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Nonlinear Analysis of Thermoviscoelasticity of Laminated Composites

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Abstract

The nonlinear thermoviscoelastic behavior of composite thin plates is investigated. An experimental program covers an achievement of creep tests under different temperatures, dimensions of specimens and distribution loads to describe the equation of creep compliance. The stress relaxation is also determined from the experimental creep compliance.

A new equation of creep compliance function D(t,σ,T) and relaxation modulus E(t,ε,T) were predicted from the experimental results to describe the nonlinear thermoviscoelastic behavior of composite thin plates. A good agreement has been observed between the proposed models of nonlinear behavior at different temperatures and experimental results and between both theoretical and FEM results.

It was found that thus the deflection is increasing at the beginning and the rate of increase is nearly constant and increase with increasing temperature from 30 C° to 60 C° with approximate rate (34.6%) for simply support plate at distributed load ($q=1.934E-3\ N/mm^2$), relative dimensions (a/b)=1.0 and time=15 min.

The results indicate that the shear stress increases with rate (50.7%), so that strain in y-axis increases with rate (19%) as a result of increase the temperature from (30 C° to 60 C°).

Keywords: Thermoviscoelastic, Stress Relaxation, Relaxation Modulus, Creep Compliance.

التحليل اللاخطي للمواد المركبة اللزجة المرنة تحت تأثير الحرارة

الخلاصة

تم بحث السلوك اللاخطي الحراري للمواد اللزجة المرنـة لمـادة مركبـة لصـفيحة نحيفة.يتضمن الجانب العملي إنجاز اختبار الزحف عند درجات مختلفة، أبعاد مختلفـة للعينـة وحمل موزع لوصف السلوك اللاخطي الحراري من خلال معادلة مطاوعـة الزحـف. إجهـاد الاسترخاء يحدد من النتائج العملية لمطاوعة الزحف.

تم اشتقاق معادلة جديدة لوصف خضوع التزحيف ومعامل الاسترخاء من النتائج العملية لوصف السلك اللاخطي الحراري اللزج – المرن لصفيحة نحيفة مركبة.

وجدت الدراسة إن معدل الزيادة الحاصلة في الانحراف عالية في البداية ثم يقل معدل الزيـادة ليقترب من الثبات كما إن الانحراف يزداد مع ازدياد درجة الحرارة من (30 C°-60 C°) ممـا يزيد معدل الانحراف بمقدار (34.6%) ومعدل الزيادة الحاصلة فـي إجهـاد لقـص بمقـدار

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(50.7%). لصفيحة العتبة البسيطة وعند حمل منتشر (q=1.9634E-3N/mm2) وأبعاد نسبية (a/b=1.0) وعند زمن (time=15 min).
وجد تطابق جيد بين النتائج السلوك اللاخطي الحراري والنتائج العملية وكلا من النتائج النظرية ونتائج طريقة العناصر المحددة.
الكلمات المرشدة:اللزوجة- المرونة الحرارية، إجهاد الاسترخاء، معامل الاسترخاء، خضوع التزحييف.

Nomenclature Notation

Symbol	Definition	Unit
a,b	Plate length and width	mm
A	Bounded area surface	mm^2
Am,Bm,Cm, Dm	Constants	----
D	Flexural rigidity of plate	N.m
D(t)	Creep compliance	1/MPa
E	Modulus of elasticity	MPa
E(t)	Relaxation modulus	MPa
n	Exponent	----
q	Distributed load	N/mm^2
t	Plate thickness	mm
T	Temperature	°C
U	Total potential energy	N.m
u	Displacement in the x-axis of plate surface	mm
v	Displacement in the x-axis of plate surface	mm
W	Strain energy	N.m
w	Deflection of plate	mm
DT	Temperature difference	°C
e	Normal strain	----
s	Normal stress	N/mm^2
n	Poisson's ratio	----
F(t)	Time dependent creep function	mm^2/N
g	Shear strain	----
t_{xy}, t_{yx}	Shear stresses	N/mm^2
W	Load potential energy	N.mm
Ds	Stress difference	MPa
qx	Rotation about x-axis	Rad
S	Summation	----
d	Deformation of creep specimen	mm
$\{\sigma\}$	Stress vector.	
$\{\varepsilon\}$	Strain vector.	
$\{T\}$	External loads applied to the boundary surfaces.	
$\{k\}$	Vector of curvature.	
$[B]$	The strain matrix	

$[N]$	The shape functions matrix
$[K]$	The stiffness matrix
$[K(e)]$	Stiffness matrix as a function of strain.

Introduction

The tress–strain relations for creep are primarily empirical. Most of the equation were developed to fit the experimental creep curves obtained under constant stress levels and constant temperature. The actual behavior of the viscoelastic materials has shown that the strain at a given time depends on all of the values of the stress in the past.

Thus, the creep phenomenon is affected by the magnitude and sequence of the stress or strain in a history of the material. Based on this fact, various methods have been suggested to present the time dependence or the viscoelastic behavior of this material [1].

Viscoelastic material as other metallic and non-metallic materials can be affected by various factors such as temperature, humidity, history variables, etc.

In this respect, literatures related to the viscoelastic behavior of composite materials under thermal effects are presented. Wenbo Luo et.al. ,2007 [2] study the creep behavior of commercial grade polycarbonate was investigated in this study. ten different constant stress ranges from 8 MPa to 50 MPa were applied to the specimen and the resultant creep strains were measured at room temperature. It was found that the creep could be modeled linearly below 15 MPa , and nonlinearly above 15 MPa. Different nonlinear viscoelastic models have been briefly reviewed and used to fit the test data. Marco Ferrari et. al. 2008 [3] the study aimed to estimate the effect of insertion length of posts with composite restoration on stress and strain distributions in central incisors and surrounding bone.

The typical , average geometries were generated in FEA environment. Dentin was considered as an elastic orthotropic material and periodontal ligament was coupled with nonlinear viscoelastic mechanical properties. B.A. Sami and H. Naima ,2009 [4] is predicted the mechanical behavior of yarns under various levels of strain , by using only their technical parameters. The study of the yarn response to tensile test and relaxation test at different strain level has permitted us to propose an analytical model predicting the entire stress-strain response of yarn. Joannie chin et. al. , 2009 [5] studied thermoviscoelastic properties of two commercial ambient cure structural epoxy adhesives were analyzed and compared. The adhesives were formulated by the same manufacturer, but one system contained accelerators to shorten its cure time.

Theoretical analysis

Bending of Simply Supported with Uniformly Loaded Rectangular Plates

is expressed in term of displacements thus[6]:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad \ldots(1)$$

The problems of bending of rectangular plates that have two opposite edges simply supported take the solution in the form of series solution as follows[6]:

$$w = \sum_{m=1}^{\infty} Y_m \sin\frac{mpx}{a} \qquad ...(2)$$

where:

Y_m : function only of the distance (y) in the y - axis.

It's assumed that the sides (x=0 and x=a) as shown in Fig.(1) are simply supported. Hence each term of Eq.(2) satisfies the boundary conditions :

$$\left.\begin{array}{l} w = 0 \\ \dfrac{\partial^2 w}{\partial x^2} = 0 \end{array}\right\} \text{on these sides}$$

It remains to determine the function Y_m in such form as to satisfy the boundary conditions on the sides of y =± b/2 and the equation of the deflection surface (1). The solution of Eq.(1) for uniform load is assumed to be in the form :

$$w = W_1 + W_2 \qquad ... (3)$$

And letting:

$$W_1 = \frac{q}{24D}(x^4 - 2ax^3 + a^3x) \qquad .... (4)$$

i.e., W_1 represents the deflection of uniformly loaded strip parallel to the x-axis. It satisfies Eq.(1) and also the boundary conditions at the edges :

x=0 and x=a.

The expression W_2 evidently has to satisfy the equation:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0 \qquad ... (5)$$

And it must be chosen in such a manner as to make the sum Eq.(3) satisfy all the boundary conditions of the plate .

Taking *w*2 in the form of series (2) in which, from symmetry (m=1, 3, 4…) and substituting into Eq.(5) we obtain :

$$\sum (Y_m^{IV} - 2\frac{m^2p^2}{a^2}Y''_m + \frac{m^4p^4}{a^4}Y_m)\sin\frac{mpx}{a} = 0 \qquad .... (6)$$

This equation can be satisfied for all values of (x) only if the function *Ym* satisfies the equation:

$$Y_m^{IV} - 2\frac{m^2p^2}{a^2}Y''_m + \frac{m^4p^4}{a^4}Y_m = 0 \qquad ...(7)$$

The general integral of this equation can take the form:

$$Y_m = \frac{qa^4}{D}(A_m\cosh\frac{mpy}{a} + B_m\frac{mpy}{a}\sinh\frac{mpy}{a} + C_m\sinh\frac{mpy}{a} + D_m\frac{mpy}{a}\cosh\frac{mpy}{a}) \qquad ...(8)$$

Observe that the deflection surface of the plate is symmetrical with respect to the x-axis Fig.(1) . In expression of (8) only even functions of (y) are kept and the integration constants (Cm=Dm=0.0) are let. The deflection surface (3) is then represented by the following expression:

$$w = \frac{q}{24D}(x^4 - 2ax^3 + a^3x) + \frac{qa^4}{D} * \sum_1^m (A_m\cosh\frac{mpy}{a} + B_m\frac{mpy}{a}\sinh\frac{mpy}{a}) * \sin\frac{mpx}{a} \qquad .....(9)$$

which satisfies Eq.(1) and also the boundary conditions at the sides (x=0.0 and x=a) . It remains now to determine the coefficients of Eq.(9) (Am and Bm) in such a manner as to satisfy the boundary conditions:

$$w=0 \quad ; \quad \frac{\partial^2 w}{\partial y^2}=0 \qquad ..(10)$$

On the sides y = ± b/2, we begin by developing expression of Eq.(4) in a trigonometric series which gives :

$$\frac{q}{24D}(x^4-2ax^3+a^3x)=\frac{4qa^4}{p^5D}\sum_{m=1}^{\infty}\frac{1}{m^5}\sin\frac{mpx}{a}$$

where:
m=1, 3, 5 …
The deflection series Eq.(9) will be represented in the form :

$$w=\frac{qa^4}{D}\sum_{m=1}^{\infty}\left(\frac{4}{p^5m^5}+Am\cosh\frac{mpy}{a}+Bm\frac{mpy}{a}\sinh\frac{mpy}{a}\right)\sin\frac{mpx}{a} \quad ....(11)$$

Substituting the boundary conditions from Eq.(10) in the expression of Eq.(11) and using the notation gaves:

$$\frac{mpb}{2a}=am \qquad ...(12)$$

We obtain the following equation for determining the constant (Am,Bm) :

$$\frac{4}{p^5m^5}+Am\cosh am+Bm\,am\sinh am=0$$
$$(Am+2Bm)\cosh am+Bm\,am\sinh am=0$$

from which :

$$\left.\begin{aligned} Am&=-\frac{2(am\tanh am+2)}{p^5m^5\cosh am}\\ Bm&=+\frac{2}{p^5m^5\cosh am}\end{aligned}\right\} \quad ....(13)$$

Substituting these values of constants in Eq.(11) we obtain the equation of the plate surface deflection ,satisfying Eq.(1) and the boundary conditions are given in the following form :

$$w=\frac{4qa^4}{p^5D}\sum_{m=1,3,5,.....}^{\infty}\frac{1}{m^5}\left(1-\frac{am\tanh am+2}{2\cosh am}\cosh\frac{2yam}{b}+\frac{1}{2\cosh am}\frac{2y}{b}\sin\frac{2yam}{b}\right)*\sin\frac{mpx}{a} \qquad (14)$$

Numerical analysis

Consider a plate subjected to a distributed load (q) normal to its mid surface Fig.(2). The stresses and strain produce work that is stored in the system as strain energy (W) [7] such that:

$$W=\frac{1}{2}\int\{s\}^T\{e\}\mathrm{dv} \qquad (15)$$

$\{s\}^T$: transposeof stress vector.

$$\{s\}^T=\{s_x, s_y, t_{xy}\} \qquad (16)$$

Where:

$\{e\}$: strain vec tor

$$\{e\}=\begin{Bmatrix} e_x\\ e_y\\ g_{xy}\end{Bmatrix} \qquad (17)$$

v : volume of plate

Using the assumption of the classical theory for plate bending, the strain displacement can be written as follows:

$$e_x=\frac{\partial u}{\partial x} \qquad e_z=\frac{\partial w}{\partial z}=0$$
$$e_y=\frac{\partial v}{\partial y} \qquad g_{xz}=\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}=0 \quad ..(18)$$
$$g_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \qquad g_{yz}=\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}=0$$

$$ex = -zkx$$
$$ey = -zky \quad ..(19)$$
$$gxy = -2\,zkxy$$

$$s_x = \frac{E}{1-v^2}(e_x + ne_y)$$
$$s_y = \frac{E}{1-v^2}(e_y + ne_x) \quad (20)$$
$$t_{xy} = G g_{xy}$$

sub Eq.(16) and Eq.(17) in Eq.(15) the strain energy (W) may be expressed as :

$$W=\frac{1}{2}\int\left\{s_x, s_y, t_{xy}\right\}\begin{Bmatrix} e_x \\ e_y \\ g_{xy}\end{Bmatrix}dv =$$
$$\frac{1}{2}\int\left(s_x e_x + s_y e_y + t_{xy} g_{xy}\right)dv \quad ..(21)$$

The applied load (q) acting normal to the plate surface area produces the load potential energy (Ω), that is:

$$\Omega = -\int\{T\}^T\{u\}dA = -\int qwdA \quad ...(22)$$

Substituting the strains and stresses from Eq. (19) and Eq. (20) into Eq. (21) and performing the integration over plate thickness (t) gives:

$$W=+\frac{1}{2}\int\{k\}^T[D]\{k\}dA=$$
$$\frac{1}{2}\int\{-kx \;\; -ky \;\; -2kxy\}[D]\begin{Bmatrix} -kx \\ -ky \\ -2kxy\end{Bmatrix}dv \quad ..(23)$$

Where for the isotropic case, the elasticity matrix [D] can be written as in [8]:

$$[D]=\frac{Et^3}{12(1-n^2)}\begin{bmatrix} 1 & n & 0 \\ n & 1 & 0 \\ 0 & 0 & 0.5(1-n)\end{bmatrix} ...(24)$$

or in another form:

$$[D]=\begin{bmatrix} Dx & D_1 & 0 \\ D_1 & Dy & 0 \\ 0 & 0 & Dxy\end{bmatrix} \quad ....(25)$$

For the nonlinear thermoviscoelastic case (E) was predicted from experimental results as following.

$$E(t,e,T)=E(e,T)*t^{n(e,T)} ...(26)$$

so that , the stress relaxation expressed as following :

$$\sigma(t,\varepsilon,T)= \quad \sigma(\varepsilon,T)* \quad t^{ns(\varepsilon,T)} \quad …(27)$$

The curvature displacement Eqs.(17) can be expressed in a matrix form as follows :

$$\{k\}=[B]w=\begin{bmatrix} -\frac{\partial^2}{\partial x^2} \\ -\frac{\partial^2}{\partial y^2} \\ -2\frac{\partial^2}{\partial x\partial y}\end{bmatrix}w \quad ... (28)$$

The nodal displacements of the plate element with (n=4) (four node) is shown in Fig.(3). Each node gives the following three degrees of freedom The transverse deflection w, rotations about x-axis θx and rotations about y-axis θy.

The element deformed shape can be approximated with a suitable set of shape functions *Ni* (x,y) :

$$w(x,y)=\sum_i Ni(x,y)ui=[N]\{u\} \quad ..(29)$$

where the nodal displacements (ui) are:

$$\{u\}^T=\begin{Bmatrix} w_1, q_{1x}, q_{1y}, w_2, q_{2x}, q_{2y}, ... \\ ...., Wn, q_{nx}, q_{ny} \end{Bmatrix} \quad (30)$$

Substituting Eq.(29) and (30) into the strain nodal displacement Eq.(28) , the matrix [*B*],Eq.(28), can be written in the expanded form :

$$[B]=\begin{bmatrix} -\frac{\partial^2 N_1}{\partial x^2} & -\frac{\partial^2 N_2}{\partial x^2} & -\frac{\partial^2 N_3}{\partial x^2} & .... & -\frac{\partial^2 Nn}{\partial x^2} \\ -\frac{\partial^2 N_1}{\partial y^2} & -\frac{\partial^2 N_2}{\partial y^2} & -\frac{\partial^2 N_3}{\partial y^2} & .... & -\frac{\partial^2 Nn}{\partial y^2} \\ -2\frac{\partial^2 N_1}{\partial x\partial y} & -2\frac{\partial^2 N_2}{\partial x\partial y} & -2\frac{\partial^2 N_3}{\partial x\partial y} & .... & -2\frac{\partial^2 Nn}{\partial x\partial y} \end{bmatrix} \quad ...(31)$$

Sub Eq.(29) in Eq.(28) and the result in Eq.(23) the strain energy equation can be written as :

$$W=\frac{1}{2}\int\{u\}^T[B]^T[D][B]\{u\}dA...(32)$$

Sub Eq. (29) in (20), the load potential equation can be written as[8]:

$$\Omega=-\int q(x,y)[N]\{u\}dA \quad ..(33)$$

the shape function [*N*] shown in appendix (A)

Since the total potential energy is given by[8]:

$$U=W+\Omega \quad ...(34)$$

Substituting of Eqs.(32,33) into the total potential energy expression Eq.(34) gives[8:

$$U=\frac{1}{2}\int\{u\}^T[B]^T[D][B]\{u\}dA-\int q(x,y)[N]\{u\}dA \quad ..(35)$$

According to the condition for equilibrium for the minimization of the total potential energy U., differentiation of Eq.(35) w.r.t. nodal displacement (ui) ,yeilds the following system of equilibrium equation [8] :

$$\left(\int[B]^T[D][B]dA\right)\{u\}-\int q(x,y)[N]dA=0 \quad ....(36)$$

where the element stiffness $[K]_e$ is given by :

$$[K]_e=\iint[B]^T[D][B]dxdy \quad ...(37)$$

and the vector of the equivalent nodal force $\{f\}_e$ is [8]:

$$\{f\}_e=\iint q(x,y)[N]dA \quad ....(38)$$

Thus, the equilibrium for plate element can be expressed in the concise form as:

$$\{f\}_e=[K]_e\{u\}_e \quad ...(39)$$

the strain matrix [*B*] can be written as follows:

$$[B]=[Q][C]^{-1} \quad ...(40)$$

The element stiffness matrix can be evaluated by substituting Eq.(40) in Eq. (37), that is:

$$[K]^e=[C^{-1}]^T\left(\iint[Q]^T[D][Q]dxdy\right)[C]^{-1} \quad ....(41)$$

An explicit expression for stiffness matrix [*K*] has been evaluated [9]. A computer program was developed use Fortran language (Visual Fortran 5.0) employing the isoparametric element and the flow chart is shown in fig(4).

The experimental programme:

Some aspects of relationship between and interconversion of creep and stress relaxation functions of nonlinear viscoelastic material are calculated from experimental results .The equipment required for experimental work is designed and built according to these requirements the planned test can be classified into two groups:

- Creep tests : in which the mechanical material properties for nonlinear viscoelastic behavior are calculated at constant applied load.
- Deflection tests : in these tests the deflection of supported plate is measured for the consequent time at different points on the plate.

There are a number of creep test specimens which are used to evaluate the property of thermoviscoelastic material. The dimensions of standard creep test specimen is shown in Fig.(5).[10]

Preparation of Creep Campsite Polyester Specimens :

Composite polyester was manufactured using one layer as, fiber in the polyester resin which was made by mixing the polyester resin with solidification (B). The percentage of type (B) was (0.8%), the final solidified material is rigid. Therefore flexible die is used to prevent the specimen cracking or fracture which may occur after removing the specimen from the die. The time required for solidification of this type of material depended on the room temperature , hence increasing this temperature reduces the required time for solidification.

Fig.(6,7) show a schematic and photograph of this specimen. The volume fraction is (0.26).

The experimental results predict the creep compliance $D(t,\sigma,T)$ as shows in fig.(8,9) and relaxation modulus $E(t,\varepsilon,T)$ shows in fig.(10,11) to describe the nonlinear thermoviscoelastic behavior of rectangular thin plate.

The results and discussion

The central deflection of plate has been analyzed for composite polyester thin plates. The results are as shown in Figs. (12,13).

The figures show the effects of three values of distributed load under thermal environment, and different relative dimension. The value of central point deflection of thin plate deflection has been tested for time range (0.05 < time <120 minute). In addition, those tests are performed at constant thickness (thick. =4 mm).

The main material properties used for the analysis of thin plate viscoelastic material and its composite are used in the experimental creep test data in the region of nonlinear behavior. Hence, the same value fraction as that for creep the test specimen is used to construct the plate material to ensure that the output data is correct for both the experimental and theoretical analysis.

Furthermore, the material properties concluded from the creep test for different temperature are used to examine the plate behavior for that different temperature.

Table (1) show the Dimensions ,temperature and boundary conditions of plates.
The results are discussed with the help of presentation the following effects:

a-The Effect of Temperatures :

Figs.(12,13) show that increase in temperature increases the central deflection value on composite thin plate if we compare between the figures under same boundary condition, distributed load, and relative dimensions (a/b). Increasing the central deflection value with increase the temperature a results in increasing in the internal elongation of the structure of composite thin plate caused by heat effect.

b-The Effect of Relative Dimension (a/b):

Fig(14) show the effect of relative dimension on the central deflection value represented by increasing the relative dimension (a/b) which will increase the deflection. The increasing in the temperature increases the central deflection, in addition the effect of temperature is greater than the relative dimension (a/b) if we compare between Fig(16).

c-Three Dimensional Plots Rotation About the x and y-axis:

The rotation of plate surface represents the slope at certain Cartesian w.r.t the x-axis or y-axis. In Figs.(15,16) a symmetrical state is observed for both rotations (θ_x,θ_y) about the centerline which has zero slope. This specimen represents a simply supported plate at all four edges with a maximum value of slope (θ_x,θ_y) at the nodes of simply supported plate.

So that, Figs.(15,16) show the effect of temperature that increases rotation of plate surface (θ_x,θ_y) and that increase appears clear where all four edges are fixed.
The comparison among theoretical, experimental and FEM gives a good agreement with limited disparity percentage.
The disparity range(3-6 %) is calculated between the theoretical and FEM

Conclusions:

The following conclusions can be drawn from the results of experimental, theoretical and FEM work for viscoelastic composite thin plate in nonlinear behavior:

1- Predict new Kernel equation $\varepsilon(\sigma,T)$ to describe the creep strain equation $\varepsilon(t,\varepsilon,T)$ by using polynomial function from fourth degree.

$$e(t,s,T)=e(s,T) * t^{n(s,T)}$$

Where:

$$e(T,s)=f_1(s)+f_2(s)*T+f_3(s)*T^2+f_4(s)*T^3+f_5(s)*T^4$$

2- Predict new stress relaxation equation $\sigma(t,\varepsilon,T)$ from the experimental results and their slope as a function of strain and temperature $ns(\varepsilon,T)$ Eq.(27).

$$s(t,e,T)=s(e,T)*t^{ns(e,T)}$$

where:

$$s(e,T)=f_{S1}(T)*e+f_{S2}(T)$$

$$ns(e,T)=f_{NS1}(e)+f_{NS2}(e)*T+f_{NS3}(e)*T^2+f_{NS4}(e)*T^3+f_{NS5}(e)*T^4$$

3- Predict new equation of creep compliance function $D(t\sigma,T)$ and their slope as a function of stress and temperature $n(\sigma,T)$ to describe nonlinear thermoviscoelastic behavior of composite thin plate at different variables by using polynomial equation.

$D(t,s,T)= D(s,T) * t^{n(s,T)}$

Where:

$D(s,T)=f_{D1}(T)*s+f_{D2}(T)$

4- Predict new equation of relaxation modulus E(t,ε,T) and their slope as a function of strain and temperature to describe nonlinear thermoviscoelastic behavior of composite thin plate at different variables by using polynomial equation.

$E(t,e,T)=E(e,T)* t^{ns(e,T)}$

Where:

$E(e,T)=f_{E1}(e)+f_{E2}(e)*T+f_{E3}(e)*T^2 +f_{E4}(e)*T^3+ f_{E5}(e)*T^4$

5- The deflection increase with rate (21%) as a result of increasing temperature from (T=30 C^o) to (T=60 C^o) for simple support plate at distributed load (q=1.943E-3 N/mm^2) , relative dimensions (a/b)=1.0 , and time = 15min.

6- The creep compliance D(tσ,T) increases with rate (32%) and relaxation modulus decreases with rate (31.4%) as a results of increase temperature from (30 C^o to 60 C^o) at (time=15 min , stress=6.7865 MPa ,strain=0.07 mm, and relative dimensions (a/b)=1.0.

7- The comparison among theoretical, experimental and FEM gives a good agreement with limited disparity percentage.

The disparity range(3-6 %) is calculated between the theoretical and FEM.

References

[1]- William N.Findley ,James S lai and Kasif onaran." Creep and Relaxation of Nonlinear Viscoelastic Material".North-Holland Publishing. 1976.

[2]- Wenbo Luo ,Said Jazouli and Toan Vu-khanh " Modeling of Nonlinear Viscoelastic Creep of Polycarbonate "e-Polymers ,NO.017,2007.

[3]- Marco Ferriri , Roberto Sorrentiuo,Fernando Zaraone,Davide Apicella,Raffaela Aversa and Antonio Apicella " Non-linear Viscoelastic Finite Element Analysis of the Effect of the Length of Glass Fiber Posts on the Biomechanical Behavior of Directly Restored Incisors and Surrounding Alveolar Bone", Dental Material Journal,Vol.4,PP185-198,2008.

[4]-B.A.Sami and H.Naima " Predication of the Mechanical Behavior of Open-End and Ring SPUN Yarns ",Journal of Applied Science ,2009.

[5]-Joannie Chin,Donald Huston and Aaron Forster" Thermoviscoelastic Analysis of Ambient Cure Epoxy Adhesives Used in Construction Applications", Material and Construction Division ,national institute of Standards\nd Technology,2009.

[6]- Stephen P. Timoshenko and S. Woinowsky-Kvierger :"Theory of Plates and Shells".1959

[7]- C.Spyrakos and John Raftoyiannis "Linear and Nonlinear Finite Element Analysis in Engineering Practice", Alogor,Inc. Publishing Division Pittsbursh,PA,1997.

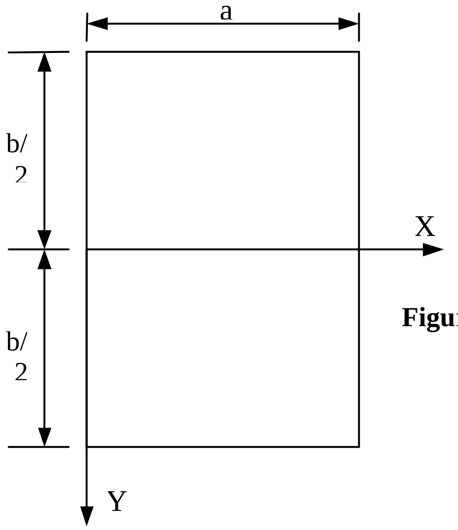
[8]- Darly L.Llogana " A first Course in the Finite Element Method ".Wadsworth Group. 2002.

[9]- O.C. Zienkiewics, Frs. "The Finite Element Method". Mc Graw. Hill Book Company (UK) limited 1977.

[10]- Standard Creep Test ASTM-D2990.

Table (1) Dimensions, Temperature and boundary condition of plates

Type	Temp. C°	a(mm)	b(mm)	(a/b)	Type of boundary conditions
A11	30	160	160	1.0	All edges simply supported.
A12	40				
A13	50				
A14	60				



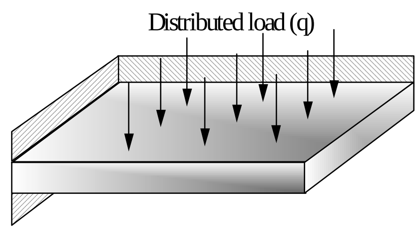


Figure (2) Plate subjected to distribution load (q) [7]

Figure (1) Simply supportedplate[6]

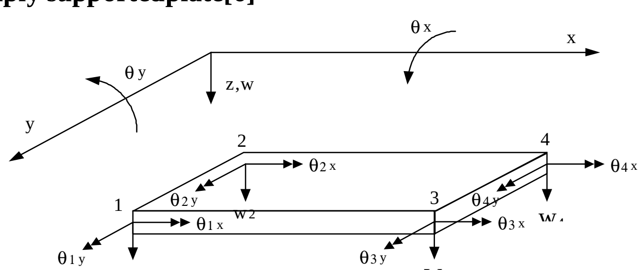


Figure (3) Nodal displacement of plate element [8]

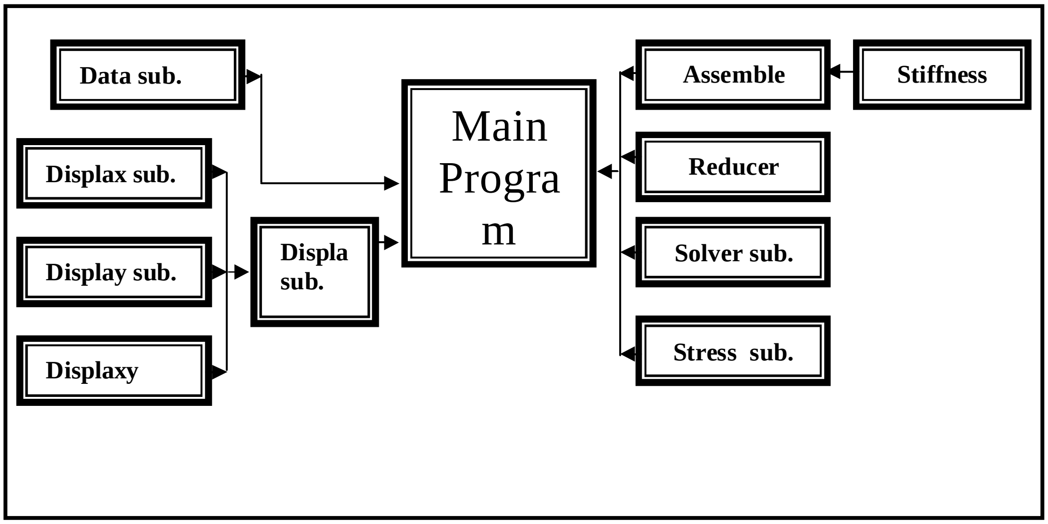


Figure (4) Flowchart for nonlinear thermoviscoelastic behavior Program by finite element method

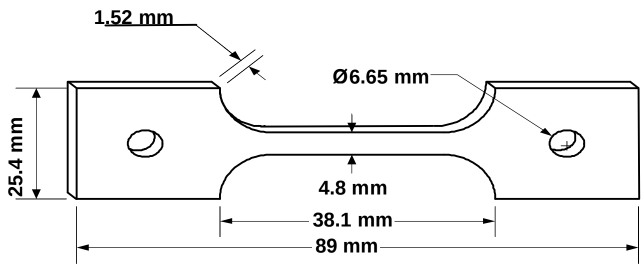


Figure (5) Standard creep test specimen[10]

Figure (6) Photograph of composite creep test specimen

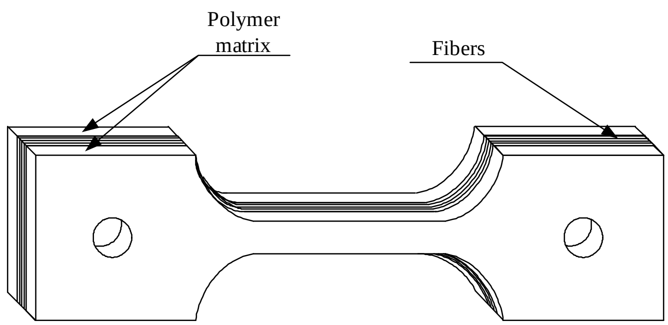


Figure (7) Schematic of composite creep test specimen

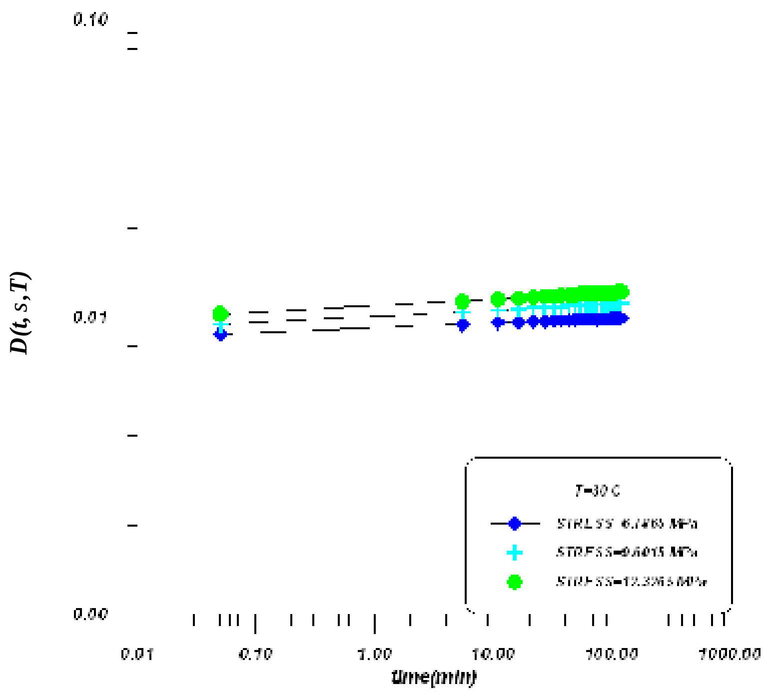


Figure (8) Log-Log Experimental nonlinear creep compliance for viscoelastic composite material at T=30 C°

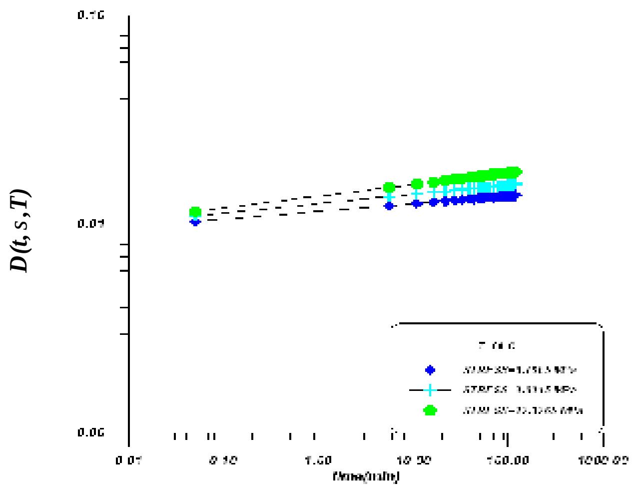


Figure (9) Log-Log Experimental nonlinear creep compliance for viscoelastic composite material at T=60 C°

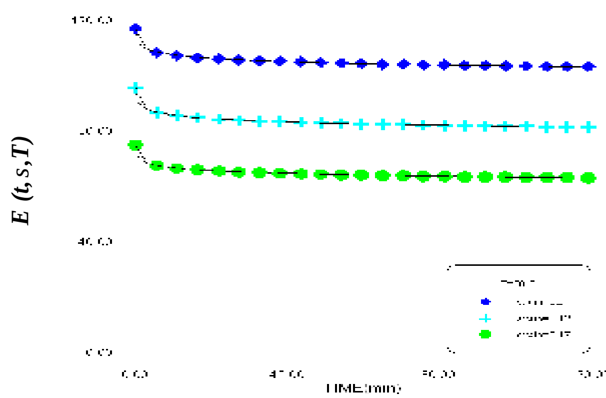


Figure (10)Stress relaxation modulus for nonlinear composite viscoelastic polyester at T=30 C°

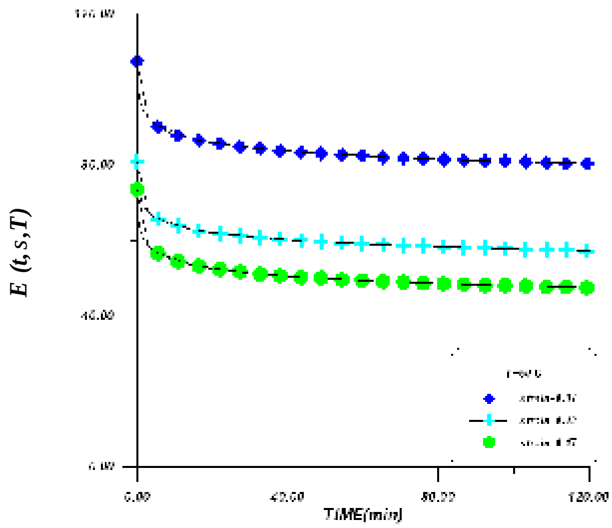


Figure (11) Stress relaxation modulus for nonlinear composite viscoelastic polyester at T=60 C^{o}

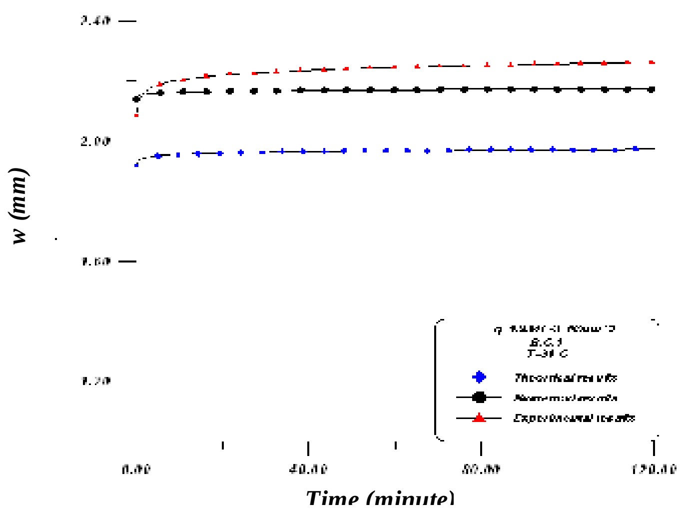


Figure (12) Variation in central point deflection with distribution load , type A11

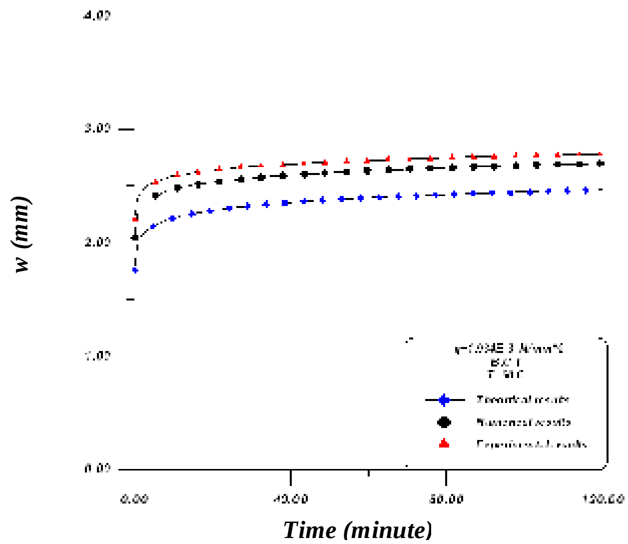


Figure (13) Variation in central point deflection with distribution load , type A14

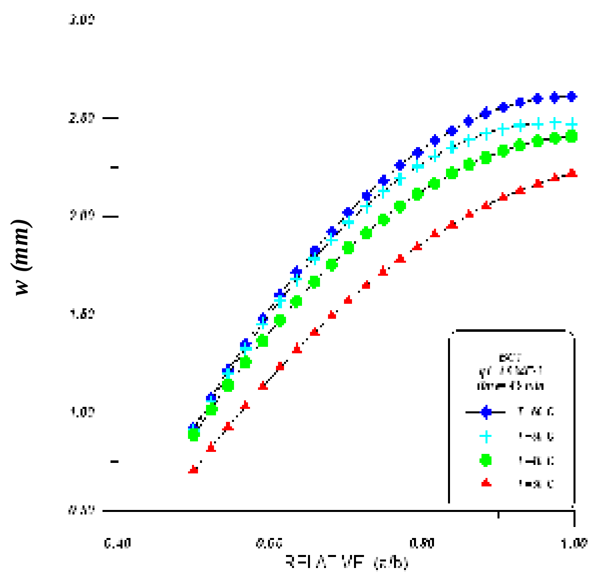
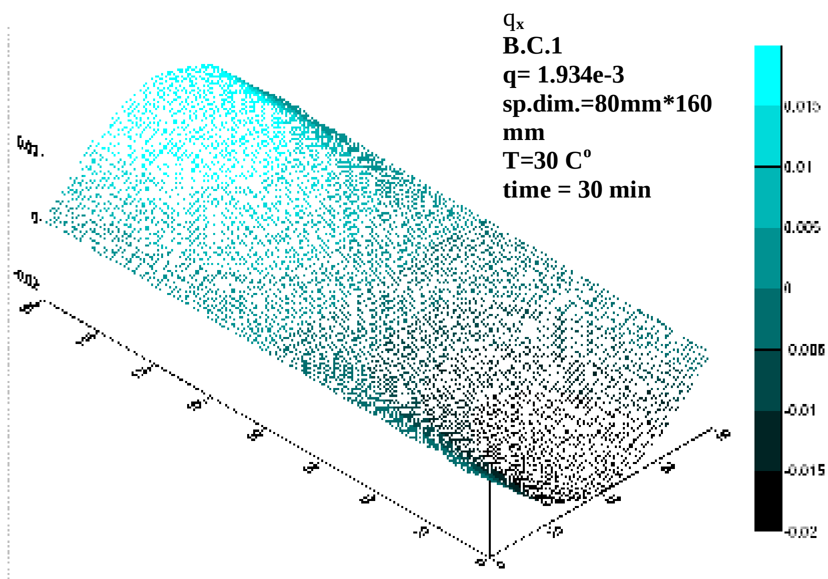


Figure (14)The relationship between central deflection and the ratio (a/b) for (B.C.1)at different temperatures



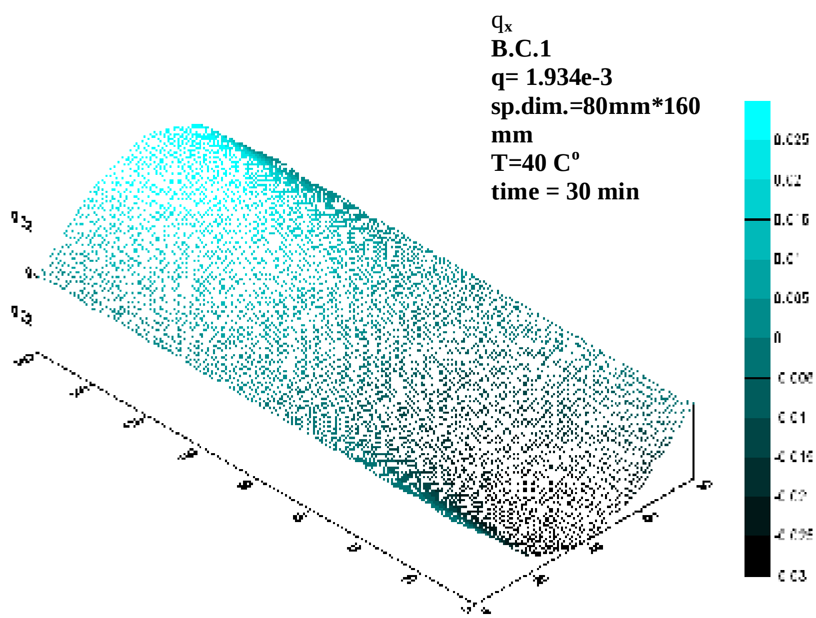
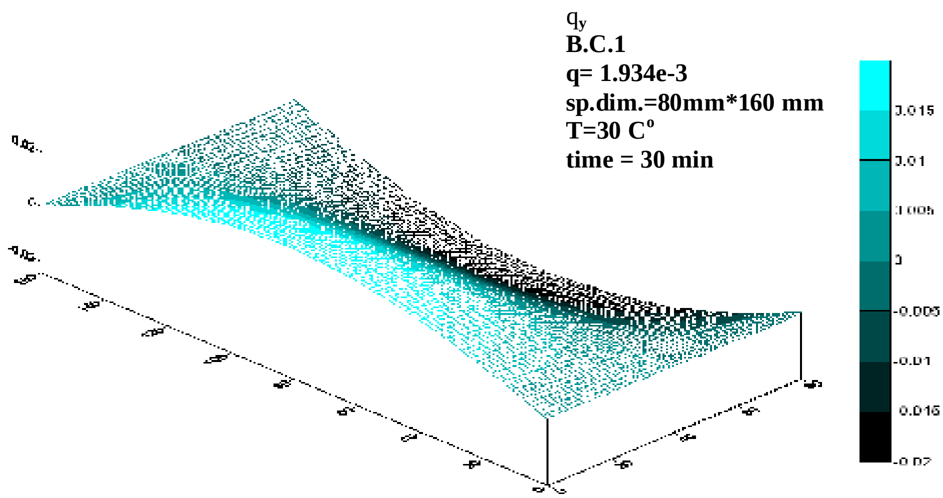


Figure (15) 3D-Rotation about x-axis ,type A31 and A32



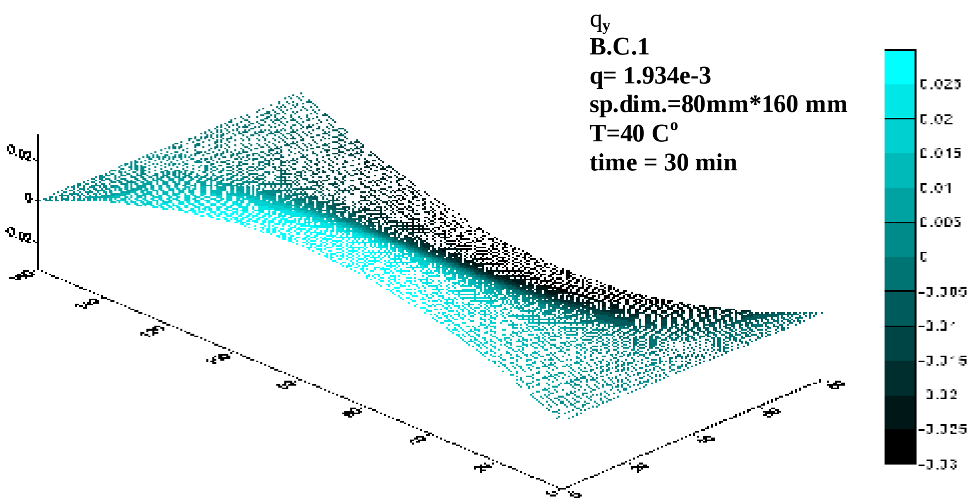


Figure (16) 3D-Rotation about y-axis ,type A31 and A32

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Appendix (A)

$$[N] = \frac{w}{\{a^e\}} \qquad \text{A-1}$$

where :

$$\{a\}^e = \begin{Bmatrix} ai \\ aj \\ al \\ ak \end{Bmatrix} \qquad \text{A-2}$$

and ;

$$ai = \begin{Bmatrix} wi \\ qxi \\ qyi \end{Bmatrix} = \begin{Bmatrix} wi \\ -\left(\frac{\partial w}{\partial y}\right)i \\ -\left(\frac{\partial w}{\partial x}\right)i \end{Bmatrix} \qquad \text{A-3}$$

A polynomial expression is used to define the shape function of the twelve parameters.

$$w = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 x^3 + a_8 x^2 y + a_9 xy^2 + a_{10} y^3 + a_{11} x^3 y + a_{12} xy^3 \qquad \text{A-4}$$