Vibration Response of Free and Force Analysis of a Rectangular Plate

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Submission date:- 8/4/2018	Acceptance date:-11/6/2018	Publication date:-27/9/2018

Abstract

This paper presents the influence of free and forced vibrations on thin the plate of (50cm length, 30 cm width, 0.5 mm thickness), made from steel 37 with two types of supports, firstly when it is fixed from two sides and other sides are free (CFCF) and secondly when one end of the plates is fixed and the other three sides are free (CFFF). As well as, this work studies the mode shapes and natural frequency of the plate. When the frequency is applied on the plate with different magnitudes (4,6,8,10) Hz, the amplitude of vibration decreased more than free vibration and when the frequency increased, the amplitude of the vibration increased or decreased but still less then at free vibration depending on the type of support. Ansys and MATLAB software were used to find the natural frequency and to enter the boundary conditions of the plate for comparison purposes.

Keywords: Boundary conditions, Rectangular cantilever thin plate, Simple supported plate, and Natural frequency.

1- Introduction:

A plate is a structural element, which is flat and thin. By the word “thin,” it’s meant that the transverse dimension or thickness of plate is small as compared with its width and length dimensions. The mathematical formula of such idea is (t /L<1) if the (t /L<1) the plate is thin and if (t /L>1) the plate is thick, **[1]**.

The vibration study of plates is a highly significant important area due to its broad diversity of engineering uses, like in aeronautical, mechanical and civil engineering. Because the members, such as plates, shells and beams are the integral parts of structures, it is necessary for design engineer to possess a previous knowledge of some modes of vibration features prior finishing the design of a specific structure.

Particularly, plates having various forms, boundary conditions at their edges, and different complicating influences, have frequently found uses in various structures, like machine design, aerospace, nuclear reactor technology, telephone industry, earthquake-resistant structures, and naval structures.

A plate is defined as a solid part limited by two parallel and flat surfaces that have two dimensions greatly larger than the third. The plate’s vibration is an old subject in which too much work has already been performed in the last decades. Various theories have been presented to deal with the problems of plate vibration. Accordingly, numerous strong recent ways have also been evolved to analyze such problems. **[2]**.

Form the analysis of cantilever and clamped supported plate, the properties of material considered are as following:

St37 Young's modulus, E =762.9 MPa, density$(\rho) = 7700 \text{kg/m}^3$, Poison’s ratio$(\vartheta) = 0.03$ and the modulus of shear, G = 0.00795 N/m^2.

Xing and Liu [3] used a new separation of variables solved new exact solutions for the free vibrations of thin orthotropic rectangular plates with the entire combinations of simply supported (S) and clamped (C) boundary conditions, and the exact solutions correctness was mathematically proved. The accurate solutions for the three cases (SSCC, SCCC, and CCCC) were progressively determined for the first time despite it was thought that they are not able to be determined. The novel accurate solutions were further confirmed by numerous numerical comparisons with the FEM solutions and those exist in the literature. Neffati M. Werfalli et al. [4] investigated the free vibration of thin isotropic rectangular plates with different edge states. This investigation concerned with the natural frequencies determination through solving the mathematical model that controls the trend of plate vibration by employing a Galerkin-based FEM. Cubic quadrilateral serendipity sub parametric elements with 12 degrees of freedom were utilized in such analysis. Although the used polynomial order was the lowest possible, the influence of method to calculate the natural frequencies exactly was illustrated by comparing the obtained solution with the available analytical outputs. The influence of aspect ratio, the elements number, and the sampling points number on the solution accuracy was also introduced.

Jiu Hui Wu et al. [5] suggested a method to provide the accurate solutions for the natural frequencies and mode shapes of a rectangular plate. Due to the high accuracy offered by the suggested method, it could be employed for verifying other analyses of free-vibration and evaluating the commercial software accuracy. The direct accurate solutions found for the most basic structural element using the suggested method will act as a base and offer an insight into the complex structures analysis. After that, the investigators have evolved the novel methods for the analysis of free and forced vibration of plate structures. Nawal H. Al Raheimy [6] This paper deals with free longitudinal vibrations of nonuniform homogeneous cantilever beams. Cantilever of rectangular cross-section with constant width and tapered thickness variation are considered. Thickness at the clamped end is estimated while it changed with different values at free end at the ratio equal to the relation (thickness at free end hf / thickness at clamped end hc) where this ratio changes from 0.05 to 0.9, From the results obtained, the main conclusion can be summarized as; the natural frequencies of the tapered thickness of cantilever beam are decreased with increasing the clamped thickness, thickness ratio and length of beam.

Moon and Choi [7] formulated the method of Transfer Dynamic Stiffness Coefficient for Frame Structures Vibration Analysis, and evolved the concept depending on the dynamic stiffness coefficient transfer, which is correlated to the force and displacement vector at every node from the left terminal to the right terminal of structure. Liew et al. [8] studied the mesh-free Galerkin method for free vibration analysis of unstiffened and stiffened corrugated plates. The analysis of this method was conducted on the stiffened corrugated plates, dealt as composite structures of equivalent orthotropic beams and plates, and the beams and plates strain energies were added up by the imposition of displacement compatible states between the beams and the plates. The matrix of stiffness of the entire structure was obtained.

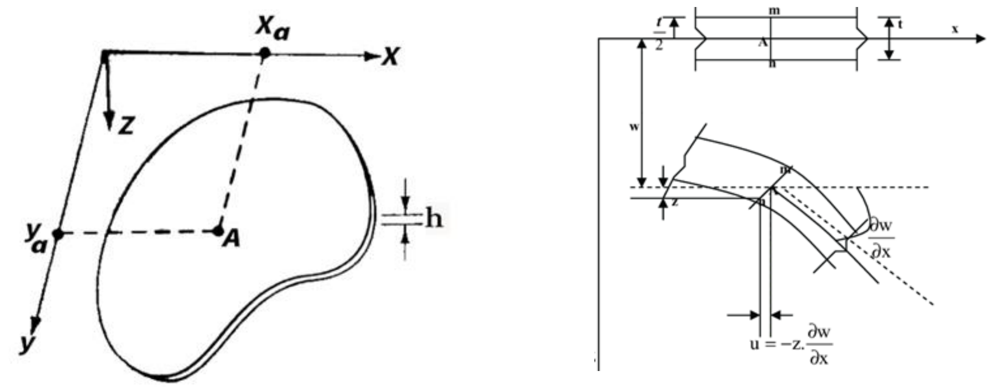
Lu et al. [9] studied the differential quadrature method for free vibration analysis of rectangular Kirchhoff plates with various boundary conditions. In the analysis of this method, the differential quadrature approach was employed in the line supports direction, whereas the accurate solution was searched in the transfer domain perpendicular to the line supports utilizing the method of state space.

2- Theoretical Model:

Consider a load-free plate depicted in Fig. (1a), which the xy plane coincides with it, the hence that the deflection is zero. The displacements at a point, taking place in the x, y, and z directions, are represented by u, v, and w, respectively. Due to the lateral loading, the deformation takes place. The mid surface point A ($x_a.y_a$) has deflection (w), Fig. (1b) [10], and the equation that is used to find the natural frequency is [11]:

$$\omega^2 = \pi^2 \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]\sqrt{\frac{D}{\rho h}}$$

Where, (*m*, *n*= 1, 2, 3…)



(a) **(b)**

Figure (1): (a) A Plate of constant thickness; (b) Part of the plate before and after deflection [9]

There are several types of mesh as the shape may be square triangle. The choice of mesh is made according to the need of the project. The square mesh is used for geometrical shapes (square, rectangle ...) where it gives higher accuracy, triangular mesh is usually used in fluid and curved shapes in this case, giving higher accuracy as shown in fig (2).

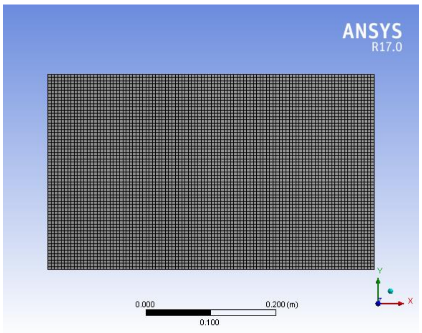


Figure (2): The mesh of the plate in ANSYS.

3- Experimental work:

This study investigates mainly the effect of different types of supporting plate to calculate the natural frequency of it, for free and forced plate with different frequencies. The results were determined analytically, numerically and experimentally.

Firstly, the derived equations were solved with a program using MATLAB13. The obtained results were compared with the obtained results from other researchers, numerical results by using ANSYS17.0 and by experimentally to verify them. The verification study was done for a rectangular plate of (30 x 50) cm. Secondly; some parameters were varied to study their effect on the natural frequencies of plate like boundary conditions, thickness ratio, type of supported plate and aspect ratio.

The experimental setup consists of a base made from a steel plate of (0.5 mm) thickness used to hold all the parts of the arrangement. Bolts were used for fixing the part on a plate without drilling it. Washers were used as an insulator, and a supported section of steel was used to support the plate. Bolts were also used to fix the plate edge as a clamped edge. The plate of experiment was fixed in its position between a plate of (10) mm thickness and bolt as shown in Fig. (3).

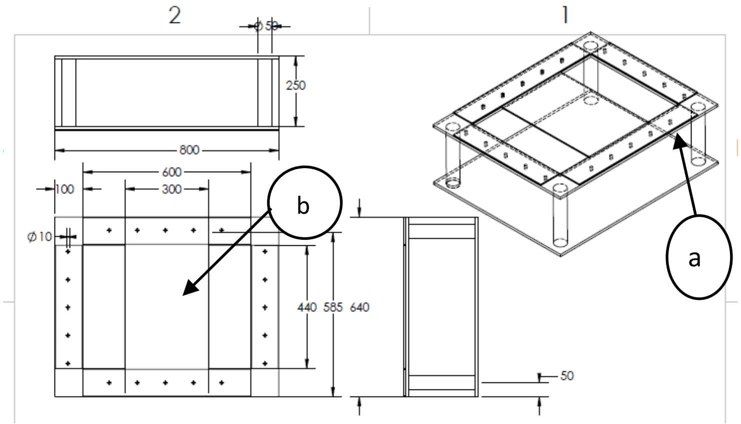


Figure (3): The system: (a) The structure that made from Steel; (b) The Plate (50*30) cm that made from Steel 37

The natural frequencies were measured using (ANSYS 17.0), (MATLAB) and accelerometer, and two kinds of supporting plate were used:

1. One edge was clamped, and the other three edges were free (CFFF).

2. Two opposite edges were free, and the others were clamped (CFCF).

To find the natural frequency for the plate with the general boundary conditions, the sensor of the vibration meter device was first pasted on the plate with the magnet that exists at the top of the four sensors, then linked to the output channel of the vibration meter and finally connected the electrical source.

Figure (4) illustrates the electronic connection of the plate to measure the displacement, velocity and acceleration using the vibration meter device. The equipment of the rig is also displayed in Fig. (4).

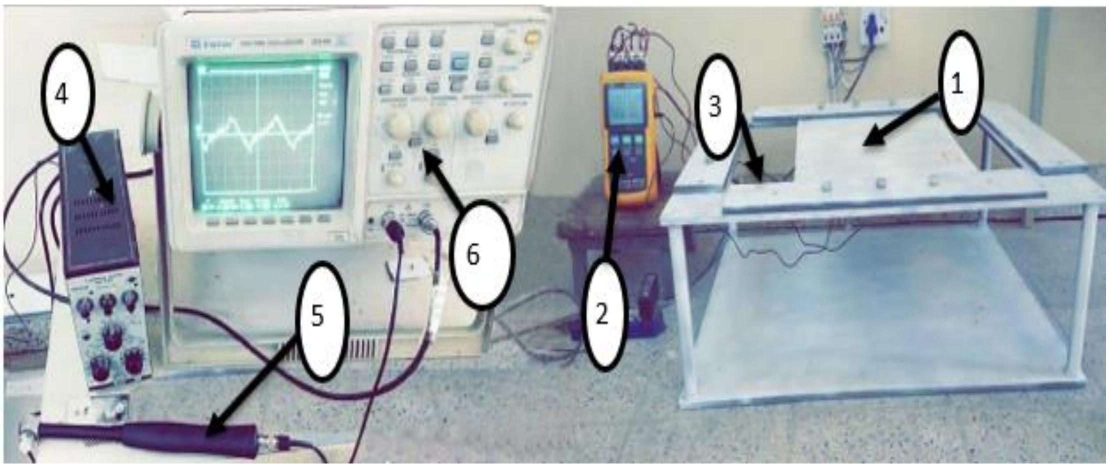


Figure (4): Measurement of the natural frequency for a free vibration in simply supported plate (CFCF): 1. Tested Plate. 2. Vibration Meters. 3. Testing structure with CFCF boundary condition. 4. Amplifier. 5. Hammer. 6. Oscilloscope.

For using the ANSYS and MATLAB programs to compare the result that found, shell ST37 is acceptable to be used for solving and analyzing thin to moderate thin shell structure [12].

For numerical study, ANSYS programming was used as stepped below:

1- Choosing the element type (shell 37).
2- Entering the material properties.
3- Creating the model.
4- Meshing the area with different sizes for convergent result.
5- Defining the boundary conditions.

6- Solving the model.
7- Reading and plotting the results.
8- Finishing the solution.

4- Result and Discussion:

The effect of supported plate on the vibration response for free and force vibration is explained for the following supported plates:

1. For CFFF supported plate:

It can be seen from Fig. (5), when the plate is a CFFF support along x-axis, the maximum amplitude occurs at a free end of the plate at x≈a, while the amplitude along y-axis that can be seen from Fig. (6) is less in magnitude in x-axis and the maximum occurs in two edges of the plate along the y-axis.

When the frequency is applied on the plate with different magnitudes (4,6,8,10) Hz, the amplitude of vibration decreases more than free vibration and when the frequency increased, the amplitude of the vibration increased but still less than at free vibration.

Because the maximum displacement is occurred at the end of the plate at x=a therefore the instantaneous stiffness of the plate will have decreased. By increasing the magnitude of frequency from (4 to 10) Hz, the location of the maximum response approaches to the left end of the plate.

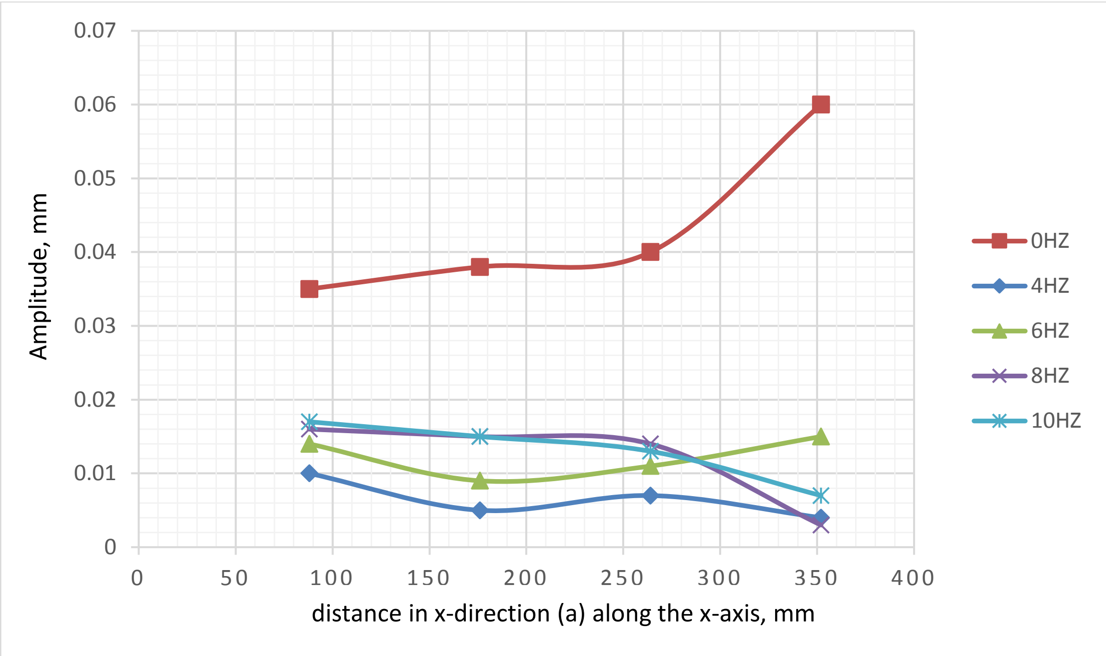


Figure (5): Relationship between Amplitude and distance in the x-direction for CFFF plate for free and force vibration (4, 6, 10, 12) Hz

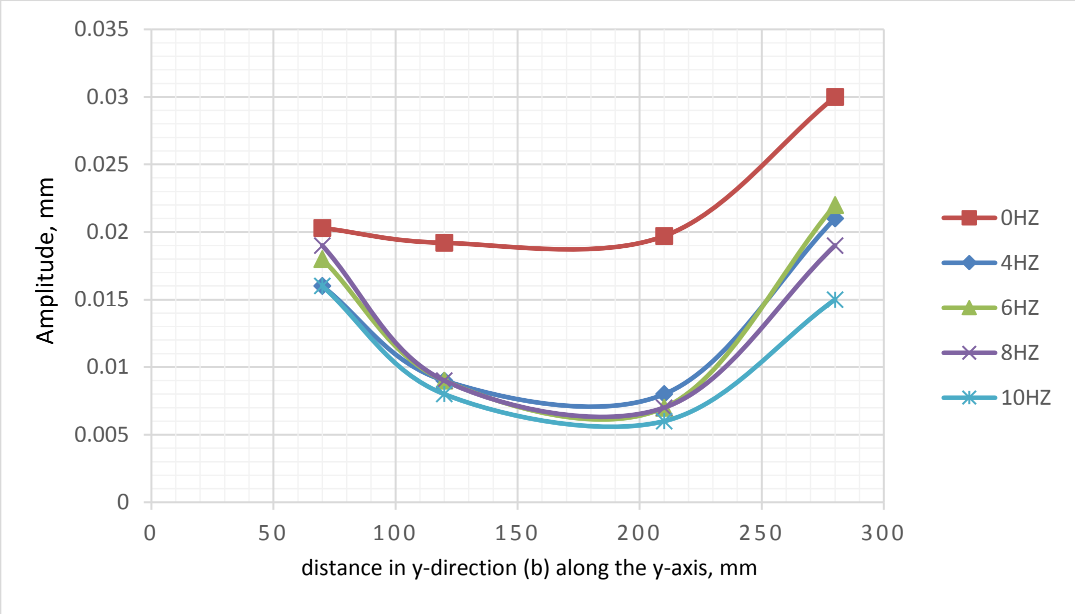


Figure (6): Relationship between amplitude and distance in the y-direction for CFFF plate for free and force vibration (4, 6, 10, 12) Hz

2. For CFCF supported plate

When the supporting of the plate is different as a clamped plate from two sides and free from the other sides shown Fig. (7), it can be seen that the amplitude of vibration along the x-axis is less than the CFFF plate. Also, when the frequency is applied at the mid of plate, the amplitude is maximum in mid plate and then when the frequency is increased, the amplitude of vibration decreased along the x- axis.

In Fig. (8), the amplitude of vibration for the plate along the y-axis is very small in magnitude compared to x- axis, and the effect of frequency (4, 6, 8, and 10) Hz is small on the amplitude along the y-axis.

The location of the maximum amplitude occurs at the mid plate because the deflection is max. at the midpoint in y=b/2 so that the stiffness is less than at the edge of plate. When the force acts on the plate with frequency, (4 to 10) Hz the amplitude will decrease than at free vibration.

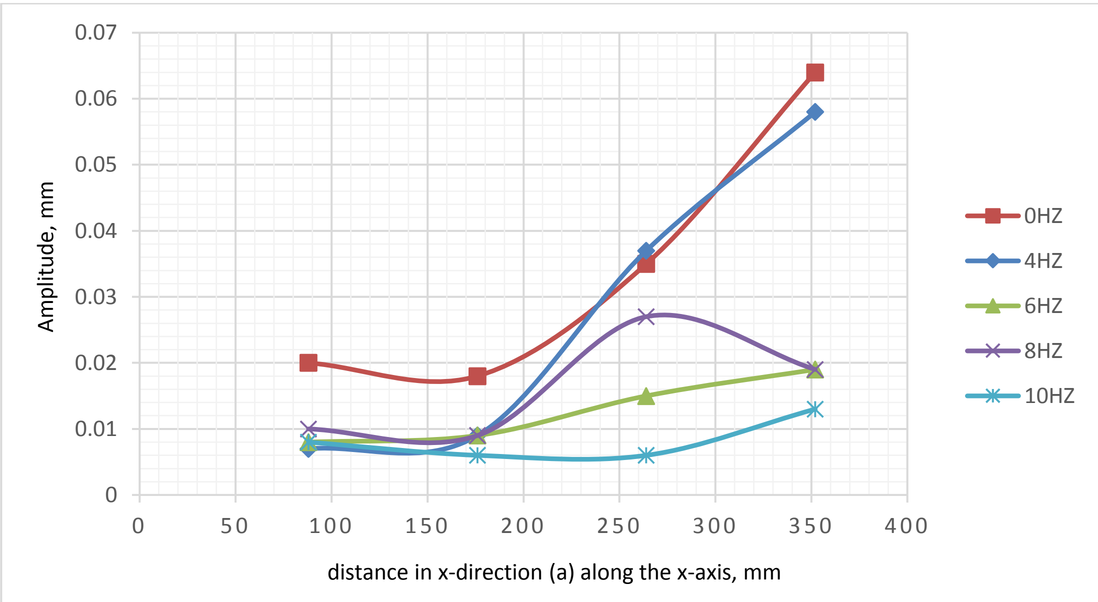


Figure (7): Relationship between amplitude and distance in the x-direction for CFCF plate for free and force vibration (4, 6, 10, 12) Hz without damping

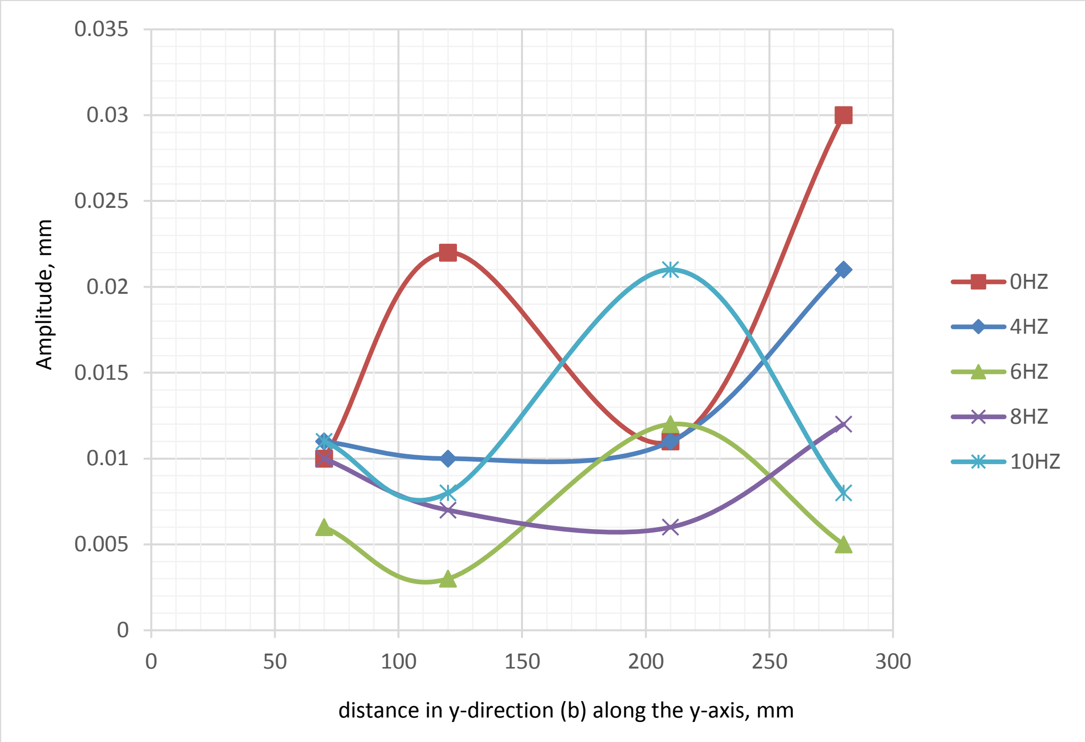


Figure (8): Relationship between amplitude and distance in the y-direction for CFCF plate for free and force vibration (4, 6, 10, 12) Hz without damping in plate.

After analyzing the results, one can observe the effect of frequency change on the amount of vibration amplitude, that the amplitude of vibration in the x-direction is higher than in the y-direction, as shown in Figs. (9) and (10). That curves are as a result of maximum magnitude of amplitude in the y-axis and frequency (4, 6, 8, 10) Hz in x-axis.

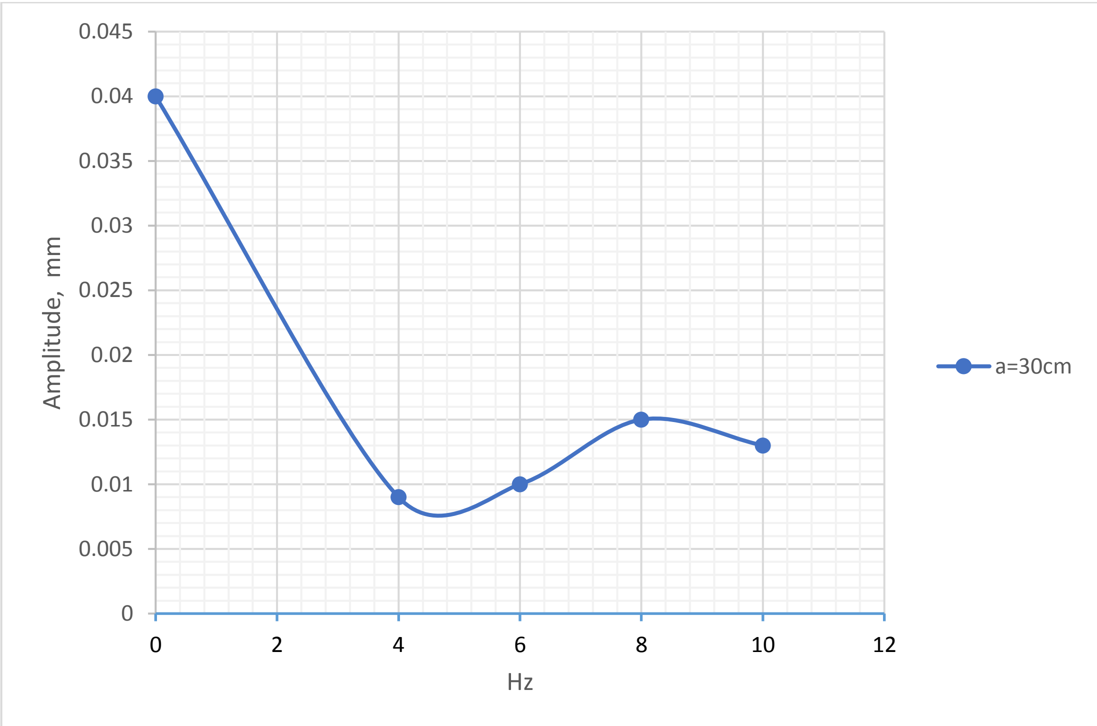


Figure (9): Relationship between amplitude and frequency (0, 4, 6, 10, 12) Hz in the x-direction at x = 30 cm for CFFF plate.

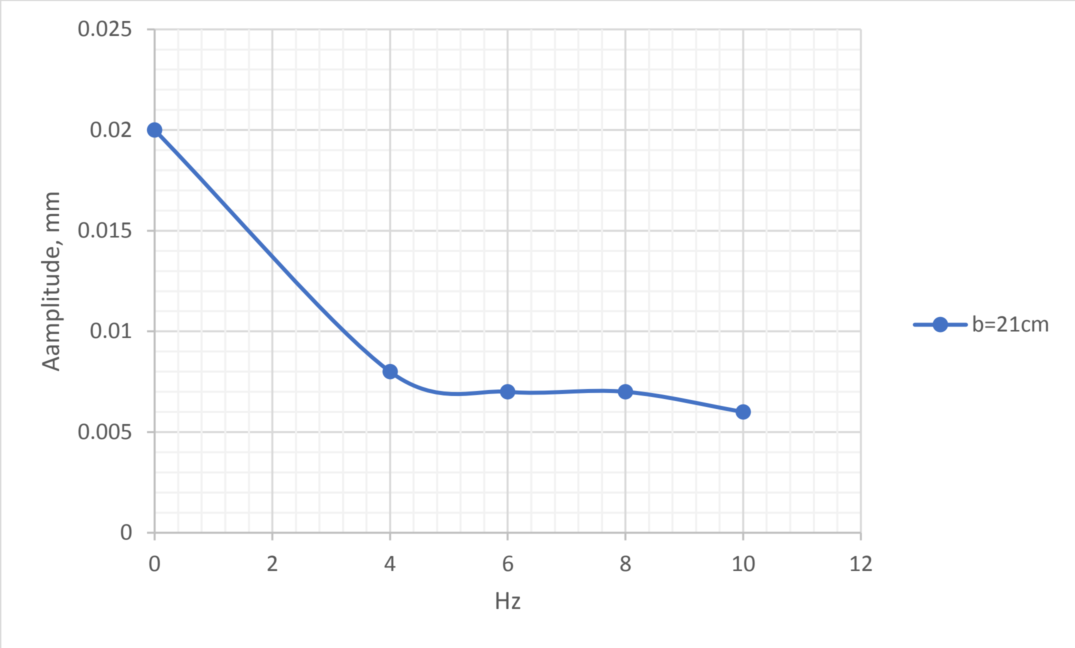


Figure (10): Relationship between amplitude and frequency (0, 4, 6, 10, 12) Hz in the y-direction at y = 21cm for CFFF plate.

The natural frequency of steel plate, with different boundary conditions, is developed from that used for isotropic plate. It is calculated from the analytical equation. The results are listed in table (1).

Table 1: Natural frequency for both supported plates in free vibration m, n, (1, 2, 3,….)

Free vibration			
CC in x	CC in y	SS in x	SS in y
177.2811	210.558	223.6068	282.8427
285.6203	250	307.3181	134.84
239.7916	225.303	226.7787	213.2007
122.4745	191.485	104.5825	72.27642

When the force of frequency equal to (4, 6, 8, 10) Hz acts on the plate at (x=50 cm) and (y=30 cm) the natural frequency of the plate x-axis can be found as in tables (2) to table (5) for system.

Table 2: Natural frequency for both supported plates in forced vibration in (4) Hz m, n, (1, 2...)

Forced vibration 4 Hz			
CC in x	CC in y	SS in x	SS in y
200	176.7767	267.2612	95.34626
346.4102	278.8867	278.8867	173.2051
239.0457	223.6068	127.3429	134.84
273.8613	169.0309	117.444	69.00656

Table 3: Natural frequency for both supported plates in forced vibration in (6) Hz m, n, (1, 2...)

Forced vibration 6 Hz			
188.9822	182.5742	250	129.0994
278.8867	298.1424	149.0712	258.1989
190.6925	239.0457	200	158.1139
81.64966	150.7557	125.6562	200

Table 4: Natural frequency for both supported plates in forced vibration in (8) Hz m, n, (1, 2...)

Forced vibration 8 Hz			
209.165	191.943	223.6068	173.2051
365.1484	298.1424	182.5742	239.0457
348.466	207.0197	149.0712	258.1989
447.2136	177.7047	125.6562	129.0994

Table 5: Natural frequency for both supported plate in forced vibration in (10) Hz m, n, (1, 2...)

Forced vibration 10 Hz			
230.0895	193.6492	223.6068	165.1446
408.2483	316.2278	223.6068	250
392.2323	258.1989	288.6751	138.0131
292.77	200	124.0347	158.1139

5- Conclusion:

The following conclusions can be drawn from the results of the present work:

1. When the plate is a CFFF support along the x-axis, the maximum amplitude occurs at a free end of the plate, while the amplitude along the y-axis is less in magnitude than in the x-axis, the amplitude of vibration decreased more than free vibration and when the frequency is increased, the amplitude of vibration increased but still less than at free vibration.

2. When the plate is a CFCF support along the x-axis, the amplitude of vibration along the x- axis is less than the CFFF plate. Also, when the frequency is applied at the mid of plate, the amplitude is maximum in mid plate and then when the frequency is increased, the amplitude of vibration decreased along the x-axis.

CONFLICT OF INTERESTS.

There are no conflicts of interest.

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تحليل سعة الاهتزاز الحر والقسري لصفيحة مستطيلة الشكل

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الخلاصة

في هذا البحث، تم دراسة تأثير الاهتزازات الحرة والقسرية على لوحة رقيقة (50 سم طول،30 سم عرض و0.5 مم سمك)، مصنوعة من الحديد 37 مع نوعين من التثبيت، أ أولا عندما تكون ثابتة الشكل وجانبي الجانبين الأخرى حرة (CFCF) وثانياً عند تثبيت أحد أطراف اللوحة وتكون الأطراف الثلاثة الأخرى حرة .(CFFF)كذلك، دراسة أشكال النمط والتردد الطبيعي للوحة. عندما يتم تطبيق التردد على الصفيحة بمقاييس مختلفة4) ،8،6، Hz (10، يقل اتساع الاهتزاز عن الاهتزاز الحر وعندما يزيد التردد يزيد او ينقص اتساع الاهتزاز لكنه لا يزال أقل عند الاهتزاز الحر، وايجاد التردد الطبيعي باستخدام برنامجي ماتلاب وانسز بادخال الشروط الحدية لتثبيت الصفيحة لغرض مقارنة النتائج.

الكلمات الداله: الشروط الحدية، صفيحة مستطيلة رقيقة مثبتة من طرف واحد، التردد الطبيعي.