Generalized Λ_{gs}-Sets and Generalized V_{gs}- Sets

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Abstract

In this paper , we introduced a new concepts of generalized Λ_{gs} - sets (breifly.g. Λ_{gs}- sets) and generalized V_{gs}- sets (breifly. g.V_{gs}- sets) and study its connection with Λ_{gs}- (resp. V_{gs}-) sets ,we give some results about that.

Keywords: Λ_{gs}-sets , V_{gs}- sets, generalized Λ_{gs}-set , generalized V_{gs}- set.

المجموعات العامة-Λ_{gs} و المجموعات العامة -V_{gs}

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الملخص: في هذا البحث سوف نقدم مفهومي المجموعات الاعم- Λ_{gs}و- V_{gs}كما

سندرس ارتباطهما مع المجموعات- Λ_{gs}و- V_{gs}سوف نعطي بعض النتائج حول ذلك.

1.Introduction

In 1986, Maki [1] continued the work of Levine [2]and Dunham [3] on generalized closed sets and closure operators by introducing the notion of a generalized Λ-set in a topological space (X,τ) and by defining an associated closure operator, i.e. the Λ-closure operator. He studied the relationship between the given topology τ and the topology τ^{Λ} generated by the family of generalized Λ-sets.Ganster and et.al. [4] introduced the notion of pre-Λ-sets and pre-V-sets and obtained new topologies defined by these families of sets . M.E. Abd El-Monsef, A.A. El-Atik and M.M. El-Sharkasy [5] are introduced the notion of b-Λ- sets and b-V-sets topological spaces and studied some of its properties. Also they proved that the topology generated by the class of b-open sets contains the topology generated by the class of pre open (resp. semi-open) sets by using the notions of Λ-sets and V-sets.

2. Preliminaries

The concept of a semi-open set in a topological space was introduced by N.Levine in 1963 [6].If (X,τ) is a topological space and $A\subset X$, then A is semi open if there exists $U\in \tau$ such that $U\subset A \subset Cl(U)$.The complement A^c of a semi-open set A,is called semi-closed and the semi-closure of a set A,denoted by SCl(A),is defined to be the intersection of all semi-closed sets containing A,SCl(A) is a semi-closed set [7]and [8].The semi-interior [8] of A, denoted by sint(A),is defined by the union of all semi-open sets contained in A. A subset A of (X,τ) is said to be generalized semi-open[9] (written as gs-open) in (X,τ) if $F\subset$ sint (A)whenever $F\subset A$, and F is closed in (X,τ), a subset A is generalized semi- closed (written as gs-closed) if its complement A^c is gs-open in (X,τ). A generalized class of closed sets was consider by Maki in 1986 [1].He investigated the sets that can be represented as union of closed sets and called them V-sets. Complements of V-sets, i.e, sets that are intersection of open sets are called Λ-sets[1].The family of all generalized semi-open(resp. generalized semi-closed)sets in (X,τ) will be denoted by $GSO(X,\tau)$ (resp.$GSC(X,\tau)$,during our work X and Y(or(X,τ) and (Y,σ)) will always denote topological spaces .No separation axioms are assumed unless stated explicitly.

Definition 2.1

Let A be a subset of a space (X,τ). We define the subsets $\Lambda_{gs}(A)$

and V_{gs} (A)as follows:

$\Lambda_{gs}(A) = \cap[U:A\subset U, U\in GSO(X,\tau)]$ and $V_{gs}(A) = U\,[F:F\subset A, F\in GSC(X,\tau)]$

Definition 2.2

A subset A of a space (X,τ) is called Λ_{gs}–(resp.V_{gs}-)set if $A=\Lambda_{gs}(A)$ (resp. $A= V_{gs}(A)$).

Lemma 2.3

Let A be subset of a space (X,τ) ,then the following properties are valid.

(1) $A\subset \Lambda_{gs}(A)$

(2) $V_{gs}(A) \subset (A)$

(3) If $A\in GSO(X,\tau)$, then $A= \Lambda_{gs}(A)$

(4) $\Lambda_{gs}(A^c) = (V_{gs}(A))^c$.

(5) $\Lambda_{gs}(\{U\, Ai : i \in I\}) =\{ U\, \Lambda_{gs}(Ai):i \in I\}$

(6) If $A \in GSC(X,\tau)$, then $A=V_{gs}(A)$.

(7) If $A\subset B$, Then $\Lambda_{gs}(A) \subset\Lambda_{gs}(B)$.

(8) $\Lambda_{gs}(\Lambda_{gs}(A))= \Lambda_{gs}(A)$.

the proof of (1),(2),(3),(4),(5) and (6) in [10 : Lemma 2.3]

For prove (7) it is clear by Definition 2.1.

Fore prove (8),first observe that by (1)and (7),we have $\Lambda_{gs}(A) \subset \Lambda_{gs}(\Lambda_{gs}(A))$.

For the converse inclusion ,let $x \notin \Lambda_{gs}(A)$ .Then there exists $G \in GSO(X,\tau)$,such that $A \subset G$, $x \notin G$.Since $\Lambda_{gs}(\Lambda_{gs}(A)) = \{G : \Lambda_{gs}(A) \subset G, G \in GSO(X,\tau)\}$ .So we have $x \notin \Lambda_{gs}(\Lambda_{gs}(A))$.Thus $\Lambda_{gs}(\Lambda_{gs}(A)) = \Lambda_{gs}(A)$.

Remark2.4

By Lemma2.3(3)and (6),we have that

(1) If $A \in GSO(X,\tau)$, then A is a Λ_{gs} –set.

(2) If $A \in GSC(X,\tau)$, then A is a V_{gs} -set.

3- The Main Results .

Definition 3.1

A subset A of a space (X, τ) is called generalized Λ_{gs}-set(briefly g. Λ_{gs}-set) if $\Lambda_{gs}(A) \subset U$ whenever $A \subset U$ and $U \in GSC(X, \tau)$.

Definition 3.2

In a space (X, τ) , a subset A is called a generalized V_{gs}- set(briefly g- V_{gs}- set) of (X, τ) if A^c is a g. Λ_{gs}-set of (X, τ).

Proposition 3.3

For a space (X, τ) the following statements hold

1- Every Λ_{gs}-set is a g. Λ_{gs}-set.

2- Every V_{gs}- set is a g. V_{gs}- set.

3- Every union of g. Λ_{gs}-sets is a g.Λ_{gs}-sets.

4- Every intersection of g. V_{gs}- sets is a g. V_{gs}- sets.

Proof : (1) and (2) Follows from Definition (2.2) and Definition (3.1)

To proof (3) let $\{A_i : i \in I\}$ is a g. Λ_{gs}-sets then by Lemma 2.3 (5)

$\Lambda_{gs}(U\{A_i : i \in I\}) = U\{\Lambda_{gs}(A_i) : i \in I\}$, Hence by hypothesis and Definition (3.1), $U\{A_I : i \in I\}$ is g. Λ_{gs}-set .

For prove (4), let $\{A_i : i \in I\}$ is a g. V_{gs}-sets then by Definition 2.3 ,$\{ A^c{}_i : i \in I \}$ is

a g. Λ_{gs}-sets.Then by(3),we obtain $U\{ A^c{}_i :i \in I \}$ is = a g. Λ_{gs}-sets.Thus by Definition (3.2), $\cap \{A_I : i \in I\}$ is g. V_{gs}-set .

Remark 3.4

The intersection of two g. Λ_{gs}-sets is not a g. Λ_{gs} -sets as shown by the following Example .

Example 3.5

Let X = { a,b,c} and $\tau = \{ \Phi,\{a,b\} , X\}$.

The family of all g. Λ_{gs}-sets = { Φ,X,{a},{b},{a,b},{a,c},{b,c} } and family of g. V_{gs}- sets ={ Φ,X,{a},{b},{c},{a,c},{b,c}} , if A={a,c} and B={b,c} . then A and B are g. Λ_{gs} -sets but A∩B={c} is not a g. Λ_{gs}-set.

Remark 3.6

The converse of Proposition 3.3 (1) (resp.(2)) is not true as in the following example .

Example 3.7

Let (X, τ) be the space in Example 3.5 , the subset A = {b,c} is a g. Λ_{gs}-set but it is not a Λ_{gs}-set.

Proposition 3.8

If A is a generalized semi closed and g. Λ_{gs}-set of a space (X, τ) , then A is Λ_{gs}-set

Proof : since $A \in GSC(X, \tau)$ and g. Λ_{gs}-set then $\Lambda_{gs}(A) \subset A$ and by Lemma 2.3 (1),

$A \subset \Lambda_{gs}(A)$, hence $\Lambda_{gs}(A) = A$, thus A is Λ_{gs}-set.

Corollary 3.9

By Remark (2.4) and Proposition(3.3),we have that:

(1) If $A \in GSO(X, \tau)$, then A is a g. Λ_{gs}-set
(2) If $A \in GSC(X, \tau)$, then A is a g. v_{gs}-set.

Proof:(1) Since $A \in GSO(X, \tau)$, then by Remak 2.4(1),A is a . Λ_{gs}-set .Thus by Proposition3.3(1),A is a g. Λ_{gs}-set.
To prove (2).by the same method we can prove that.

Proposition 3.10

Let(X, τ) be a space and $x \in X$, then

(1) $\{x\}$ is a generalized semi open or $\{x\}^c$ is a g. Λ_{gs}-set of (X, τ)
(2) $\{x\}$ is a generalized semi open or $\{x\}$ is a g. v_{gs}-set of (X, τ)

Proof : (1) Suppose that $\{x\}$ is not generalized semi open , then X is the only generalized semi closed set containing $\{x\}^c$ we have

$\Lambda_{gs}(\{x\}^c) \subset X$ holds . This implies $\{x\}^c$ is a g. Λ_{gs}-set of (X, τ) .

(2)Follows from (1) and Definition 3.2.

Proposition 3.11

If $A \subset B \subset \Lambda_{gs}(A)$ and A is a g. Λ_{gs}-set of a space (X, τ) , then B is a g. Λ_{gs}-set of (X, τ).

Proof : Since $A \subset B \subset \Lambda_{gs}(A)$ then by Lemma 2.3 (7) we have

$\Lambda_{gs}(A) \subset \Lambda_{gs}(B) \subset \Lambda_{gs}(\Lambda_{gs}(A))$, then by Lemma 2.3(8),we have $\Lambda_{gs}(A) \subset \Lambda_{gs}(B) \subset \Lambda_{gs}(A)$.Thus, we get $\Lambda_{gs}(A) = \Lambda_{gs}(B)$ let F be any generalized semi closed subset of (X, τ) such that $B \subset F$. Since $A \subset B$ and A is a g. Λ_{gs}- set. Then we have $\Lambda_{gs}(B)= \Lambda_{gs}(A) \subset F$

In the following Propositions we give a characterization of g. v_{gs}-sets (Definition 3.2) By using v_{gs}- operations and we obtain results concerning subsets.

Proposition 3.12

Subset A of a space (X, τ) is a g. v_{gs}-set if and only if $U \subset v_{gs}(A)$ whenever $U \subset A$ and $U \in GSO(X, \tau)$

Proof: Necessity . Let U be a generalized semi open subset of (X, τ) such that $U \subset A$. Then since U^c is generalized semi closed and $A^c \subset U^c$, then by Definition 3.2 , A^c is a g. Λ_{gs}- set , thus by Definition 3.1 $\Lambda_{gs}(A^c) \subset U^c$, hence by Lemma 2.3 (4) $(v_{gs}(A))^c \subset U^c$. thus $U \subset v_{gs}(A)$

Sufficiency Le F be a generalized semi closed subset of (X, τ) such that $A^c \subset F$. Since F^c is generalized semi open and $F^c \subset A$, by assumption we have $F^c \subset v_{gs}(A)$. Then by Lemma 2.3 (4), $(v_{gs}(A))^c = \Lambda_{gs}(A^c) \subset F$

thus A^c is a g. Λ_{gs}- set , i.e , A is a g. v_{gs} – set .

Corollary 3.13

Let A be a g.v_{gs}- set in a space (X, τ) , then for every generalized semi closed set F such that $v_{gs}(A) \cup A^c \subset F$, F=X holds.

Proof : The assumption $v_{gs}(A) \cup A^c \subset F$ implies $F^c \subset (v_{gs}(A))^c \cap A$. where F^c is generalized semi open set , since A is a g. v_{gs}- set , then by Proposition 3.12,

we have $F^c \subset v_{gs}(A)$ and hence $(v_{gs}(A))^c \subset F$ and

$F^c \subset (v_{gs}(A))^c \cap v_{gs}(A) = \Phi$. Therefore , we have F = X

Corollary 3.14

Let A be a g. v_{gs}- set of a space (X , τ)

Then $v_{gs}(A) \cup A^c$ is a generalized semi closed set if and only if A is a v_{gs}-set

Proof : To prove A is a v_{gs}-set. By Corollary 3.13, $v_{gs}(A) \cup A^c = X$. Thus $(v_{gs}(A))^c \cap A = \Phi$ .Therefore , $A \subset v_{gs}(A)$ and by Lemma 2.3 (2) , we get $v_{gs}(A)=A$. Hence A is a $v_{gs}(A)$, sufficiency is obvious .

Proposition 3.15

Let A be a subset of space (X, τ) such that $v_{gs}(A)$ is generalized semi closed , if X=F holds for every generalized semi closed subset F such that $v_{gs}(A) \cup A^c \subset F$ then A is a g. v_{gs}-set.

Proof : Let $U \subset A$ where U is generalized semi open.According to assumption $v_{gs}(A) \cup U^c$ is generalized semi closed such that

$v_{gs}(A) \cup A^c \subset v_{gs}(A) \cup U^c$, it follows that $v_{gs}(A) \cup U^c = X$ and hence

$U \subset v_{gs}(A)$ then by proposition 3.12 , A is a g. v_{gs}set

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