Design of Trajectory Tracking Controller for a Differential-Drive Mobile Robot Platform Based on Integral Control

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ABSTRACT

Atrajectory tracking controller based on integral control technique for the wheeled differential mobile robot using the kinematic model is proposed in this paper. The proposed controller design is achieved by using the nonlinear kinematics model of the mobile robot which is being transformed to a nonlinear error model by shifting its states to the origin and by adding a dummy integral control state variables that are augmented with the nonlinear error model, the model became ready to be used to design a controller. The nonlinear controller which will enforce the system dynamics to follow the desired trajectory guarantee that the steering control system of the mobile robot will behave as a second order reference model with specified natural frequency and damping ratio is being selected by the designer, and the average speed control system will behave as a first order reference model with specified time constant chosen by the designer. The simulation results which is achieved by using MATLAB Rev. (14.9 2009b) show the potential of the proposed controller to track the mobile robot to the desired trajectory with very slight error.

Keywords: Differential-Drive Mobile Robot, Integral Control

تصميم مسيطر متتبع للمسار لمنصة روبوت متحركة ذات محرك فرقي بالاعتماد على السيطرة التكاملية

الخلاصة:

في هذا البحث تم تصميم مسيطر متتبع للمسار بالاعتماد على تقنية السيطرة التكاملية للسيطرة على منصة روبوت متحركة ذات محرك فرقي. وقد تم إنجاز تصميم المسيطر باستخدام النموذج الرياضي الحركي اللا خطي المجرد لمنصة الروبوت المتحرك والذي تم تحويله إلى نموذج الخطأ اللا خطي وذلك بتحريك الحالات إلى نقطة الأصل ، ومن ثم بإضافة حالات السيطرة التكاملية الوهمية التي تم دمجها مع

النموذج الرياضي أصبح النموذج الرياضي جاهزاً لغرض تصميم المسيطر. أن المسيطرة اللا خطي الذي سوف يجبر ديناميكية نظام الروبوت المتحرك لإتباع المسار المطلوب سوف يضمن أن نظام السيطرة الموجه لمنصة الروبوت المتحرك سوف يتصرف كنموذج رياضي من الدرجة الثانية مع تحديد التردد الطبيعي ونسبة التخميد من قبل المصمم ، وكذلك يضمن أيضاً أن نظام السيطرة على معدل السرعة لمنصة الروبوت سوف يتصرف كنموذج من الدرجة الأولى مع تحديد الثابت الزمني من قبل المصمم أيضاَ. أن نتائج المحاكاة التي تم انجازها باستخدام برنامج ماتلاب (14.9 2009b) تظهر إمكانية الروبوت لجعل منصة الروبوت المتحرك يتبع المسار المطلوب مع نسبة خطأ جداً قليلة.

INTRODUCTION

The mobile robot was modeled as a rigid body that satisfies a nonholonomic constraint, which means the motion of the system is not completely free. In a nonholonomic systems, the instantaneous velocities of system components are restricted, thereby the local movement of the system is limited. This means, for example, that the mobile robot cannot move sideways. Common examples of vehicles with nonholonomic motion constraints are automobiles and vehicles with trailers. Parallel parking is a familiar illustration of the type of difficulty associated with even this simple path planning problem. Other contexts in which nonholonomic constraints occur include when there is a rolling contact, such as with a fingered hand on a surface, or when conservation of angular momentum is a significant factor, as in the case of free-flying robots. Nonholonomic systems are not locally controllable, yet they are in many cases globally controllable [1].

The problem of differential-drive trajectory tracking is being extensively reviewed in survey and a lot of research efforts are being done for achieving a suitable solution to this problem and due to its importance it is still have a great importance and the researchers and the designers are still have interest with this field. Back to 1990, Y.Kanayamaet. al. [2], proposed a stable tracking control rule fornon-holonomic vehicles. The input to the vehicle are a reference posture and reference velocities. The major objective of their work was to propose a control rule to find a reasonable target linear and rotational velocities. In order to achieve this objective they perform a Linearization to the system's differential equation which was useful to decide parameters for critical dumping for a small disturbance. In order to avoid any slippage, a velocity/acceleration limitation scheme was introduced. Hazry et.al. [3], in 2006used the proportional-integral-derivative (PID)controllers to achieve the control of the differential-drive mobile robot trajectory tracking. They presented a new development for proportional parameter estimation in mobile robot for stable tracking control system. Proportional control parameters are decided by determining the minimal root mean square error(RMSE) of deviation in wheel rotations for the right wheel and the left wheel in the real environment. The selected minimal RMSE are use in the developed proportional controller which consists of two different system that are individually control the two D.C motors to generate the PWS (power wheel steering) of the mobile robot. The two system works concurrently with different value of proportional control parameters to perform stable movement intrajectory straight line tracking. In 2007, M. Defoortet. al. [4], applied a robust sliding mode control algorithm using an integral sliding variable to solve both the stabilization and tracking problems of the wheeled mobile robot. In their work they highlighted an interesting property of the integral sliding mode where they found that it is possible to avoid some control singularities that could appear during the reaching phase with classical sliding mode controls. To demonstrate the advantages of the proposed controller, experimental results were given.

In the field of intelligent control systems applications; Y. Lianget. al. in 2010 [5] designed an adaptive fuzzy control algorithm for trajectory tracking of mobile robot. Their proposed control scheme combined with the fuzzy PD(Proportional and Differential) control and the separate integral control. The control scheme can not only make full use of the advantage of the fuzzy control, but also have the good steady state tracking ability of the integral control. The control scheme introduces so many parameters which are difficult to optimize. In order to realize the online adaptive learning of the control parameters, the modified VFSA (Very Fast Simulated Annealing) is used. The simulation results show that the method is feasible, and can quickly approach the desired trajectory. The adaptive neuro fuzzy inference system (ANFIS) is used to achieve the trajectory tracking controller design of a mobile robot by M.Imenet. al. [6] in 2011, the proposed tracking control of mobile robot used two cascade controllers. The first fuzzy controller produces a variable which shows curvature of the path and is considered as one of the inputs of the second fuzzy controller. Adaptive Neuro Fuzzy Inference System(ANFIS) is applied as second stage controller for the solution of the path tracking problem of mobile robots. Agradient descent learning algorithm is used to adjust the parameters. That presented controller is compared with previous work to confirm its effectiveness.

In this paper the design of trajectory controller is done by using the nonlinear kinematics model without an approximation to this model to design an integral controller that deliver the required velocities to the left and right motors of the mobile robot. The necessary simulations are used to validate the controller and the results show the effectiveness of the controller.

Mathematical Model of Differential-Drive Mobile Robot

The mobile robot which is located in a 2D plane in which a global Cartesian coordinate system is defined. The robot platform in a world that possesses three degree of freedom in its positioning which are represented by a posture [2];

$$p = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad \ldots..(1)$$

Where the heading direction $\theta$is taken counterclockwise from the x-axis. The entire locus of the point $(x(t), y(t))$ is called the **path** or **trajectory**. The mobile robot is controlled by its linear velocity υ and the rotational velocity ω, which are a function of time. The mobile robot kinematics is defined by a Jacobian matrix J:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \dot{p} = Jq = \begin{bmatrix} -\cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} q \qquad \ldots\textbf{(2)}$$

Where; $q = [v \quad \omega]^T$

The motion of a typical mobile robot can be controlled by setting the velocities (v_L, v_R) of each of the two main wheels. With constant v_L and v_R the center of the robot moves with average speed, $v = \frac{(v_L+v_R)}{2}$ on a circle that has its center on the wheel axis, as shown in the following Figure [1].

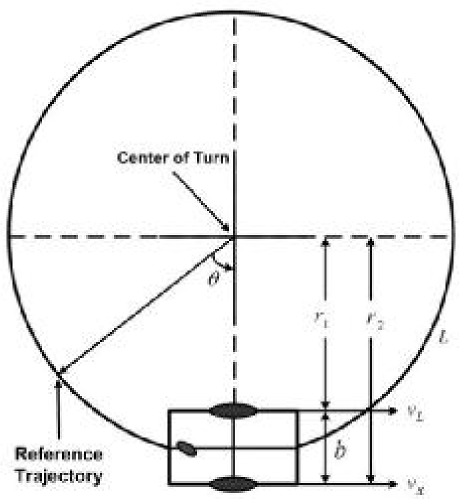

Figure (1): Rotational Motion of Robot.

Where :

$v_L = r_1\omega$; is left wheel speed; and,

$v_R = r_2\omega$; is the right wheel speed

The speed of rotation $\omega = \dot{\theta}$ of the robot is thus given by:

$$\dot{\boldsymbol{\theta}} = \frac{\mathbf{v_R - v_L}}{\mathbf{r_2 - r_1}} = \frac{\mathbf{v_R - v_L}}{\mathbf{b}} \qquad \ldots\textbf{(3)}$$

Where b is the wheel base, by assuming the average wheel velocity is v and introduce the control variables $u_1$and $u_2$in the following integral form;

$$\mathbf{v_R} = \int_{\mathbf{t_o}}^{\mathbf{t}} \mathbf{u_1}\, \mathbf{dt} \qquad \ldots\textbf{(4)}$$

$$\mathbf{v_L} = \int_{\mathbf{t_o}}^{\mathbf{t}} \mathbf{u_2}\, \mathbf{dt} \qquad \ldots\textbf{(5)}$$

To get a full description of the robot motion, the translational motion also needs to be considered, which is depicted in Figure (2). Here v will be imposed to behave like a desired average velocity v_{des}, by using the designed controller. As a result to this, the robot will follow the desired trajectory that resulted from navigation system.

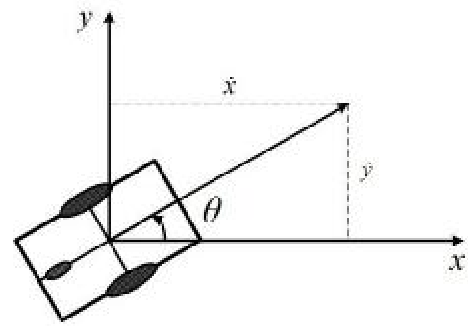

Figure (2): Translation motion of Robot.

Here the state vector $x = (x_1, x_2, x_3) = (x, y, \theta)$ is chosen to be;

$$\dot{x}_1 = -\frac{v_R+v_L}{2}cos(x_3) \quad ..(6)$$

$$\dot{x}_2 = \frac{v_R+v_L}{2}sin(x_3) \quad ..(7)$$

$$\dot{x}_3 = \frac{v_R-v_L}{b} \quad ..(8)$$

Eq. (6) – (8) is the nonlinear state space model of the differential mobile robot system, where b; is a given constant depends on robot geometry.

Trajectory Tracking Controller Design

In the subsequent manipulations, the controller will be designed using the nonlinear state space model (differential mobile robot kinematic model) without using any kind of linear approximations of the equation of motion. In order to achieve the controller design, it is needed as a first step to shift the nonlinear state space coordinates in the $(x - y)$ space (Eq. (6)-(7)), so that the desired location or target is at origin, defining the new coordinates in the following way:

$$\begin{aligned} z_1 &= x_1 - x_{1d} \\ z_2 &= x_2 - x_{2d} \end{aligned} \quad ...(9)$$

Where x_{1d}: is the desired x-axis displacement;

x_{2d}: is the desired y-axis displacement.

So the nonlinear state space model (Eq. (6)-(7)), is transformed in the following way:

$$\dot{z}_1 = -\frac{v_R+v_L}{2}\cos(z_3) - \dot{x}_{1d}$$

$$\dot{z}_2 = \frac{v_R+v_L}{2}\sin(z_3) - \dot{x}_{2d} \quad ... (10)$$

Where $z_3 = x_3$, in this analysis as the shifiting will be done by imposing a reference model to behave like.

Hint: it is important that x_{1d}, and x_{2d} should be continuously differentiable.

Now, without lose of generality we consider the desired location is fixed, thus;

$$\dot{x}_{1d} = \dot{x}_{2d} = 0 \qquad \text{...(11)}$$

The next step is to assume the following dummy integral states to be;

$$z_4 = v_R \,, z_5 = v_L \qquad \text{...(12)}$$

By utilizing the above assumption, the new augmented state space model of the differential mobile robot will result in;

$$\begin{aligned} \dot{z}_1 &= -\frac{z_4 + z_5}{2}\cos(z_3) \\ \dot{z}_2 &= \frac{z_4 + z_5}{2}\sin(z_3) \\ \dot{z}_3 &= \frac{z_4 - z_5}{b} \\ \dot{z}_4 &= u_1 \\ \dot{z}_5 &= u_2 \end{aligned} \qquad \text{...(13)}$$

The above model (Eq. (13) will be used to design the nonlinear controller which will enforce the system dynamics to follow the desired trajectory. The steering control system [(θ) or (x_3)] here is designed to behave as a second order reference model with specified natural frequency ω_n and damping ratio ζ, namely

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = \omega_n^2\theta_{des} \qquad \text{...(14)}$$

While the average speed control system is designed to behave as a first order reference model with specified time constant τ, as follows

$$\tau\dot{v} + v = v_{des} \qquad \text{...(15)}$$

Now our objective is to design a control laws u_1 and u_2 that is able to regulate the shifted coordinates to the origin $(z_1, z_2 \rightarrow 0)$, which means that ($x_1 = x_{1d}$and $x_2 - x_{2d}$), and should implicitly mean that the [(θ) or (x_3)] should act like θ_{des} and the average speed v should act like v_{des}. The design procedure will be achieved by using a parallel fashion technique to compute the control laws that are aimed to replace the undesired system dynamics with a desired one selected by the designer (the desired dynamics are selected by choosing the desired parameters ($\zeta, \omega_n$and τ).

Namely, by proceeding in parallel computing fashion we first start with Eq. (14) and rewriting the equation in the following way;

$$\ddot{\theta} = -2\zeta\omega_n\dot{\theta} + \omega_n^2[\theta_{des} - \theta] \quad \text{...(16)}$$

Now, $\ddot{\theta}$ can be found by differentiation of Eq. (3) with respect to time t and substituting the result into Eq. (16), yields;

$$\frac{u_1 - u_2}{b} = -2\zeta\omega_n\dot{\theta} + \omega_n^2[\theta_{des} - \theta] \quad \text{...(17)}$$

For the average speed system, we rewrite the Eq. (15) as follows;

$$\dot{v} = \frac{1}{\tau}[v_{des} - v] \quad \text{...(18)}$$

As we mention before that $v = \frac{(v_L + v_R)}{2}$, so again by differentiation of this formula with respect to time t, and by substituting back into Eq. (18), we find;

$$\frac{u_1 + u_2}{2} = \frac{1}{\tau}\left[v_{des} - \left(\frac{z_4 + z_5}{2}\right)\right] \quad \text{...(19)}$$

By utilizing the resultant Eq. (17) and Eq. (19), the control laws u_1 and u_2 can be derived as;

$$u_1 = \frac{[v_{des} - ([z_4 + z_5]/2)]}{\tau} - \zeta\omega_n(z_4 + z_5) + \frac{\omega_n^2\, b}{2}[\theta_{des} - z_3] \quad \text{...(20)}$$

$$u_2 = \frac{[v_{des} - ([z_4 + z_5]/2)]}{\tau} + \zeta\omega_n(z_4 + z_5) - \frac{\omega_n^2\, b}{2}[\theta_{des} - z_3] \quad \text{...(21)}$$

Analysis of Simulation Results

In the simulation results analysis it is needed first to achieve the control law calculations, The maximum left velocity v_L and right velocity v_R will be taken to as ($0.27778\, m/sec$(i.e. 1 $km/hour$)) and the width between the two wheels b will be ($0.277\, m$) (these specifications are taken from Rover 5 Robot specification datasheet [7] which taken here as a case study for our design). The simulation results are obtained using MATLAB Rev. (14.9 2009b) with the Simulink model shown in Figure(3) below. In order to examine the designed control laws capability on tracking different trajectories, the simulation will be done with the following four testing desired trajectories:

Trajectory Path I: $x_d = 2\sin(2\pi t)$

$$y_d = 2\sin\left(2\pi t + {}^{pi}/_{2}\right) \quad \text{...(22)}$$

Trajectory Path II: $x_d = 2\sin\left(\pi t + {}^{pi}/_{2}\right)$

$$y_d = 2\sin\left(2\pi t + {}^{pi}/_{4}\right) \quad \text{...(23)}$$

Trajectory Path III: $x_d = 2\sin\left(2\pi t + {pi}/{4}\right)$

$$y_d = 2\sin\left(\pi t + {pi}/{2}\right) \quad \text{...(24)}$$

The chosen desired trajectories are taken here with main important property that all these trajectories are **continuously differential**. It is needed to obtain θ_{des}(i.e. the desired orientation angle), actually this can be easily obtained by rewriting the formula [8]in the following way:

$$\theta_{des} = tan^{-1}\left(\frac{\dot{y}_d}{\dot{x}_d}\right) \quad \text{...(25)}$$

It can be noticed clearly that in order to calculate the dummy control variables u_1 and u_2 (Eq.(19) and Eq.(20), respectively) the desired controller parameters should be selected first by the designer and can be found in Table (1).

After calculating the control laws (Eq. (20) and Eq.(21) parameters , the above desired trajectories is imposed to obtain the simulation results which will be used to show the potential of the designed trajectory tracking controller .The simulation results for the mobile robot trajectory tracking for the first trajectory path I are shown in Fig. (4), while Figures (5), (7) and (9) show the tracking response for the x-axis, y-axis, and the orientation angle θ respectively. It can be noticed that the controller is efficient to track the desired trajectory with a very slight error that can be considered as tolerable, this can be obviously shown in Fig. (6) and (8) for the error states z_1 and z_2, respectively. This slight error values are justified to the integral part of the controller where it is well-known that the integral usually added to counteract the error. In Fig. (10), the profile of differential mobile robot left and right motor velocities v_L, and v_R are shown and the reader can see that although the right and left velocities are constrained to simulate the Rover 5 differential mobile robot model which is taken as a case study but the designed controller is able to achieve the imposed trajectories within the actuator limits. The rest of figures are dedicated to show the simulation results for the desired trajectories (I and II). All the tests performed show the ability of the design control law to track the desired trajectories in an efficient manner within the design specifications. In the same way, it can be seen that at every imposed trajectory the controller still have the ability to achieve the desired performance with similar behavior. It is worth mentioning to say that at the three cases which corresponds to the desired paths (I through III), the corresponding error behavior is of order of (10^{-6}or 10^{-5}). From theoretical point of view, the error at every case should approach zero but due to the simulation context (using numerical solver), the error values will appear very small (note the Figures (6), and (8), Figures (13), and (15), and Figures (20), and (22) for corresponding desired paths I, II, and III, respectively).

CONCLUSIONS

An integral controller is designed to track a differential-drive mobile robot to a desired trajectory. To apply control design algorithm, the nonlinear kinematics model of the mobile robot is first being adapted by shifting its coordinates to the origin and then adding a dummy integral control state variables that are augmented with the nonlinear error model. A reference second order model is chosen by assigning a specified natural frequency and damping ratio to make the steering control system of the mobile robot to behave as a reference model, for the average speed control system, a first order reference model with specified time constant that is chosen by the designer to make the average speed system behave like. To show the effectiveness of the proposed controller simulation results is achieved and the system is being tracked to the desired trajectory with very small error.

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Table (1): Controller desired Parameters

Controller Desired Parameter	Chosen Value	Units
ω_n	100	rad/sec
ζ	0.7	-
τ	0.1	sec
v_{des}	0.01	m/sec

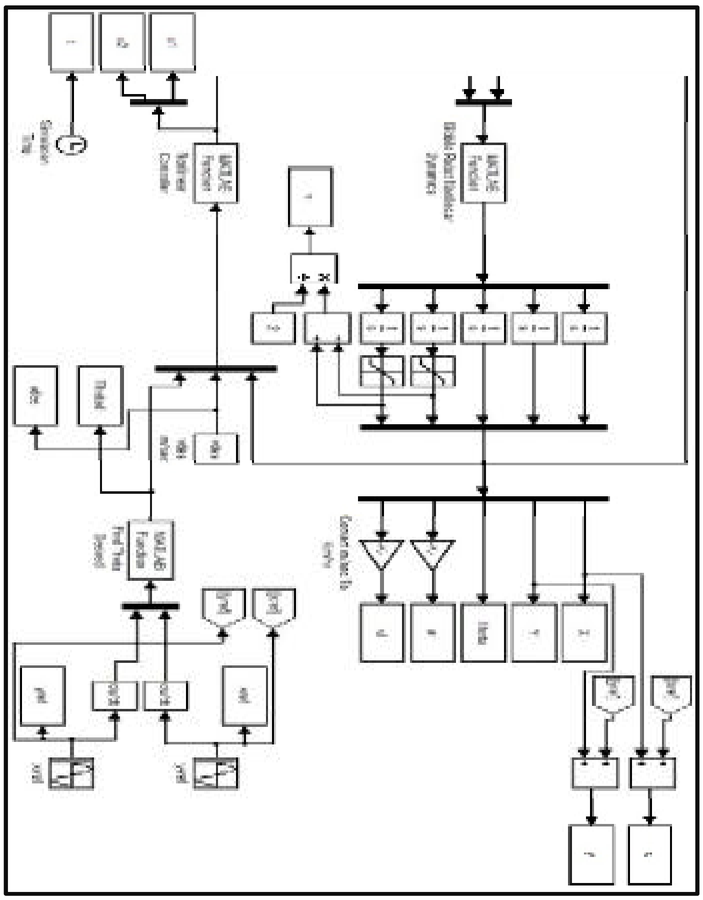

Figure (3): Matlab/Simulink Model for Mobile Robot Trajectory Tracking Controller.

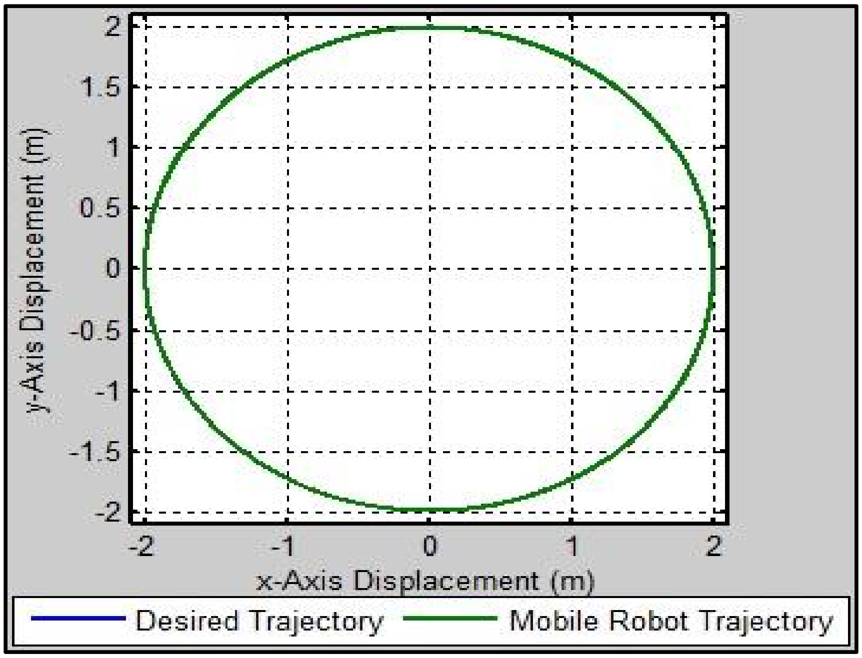

Figure (4): Simulation Results of Mobile Robot Tracking on Trajectory Path I

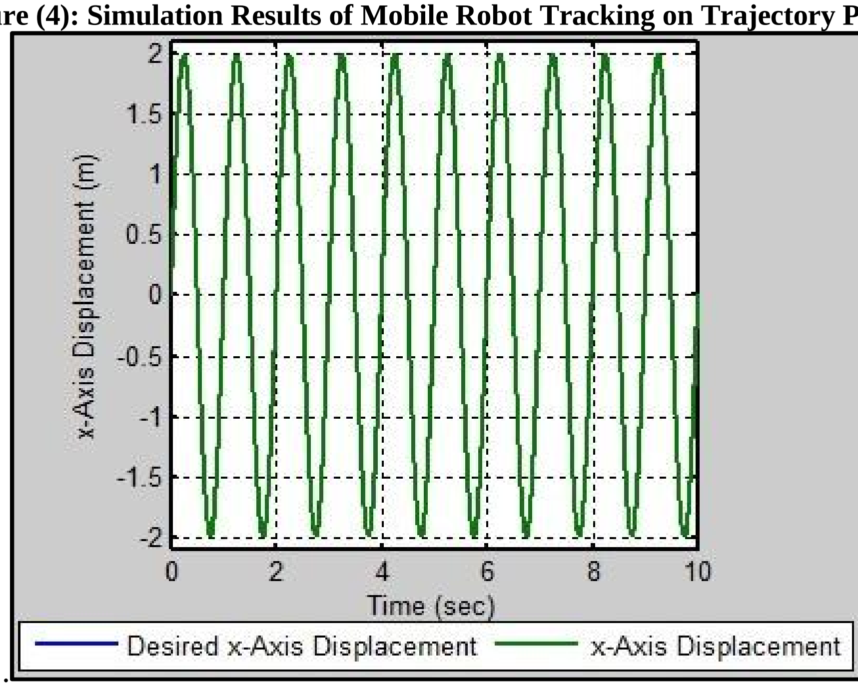

Figure (5): Simulation Results of x-axis variable response on Trajectory Path I.

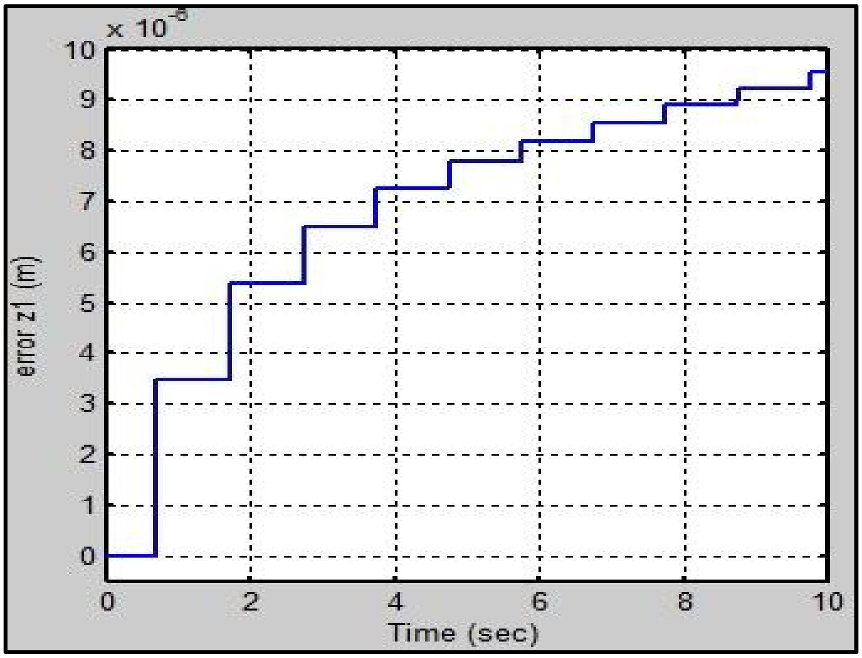

Figure (6): Simulation Results of x-axis error variable response on Trajectory Path I.

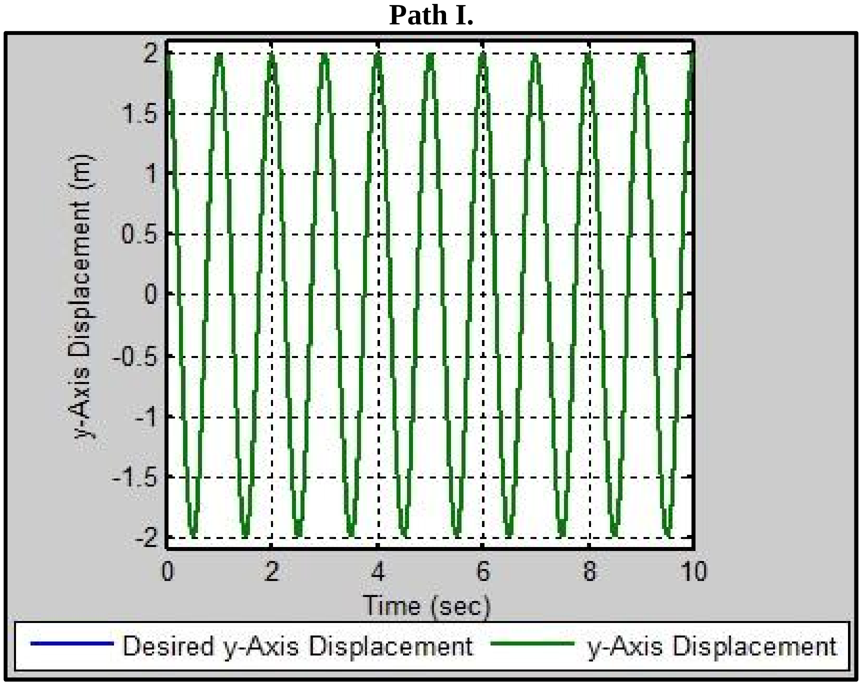

Figure (7): Simulation Results of y-axis variable response on Trajectory Path I.

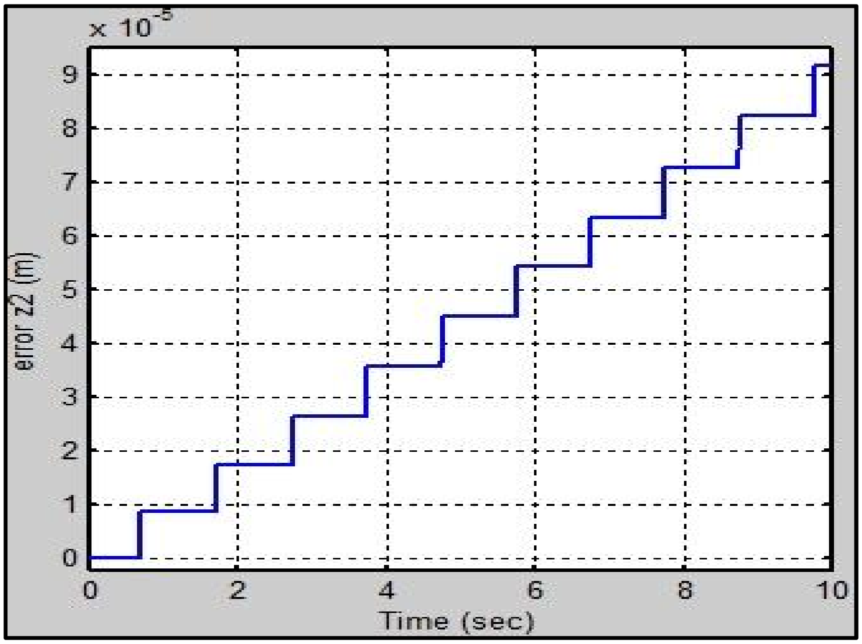

Figure (8): Simulation Results of y-axis error variable response on Trajectory Path I.

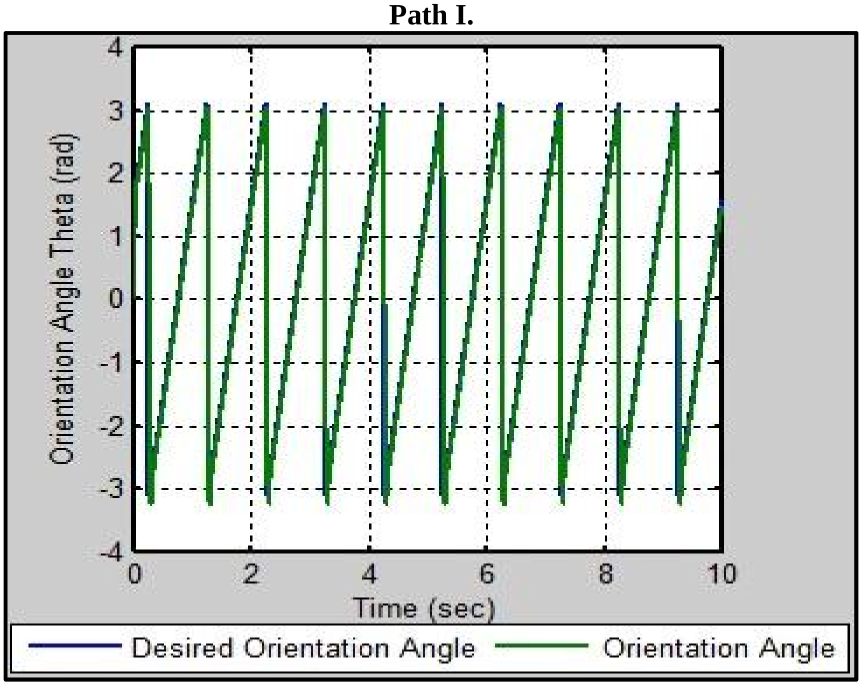

Figure (9): Simulation Results of Orientation Angle θ variable response on Trajectory Path I.

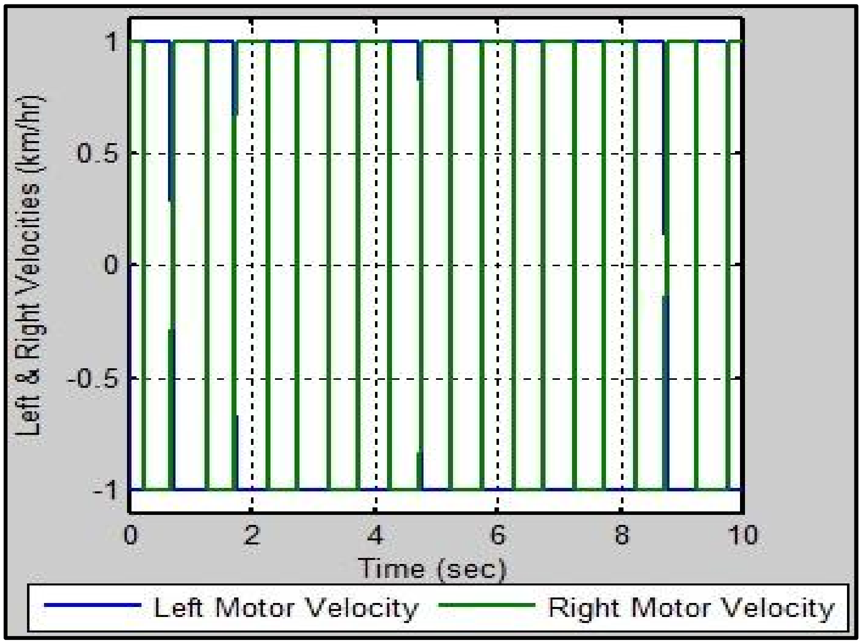

Figure (10): Simulation Results of Left and Right Motor Velocities on Trajectory Path I.

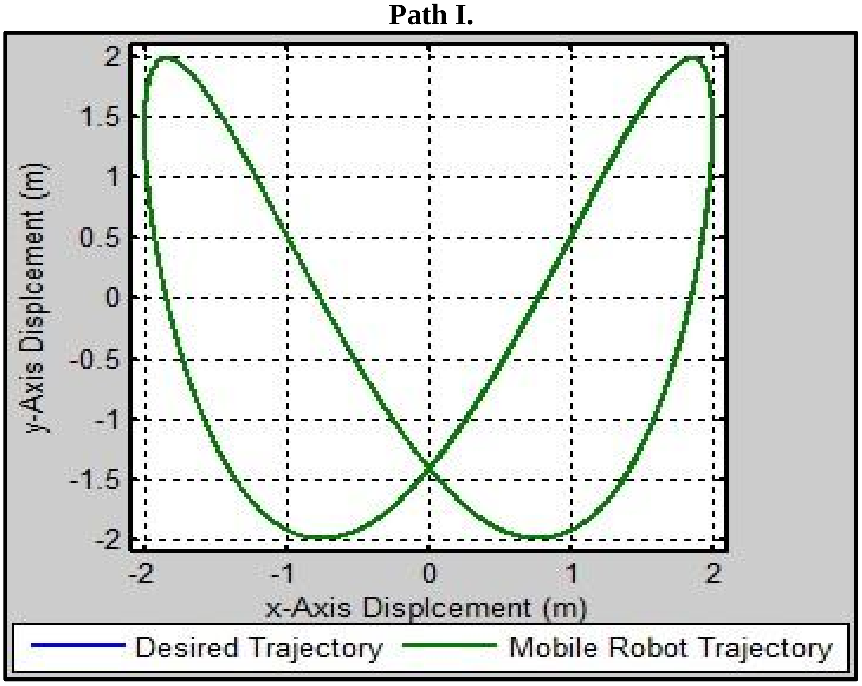

Figure (11): Simulation Results of Mobile Robot Tracking on Trajectory Path II.

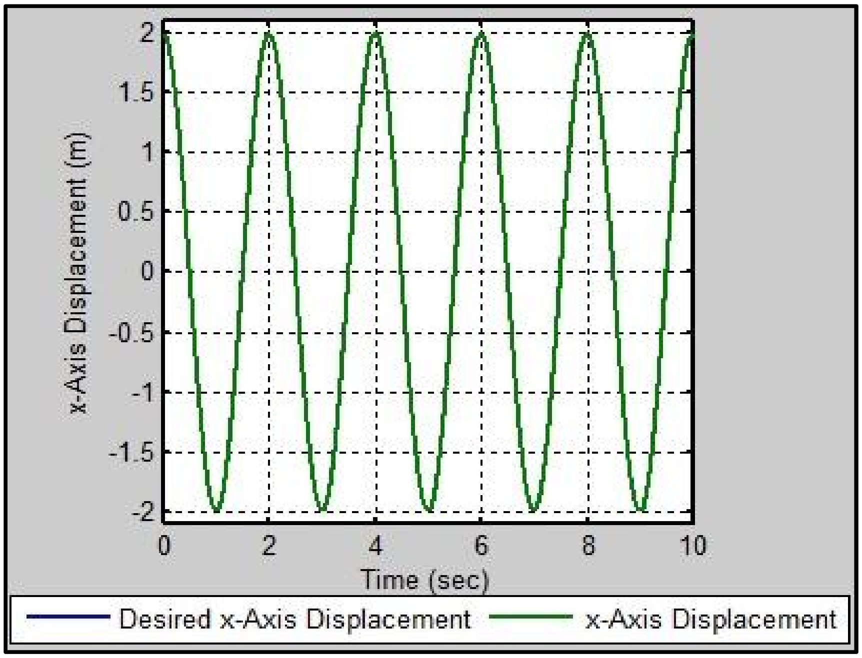

Figure (12): Simulation Results of x-axis variable response on Trajectory Path II.

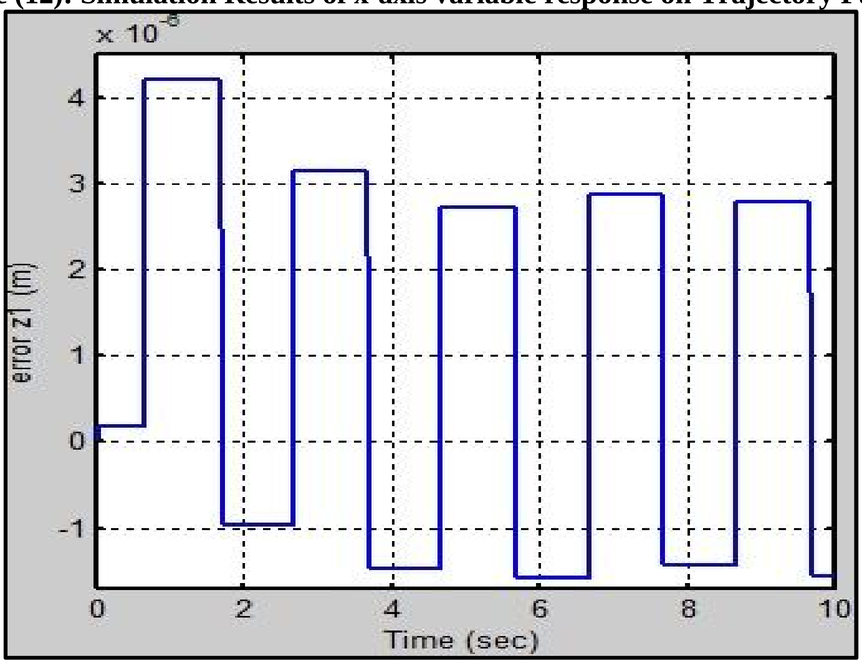

Figure (13): Simulation Results of x-axis error variable response on Trajectory Path II.

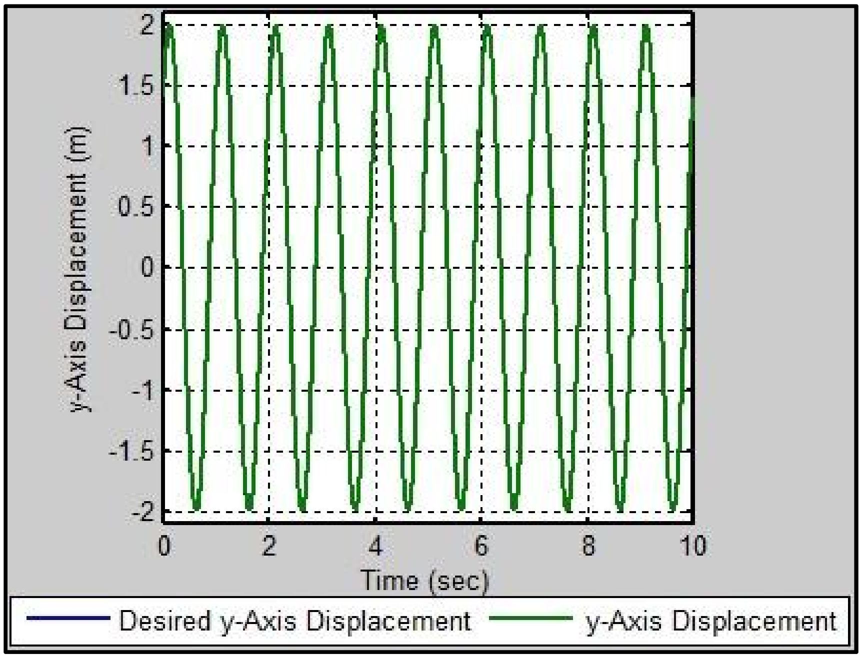

Figure (14): Simulation Results of y-axis variable response on Trajectory Path II.

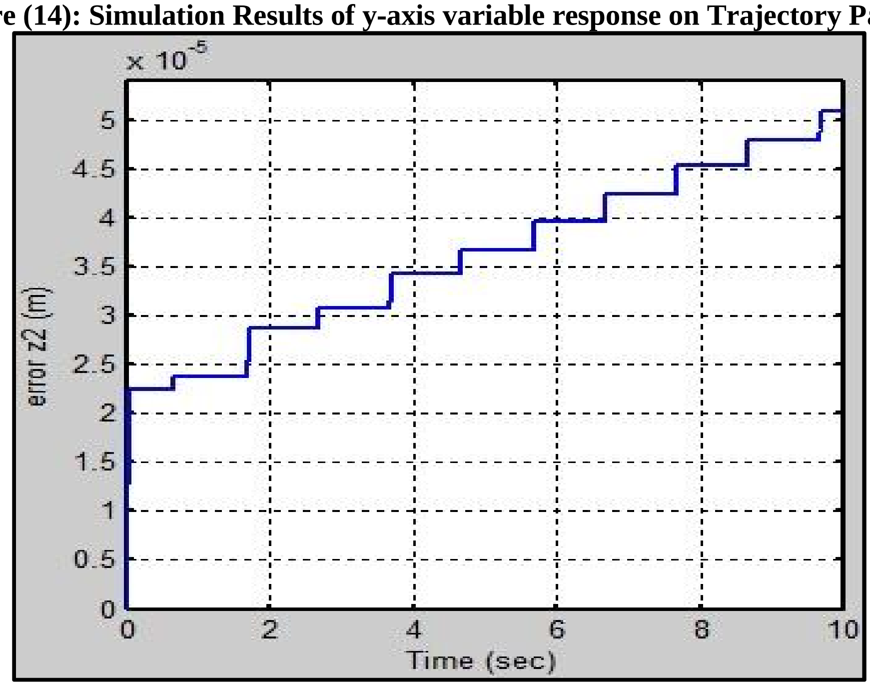

Figure (15): Simulation Results of y-axis error variable response on Trajectory Path II.

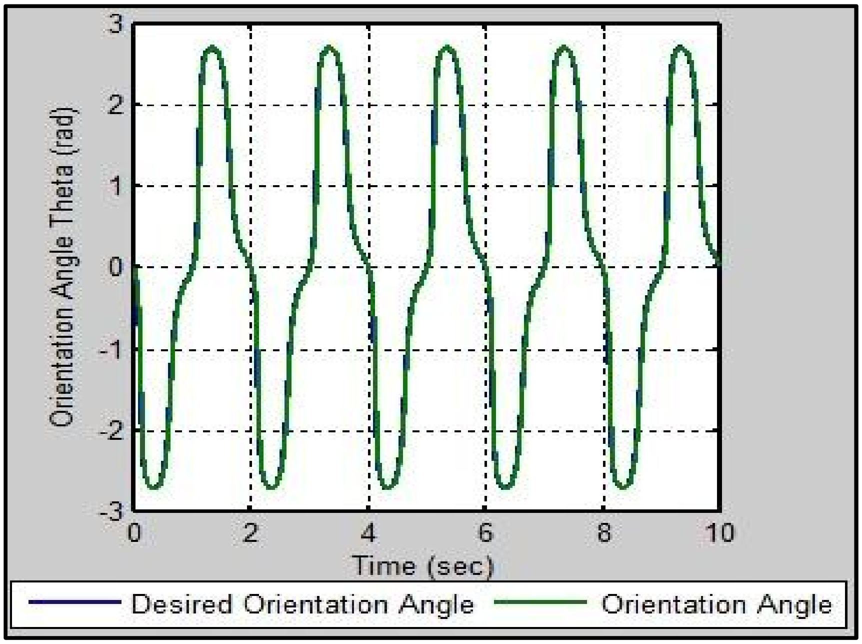

Figure (16): Simulation Results of Orientation Angle θ variable response on Trajectory Path II.

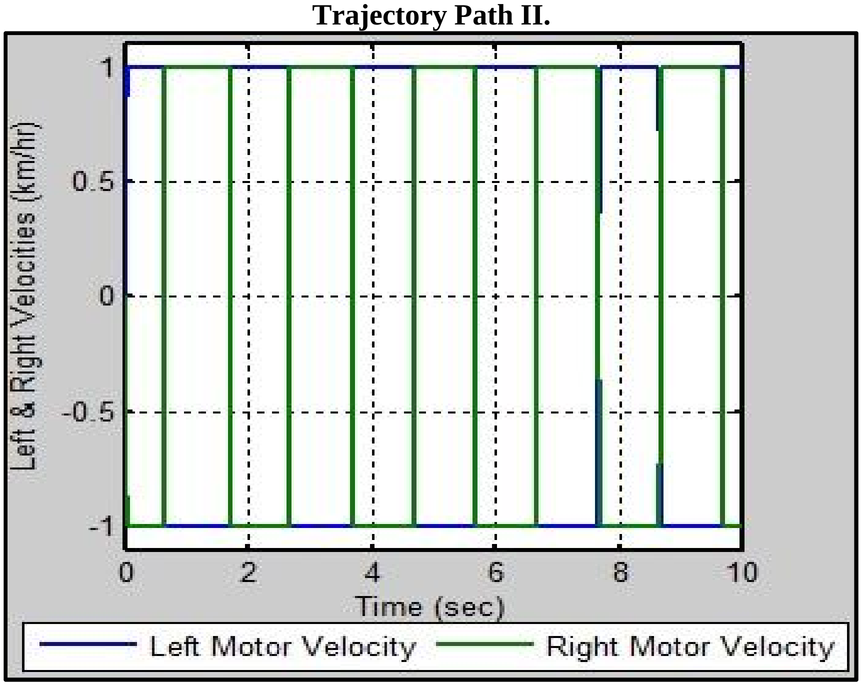

Figure (17): Simulation Results of Left and Right Motor Velocities on Trajectory Path II.

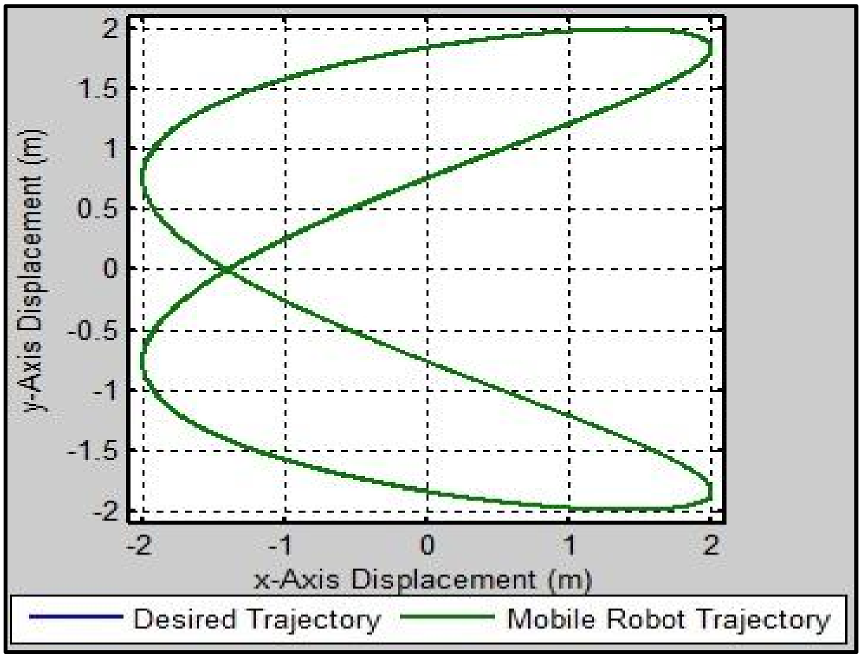

Figure (18): Simulation Results of Mobile Robot Tracking on Trajectory Path III.

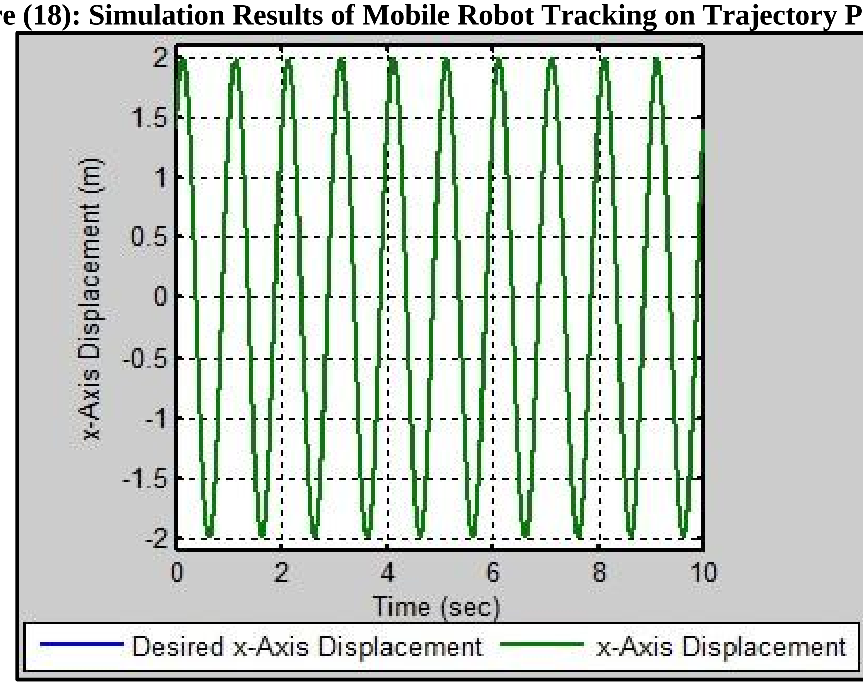

Figure (19): Simulation Results of x-axis variable response on Trajectory Path III.

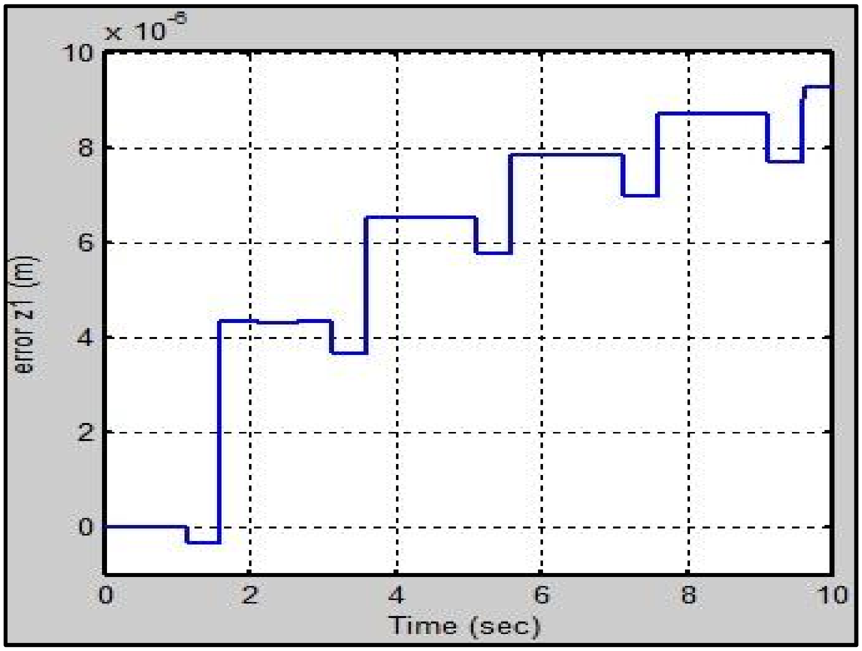

Figure (20): Simulation Results of x-axis error variable response on Trajectory Path III.

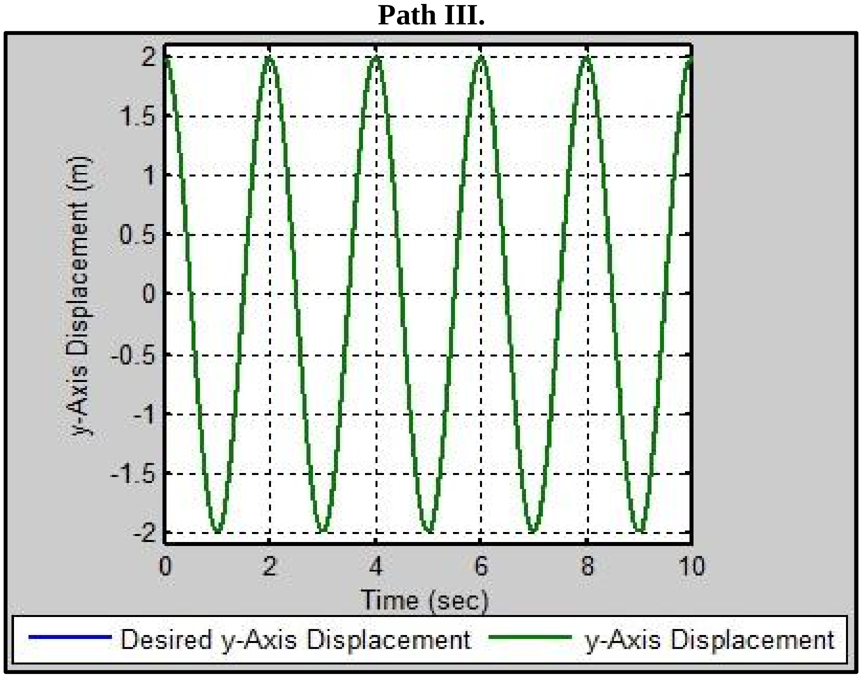

Figure (21): Simulation Results of y-axis variable response on Trajectory Path III.

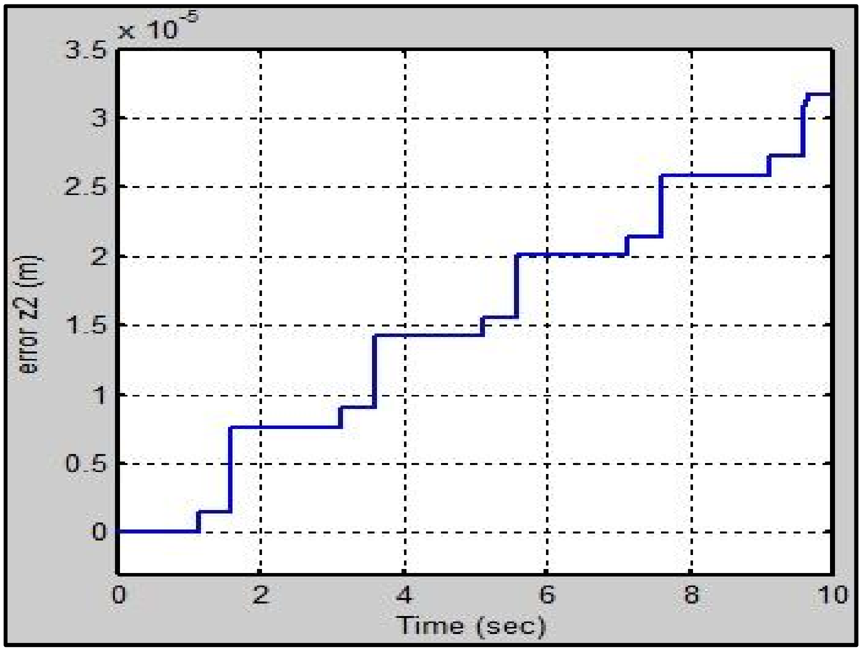

Figure (22): Simulation Results of y-axis error variable response on Trajectory Path III.

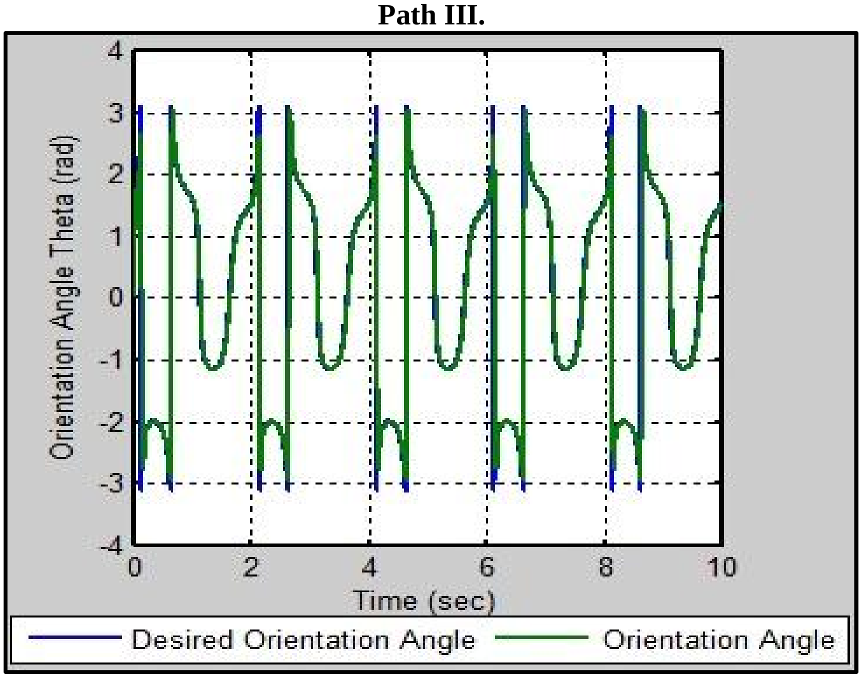

Figure (23): Simulation Results of Orientation Angle θ variable response on Trajectory Path II.

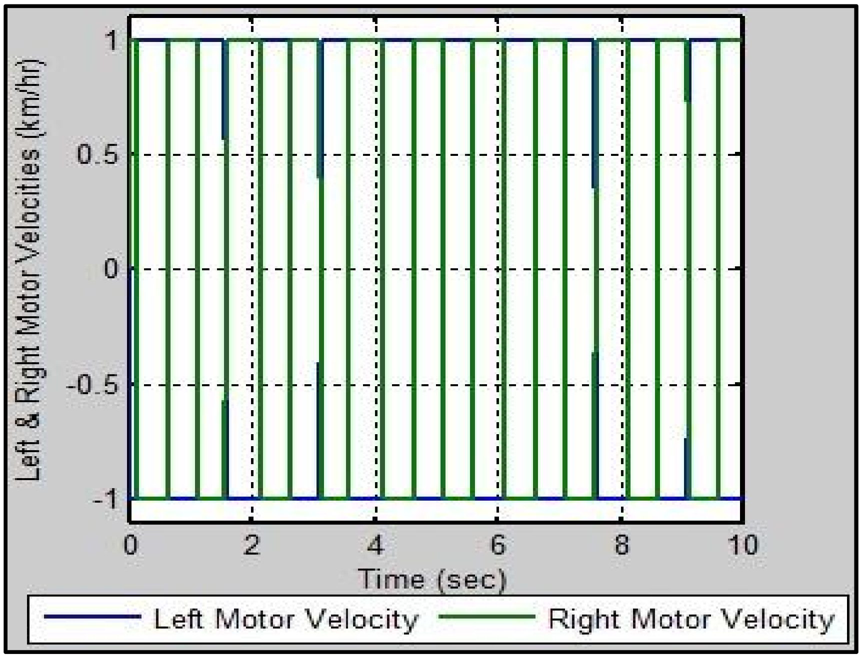

Figure (24): Simulation Results of Left and Right Motor Velocities on Trajectory Path III.