Bifurcation and The Influence parameters On A Novel Chaotic Dynamic System

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Abstract

In this paper introduces a novel nonlinear ten-dimensional system which has twelve positive real parameters, We analyzed the novel system behavior by means of phase portraits , equilibrium points , calculate Lyapunov exponents, fractional dimension and attractors of the system. Bifurcation and the influence parameters on a novel chaotic dynamic system . Numerical simulations using MATEMATICA are provided to illustrate phase portraits and the qualitative properties of the novel chaotic system.

Keywords: numerical simulation, ten-dimensional chaotic system, chaotic attractor , Bifurcation and Lyapunov exponents.

التشعب وتأثير المعلمات على نظام فوضوي مبتكر

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المستخلص

في هذا البحث تم تقديم نظام مبتكر لا خطي ذات عشرة ابعاد ويحتوي اثنا عشرة معلمة حقيقية موجبة ، قمنا تحليل سلوك النظام المبتكر عن طريق صور الطور او الحالة ، نقاط التوازن ، حساب قوى لابنوف ، البعد الكسوري للنظام ، والجاذبون للنظام ، والتشعب وتأثير المعلمات على النظام الديناميكي الفوضوي ، المحاكاة العددية تمت باستخدام برنامج الـMATHEMATICA لتوضيح صور الطور والخصائص النوعية للنظام الفوضوي المبتكر .

1. Introduction

Chaos analysis and applications in dynamical systems are observed in many practical applications in engineering, computer cryptography [1-2].Since Lorenz found the first chaotic attractor in a three dimensional autonomous system in 1963, the three-dimensional chaotic system has been a focal point of study for many researchers in the past few decades. The chaotic characteristics of the hyperchaotic system are more complex Thus, to use the hyperchaotic system signal as the encrypted signal has more extensive application prospect [3-6]. In [7-10], the four-dimensional system, five-dimensional system, six-dimensional, seven-dimensional system and their realizing circuits are given, which lays a foundation of the construction of the higher dimensional hyperchaotic system[11] . In this paper, we introduce a novel 10-D chaotic system with ten quadratic nonlinearities and discuss its qualitative properties. The novel ten-dimensional chaotic system has three unstable equilibrium points. It is organized as follows: Section 2 Establishment of the Novel ten-dimensional Dynamic System; Section 3. The influence of system parameters .Section 4. Bifurcation Diagram.

2. Establishment of the Novel ten-dimensional Dynamic System

The novel ten-dimensional autonomous system is obtained as follows:

$$
\begin{aligned}
\frac{dx}{dt} &= \sigma(y-x)-\rho(zs+w) \\
\frac{dy}{dt} &= \delta x - xz - y \\
\frac{dz}{dt} &= xy - \eta z \\
\frac{du}{dt} &= -v + \eta sq - u \\
\frac{dv}{dt} &= -\lambda(v+wq) - u \qquad\qquad (1) \\
\frac{dw}{dt} &= \gamma x + zx \\
\frac{dp}{dt} &= \mu r \\
\frac{dq}{dt} &= -\mu(q+pr) \\
\frac{dr}{dt} &= -\varphi p + qx + \xi qs \\
\frac{ds}{dt} &= -\beta pr - \omega(s-p)
\end{aligned}
$$

Where x, y, z, u, v, u, w, p, q, r and s are the states of system and

$\sigma, \rho, \delta, \gamma, \eta, \lambda, \gamma, \mu, \varphi, \xi, \beta$ *and* ω are real positive parameters of the system.

The 10-D system (1) exhibits a chaotic attractor, when the system parameter values are chosen as:

$$\sigma = 20, \rho = 2.1, \delta = 15, \eta = 2, \lambda = 8, \gamma = 10, \mu = 5, \varphi = 25, \xi = 5.1, \beta = 1, \omega = 1.9 \qquad (2)$$

We take the initial conditions as:

$$x(0) = -1, y(0) = 4, z(0) = 1, u(0) = 0, v(0) = 0, w(0) = 1, p(0) = 0,$$
$$q(0) = -1, r(0) = 8, s(0) = 5.$$

This a novel ten-dimensional nonlinear system. Some basic properties of the system have been investigated .The new 10-D chaotic system has three unstable equilibrium points and calculated Lyapunov exponents, the Lyapunov exponents of the system are : [12]

L_1= 18.94059 , L_2= 9.96383 , L_3= 1.00877,L_4= 0.828434 ,L_5= 0.0490522 ,L_6= -0.0132193 ,L_7= -0.0646262 , L_8= -1.00973 , L_9= -28.8771 , L_{10}= -39.729, the maximal Lyapunov exponent (MLE) of the novel system is L_1= 18.94059. In addition the Lyapunov dimension of the novel chaotic system is obtained as D_{KY} =9.04596.

Using MATHEMATICA program, the numerical simulation have been completed. This nonlinear system exhibits the complex and abundant chaotic dynamics behaviors, the strange attractors are shown in Figs.1&2. [12]

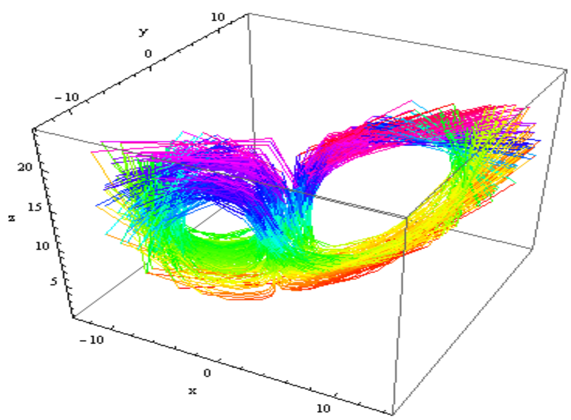

Fig.1. Chaotic attractors ,three- dimensional view (x-y-z)

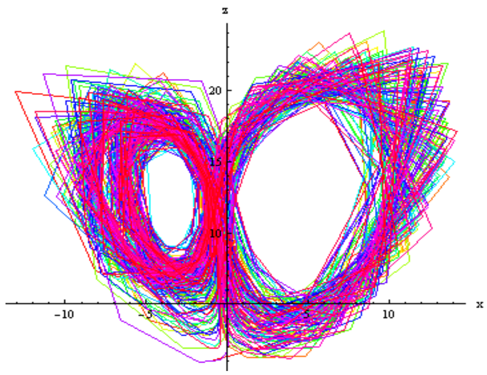

Fig.2 ***Chaotic attractors*** **,** ***z–x*** **phase plane**

3. The influence of system parameters

From the above analysis, it is visible that the stability of system equilibria will be changed along with the change of system parameters, and the system will also be in different state.

By using numerical simulation method using Mathematica program , the change of system parameters and system conditions are analyzed below. We let δ increasing when other parameters are fixed. While $\boldsymbol{\delta}$ increases, the system is undergoing some representative dynamical routes, such as stable fixed points, period-doubling loops, chaos and hyper-chaotic bifurcation, which are summarized as follows:

• $\delta = 1$, system (1) is stable, as shown three-dimensions in Figure 3, and shown two-dimensions in Figure 4.

• $\delta = 2$, system (1) is stable, as shown three-dimensions in Figure 5, and shown two-dimensions in Figure 6.

• $\delta = 2.3$, there is a period-doubling bifurcation window too, as shown three-dimensions in Figure 7, and shown two-dimensions in Figure 8.

• $\delta = 2.9$, there is a period-doubling bifurcation window too, as shown three-dimensions in Figure 9, and shown two-dimensions in Figure 10.

• $\delta = 3$, there is a chaotic bifurcation window too, as shown three-dimensions in Figure 11, and shown two-dimensions in Figure 12.

• δ = 15, there is a hyper-chaotic bifurcation window too, as shown three-dimensions in Figure 13, and shown two-dimensions in Figure 14.

• δ = 30, there is a hyper-chaotic bifurcation window too, as shown three-dimensions in Figure 15, and shown two-dimensions in Figure 16.

• δ = 40, there is a hyper-chaotic bifurcation window too, as shown three-dimensions in Figure 17, and shown two-dimensions in Figure 18.

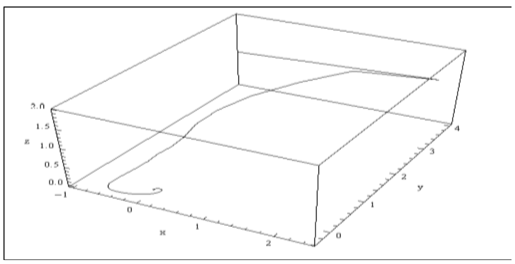

Figure 3: Phase portraits of system (1) in x-y-z plane with δ = 1

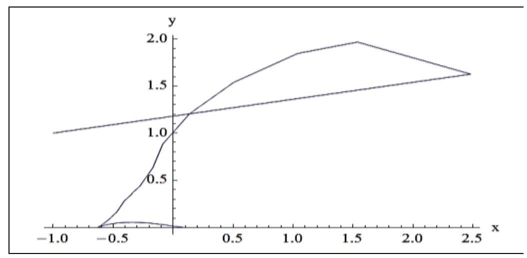

Figure 4: Phase portraits of system (1) in x-y plane with δ = 1

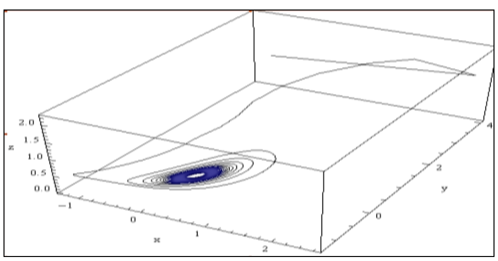

Figure 5: Phase portraits of system (1) in x-y-z plane with δ = 2

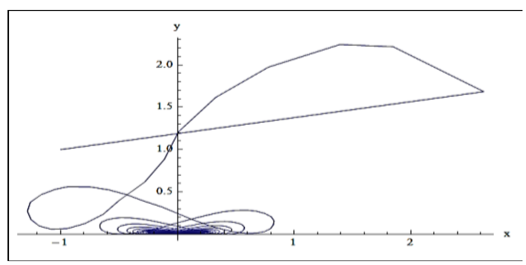

Figure 6: Phase portraits of system (1) in x-y plane with δ = 2

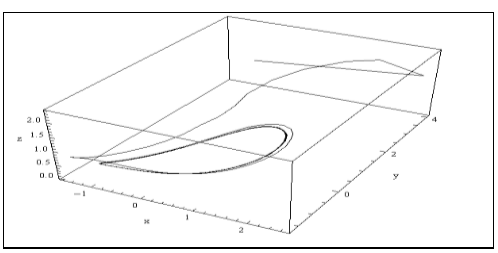

Figure 7: Phase portraits of system (1) in x-y-z plane with δ = 2.3

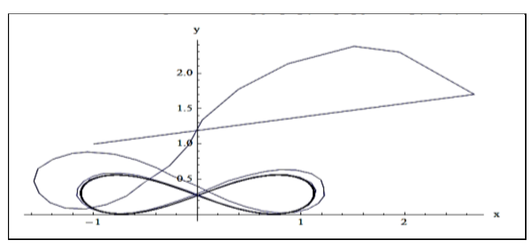

Figure 8: Phase portraits of system (1) in x-y plane with δ =2.3

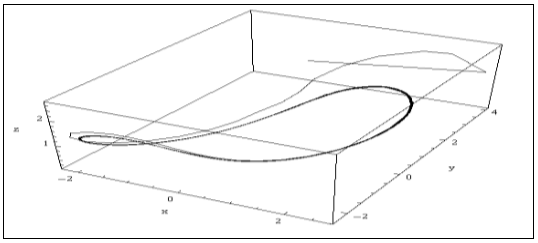

Figure 9: Phase portraits of system (1) in x-y-z plane with δ = 2.9

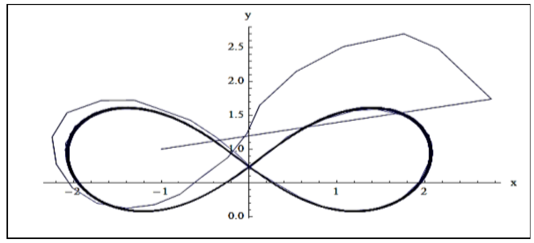

Figure 10: Phase portraits of system (1) in x-y plane with δ = 2.9

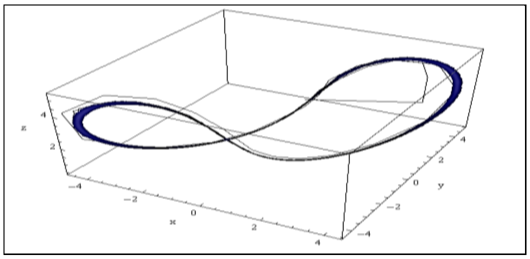

Figure 11: Phase portraits of system (1) in x-y-z plane with δ = 3

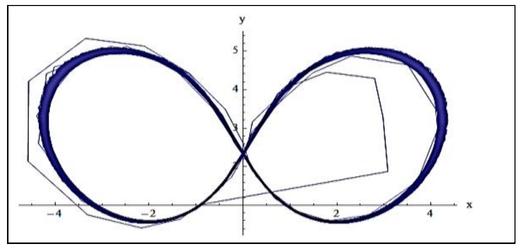

Figure 12: Phase portraits of system (1) in x-y plane with δ = 3

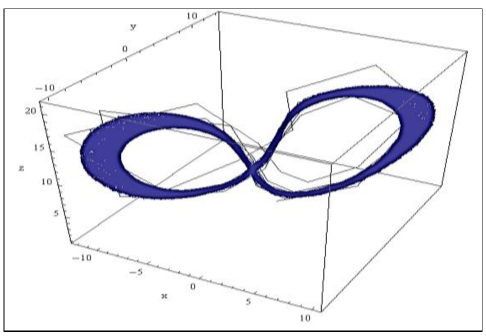

Figure 13: Phase portraits of system (1) in x-y-z plane with δ = 15

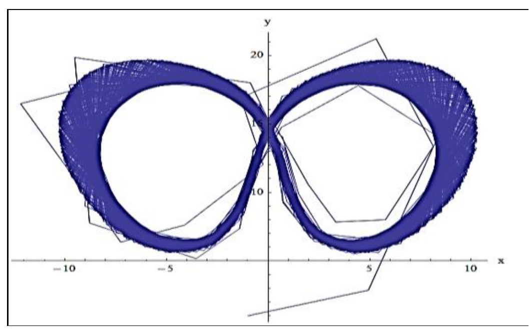

Figure 14: Phase portraits of system (1) in x-y plane with δ = 15

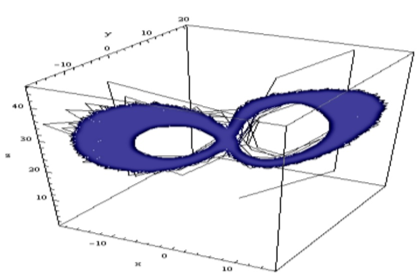

Figure 15: Phase portraits of system (1) in x-y-z plane with δ = 30

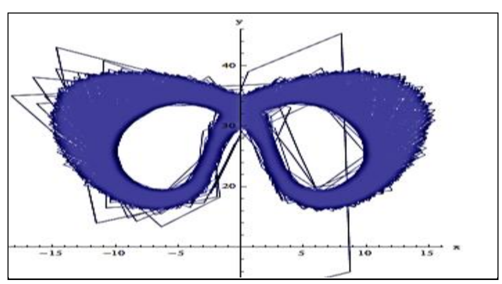

Figure 16: Phase portraits of system (1) in x-y plane with δ = 30

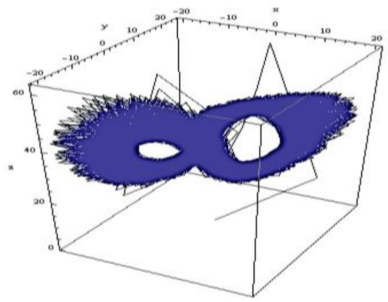

Figure 17: Phase portraits of system (1) in x-y-z plane with δ = 40

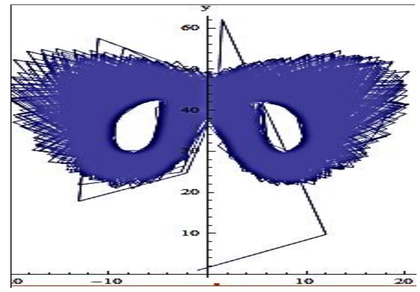

Figure 18: Phase portraits of system (1) in x-y plane with δ =40

4. Bifurcation Diagram

Up until now, behavior has been shown for only a few different values of $\boldsymbol{\delta}$. To see what happens for a large range of $\boldsymbol{\delta}$, we can construct a bifurcation diagram numerically.

The system (1) is solved numerically by Mathematica program simulation and the values of z for the maximums are kept track of after transients are discarded. we can look at the interesting region of the bifurcation diagram between $\boldsymbol{\delta}$ = 26 and $\boldsymbol{\delta}$ = 28. The plot points to limit cycle behavior. As $\boldsymbol{\delta}$ is decreased, it appears there is period doubling, which occurs between $\boldsymbol{\delta}$ = 27.65 and $\boldsymbol{\delta}$ = 26.95. This is shown The period doubling can be seen in a plot of z max for a range of $\boldsymbol{\delta}$. This is shown in Figure 19. We can see that for a range of $\boldsymbol{\delta}$ values, the maxima only take certain values. We can also see clearly that the top branch splits as $\boldsymbol{\delta}$ is decreased. Figure 20 zooms in on this area, giving a better view of the period doublings. This period doubling is a feature of many chaotic systems, and has been observed in physical experiments [1]. From the figure we can see that as $\boldsymbol{\delta}$ is decreased at a constant rate, the doublings occur more rapidly. In other words, the range of $\boldsymbol{\delta}$ needed to be covered before the next period doubling occurs decreases. The Feigenbaum constant, given by:

$$\boldsymbol{Feig.} = \lim_{n\to\infty} \frac{\delta_n - \delta_{n-1}}{\delta_{n+1} - \delta_n} = 4.669..., \qquad (3)$$

tells us approximately where the next period doubling will occur as a parameter $\boldsymbol{\delta}$ is varied for the chaotic system . Figure 19 can be used to obtain a rough estimate of this constant for the wheel system. Using the values $\boldsymbol{\delta}$ = 27.19, 26.88, and 26.81 for the locations of the period doublings, the estimate for d comes out to 4.43. Figure 21 shows another plot of z max against δ for a larger range of δ. In this figure we can see two regions with limit cycle behavior. As δ is increased, there is period halving in the gap in the region $26 < \delta < 28$ and period doubling in the gap in the region $39 < \delta < 43$.

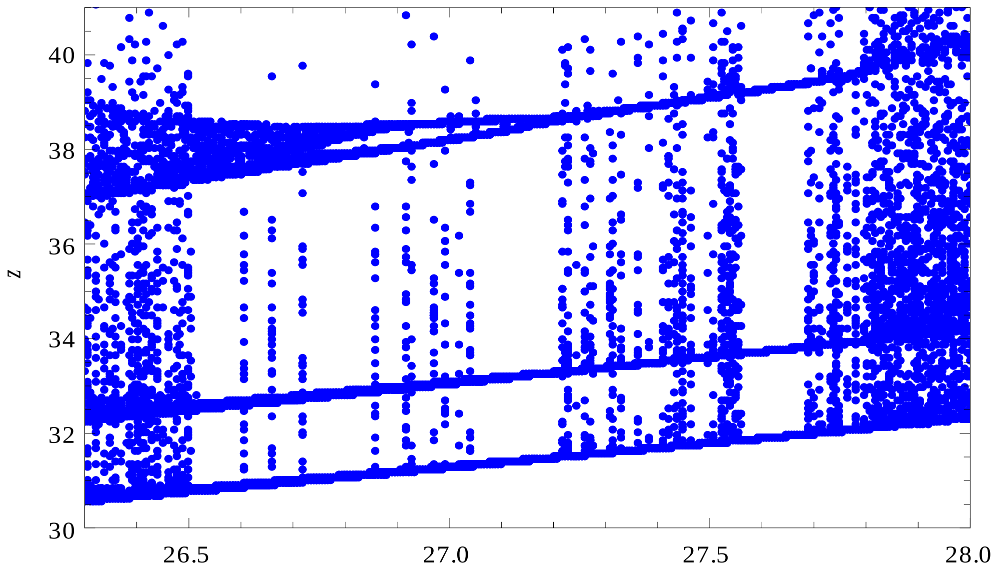

Figure 19 : z Bifurcation diagram for increasing δ

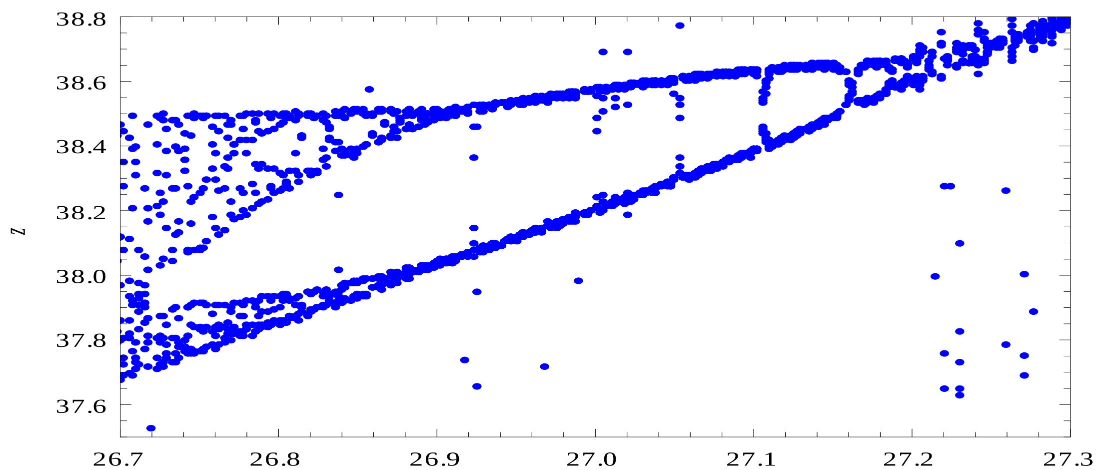

Figure 20: Zoomed in z orbit diagram. This plot zooms into the region of Figure 19 showing limit cycle period doubling. This plot can be used to obtain an estimate of the Feigenbaum constant.

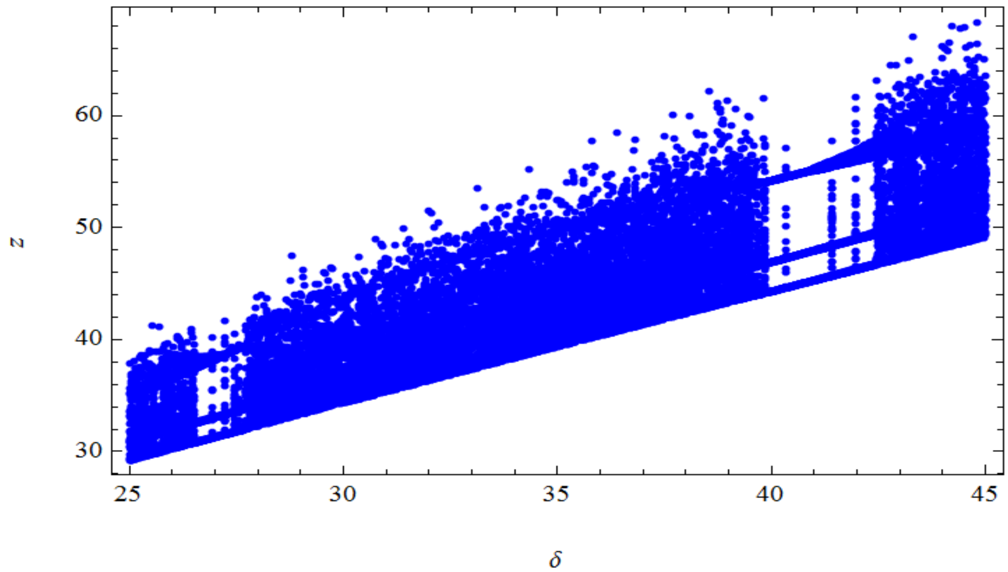

Figure 21 : Z orbit diagram for a larger range of δ. The region investigated earlier, with δ < 28, is shown on the left side of the plot. The plot shows both period halving (δ ~ 27)and period doubling (δ ~41) as δ is increased.

Conclusion

In this paper presented a novel ten-dimensional nonlinear system. Some basic properties of the system have been investigated. By using numerical simulation method in Mathematica program , the change of system parameters and system conditions are analyzed below. We let δ increasing when other parameters are fixed. While δ increases, the system is undergoing some representative dynamical routes, such as stable fixed points, period-doubling loops, chaos and hyper-chaotic bifurcation.

References

[1] C. W. Chang-Jian and S. M. Chang, “Bifurcation and chaos analysis of spur gear pair with and without nonlinear suspension”, Nonlinear Analysis: Real World Applications, vol. 12, **(2011)**, pp. 979-89.

[2] G. Qiang, Y. Hua and Y. Hongye. The Research of Chaos-based M-ary Spreading Sequences. *TELKOMNIKA*. Vol. 10, **(2012)**; pp.2151-2158.

[3] J. Lv, S. M. Yu, H. Leung and G. Chen, “Experimental verification of multi-directional multi-scroll chaotic attractors”, IEEE Trans. Circuits Syst. I, vol. 53, no. 1, **(2006)**, pp. 149-165.

[4] S. M. Yu, J. Lv, H. Leung and G. Chen, “Design and implementation of n-scroll chaotic attractors from a general Jerk circuit”, IEEE Trans. Circuits Syst. I, vol. 52, no. 7, **(2005)**, pp. 1459-1476.

[5] J. Lv, G. Chen, X. H. Yu and H. Leung, “Design and analysis of multi-scroll chaotic attractors from saturated function series”, IEEE Trans. Circuits Syst. I, vol. 51, no. 12, **(2004),** pp. 2476-2490.

[6] G. Y. Wang, S. S. Qiu and Z. Xu, “A new three-dimensional quadratic chaotic system and its circuitry implementation”, Acta Physica Sinica, vol. 55, no. 7, **(2006),** pp. 3289-3301.

[7] J. Lv, B. Zhang and J. Zhu, “Four-dimensional Secondary Hyperchaotic System and its Hardware Implementation”, 2013 journal of haerbin university of science and technology, **(2013),** pp. 95-98.

[8] J. L. Zhu and H. C. Zhao, “Five-dimensional chaotic system and its circuitry implementation,” 2009 2nd International Congress on Image and Signal Processing, Tianjin, China, Oct.17-19, **(2009)**, pp. 1-5.

[9] J. Zhu, Y. Wang, S. Kang, “Six-dimensional Chaotic System and Its Circuitry Implementation,” 2011 Proc IEEE CEBM, **(2011),** pp. 2018-2022.

[10] D. Liu, J. Zhu and H. Sun, “Design and Application of a NEW Seven-Dimensional Hyperchaotic System”, International Journal of Control and Automation Vol.7, No.4 **(2014)**, pp.385-394

[11] B. Bao, Z. Liu and J. Xu, “New chaotic system and its hyperchaos generation”, Journal of Systems Engineering and Electronics, vol. 20, no. 6, **(2009),** pp. 1179-118.

[12] Sadiq A. Mehdi , Abid Ali H. Alta’ai and Salim Ali ABBAS,” Analysis of A Novel Chaotic Dynamic System with ten quadratic nonlinearities, IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 2 Ver. IV (Mar - Apr. 2015), PP 40-46.