1} & \cdots & \frac{\partial \Psi_{\Pi 1}}{\partial j_1} \\ \cdots & \cdots & \cdots \\ \frac{\partial \Psi_{\Pi 6}}{\partial x_1} & \cdots & \frac{\partial \Psi_{\Pi 6}}{\partial j_1} \end{pmatrix}$$

Thus the controlled forces (14) can be formulated as follows:

$$Q = \left[\left(\frac{\partial \Psi_{\Pi}}{\partial r} \right)^{-1} \left(\frac{\partial \Psi_{\Pi}}{\partial q} \right) \right]^T K_1^T F \quad (15)$$

From the above equation can evaluate all the controlled forces which effected on the prismatic joints of the manipulator.

7. Example of analysis:

In this example a manipulator with the following parameters is considered:

a. Coordinates of points A_i of the base (in the Global coordinates system) are as in the matrix:

$$r_{Ai} = \begin{pmatrix} 2 & 1.414 & 0 & -1.414 & -2 & -1.414 & 0 & 1.414 \\ 0 & 1.414 & 2 & 1.414 & 0 & -1.414 & -2 & -1.414 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

.

b. Coordinates of points B_i in the platform coordinates systems are as in the matrix:

$$r_{Bi} = \begin{pmatrix} 1.6 & 1.45 & -0.278 & -0.917 & -1.575 & -1.31 & 0.278 & 0.917 \\ 0 & 0.676 & 1.575 & 1.31 & -0.278 & -0.917 & -1.575 & -1.31 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

c. It can be assumed that the platform moves from the point 1 to the point 6 throw the track points 2,3,4,5. The coordinates of the center of the platform P (r_0) and Euler angles of its *orientation* (y, q, j) *are as shown in Table1*.

d. At any point of the track it can be found that:

$$q_i = r_0 + A_{0,1} r_{Bi} - r_{Ai} \,; i = 1,2,...,6$$

.

The results for all points are as explained in Table 2.

e. The platform mass is m = 15Kg, tenser of platform Inertia is

$$J = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix} \text{Kg m}^2$$

and the forces and moments vector which applied to the platform is:

$$P = (0.5N \quad 0.25N \quad -0.75N \quad 0.5N.m \quad 1N.m \quad 1.25N.m)^T$$

.

f. The movement of the platform throw the track points (1,2,3,4,5,6) with acceleration and angular velocity of point P as are follow:

$$\ddot{x} = \begin{bmatrix} 0.1 m/s^2 \\ 0.01 m/s^2 \\ 0.002 m/s^2 \\ 0.15 \deg/s^2 \\ 0.1 \deg/s^2 \\ 0.01 \deg/s^2 \end{bmatrix}$$

And $$w = \begin{bmatrix} 0.8\deg/s \\ 0.16\deg/s \\ 0.01\deg/s \end{bmatrix}$$

.The results of the motorized forces are as shown in the Table 3. In this table it can be found that there is a singularity position in the point 4, it can be seen that, the forces are very high in this position.

8. Conclusions

In this paper, an approach for the dynamic analysis of the Gough-Stewart platform manipulator is proposed based on the virtual work. The mathematical simulation for the robot arms in this paper is a novel method and it has been derive by the authors. The stiffness of the arms caused by the motion of the platform was neglected. The joints of the manipulator's arms have been assumed as ideal joints (without friction forces). It has been proved that all the controlled forces can be evaluated without using the reactions in the kinematic joints

9. References

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Table (1) Platform center coordinates and its angles of orientation

point / coord.	1	2	3	4	5	6
X (m)	0.25	0.5	0.85	1	1.2	1.4
Y (m)	-0.2	-0.45	-0.8	-1	-1.3	-1.5
Z (m)	2.2	2.35	2.7	3	3.2	3.5
ψ^o	-10	-20	-30	-30	-40	-45
θ^o	12	18	35	30	35	40
φ^o	8	15	26	31.95832	35	40

Table (2) Legs length

point / q_i in (m)	1	2	3	4	5	6
q1	2.2664	2.5530	3.2884	3.6198	4.1003	4.6117
q2	2.6035	3.0504	3.9605	4.2101	4.7743	5.2914
q3	2.5986	2.9654	3.7631	3.9677	4.3756	4.7763
q4	2.5846	2.9322	3.5634	3.7983	4.1697	4.5303
q5	2.2393	2.3956	2.6738	3.1315	3.3431	3.6687
q6	2.0335	2.0585	2.1544	2.6063	2.7686	3.0855

Table (3) Controlled forces

Force In (N) \ point	1	2	3	4	5	6
Q_1	-1.832	-2.760	12.306	4.58E7	10.507	5.258
Q_2	0.057	1.082	-13.461	-4.26E7	-12.351	-7.239
Q_3	3.943	4.478	-4.062	-2.49E7	-0.546	2.234
Q_4	-3.822	-4.945	9.763	3.85E7	4.538	-0.069
Q_5	-0.867	-0.874	3.758	2.34E7	6.022	3.860
Q_6	1.640	2.133	-9.906	-4.09E7	-9.394	-5.173
$\sum Q^2 (N^2)$	36.974	58.619	556.73	8.25E15	408.367	126.723

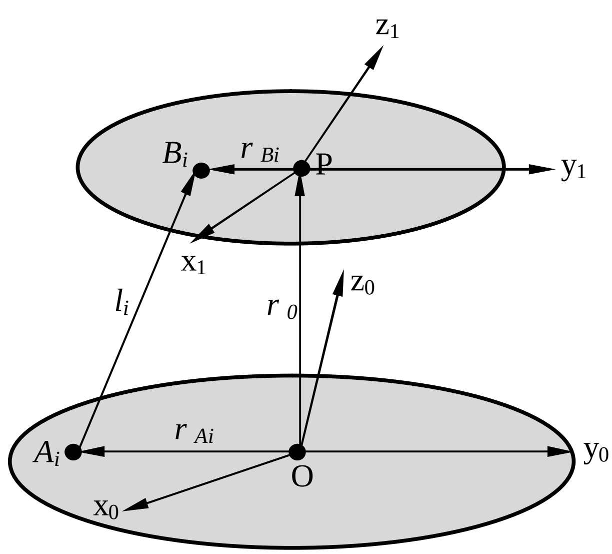

Figure (1) Position of the robot joints

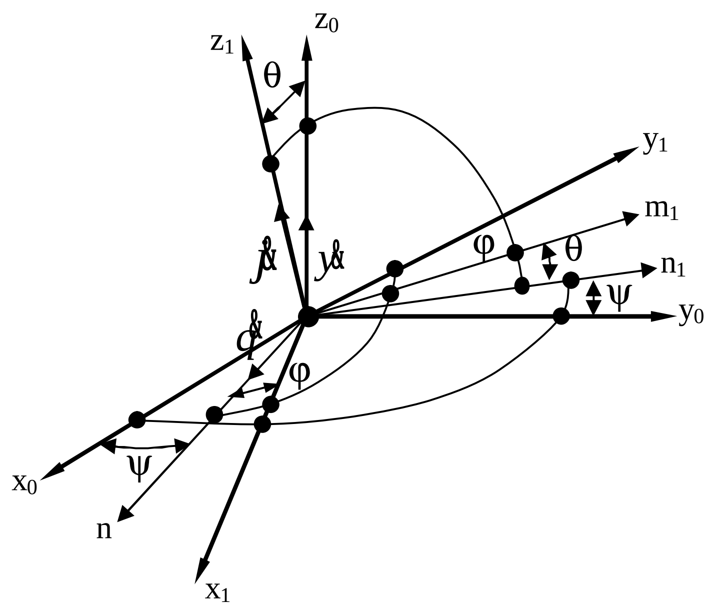

Figure (2) The rotation of the axis