BEDDINGTON-DEANGELIS PREDATOR-PREY SYSTEM WITH STAGE STRUCTURE FOR PREY

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Abstract:
In this paper, a stage predator - prey model with Beddington - DeAngelis functional response proposed and analyzed, sufficient conditions dissipativeness, and uniform persistence the dynamical analysis, by constructing appropriate Lyapunov functions, a set of easily verifiable-structured are derived for the Existence sufficient conditions are obtained for the global asymptotic stability of nonnegative equilibrium of the model. Numerical simulations are presented to illustrate the stability.

١.Introduction:

The traditional Lotka–Volterra type predator–prey systems are very important in the models of multi-species population dynamics. Standard Lotka-Volterra type models, on which a large body of existing predator–prey theory is built by assuming that the per capita rate of predation depends on the prey numbers only. Recently, the traditional prey-dependent, predator–prey models have been challenged by several biologists based on the fact that functional and numerical response over typical ecological time scales in recent years [٢].

The life cycle of most, if not all, animals and insects consists of at least two stages, immature and mature, and the individuals in the first stage often can neither hunt nor reproduce, being raised by their mature parents. Further immediately recognizable morphological and behavioral differences may exist between these stages and other adaptive stages, such as dormancy stages, may exist for immediate survival purposes.[٤]

Stage-structured models have received much attention. Aiello and Freedman proposed and investigated a stage-structured model of single species growth, where the transformation rate of mature is proportional to the existing immature species. The pioneering work of Aiello and Freedman represents a mathematically more careful and ecologically

meaningful formulation approach. Many authors have joined the studies on different kinds of stage-structured models and have made much more significant progress. In particular The dynamic interactions between the predator and the prey have long been one of the dominant themes in mathematical biology due to their universal existence and importance. In the description of the dynamic interactions, a crucial element of all models is the classic definition of predator's functional responses. There have been several famous functional response types: Holling types I–III , Hassel-Varley type, Beddington-DeAngelis types and the recent well-known ratio-dependence type , etc.[٣]
The organization of this paper is as follows:
In the next section, sufficient conditions are derived for the Existence and dissipativeness of system(٢,١), in section(٣), the dynamical analysis and by means of suitable Lyapunove function, sufficient condition are obtained for the global stability of non-negative equilibrium point of the system (٤), and the numerically studies

٢.The mathematical model

$$x_1'(t) = ax_2(t) - r_1x_1(t) - bx_1(t)$$
$$x_2'(t) = bx_1(t) - b_1x_2^2(t) - \frac{a_1x_2(t)y(t)}{(my(t)+x_2(t)+c)} \quad \ldots\ldots\ldots\ldots\ldots\ldots..(٢,١)$$
$$y'(t) = -ry(t) + \frac{a_2x_2(t)y(t)}{(my(t)+x_2(t)+c)}$$

Where $x_1(t), x_2(t)$ is the density of immature prey and mature prey at time t, respectively $y(t)$ is density of predator at time t. The module satisfies the following assumptions

(H١): The prey population: the birth rat into the immature population is proportional to the existing mature population with a proportionality constant $a > 0$;the death rate is proportional to the existing immature prey population with proportionality $r_1 > 0$, b_1 is the death and overcrowding rat of the population; the transformation rate from the immature prey individuals to mature prey individuals is proportional to the existing immature prey population with proportionality b .

(H٢): the predator population: the predator feed only on the mature prey, r is the death rat of the predator; a_1 is the capturing rate of the predator; a_2/a_1 is the rate of the conversion of nutrients in to the reproduction of the predator; m is the half saturation rat of predator. The positive constant c is the saturating functional response parameters.
The initial conditions for system () take the form of $x_1(t) > 0, \quad x_2(t) > 0, \quad y(t) > 0$ ……………(٠

It is easy to show that system () has a unique solution $z(t) = (x_1(t), x_2(t), y(t))$ satisfying initial condition

٣. Existence and dissipativeness

Definition(٣،١):A system of differential equation is said to be dissipative if there is a bounded subset D of R^n such that for any $x_0 \in R^n$ there is a time t_0, which depends on x_0 and D so that the solution $\varphi(t, x_0) \in$ D for $t \geq t_0$.

$$x_1'(t) = ax_2(t) - r_1x_1(t) - bx_1(t) \qquad = G_2(x_1, x_2, y)$$

$$x_2'(t) = bx_1(t) - b_1x_2^2(t) - \frac{a_1x_2(t)y(t)}{(my(t) + x_2(t) + c)} = G_2(x_1, x_2, y) \qquad \ldots\ldots\ldots\ldots\ldots$$

$$y'(t) = -ry(t) + \frac{a_2x_2(t)y(t)}{(my(t) + x_2(t) + c)} \qquad = G_3(x_1, x_2, y)$$

…………….(٣،١)

obviously, the interaction function $G_i; i = (1,2,3)$ of system (٣،١) are continuous and continuous partial derivatives on state space $R_+^3 = \{(x, x, y) : x_1 \geq 0, x_2 \geq 0, y \geq 0\}$. Therefore the solution of the system (٣،١) with non-negative initial condition exists and is unique, as the solution of system (٣،١) initiating in the non-negative octant is bounded. Further more, the following theorem establishes the dissipativeness of the system (٣،١). The system is said to be dissipative if all population initiating in R_+^3 are uniformly limited by their environment [١]

Theorem ٣،١: system (٢،١) is dissipative

Proof: let $(x_1(t), x_2(t), y(t))$ be any solution of the system with non _ negstive initial condition

By the second equation of system (٢،١), we have

$x_2'(t) \le bx_1(t) - b_1x_2^2(t)$ by

Lemma ٢،١ of paper [٨] , we have $\lim_{t\to\infty} x_2(t) \le \frac{1}{4b_1}$ for large enough t .

Let $W(t) = c_1x_1(t) + c_2x_2(t) + c_3y(t)$

$$W'(t) = c_1x_1'(t) + c_2x_2'(t) + c_3y'(t)$$

$$W'(t) = c_1(ax_2(t) - (r_1+b)x_1(t)) + c_2(bx_1(t) - b_1x_2^2(t) - \frac{a_1x_2(t)y(t)}{(my(t)+x_2(t)+c)}) + c_3(-ry(t) + \frac{a_2x_2(t)y(t)}{(my(t)+x_2(t)+c)})$$

Let $c_1 = a_2, c_2 = a_2, c_3 = a_1$

$$W'(t) = a_2ax_2(t) - a_2(r_1+b)x_1(t) + a_2bx_1(t) - a_2b_1x_2^2(t) - \frac{a_2a_1x_2(t)y(t)}{(my(t)+x_2(t)+c)} - a_1ry(t) + \frac{a_1a_2x_2(t)y(t)}{(my(t)+x_2(t)+c)}$$

$$W'(t) = a_2ax_2(t) - a_2r_1x_1(t) - a_2b_1x_2^2(t) - a_1ry(t)$$

Let $S = \min\{r, r_1\}$

$$W'(t) = aa_2x_2(t) - a_2b_1x_2^2(t) - S\{a_2x_1(t) + a_1y(t)\} - Sa_1x_2 + a_1Sx_2$$

$$W'(t) \le (a_2a + Sa_1)x_2(t) - b_1x_2^2(t) - S\{a_2x_1(t) + a_2x_2(t) + a_1y(t)\}$$

$$W'(t) \le (a_2a + S)x_2(t) - b_1x_2^2(t) - SW(t)$$

$$W'(t) + SW(t) \le \frac{a_2a + S}{4b_1}$$

$$W(t) \le \frac{a_2a + S}{4b_1} e^{-St}$$

$$\lim_{t\to\infty}(W(t)) = \frac{a_2a + S}{4b_1} = Z$$

So for at $t \ge 0$ all species are bounded.
Not that, for a biologically realistic model, it is required that system (٣،١) has to be dissipative (i.e. all population are uniformly limited in time by their environments). Hence system is dissipative.

Theorem ٣،٢: A necessary condition for predator species y to survive is

$a_2 > r$ ....(٣،١)

proof:

$$y'(t) = y\left(r + \frac{a_2 x_2 y}{my + x_2 + c}\right)$$

$$y'(t) \le y\left(r + \frac{a_2 x_2 y}{my + x_2}\right)$$

$$y'(t) \le y\left(r + \frac{a_2 x_2 y}{x_2}\right)$$

$$y'(t) \le y\left(r + a_2\right)$$

$y' \le Vy$ where $V = (r + a_2)$

Clear if $V < 0$ then $\lim_{t \to \infty}(y(t)) = 0$

In this case, the predator population will go to extinction. Hence, V should be positive.

(i.e. $a_2 > r$) is the necessary condition for survival of a predator.

Theorem ٣،٣: System (٢،١) is uniformly persistent

Proof : see[٣،٨]

٤. The dynamical analysis and stability

In this section the existence of the equilibrium point, for a three species system (٣،١), and then stability analysis are investigated, the persistence condition of all the species are established.

At most there are three possible non-negative equilibrium point for system (٣،١). The existence conditions and local stability analysis for them are given sa following

١- The equilibrium point $F_0(0, 0, 0)$ always exists.

٢- The equilibrium point $F_1(\tilde{x}_1, \tilde{x}_2, 0)$ always exists where

$$\tilde{x}_1 = \frac{a^2 b}{b_1(r_1 + b)^2}, \quad \tilde{x}_2 = \frac{a\ b}{b_1(r_1 + b)}$$

٣-The positive equilibrium point $F_2(\dot{x}_1, \dot{x}_2, \dot{y})$ exists in the Int. R_+^2 if there is a positive solution to the flowing set algebraic equations

$$G_2(x_1,x_2,y) = ax_2(t) \quad r_1x_1(t) \quad bx_1(t) \qquad = 0$$

$$G_2(x_1,x_2,y) = bx_1(t) \quad b_1x_2^2(t) \quad \frac{a_1x_2(t)y(t)}{(my(t)+x_2(t)+c)} = 0$$

$$G_3(x_1,x_2,y) = \quad ry(t) + \frac{a_2x_2(t)y(t)}{(my(t)+x_2(t)+c)} \qquad = 0$$

$$\dot{x}_1 = \frac{a\dot{x}_2}{r_1+b}, \quad y = \frac{(a_2 \quad r)\dot{x}_2 \quad cr}{mr}$$

$$\dot{x}_2 = \frac{Z_2+\sqrt{Z_2^2+4Z_1Z_3}}{2Z_1} \ldots\ldots, Z_1 = \frac{a_2b_1}{a_1}, \quad Z_2 = \frac{a_2 \quad r}{m} \quad \frac{a_2ab}{a_1(r_1+b)}, \quad Z_3 = \frac{cr}{m}$$

Obviously, $\dot{x}_1 > 0$, $\dot{x}_2 > 0$, $\dot{y} > 0$ if and only if the following condition holds

……………………(٤،١)

Now the local dynamical behavior of the system (٣،١) around each of these equilibrium point is discussed by computing the jacobian matrix $J(x_1, x_2, y)$ of the system (٣،١) at the point (x_1, x_2, y)

$$J(x_1, x_2, y) = \begin{pmatrix} \frac{\partial G_1}{\partial x_1} & \frac{\partial G_1}{\partial x_2} & \frac{\partial G_1}{\partial y} \\ \frac{\partial G_2}{\partial x_1} & \frac{\partial G_2}{\partial x_2} & \frac{\partial G_2}{\partial y} \\ \frac{\partial G_3}{\partial x_1} & \frac{\partial G_3}{\partial x_2} & \frac{\partial G_3}{\partial y} \end{pmatrix}$$

Where

$$\frac{\partial G_1}{\partial x_1} = \quad (r_1+b) < 0, \quad \frac{\partial G_1}{\partial x_2} = a > 0, \quad \frac{\partial G_1}{\partial y} = 0$$

$$\frac{\partial G_2}{\partial x_1} = b > 0, \quad \frac{\partial G_2}{\partial x_2} = \quad 2b_1x_2 \quad \frac{a_1(my+c)y}{(my+x_2+c)^2} < 0, \quad \frac{\partial G_2}{\partial y} = \frac{a_1(x_2+c)x_2}{(my+x_2+c)^2}$$

$$\frac{\partial G_3}{\partial x_1} = 0, \quad \frac{\partial G_3}{\partial x_2} = \frac{a_2(my+c)y}{(my+x_2+c)^2} > 0, \quad \frac{\partial G_3}{\partial y} = \frac{a_2(x_2+c)x_2}{(my+x_2+c)^2}$$

Therefore, the jacobian matrix of the system (٣،١) at the equilibrium point $F_0(0, 0, 0)$ is written by:

$$J(F_0) = \begin{matrix} (r_1+b) & a & 0 \\ b & 0 & 0 \\ 0 & 0 & r \end{matrix}$$

Hence, by using straight forward computation it is easy to verify that, the eigenvalues of $J(F_0)$ satisfy the following relations

$\lambda_{03} = r < 0$

$\lambda_{01} + \lambda_{02} = (r_1 + b)$

$\lambda_{01}.\lambda_{02} = ab$

$(0, 0, 0)$ is unstable saddle point with locally stable manifold of dimension two and unstable manifold of dimension one, is unstable in the x_1- direction and asymptotically stable in the x_3- direction.

The jacobian matrix of the system (٣،١) at the equilibrium point $F_1(\tilde{x}_1, \tilde{x}_2, 0)$ is written by:

$$J(F_1) = \begin{matrix} (r_1 + b) & a & 0 \\ b & 2b_1\tilde{x}_2 & \dfrac{a_1\tilde{x}_2}{(\tilde{x}_2 + c)} \\ 0 & 0 & r + \dfrac{a_2\tilde{x}_2}{(\tilde{x}_2 + c)} \end{matrix}$$

$$\lambda_{11} = r + \frac{a_2\tilde{x}_2}{(\tilde{x}_2 + c)}$$

$$\lambda_{12} + \lambda_{13} = (r_1 + b + 2b_1\tilde{x}_2)$$

$$\lambda_{12}.\lambda_{13} = 2b_1(r_1 + b)\tilde{x}_2 \quad ab = ab$$

Clearly $(\tilde{x}_1, \tilde{x}_2, 0)$ is locally a asymptotically stable in R^3_+ if and only if

$$r > \frac{a_2\tilde{x}_2}{(\tilde{x}_2 + c)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(٤,٢)$$

Further, it is clear that whenever the harvesting efforts of the susceptible prey remains below of its intrinsic growth rate, the susceptible population will persist.

The Jacobian matrix at the planer equilibrium point $F_2(\dot{x}_1, \dot{x}_2, \dot{y})$ can be written as:

$a_{11}=(r_1+b)<0,\ a_{12}=a>0,\quad a_{13}=0$

$a_{21}=b>0,\quad a_{22}=2b_1\dot{x}_2\ \frac{a_1D}{K^2}<0,\ a_{23}=\frac{a_1R_2}{K^2}\quad J(F_{21})=\begin{matrix} a_{11} & a_{13} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{matrix}$

$a_{31}=0,\quad a_{32}=\frac{a_2D}{K^2}>0,\quad a_{33}=\frac{a_2R}{K^2}$ Where

$D=(m\dot{y}+c)\dot{y},\quad R=(\dot{x}_2+c)\dot{x}_2,\quad K=(m\dot{y}+\dot{x}_2+c)$

Then the characteristic equation of the Jacobian matrix is given by:

$\lambda^3+A_1\lambda^2+A_2\lambda+A_3=0$

here λ is the eigenvalue associated with F_2, and

$A_1=\ (a_{11}+a_{22}+a_{33})$

$A_2=a_{11}a_{22}+a_{11}a_{33}+a_{22}a_{33}\quad a_{21}a_{12}\quad a_{23}a_{32}$

$A_3=\ a_{11}a_{22}a_{33}+a_{12}a_{21}a_{33}+a_{11}a_{23}a_{32}$

Therefore be substituting the values of a_{ij}, and then simplifying the resulting terms we obtain that:

$A_1=\ (\ (r_1+b)\quad 2b_1\dot{x}_2+\frac{a_1D}{K^2}\quad r+\frac{a_2R}{K^2})$

$A_1=r_1+b+2b_1\dot{x}_2+\frac{a_1D}{K^2}+r\quad\frac{a_2R}{K^2}$

$A_3=(r_1+b)(2b_1\dot{x}_2+\frac{a_1D}{K^2})(r\quad\frac{a_2R}{K^2})+((r_1+b)\frac{a_1a_2RD}{K^4})\quad ab(r\quad\frac{a_2R}{K^2}))$

Let $M=(+r\quad\frac{a_2R}{K^2}),\quad N=(2b_1\dot{x}_2+\frac{a_1D}{K^2}),\quad E=\frac{a_1a_2DR}{K^4}$

Therefore, an application of Routh-Hurwitz criterion

Then

$A_1 = r_1 + b + M + N$

$A_2 = (r_1 + b)M + (r_1 + b)N + NM \quad ab + E$

$A_3 = (r_1 + b)MN + (r_1 + b)E \quad abM$

Þ $= A_1 A_2 \quad A_3$

Þ $= (r_1 + b + M + N)((r_1 + b)M + (r_1 + b)N + NM \quad ab + E) \quad ((r_1 + b)MN + (r_1 + b)E \quad abM)$

Þ $= E(r_1 + b + N + M \quad r_1 \quad b) + [(r_1 + b)(N + M)(r_1 + b + N + M) + (r_1 + b + N + M)MN$
$\quad ab(r_1 + b + N + M) \quad (r_1 + b)NM + abM$

Þ $= E(N + M) + [(r_1 + b)(N + M)(r_1 + b + N + M) + (r_1 + b + N + M)MN$
$\quad ab(r_1 + b + N + M) \quad (r_1 + b)NM + abM$

Þ $= E(M + N) \quad [ab(r_1 + b + N) \quad [(r_1 + b)(r_1 + b + M + N)(M + N) + MN(N + M)]$

$$L2 = N + M \qquad \cdots(4.3a)$$

$$L1 = [ab(r_1 + b + N) \quad [(r_1 + b)(r_1 + b + M + N)(M + N) + MN(N + M)] \qquad \cdots(4.3b)$$

Then

Þ $= EL2 \quad L1 > 0 \text{ if } E > \frac{L1}{L2}$

Therefore, according to the above analysis, the following theorem can be easily proved

Theorem: Assume that the positive equilibrium point $F_2(\dot{x}_1, \dot{x}_2, \dot{y})$ exists in Int. R^3_+. Then F_2 is locally asymptotically stable if and only if the condition hold

$$r > \frac{a_2 R}{K^2} \text{...}(4.4a)$$

$$(r + b)N > ab$$

$$ab(r_1 + b + N) > [(r_1 + b)(r_1 + b + M + N)(M + N) + MN(N + M) \text{.....}(4.4b)$$

$$E > \frac{L1}{L2} \text{..}(4.4c)$$

Definition [٦]**:** Assume that the equilibrium point $\dot{x}$ depends smoothly on some parameter μ in an open interval I of R. If there exist a $\mu \in$ I such that:

١. a simple pair of complex point eigenvalues of the Jacobian matrix $J(\dot{x})$ at the equilibrium point $\dot{x}$ exists, say $\alpha(\mu \pm i\beta(\mu))$ such that they becomes purely imaginary at $\mu = \dot{\mu}$, whereas all the other eigenvalues remain real and negative: and

٢. $\left.\frac{d\alpha}{d\mu}\right|_{\mu=\dot{\mu}} \neq 0$, then at $\dot{\mu}$ we have simple Hopf bifurcation

The conditions (١) and (٢) for a simple Hopf Bifurcation are equivalent to the following condition on the coefficients of the characteristic polynomial:

$$p_n(\lambda:\mu) = \lambda^n + a_1(\mu)\lambda^{n-1} + a_2(\mu)\lambda^{n-2} + \dots\dots + a_{n-1}(\mu)\lambda + a_n(\mu)$$

$(C1)$ $a_n(\mu^*) > 0, D_1(\mu^*) = a_1(\mu^*) > 0,$

$$D_2(\mu^*) = \det \begin{pmatrix} a_1(\mu^*) & a_3(\mu^*) \\ 1 & a_2(\mu^*) \end{pmatrix} > 0, \dots, D_{n-2}(\mu^*) > 0, D_{n-1}(\mu^*) = 0$$

$(C2)$ $\left.\frac{dD_{n-1}}{d\mu}\right|_{\mu=\mu^*} \neq 0$

Theorem: Assume that the positive equilibrium point $F_2(\dot{x}_1, \dot{x}_2, \dot{y})$ exists in Int. R_+^3.and let the condition (٤،٤a), (٤،٤b), (٤،٤c) hold together with $E = \frac{L1}{L2}$

Where $L1$ and $L2$ are given in equation (). A simple Hopf bifurcation occurs at the value $E = \overset{*}{E}$

Proof: According to Liu criterion[], a simple Hopf bifurcation occurs if and only if $A_1(\overset{*}{E}) > 0, A_3(\overset{*}{E}) > 0, \Delta(\overset{*}{E}) = 0$ and $\left.\frac{d\Delta}{dE}\right|_{E=\overset{*}{E}} \neq 0$

Now, since we have

$$A_1(\overset{*}{E}) = r_1 + b + M + N > 0$$

$$A_3(\overset{*}{E}) = (r_1 + b)MN - abM + (r_1 + b)\frac{[ab(r_1 + b + N) - [(r_1 + b)(r_1 + b + M + N)MN + MN(N + M)]]}{N + M} > 0$$

$\Phi(\overset{*}{E}) = EL2 \quad L1 = 0$

and

$$\left.\frac{d\Phi}{dE}\right|_{E=\overset{*}{E}} = L2 \,, 0 .$$

Theorem: Let

$x_2 b_1(r_1 + b) \quad ab$ ……………………………………….(٤,٥)

Then non – negative equilibrium point $(\tilde{x}_1, \tilde{x}_1, 0)$ is globally asymptotically stable

Proof:

Consider the following Lyapunove function

$$L(x, x, y) = c_1(x_1 \quad \dot{x}_1 \quad \dot{x}_1 \ln\frac{x_1}{\dot{x}_1}) + c_2(x_2 \quad \dot{x}_2 \quad \dot{x}_2 \ln\frac{x_2}{\dot{x}_2}) + c_3(y \quad \dot{y} \quad \dot{y} \ln\frac{y}{\dot{y}})$$

Where c_1, c_2, c_3 are positive constants

$$L'(t) = c_1(\frac{x_1 \quad \tilde{x}_1}{x_1})x_1'(t) + c_2(\frac{x_2 \quad \tilde{x}_2}{x_2})x'(t)_2 + c_3 y'(t)$$

$$L'(t) = c_1(\frac{x_1 \quad \tilde{x}_1}{x_1})(ax_2(t) \quad r_1x_1(t) \quad bx_1(t)) + c_2(\frac{x_2 \quad \tilde{x}_2}{x_2})(bx_1(t) \quad b_1x_2^2(t) \quad \frac{a_1x_2(t)y(t)}{(my(t) + x_2(t) + c)})$$
$$+ c_3(\quad ry(t) + \frac{a_2x_2(t)y(t)}{(my(t) + x_2(t) + c)})$$

$$L'(t) = c_1(\frac{x_1 \quad \tilde{x}_1}{x_1})(ax_2(t) \quad r_1x_1(t) \quad bx_1(t)) + c_2(\frac{x_2 \quad \tilde{x}_2}{x_2})(bx_1(t) \quad b_1x_2^2(t) \quad \frac{a_1x_2(t)y(t)}{(my(t) + x_2(t) + c)})$$
$$+ c_3(\quad ry(t) + \frac{a_2x_2(t)y(t)}{(my(t) + x_2(t) + c)}) \quad [c_1(\frac{x_1 \quad \tilde{x}_1}{x_1})(a\tilde{x}_2 \quad r_1\tilde{x}_1 \quad b\tilde{x}_1) + c_2(\frac{x_2 \quad \tilde{x}_2}{x_2})(b\tilde{x}_1 \quad b_1\tilde{x}_2^2)]$$

$$L'(t) = c_1(\frac{x_1 \quad \tilde{x}_1}{x_1})(a((x_2(t) \quad \tilde{x}_2) \quad r_1(x_1(t) \quad \tilde{x}_1) \quad b(x_1(t) \quad \tilde{x}_1)) + c_2(\frac{x_2 \quad \tilde{x}_2}{x_2})(b(x_1(t) \quad \tilde{x}_1) \quad b_1(x_2^2(t) \quad \tilde{x}_2^2)$$
$$\frac{a_1x_2(t)y(t)}{(my(t) + x_2(t) + c)}) + c_3(\quad ry(t) + \frac{a_2x_2(t)y(t)}{(my(t) + x_2(t) + c)})$$

Let $c_2 = 1, \quad c_3 = \dfrac{x_2}{x_2 \quad \tilde{x}_2} = \dfrac{x_2 b_1(r_1 + b)}{x_2 b_1(r_1 + b) \quad ab}$

$$L'(t) = \frac{c_1 a}{x_1}(x_2 \quad \tilde{x}_2)(x_1 \quad \tilde{x}_1) \quad \frac{c_1(r_1+b)}{x_1}(x_1 \quad \tilde{x}_1)^2 + \frac{b}{x_2}(x_1 \quad \tilde{x}_1)(x_2 \quad \tilde{x}_2) \quad \frac{b_1}{x_2}(x_2 \quad \tilde{x}_2)^2(x_2+\tilde{x}_2)$$
$$\frac{a_1 x_2 b_1(r_1+b)}{a_2(x_2 b_1(r_1+b) \quad ab)} ry(t)$$

$$L'(t) = (\frac{c_1 a}{x_1}+\frac{b}{x_2})(x_2 \quad \tilde{x}_2)(x_1 \quad \tilde{x}_1) \quad \frac{c_1(r_1+b)}{x_1}(x_1 \quad \tilde{x}_1)^2 \quad \frac{b_1}{x_2}(x_2 \quad \tilde{x}_2)^2(x_2+\tilde{x}_2)$$
$$\frac{a_1 x_2 b_1(r_1+b)}{a_2(x_2 b_1(r_1+b) \quad ab)} ry(t)$$

$$L'(t) = \quad [\sqrt{\frac{c_1(r_1+b)}{x_1}}(x_1 \quad \tilde{x}_1) \quad \sqrt{\frac{b_1}{x_2}(x_2+\tilde{x}_2)}(x_2 \quad \tilde{x}_2)]^2$$
$$\frac{a_1 x_2 b_1(r_1+b)}{a_2(x_2 b_1(r_1+b) \quad ab)} ry(t)$$

If

$$c_1 = \frac{R_1 + \sqrt{R_2^2 \quad 4R_1R_3}}{2R_1}, \quad R_1 = (\frac{a}{x_1})^2, \quad R_2 = \frac{2}{x_1x_2}(ab + 2b_1(r_1+b)x_2), \quad R_3 = (\frac{b}{x_2})^2$$

Then $L'(t) < 0$ under given condition (٤،٥). Therefore, F_2 is a globally asymptotically stable in the Int. R_+^3.

Theorem: Let

$c > x_2$

$2(r+b)mayy\dot{y} < abR$

$K_2^2 > 4K_2K_3$

Then non – negative equilibrium point $(\dot{x}_1, \dot{x}_2, \dot{y})$ is globally asymptotically stable

Proof:

Consider the following Lyapunove function

$$L(x,x,y) = c_1(x_1 \quad \dot{x}_1 \quad \dot{x}_1 \ln\frac{x_1}{\dot{x}_1}) + c_2(x_2 \quad \dot{x}_2 \quad \dot{x}_2 \ln\frac{x_2}{\dot{x}_2}) + c_3(y \quad \dot{y} \quad \dot{y}\ln\frac{y}{\dot{y}})$$

Where c_1, c_2, c_3 are positive constants

$$L'(t)=c_1\left(\frac{x_1-\dot{x}_1}{x_1}\right)x_1'+c_2\left(\frac{x_2-\dot{x}_2}{x_2}\right)x_2'+c_3\left(\frac{y-\dot{y}}{y}\right)y'$$

$$L'(t)=c_1\left(\frac{x_1-\dot{x}_1}{x_1}\right)(ax_2-r_1x_1-bx_1)+c_2\left(\frac{x_2-\dot{x}_2}{x_2}\right)\left(bx_1-b_1x_2^2-\frac{a_1x_2y}{(my+x_2+c)}\right)+c_3\left(\frac{y-\dot{y}}{y}\right)\left(-ry+\frac{a_2x_2y}{(my+x_2+c)}\right)$$

$$L'(t)=c_1\left(\frac{x_1-\dot{x}_1}{x_1}\right)(ax_2-(r_1+b)x_1)+c_2\left(\frac{x_2-\dot{x}_2}{x_2}\right)\left(bx_1-b_1x_2^2-\frac{a_1x_2y}{(my+x_2+c)}\right)+c_3\left(\frac{y-\dot{y}}{y}\right)\left(-ry+\frac{a_2x_2y}{(my+x_2+c)}\right)$$

$$-\left[c_1\left(\frac{x_1-\dot{x}_1}{x_1}\right)(a\dot{x}_2-(r_1+b)\dot{x}_1)+c_2\left(\frac{x_2-\dot{x}_2}{x_2}\right)\left(b\dot{x}_1-b_1\dot{x}_2^2-\frac{a_1x_2y}{(my+x_2+c)}\right)+c_3\left(\frac{y-\dot{y}}{y}\right)\left(-r\dot{y}+\frac{a_2\dot{x}_2\dot{y}}{(m\dot{y}+\dot{x}_2+c)}\right)\right]$$

$$L'(t)=\frac{c_1a}{x_1}(x_2-\dot{x}_2)(x_1-\dot{x}_1)-\frac{c_1(r_1+b)}{x_1}(x_1-\dot{x}_1)^2+\frac{c_2b}{x_2}(x_1-\dot{x}_1)(x_2-\dot{x}_2)-\frac{c_2b_1}{x_2}(x_2^2-\dot{x}_2^2)^2(x_2-\dot{x}_2)$$

$$-\frac{c_2a_1}{x_2}(x_2-\dot{x}_2)\frac{my\dot{y}(x_2-\dot{x}_2)-\dot{x}_2x_2(y-\dot{y})+cy(x_2-\dot{x}_2)+c\dot{x}_2(y-\dot{y})}{(my+x_2+c)(m\dot{y}+\dot{x}_2+c)}$$

$$+c_3a_2(y-\dot{y})\left(\frac{m\dot{y}(x_2-\dot{x}_2)-m\dot{x}_2(y-\dot{y})+c(x_2-\dot{x}_2)}{(my+x_2+c)(m\dot{y}+\dot{x}_2+c)}\right)$$

Let $P=(my+x_2+c)(m\dot{y}+\dot{x}_2+c)$

$$L'(t)=\frac{c_1a}{x_1}(x_2-\dot{x}_2)(x_1-\dot{x}_1)-\frac{c_1(r_1+b)}{x_1}(x_1-\dot{x}_1)^2+\frac{c_2b}{x_2}(x_1-\dot{x}_1)(x_2-\dot{x}_2)-\frac{c_2b_1}{x_2}(x_2+\dot{x}_2)(x_2-\dot{x}_2)^2$$

$$-\frac{c_2a_1}{x_2P}(m\dot{y}+cy)(x_2-\dot{x}_2)^2+\frac{c_2a_1\dot{x}_2}{P}(y-\dot{y})(x_2-\dot{x}_2)-\frac{c_2a_1c\dot{x}_2}{x_2P}(y-\dot{y})(x_2-\dot{x}_2))$$

$$+\frac{c_3a_2}{P}(m\dot{y}+c)(x_2-\dot{x}_2)(y-\dot{y})-\frac{c_3a_2m\dot{x}_2}{P}(y-\dot{y})^2$$

$$P=(my+x_2+c)(m\dot{y}+\dot{x}_2+c)$$

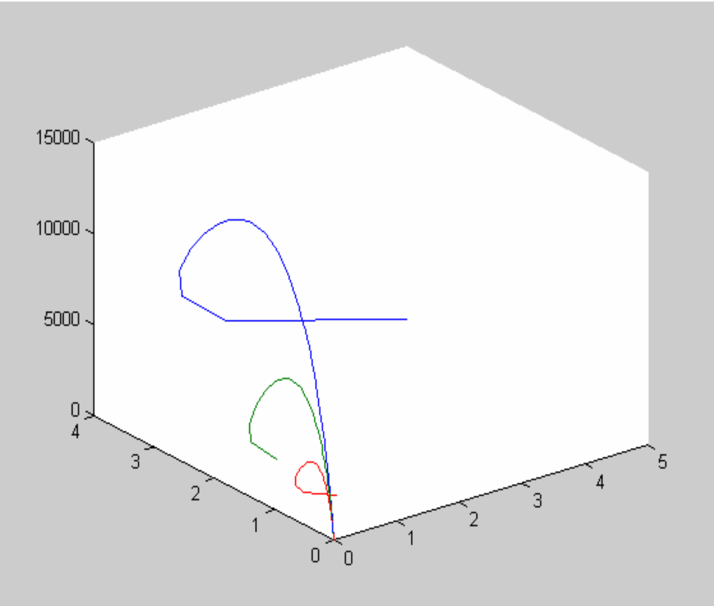
$$c_1=\frac{P_2+\sqrt{P_2^2-4P_1P_3}}{2P_2};P_1=\left(\frac{a}{x_1}\right)^2;P_2=\frac{ab}{x_1x_2}-\frac{4ma_1y\dot{y}(r_1+b)}{Px_1x_2};P_1=\left(\frac{b}{x_2}\right)^2$$

$$c_2=1;c_3=\frac{a_1x_2y(c-x_2)}{x_2(ma_2y+a_2)}$$

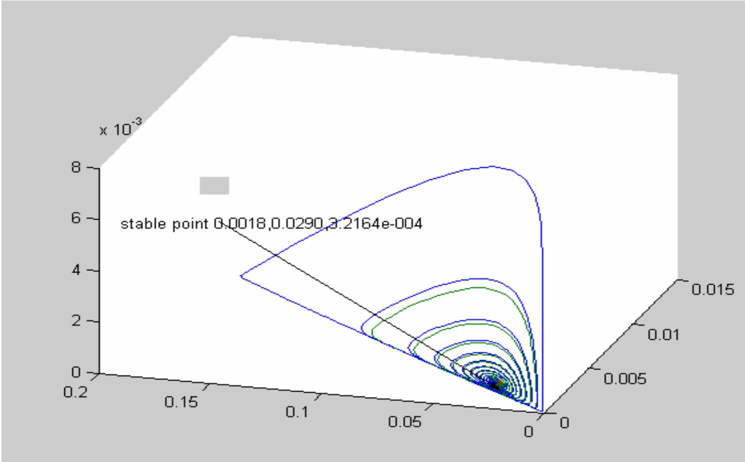
$$L'(t)=-\left[\sqrt{\frac{P_2+\sqrt{P_2^2-4P_1P_3}}{2P_2}\cdot\frac{(r_1+b)}{x_1}}(x_1-\dot{x}_1)-\sqrt{\frac{b_1}{x_2}(x_2+\dot{x}_2)}(x_2-\dot{x}_2)\right]^2-\frac{a_2m\dot{x}_2a_1x_2y(c-x_2)}{Px_2(ma_2y+a_2)}(y-\dot{y})^2$$

Numerical analysis

The global dynamical behavior of the non-linear system)٢،١), in the positive octant, is investigated numerically. A numerically integration for system (٢،١) is carried out for various choices of biologically feasible parameters value and for different set of initial condition. System (٢،١) is solved numerically using the matlap (simulink,odu٤٥)
We use the following parameter values
$a = a_1 = r = b = m = 1, r_1 = c = 10, a_2 = 100000$
$F_2 = (1.000097908399766e-004, 0.00110010769924, 0.99997688469819)$
, the positive steady F_2 state is global asymptotically stable see fig (٤،١). Numerical simulations of trajectories starting at various initial populations seem to indicate that the stability is also global for the parameter values we used.

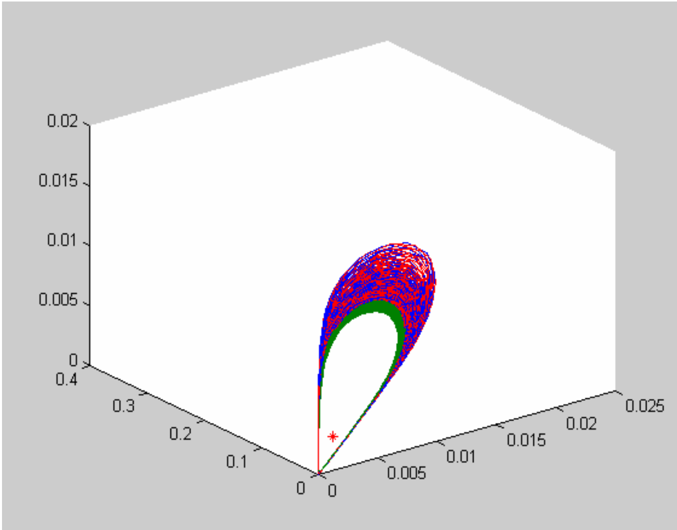


Fig(٤،١)



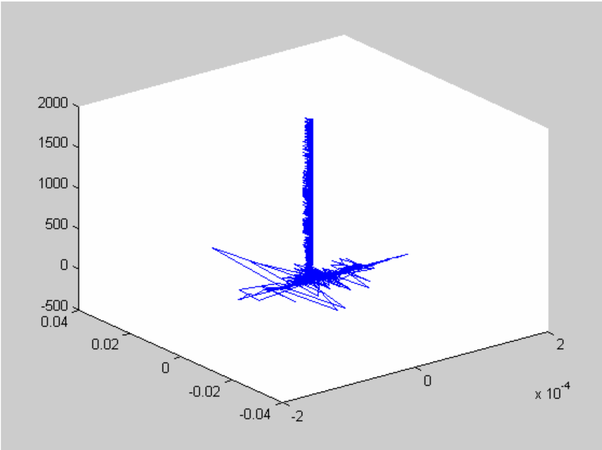
fig(٤,٢) stable limit cycle approach from inside

$a = .2, a_1 = 1000, r = 1, b = 3, m = .01, r_1 = .2, c = 2, a_2 = 70$

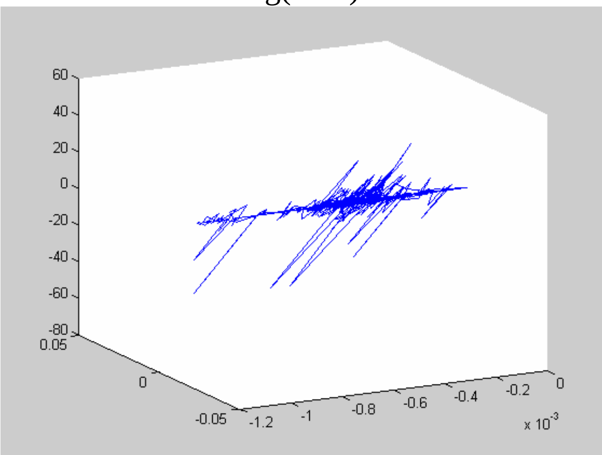


fig(٤,٣) stable limit cycle approach from outside

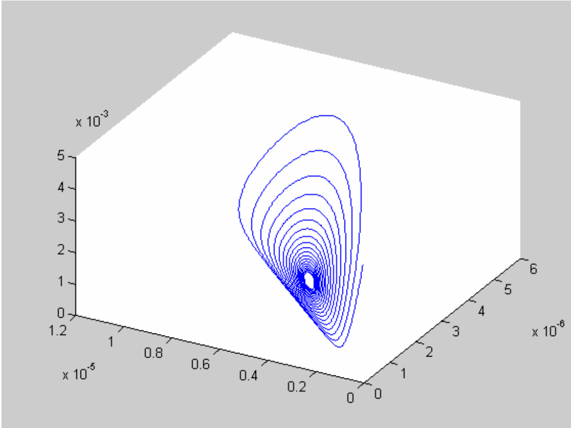
$a = .9, a_1 = 1000, r = 1, b = 10, m = .1, r_1 = .001, c = 2, a_2 = 70$



fig(٤,٤a)



fig(٤,٤b)



fig(٤،٤c)

In this case, take

$a = .2, r_1 = .3, b = .1, b_1 = .7, c = 1, m = .2, a_1 = 70, a_2 = 100000$ if $r < .5$ the system (٢،١)still chaotic , see fig (٤،٤a),(٤،٤b) if r ¡Ý5 , the system (٢،١) still stable, see fig (٤،٤c) fig(٤،٤b)

References

١. A.T.Balasim, " on the Dynamical Behavior of Multispecies Ecological system" D.M.thesis, science university of Baghdad, ٢٠٠٥

٢. M.A.j. Chaplain, F.A.Davidson, Ruixu, "persistence and global stabilitiy of a ratio-dependent predator-prey model with stage structure", Applied mathematics and computation,٢٠٠٣

٣. G.jingyuan, Yu.Guangtian,: A stage structure predator-prey model with Beddington-DeAngelis functional response ",Applied mathematics and cmputation,٢٠٠٨

٤. Paul Georgescu Submitted , "On the validation, stability and control of certain biological systems By to" Mathematics and Its Applications Central European University ٢٠٠٨

٥. H.A.Ibrahim, " on the dynamics of the harvested prey-predator models with infectious disease in prey"D.M.thesis, science university of Baghdad, ٢٠٠٨

٦. jack k.," criterion of Hopf Bifurcation without using Eigenvalues ", journal of mathematical analysis and application, ١٩٩٤

٧. Y.N.XiAo, lan Sum, " global stability of a predator-prey system with stage structure for the predator" Acta mathematical since, ٢٠٠٣

٨. Xinyu song, Limung and Avidan U.N eumann " Ratio-Dependent Predator-Prey with stage structure for prey' Applied Mathematics and Computation,٢٠٠٤