P- ISSN 1991-8941 E-ISSN 2706-6703 2023,(17), (2):245– 253
Journal of University of Anbar for Pure Science (JUAPS) Open Access

On Restricted Shrinkage Jackknife Biased Estimator for Restricted Linear Regression Model

Ahmed A. Mohammed *, Feras Sh. M. Batah

Department of Mathematics, College of Education for Pure Science, University of Anbar, Anbar, Iraq;

ARTICLE INFO

Received: 07 / 05 /2023
Accepted: 19 / 06 / 2023
Available online: 18 /12 / 2023

DOI: 10.37652/juaps.2023.181574

Keywords:
Restricted regression model,
jackknifed biased estimator,
Multicollinearity problem,
Two parameters estimator,
Simulation study**.**

ABSTRACT

In restricted linear regression model, more methods proposed to address the Multicollinearity problem and the high variance. For example, shrinkage biased estimation and optimization (Lagrange function). In this paper, we propose new biased estimator based on philosophy of Jackknife with the restricted least squares estimator. A new estimator called Restricted Shrinkage Jackknife estimator (RSJ). Also, we show that the statistical properties of new estimator with some theorems to compare the performance of new estimator with some restricted estimators and we make simulation study of these estimators. Finally, a real data has been taken into consideration to demonstrate how well the estimators perform.

1. Introduction:

The multiple linear regression model is given by the following equation:

$$A = B\nu + \xi, \quad (1)$$

where v is a ($p \times 1$) vector of the unknown parameters, p is the number of explanatory variables, A is an $(n \times 1)$ vector of the responses, B is an $(n \times p)$ matrix of the explanatory variables, p is the number of the explanatory variables, and ξ is an $(n \times 1)$ vector of the random errors with $E(\xi) = 0$ and $Var(\xi) = \sigma^2 I_n$. In some cases, the linear limitation is satisfied as follows:

$$R\nu = r, \quad (2)$$

where r is a vector $m \times 1$ and R is a nonzero $m \times p$ matrix with rank(R)= $m < p$

Because it reduces variance, the restricted least square estimator (RLS) is regarded as one of the more significant unbiased estimators to handle the Multicollinearity problem and high variance see [1]. The following formula provides the (RLS) estimator:

$$\hat{\nu}_{RLS} = \hat{\nu} + S^{-1}R'(RS^{-1}R')^{-1}(r - R\hat{\nu}) \ , \quad (3)$$

where $\hat{\nu} = S^{-1}B'A$, and $S^{-1} = (B'B)^{-1}$.

Multicollinearity is a problem that researchers frequently deal against. In other words, because of the linear relationship between the Regressors of the B matrix, B'B is always ill-conditioned. Therefore, RLS's estimation of the unknown coefficient. In order to solve this issue, the researcher employed restricted biased estimating. The shrinkage restricted estimator (RRR) was first presented by [2] to address the Multicollinearity issue. Using the ORR philosophy, the RRR estimator modified the RLS estimator. This is how the RRR estimator is provided:

$$\hat{\nu}^*(k) = M\hat{\nu}_{RLS}, \quad (4)$$

where $M = (I + kS^{-1})^{-1}$. See [3] introduced the restricted two parameter estimator (RTPE) as the follows:

$$\hat{\nu}_{RTPE}(k, d) = M_{kd}B'A, \quad (5)$$

where $M_{kd} = L_{kd}^{-1} - L_{kd}^{-1}R'\left(RL_{kd}^{-1}R'\right)^{-1}L_{kd}^{-1}R'$, $L_{kd}^{-1} = (S + kI)^{-1}(I + kdS^{-1})$, and $d > 0$.

[4] introduced the restricted (k – d) class estimator in linear regression model as follows:

*Corresponding author at: Department of BBBBB, College of SSSSSS , University of A , A , Iraq;
ORCID:https://orcid.org/0000-0000-0000-0000;Tel:+964000000000000
E-mail address: ssssssssssss@ uuuuuuuuuu,ee.iq

$$\hat{v}(k,d) = (\mathrm{B}'\mathrm{B} + I)^{-1}(\mathrm{B}'\mathrm{B} + (k+d)I)(\mathrm{B}'\mathrm{B} + kI)^{-1}(\mathrm{B}'\mathrm{B})\hat{v}_{RLS}, \quad (6)$$

[5] proposed restricted almost unbiased ridge regression estimator (RAURE) based on the RRR estimator.

$$\hat{v}_{RAURE}(k) = \left(I - k^2 S_k^{-2}\right)\hat{v}_{RLS}\,. \quad (7)$$

Based on the RTPE estimator, [6] suggested the restricted almost unbiased two-parameter estimator (RAUTPE) as

$$\hat{v}_{RAUTPE}(k,d) = [I - (I - M_{kd}S)]^2\hat{v}. \quad (8)$$

Mohammed and Alheety presented the following shrinkage restricted ridge regression estimator in 2023: [7]

$$\hat{v}_{SRRE}(k) = W\hat{v}_{RLS}, \quad (9)$$

where $W = \left(I - kS_k^{-1}\right)$.

Authors in [8] introduced a new two parameter estimator to control the Multicollinearity using the first parameter k in two parameter estimator, which is the ridge estimator's biasing parameter, by treating k as a Lagrangian multiplier. However, Batah proposed a mathematical formula which it is quicker than the ridge regression estimator, depends on reducing the variance, [9] and the Jackknifed ridge estimator depends on minimizing the bias, in order to construct the above estimators quickly based on the least squares estimator [10]. Numerous researchers focused on linear regression estimation have chosen the restricted least squares and ridge estimators due to their attractive ability to decrease Multicollinearity. Each estimator was created by focusing on a specific biasing parameter. As a result, the SRRE estimator violates the linear restrictions.

Now, we believe that defining a new restricted estimator with jackknifed technique employing both the two biasing parameters of the ridge and Liu estimators will be a good idea. One of our goals in constructing our suggested estimator is to control Multicollinearity by handling k and d in the jackknife technique, which uses the two parameters k, the biasing parameter of the ridge estimator, and d, the shrinkage parameter in the Liu estimator. Because the RLS and jackknife of parameter estimation are based on different methodologies, we propose a new restricted biased estimator called the restricted shrinkage jackknife estimator (RSJ). We demonstrate the statistical characteristics of the (RSJ) estimator in Section 2. In section 3, just several theorems of that RSJ comparisons. We simulate a study of the RSJ using a few restricted biased estimators in section 4. A numerical example was taken into consideration to demonstrate how well the estimators performed in section 5. Finally, section 6 concludes with some final thoughts.

2. The proposed estimator (RSJ) and its statistical properties

By integrating the jackknife approach with the RLS estimator, we introduce a new jackknife biased estimator restricted linear regression model termed restricted shrinkage jackknife estimator (RSJ) in this section as:

$$\hat{v}_{SRJ}(k,d) = N_{kd}\hat{v}_{RLS}. \quad (10)$$

Where $N_{kd} = [I - (k+d)^2(S+kI)^{-2}][I - (k+d)(S+kI)^{-1}]$. The RJS and its statistical properties (the mean, the variance, and the mean squared error), respectively are given by :

$$Bais\left(\hat{v}_{SRJ}(k,d)\right) = -(k+d)(S+kI)^{-1}\delta(S+kI)^{-1}\nu. \quad (11)$$

Where $\delta = I + F_{kd} - (F_{kd})^2$ and $F_{kd} = ((X'X + kI)^{-1}(X'X - dI)$. The variance is

$$Var\left(\hat{v}_{SRJ}(k,d)\right) = \sigma^2\varphi\Lambda^{-1}\varphi'. \quad (12)$$

Where $\varphi = (2I - F_{kd})(F_{kd})^2$, $\Lambda = M_0 S M_0{}'$ and $M_0 = S^{-1} - S^{-1}R'(RS^{-1}R')^{-1}RS^{-1}$.

The mean squares error of RJS is

$$MSE\left(\hat{v}_{SRJ}(k,d)\right) = \sigma^2\varphi\Lambda^{-1}\varphi' + (k+d)^2(S+kI)^{-1}\delta(S+kI)^{-1}\nu'\nu\,[(S+kI)^{-1}\delta(S+kI)^{-1}]', \quad (13)$$

Thus, the scalar mean square error (SMSE) of the RSJ is given by as follows:

$$SMSE\left(\hat{v}_{SRJ}(k,d)\right) = \sigma^2 tr(\varphi\Lambda^{-1}\varphi') + (k+d)^2 tr((S+kI)^{-1}\delta(S+kI)^{-1}\nu'\nu\,[(S+kI)^{-1}\delta(S+kI)^{-1}]'. \quad (14)$$

The proposed estimate's goal is to tackle the Multicollinearity problem and large variance by combining the jackknife approach and the RLS estimator.

3. The RSJ's Performance in Comparison to Other Restricted Estimators.

We need some lemmas to demonstrate the performance of the RSJ estimator in comparison to some restricted biased estimators. We employ the following lemmas for comparing the underlying estimators:

Lemma1: See[11] suppose that $\hat{v}_i^* = A_i Y$, $i = 1,2$ be two linear homogeneous estimators of β such that $G = M_1 M_1' - M_2 M_2'$ is positive definite (p.d). If $C_2' G^{-1} C_2 < \sigma^2$ then Δ is p.d .

Lemma 2: See [10] Let W is a positive definite matrix (p.d.) and Z is a nonnegative definite matrix (n.n.d.). Then $W - Z \geq 0 \leftrightarrow \lambda_{max}(ZW^{-1}) \leq 1$.

Lemma 3: See [12] Suppose that Y a positive definite matrix and X a nonnegative matrix and $\Lambda = diag\left(\lambda_i^A(X)\right)$ is the diagonal matrix of the Eigen values of Y in the matrix X. There exists a singuler matrix W such that $Y = W'W$ and $X = W\Lambda W'$.

3.1 A comparison of the RSJ and RLS estimators.

The different MSE between of the RSJ and the RLS estimators is given by as follows:

$$MSE(\hat{v}_{RLS}) - MSE(MSE\left(\hat{v}_{RSJ}(k,d)\right) = \sigma^2[\Lambda - \sigma^2\varphi\Lambda\varphi'] - C_1C_1',$$

where $C_1C_1' = (k+d)^2 tr((S+kI)^{-1}\delta(S+kI)^{-1}v'v[(S+kI)^{-1}\delta(S+kI)^{-1}]'$

We can now express the following theorem.

Theorem 1: If $k > 0$,. $-\infty < d < \infty$, the RSJ estimator outperforms the RLS estimator using the MSE if and only if $MSE(\hat{v}_{RLS}) - MSE(MSE\left(\hat{v}_{RSJ}(k,d)\right) > 0$, indicating that $[\Lambda - \sigma^2\varphi\Lambda\varphi']$ positive definite.

Proof: This provides the variance difference between the RSJ and RLS estimators:

$$Var(\hat{v}_{RLS}) - Var\left(\hat{v}_{RSJ}(k,d)\right) = \sigma^2[\Lambda - \sigma^2\varphi\Lambda\varphi']$$

$$Var(\hat{v}_{RLS}) - Var\left(\hat{v}_{RSJ}(k,d)\right) = \sigma^2 diag\left\{\frac{(\lambda_i - r_{ii})^2}{\lambda_i^3} - \frac{(\lambda_i+k+2d_i)^2(k+d)(2\lambda_i+k+d_i)(\lambda_i-r_{ii})^2}{\left(\lambda_i^3(\lambda_i+k)^4\right)}\right\}_{i=1}^p$$

$$= \sigma^2 diag\frac{(\lambda_i - r_{ii})^2}{\lambda_i^3}\left\{1 - \frac{(\lambda_i+k+2d_i)^2(k+d)(2\lambda_i+k+d_i)}{((\lambda_i+k)^4}\right\}_{i=1}^p,$$

where $H^* = TR'(RS_k^{-1}R')RT', the\ diag(H^*) = r_{ii}$.Therefore $\sigma^2[\Lambda - \sigma^2\varphi\Lambda\varphi']$ is (p.d) if and only if

$$1 - \frac{(\lambda_i+k+2d_i)^2(k+d)(2\lambda_i+k+d_i)}{((\lambda_i+k)^4} > 0$$

So that by Lemma (1 – 3), we get $\Lambda - \sigma^2\varphi\Lambda\varphi'$is positive definite. Now, the theorem is established.

3.2 A comparison of the RSJ and RRR estimators.

We compare the RSJ and RRR estimators using the MSE in the following way:

$$MSE\left(\hat{v}^*(k)\right) - MSE\left(\hat{v}_{RSJ}(k,d)\right) = \sigma^2[M\Lambda M' - \sigma^2\varphi\Lambda\varphi'] + C_2C_2' - C_1C_1'$$

where $C_2C_2' = k^2 v' S_k^{-2} v$ the bias of RRR estimator. The following theorem can be stated.

Theorem 2: The RSJ estimator is superior to the RRR estimator if and only if $MSE\left(\hat{v}^*(k)\right) - MSE\left(\hat{v}_{RSJ}(k,d)\right) > 0$ that means, the SRJ estimator has minimum MSE.

Proof : This provides the variance difference between the RSJ and RRR estimators:

$$Var\left(\hat{v}^*(k)\right) - Var\left(\hat{v}_{RSJ}(k,d)\right) = \sigma^2\Lambda[MM' - \sigma^2\varphi\varphi']$$

$$Var\left(\hat{v}^*(k)\right) - Var\left(\hat{v}_{RSJ}(k,d)\right) = \sigma^2 diag\frac{(\lambda_i - r_{ii})^2}{\lambda_i^3}\left\{\frac{\lambda_i^2}{(\lambda_i+k)^2} - \frac{(\lambda_i+k+2d_i)^2(k+d)(2\lambda_i+k+d_i)}{((\lambda_i+k)^4}\right\}_{i=1}^p.$$

Therefore, the $\sigma^2\Lambda[MM' - \sigma^2\varphi\varphi'] > 0$ is positive definite if and only if

$$\left\{\frac{\lambda_i^2}{(\lambda_i+k)^2} - \frac{(\lambda_i+k+2d_i)^2(k+d)(2\lambda_i+k+d_i)}{((\lambda_i+k)^4}\right\} > 0 \text{ if}$$

$$\lambda_i^2((\lambda_i+k)^4 - (\lambda_i+k)^2(\lambda_i+k+2d_i)^2(k+d)(2\lambda_i + k + d_i) > 0$$

$$(\lambda_i+k)^2\left[\lambda_i^2(\lambda_i+k)^2 - (\lambda_i+k+2d_i)^2(k+d)(2\lambda_i+k + d_i)\right] > 0.$$

So that by Lemma (1 – 3), we get $\sigma^2\Lambda[MM' - \sigma^2\varphi\varphi'] > 0$, then the proof is established.

3.3 A comparison of the RSJ and SRRE estimators.

To determine the performance of the RSJ estimator in comparing to the SRRE estimator, we compare the RSJ and SRRE estimators using the MSE criteria as follows:

$$MSE\left(\hat{v}_{SRRE}(k)\right) - MSE\left(\hat{v}_{RSJ}(k,d)\right) = \sigma^2 \Lambda[WW' - \sigma^2\varphi\varphi'] + C_3C_3 - C_1C_1'$$

Where $C_3C_3 = k^2 S_k^{-1}\beta\beta' S_k^{-1}$. Therefore, the theorem is established.

Theorem 3: if $k > 0$, $d > 0$ the RSJ estimator is the best comparison with SRRE estimator if and only if $MSE\left(\hat{v}_{SRRE}(k)\right) - MSE\left(\hat{v}_{RSJ}(k,d)\right) > 0$.

Proof : These are the formulas for the difference variance between the RSJ and RRR estimators:

$$Var\left(\hat{v}_{SRRE}(k)\right) - Var\left(\hat{v}_{RSJ}(k,d)\right) = \sigma^2 \Lambda[WW' - \sigma^2\varphi\varphi']$$

$$Var\left(\hat{v}_{SRRE}(k)\right) - Var\left(\hat{v}_{SRJ}(k,d)\right) = \sigma^2 diag \frac{(\lambda_i - r_{ii})^2}{\lambda_i^3}\left\{\frac{1}{(\lambda_i + k)^2} - \frac{(\lambda_i + k + 2d_i)^2 (k+d)(2\lambda_i + k + d_i)}{((\lambda_i + k)^4}\right\}_{i=1}^{p}.$$

Therefore $\sigma^2 \Lambda[WW' - \sigma^2\varphi\varphi']$ is positive definite if and only if

$$\frac{1}{(\lambda_i + k)^2} - \frac{(\lambda_i + k + 2d_i)^2 (k+d)(2\lambda_i + k + d_i)}{((\lambda_i + k)^4} > 0,$$

$$((\lambda_i + k)^4 - (\lambda_i + k)^2(\lambda_i + k + 2d_i)^2(k+d)(2\lambda_i + k + d_i) > 0$$

$$(\lambda_i + k)^2\{(\lambda_i + k)^2 - (\lambda_i + k + 2d_i)^2(k+d)(2\lambda_i + k + d_i)\} > 0.$$

So that by Lemma (1 – 3), we have $\sigma^2 \Lambda[WW' - \sigma^2\varphi\varphi'] > 0$. The proof is established.

4. Simulation Study

To compare the performance of the proposed estimator with other jackknife biased estimators by MATLAB software in order to demonstrate how well it performs. The goal of this study is to compare the performance of the MJE estimator with a few other constrained estimators already in use. When the Regressors are highly correlated, this simulation is intended to assess how well the estimators RLS, RRR, SRRE, and RSJ perform. The following equation was used to generate the matrix B, according to [13], [14]:

$$B_{ij} = (1-\mu^2)^{1/2} Z_{ij} + \mu Z_{ip}, \quad i = 1,2,\dots n \quad and \quad j = 1,2,\dots p \qquad (15)$$

where μ stands for any two variables' correlation with one another and Z_{ij} independent standard normal pseudo-random numbers. The standardized nature of these variables allows for the correlation form of B'B. Moreover, $p = 5$ while σ are chosen as the explanatory values (1, 5, 10). The correlation coefficient μ will be set at (0.85, 0.95, and 0.99) with the sample size is (n = 50, 100, 150). According to the condition $\nu'\nu = 1$ that the matrix's largest eigenvalue must be greater than one, the coefficients $\nu_1, \nu_2, \dots, \nu_p$ are chosen as the eigenvectors corresponding to that value. Thus, sets of B`s are created for all n , σ , p , ν and μ. By creating new error terms, the experiment was repeated 5000 times. Here is how to calculate estimated mean square error (EMSE):

$$EMSE(\nu^{**}) = \frac{1}{5000}\sum_{i=1}^{5000}(\nu^{**} - \nu)'(\nu^{**} - \nu),$$

Hence, ν^{**} any estimators would be (RLS, RRR, SRRE or RSJ).

Table1: Calculated MSE under the conditions of n=50, =0.85, and p=5.

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.10685	0.246982	0.247002	0.244241
	$k_{S_{arith}}$	0.10685	0.246651	0.247326	0.219607
	k_{SMD}	0.10685	0.246984	0.247	0.244645
	k_{MU1}	0.10685	0.246935	0.247049	0.23404
	k_{MU2}	0.10685	0.246968	0.247016	0.240765
	k_{MU3}	0.10685	0.243766	0.249737	0.319041
	k_{MU4}	0.10685	0.243556	0.249889	0.322946
5	k_{KS}	0.512474	0.19412	0.194198	0.176972
	$k_{S_{arith}}$	0.512474	0.193712	0.195012	0.134945
	k_{SMD}	0.512474	0.194147	0.19417	0.188039
	k_{MU1}	0.512474	0.194065	0.194261	0.158858
	k_{MU2}	0.512474	0.193709	0.195027	0.135155
	k_{MU3}	0.512474	0.194694	0.197724	0.177558
	k_{MU4}	0.512474	0.198112	0.200337	0.208239
10	k_{KS}	0.355371	0.300073	0.300317	0.298853
	$k_{S_{arith}}$	0.355371	0.28978	0.308	0.327589
	k_{SMD}	0.355371	0.300143	0.300248	0.299567
	k_{MU1}	0.355371	0.300068	0.300322	0.298811
	k_{MU2}	0.355371	0.297138	0.302979	0.29958
	k_{MU3}	0.355371	0.299599	0.30078	0.295502
	k_{MU4}	0.355371	0.292757	0.306199	0.317773

Table2: Calculated MSE under the conditions of $n = 50$, $\mu = 0.95$, $p = 5$

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.181346	0.15362	0.153748	0.147698
	$k_{S_{arith}}$	0.181346	0.152208	0.155245	0.110728
	k_{SMD}	0.181346	0.153657	0.15371	0.150525
	k_{MU1}	0.181346	0.153517	0.153851	0.140627

	k_{MU2}	0.181346	0.153541	0.153827	0.142185
	k_{MU3}	0.181346	0.147715	0.165045	0.140508
	k_{MU4}	0.181346	0.472331	0.184199	0.20667
5	k_{KS}	0.708076	0.328255	0.325002	0.228588
	$k_{s_{arith}}$	0.708076	0.344938	0.311709	0.098866
	k_{SMD}	0.708076	0.326769	0.326454	0.310494
	k_{MU1}	0.708076	0.328487	0.324779	0.218932
	k_{MU2}	0.708076	0.39729	0.289167	0.118734
	k_{MU3}	0.708076	0.457936	0.277112	0.137948
	k_{MU4}	0.708076	0.744972	0.260376	0.180052
10	k_{KS}	0.48228	0.179731	0.180168	0.142381
	$k_{s_{arith}}$	0.48228	0.176858	0.184118	0.110391
	k_{SMD}	0.48228	0.179922	0.179973	0.172601
	k_{MU1}	0.48228	0.179773	0.180125	0.147777
	k_{MU2}	0.48228	0.176186	0.185738	0.117129
	k_{MU3}	0.48228	0.215057	0.203739	0.18491
	k_{MU4}	0.48228	0.179395	0.194433	0.151649

Table3: Calculated MSE under the conditions of $n = 50$, $\mu = 0.99$, $p = 5$

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.193258	0.44448	0.442355	0.368877
	$k_{s_{arith}}$	0.193258	0.500688	0.410371	0.311636
	k_{SMD}	0.193258	0.443504	0.443316	0.421171
	k_{MU1}	0.193258	0.443828	0.442994	0.399863
	k_{MU2}	0.193258	0.448986	0.438206	0.314888
	k_{MU3}	0.193258	1.050575	0.346353	0.335315
	k_{MU4}	0.193258	15.97594	0.370501	0.322404
5	k_{KS}	1.202945	0.439098	0.413033	0.039219
	$k_{s_{arith}}$	1.202945	0.455415	0.399188	0.02562
	k_{SMD}	1.202945	0.425984	0.425468	0.308213
	k_{MU1}	1.202945	0.438145	0.413895	0.041755
	k_{MU2}	1.202945	0.571273	0.333742	0.018274
	k_{MU3}	1.202945	0.637118	0.201988	0.153542
	k_{MU4}	1.202945	396.8677	0.145695	0.004939
10	k_{KS}	0.847195	0.758035	0.724439	0.316658
	$k_{s_{arith}}$	0.847195	0.88295	0.654594	0.241511
	k_{SMD}	0.847195	0.741003	0.740179	0.674506
	k_{MU1}	0.847195	0.747104	0.734265	0.45582
	k_{MU2}	0.847195	46.91659	0.455496	0.272379
	k_{MU3}	0.847195	11.13553	0.433725	0.282068
	k_{MU4}	0.847195	160.1004	0.486693	0.262777

Table4: Calculated MSE under the conditions of $n = 100$, $\mu = 0.85$, $p = 5$

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.121184	0.346436	0.346421	0.343506
	$k_{s_{arith}}$	0.121184	0.347437	0.345469	0.292956
	k_{SMD}	0.121184	0.346434	0.346422	0.343957
	k_{MU1}	0.121184	0.34648	0.346377	0.329751
	k_{MU2}	0.121184	0.346473	0.346384	0.331859
	k_{MU3}	0.121184	0.346868	0.345998	0.287938
	k_{MU4}	0.121184	0.352904	0.341554	0.359904
5	k_{KS}	0.10337	0.30283	0.302831	0.300446
	$k_{s_{arith}}$	0.10337	0.302811	0.302849	0.253849
	k_{SMD}	0.10337	0.30283	0.302831	0.300907
	k_{MU1}	0.10337	0.302828	0.302833	0.288531
	k_{MU2}	0.10337	0.302829	0.302832	0.294713
	k_{MU3}	0.10337	0.302789	0.302871	0.260426
	k_{MU4}	0.10337	0.302412	0.303106	0.353834
10	k_{KS}	0.381115	0.277322	0.277388	0.277159
	$k_{s_{arith}}$	0.381115	0.272058	0.282228	0.32097
	k_{SMD}	0.381115	0.277345	0.277365	0.277278
	k_{MU1}	0.381115	0.277335	0.277375	0.277226
	k_{MU2}	0.381115	0.273355	0.281108	0.311376
	k_{MU3}	0.381115	0.275846	0.278827	0.288286
	k_{MU4}	0.381115	0.273667	0.280832	0.308823

Table5: Calculated MSE under the conditions of $n = 100$, $\mu = 0.95$, $p = 5$

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.130583	0.290199	0.29019	0.285976
	$k_{s_{arith}}$	0.130583	0.290814	0.289686	0.23746
	k_{SMD}	0.130583	0.290197	0.290193	0.287744
	k_{MU1}	0.130583	0.290209	0.290181	0.279783
	k_{MU2}	0.130583	0.290231	0.290158	0.267187
	k_{MU3}	0.130583	0.297449	0.288206	0.305602
	k_{MU4}	0.130583	0.301043	0.288044	0.31803
5	k_{KS}	0.478876	0.220756	0.220511	0.194917
	$k_{s_{arith}}$	0.478876	0.222461	0.218944	0.099013
	k_{SMD}	0.478876	0.22065	0.220616	0.214432
	k_{MU1}	0.478876	0.220952	0.220318	0.165949
	k_{MU2}	0.478876	0.22179	0.219533	0.110828
	k_{MU3}	0.478876	0.243298	0.209496	0.14029
	k_{MU4}	0.478876	0.285281	0.204841	0.192147
10	k_{KS}	0.470292	0.458185	0.457568	0.4316
	$k_{s_{arith}}$	0.470292	0.464303	0.451909	0.303487
	k_{SMD}	0.470292	0.457918	0.457833	0.451867
	k_{MU1}	0.470292	0.458438	0.457317	0.41436
	k_{MU2}	0.470292	0.466014	0.450463	0.300297
	k_{MU3}	0.470292	0.463826	0.452323	0.304999
	k_{MU4}	0.470292	0.55745	0.412437	0.33325

Table6:Calculated MSE under the conditions of $n = 100$, $\mu = 0.99$, $p = 5$

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.171462	0.535428	0.534644	0.502161
	$k_{s_{arith}}$	0.171462	0.556137	0.517044	0.410972
	k_{SMD}	0.171462	0.535079	0.534992	0.522411
	k_{MU1}	0.171462	0.53541	0.534662	0.503098
	k_{MU2}	0.171462	0.537351	0.532762	0.442926
	k_{MU3}	0.171462	0.684373	0.467836	0.409346
	k_{MU4}	0.171462	1.666934	0.424028	0.407612
5	k_{KS}	0.30281	0.544302	0.541856	0.505601
	$k_{s_{arith}}$	0.30281	0.695292	0.481294	0.475919
	k_{SMD}	0.30281	0.543101	0.543043	0.535562
	k_{MU1}	0.30281	0.543171	0.542973	0.532634
	k_{MU2}	0.30281	0.544388	0.541773	0.504572
	k_{MU3}	0.30281	2.568749	0.442426	0.460855
	k_{MU4}	0.30281	0.596708	0.507707	0.483359
10	k_{KS}	0.527251	0.714803	0.708544	0.568994
	$k_{s_{arith}}$	0.527251	0.730627	0.69433	0.514137
	k_{SMD}	0.527251	0.711712	0.711585	0.685797
	k_{MU1}	0.527251	0.714026	0.709299	0.586036
	k_{MU2}	0.527251	0.960609	0.601085	0.499343
	k_{MU3}	0.527251	40.4152	0.473358	0.463604

	k_{MU4}	0.527251	26.26582	0.502768	0.47492

Table7: Calculated MSE under the conditions of $n = 150$, $\mu = 0.85$, $p = 5$

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.107817	0.331163	0.331158	0.329103
	$k_{S_{arith}}$	0.107817	0.3315	0.330827	0.263883
	k_{SMD}	0.107817	0.331162	0.331158	0.329339
	k_{MU1}	0.107817	0.331189	0.331132	0.315825
	k_{MU2}	0.107817	0.331172	0.331149	0.324101
	k_{MU3}	0.107817	0.331623	0.330708	0.263384
	k_{MU4}	0.107817	0.341217	0.324438	0.370429
5	k_{KS}	0.245266	0.321986	0.32199	0.321294
	$k_{S_{arith}}$	0.245266	0.321701	0.322272	0.33098
	k_{SMD}	0.245266	0.321987	0.321988	0.321687
	k_{MU1}	0.245266	0.321985	0.32199	0.321041
	k_{MU2}	0.245266	0.321959	0.322017	0.315422
	k_{MU3}	0.245266	0.321679	0.322293	0.33263
	k_{MU4}	0.245266	0.32159	0.32238	0.338809
10	k_{KS}	0.335675	0.412843	0.412828	0.412508
	$k_{S_{arith}}$	0.335675	0.413981	0.411754	0.40562
	k_{SMD}	0.335675	0.412838	0.412833	0.412705
	k_{MU1}	0.335675	0.412844	0.412828	0.412488
	k_{MU2}	0.335675	0.413058	0.412616	0.40832
	k_{MU3}	0.335675	0.413671	0.412034	0.406178
	k_{MU4}	0.335675	0.414417	0.411372	0.404969

Table8: Calculated MSE under the conditions of $n = 150$, $\mu = 0.95$, $p = 5$

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.118698	0.275765	0.275769	0.271646
	$k_{S_{arith}}$	0.118698	0.275086	0.276405	0.237175
	k_{SMD}	0.118698	0.275767	0.275768	0.273269
	k_{MU1}	0.118698	0.275761	0.275774	0.266346
	k_{MU2}	0.118698	0.275746	0.275788	0.252432
	k_{MU3}	0.118698	0.272679	0.278124	0.291667
	k_{MU4}	0.118698	0.270877	0.279046	0.314314
5	k_{KS}	0.337373	0.283569	0.283591	0.279914
	$k_{S_{arith}}$	0.337373	0.283306	0.283865	0.262864
	k_{SMD}	0.337373	0.283579	0.283581	0.282697
	k_{MU1}	0.337373	0.28356	0.2836	0.277586
	k_{MU2}	0.337373	0.283315	0.283855	0.262783
	k_{MU3}	0.337373	0.283088	0.29041	0.337837
	k_{MU4}	0.337373	0.281462	0.28778	0.31686
10	k_{KS}	0.414478	0.205668	0.205736	0.200485
	$k_{S_{arith}}$	0.414478	0.20477	0.206632	0.185961
	k_{SMD}	0.414478	0.205699	0.205705	0.204519
	k_{MU1}	0.414478	0.205658	0.205746	0.199256
	k_{MU2}	0.414478	0.20353	0.207861	0.197201
	k_{MU3}	0.414478	0.196765	0.214447	0.253159
	k_{MU4}	0.414478	0.198761	0.21252	0.238418

Table9:Calculated MSE under the conditions of $n = 150$, $\mu = 0.99$, $p = 5$

σ	k	RLS	RRR	SRRE	RSJ
1	k_{KS}	0.163173	0.504736	0.504445	0.467734
	$k_{S_{arith}}$	0.163173	0.512162	0.497527	0.380918
	k_{SMD}	0.163173	0.504603	0.504578	0.485814
	k_{MU1}	0.163173	0.504761	0.50442	0.464706
	k_{MU2}	0.163173	0.505461	0.503727	0.413275
	k_{MU3}	0.163173	0.622647	0.448879	0.383602
	k_{MU4}	0.163173	1.092348	0.414373	0.386683
5	k_{KS}	0.387535	0.146185	0.146337	0.11604
	$k_{S_{arith}}$	0.387535	0.145109	0.147587	0.10005
	k_{SMD}	0.387535	0.146259	0.146262	0.139046
	k_{MU1}	0.387535	0.146226	0.146295	0.126503
	k_{MU2}	0.387535	0.145306	0.147332	0.099249
	k_{MU3}	0.387535	3.24117	0.19304	0.214784
	k_{MU4}	0.387535	0.170024	0.167752	0.154983
10	k_{KS}	0.501696	0.428543	0.427926	0.366235
	$k_{S_{arith}}$	0.501696	0.438427	0.419963	0.324566
	k_{SMD}	0.501696	0.42824	0.428227	0.415885
	k_{MU1}	0.501696	0.428299	0.428168	0.403022
	k_{MU2}	0.501696	13.7916	0.379008	0.370437
	k_{MU3}	0.501696	0.444604	0.416314	0.32651
	k_{MU4}	0.501696	0.476074	0.405332	0.333637

Through the simulation study, we show that, the performance of the RSJ compared with some restricted biased estimators. From Table 1 to 9, we show that the performance of new restricted biased estimator for all cases of , σ and μ.

1. From Tables 1 – 3, when ($n = 50$, $\mu = .85$, $\sigma = 1$), the RLS estimator has an EMSE of the minimum mean square error. While, the RSJ's performance is the best when compared to other estimators when($\sigma = 5, 10$, and $\mu = 0.95$, 0.99).
2. From Tables 4 – 6, when ($n = 50$, $\mu = .85$, $\sigma = 1$), the RLS estimator is superior to of any estimator biased estimators. While, the RSJ estimator is better than of any restricted biased estimator because has minimum EMSE when($\sigma = 5, 10$, and $\mu = 0.95$, 0.99).
3. From Tables 7 – 9, when ($n = 150$, $\mu = .85$, $\sigma = 1$) the performance the RLS estimator is the best. While($\sigma = 5,, 10$, $\mu = .95$, $.99$), when compared to other estimators, the RSJ estimator has the lowest EMSE. The RSJ estimator is therefore superior to all restricted estimators.

Through the simulation study in this section, it becomes evident that, when the sample size increases the performance of jackknife-biased estimators becomes the best.

5. Numerical Example:

Numerical examples are provided to demonstrate the RSJ estimator's performance utilizing real data. The dataset of acetylene that Bashtain employed is applied on a big scale (2011). Finding the performance of the RSJ estimator in comparison to the RRR, SRRE, and RLS estimators is the purpose of the difference in SMSE. The values of R and r are provided for the linear constraints in equation (2) as follows:

$$R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \end{bmatrix} \quad , r = [1.2170 \quad 1.0904].$$

The RLS, RRR, SRRE, and RSJ estimators are compared using the Scalar Mean Square Error (SMSE) criteria.

Table 10: The SMSE for different estimators under different estimated ridge parameters k and d.

k	***RLS***	***RRR***	***SRRE***	***RSJ***
0.0161	**135.7475**	**950.2154**	**12.2229**	**10.8769**
0.0243	**135.7475**	**582.9747**	**9.5455**	**7.8207**
0.050	**135.7475**	**395.2216**	**7.8989**	**5.4224**
0.020	**135.7475**	**286.1091**	**6.7818**	**4.0506**
0.50	**135.7475**	**217.0027**	**5.9080**	**3.7911**
0. 10	**135.7475**	**170.4333**	**5.3614**	**2.5568**
0.15	**135.7475**	**137.5420**	**4.7867**	**2.0448**

According to the previously discussed statistics. Table 10 demonstrates that, for all k values, the RSJ estimator has the lowest calculated SMSE, depends on two parameter k and d. Particularly, when the ridge parameter k lies between (0 – 0.5), the RSJ estimator performs better than all estimators under consideration this paper. Otherwise, the SRRE performs better with fixed (d = 0.01 or 0.1). These results are shown in Figures 1, 2, and 3. Moreover, if the parameter d is smaller than 0.6 with fixed (k = 0.01 or 0.1), then the RSJ estimator performs better than any biased estimator. Otherwise, the RRRE performs better, as shown by Figures 4 and 5.

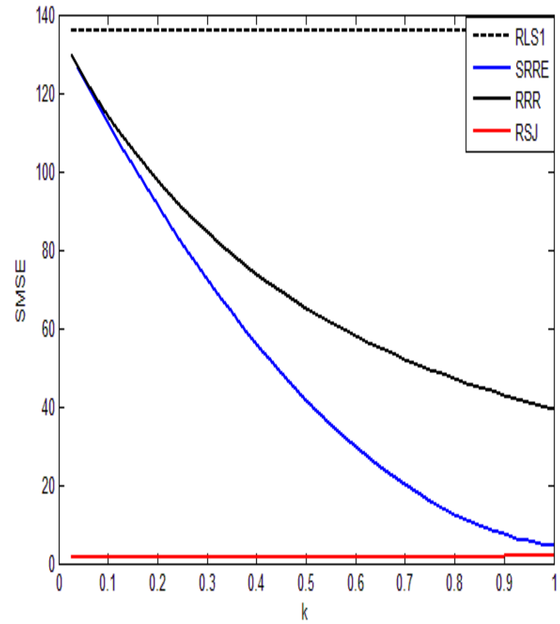

Figure 1: scalar mean square error RLS, RRR, SRRE and RSJ estimators for different k where d=0.1.

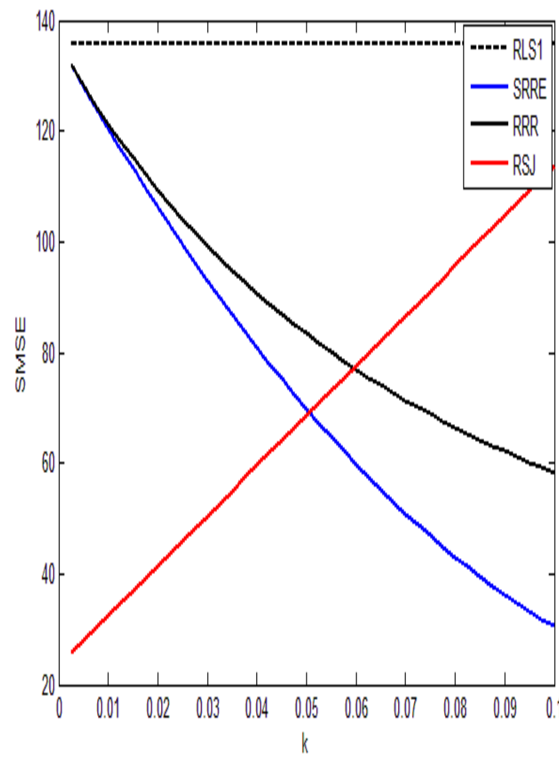

Figure 2: scalar mean square error RLS, RRR, SRRE and RSJ estimators for different k where d=0.01

P- ISSN 1991-8941 E-ISSN 2706-6703 *Journal of University of Anbar for Pure Science (JUAPS)* Open Access
2023,(17), (2):245– 253

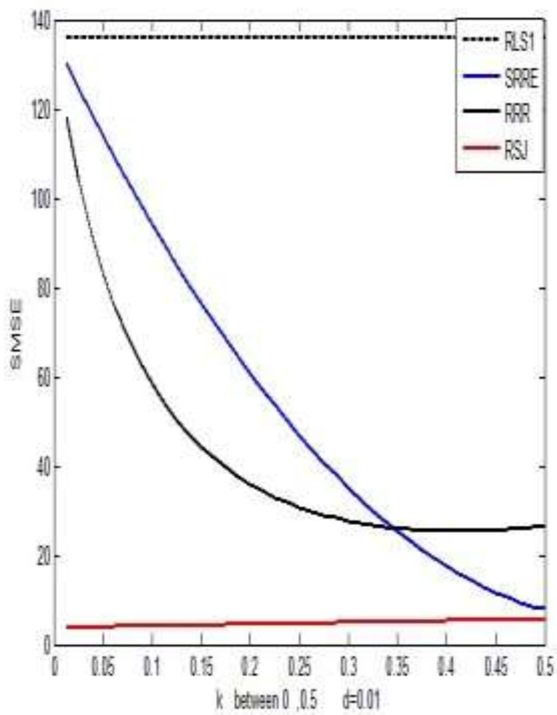

Figure 3: SMSE of RLS, RRR, SRRE and RSJ estimators for different k between 0 and 0.5 where d=0.01

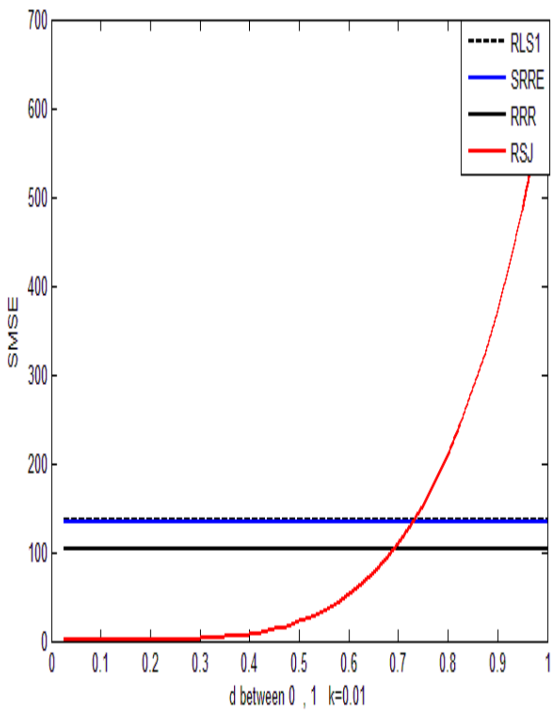

Figure 4: scalar mean square error RLS, RRR, SRRE and RSJ estimators for different d where k=0.01

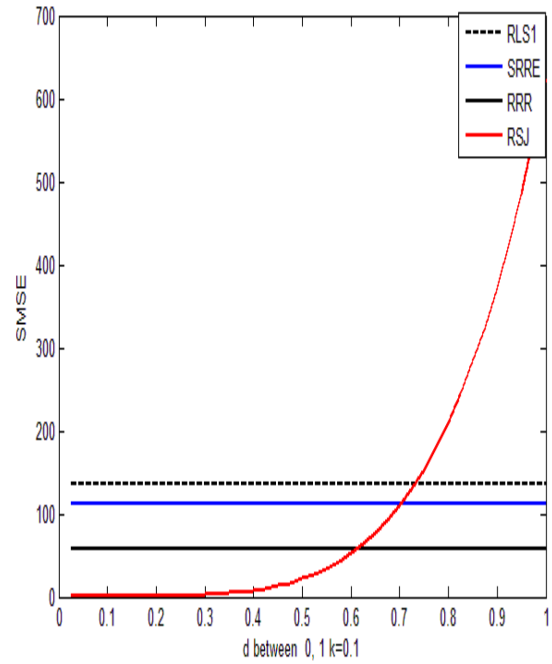

Figure 5: scalar mean square error RLS, RRR, SRRE and RSJ estimators for different d where k=0.1

5. Conclusion

We present a new jackknife biased estimator restricted linear regression model called the restricted shrinkage jackknife estimator (RSJ) by combining the jackknife technique and the RLS estimator. The RSJ estimator, which is dependent on the two parameters k and d, has the lowest computed SMSE. When the ridge parameter k is between (0 and 0.5), the RSJ estimator outperforms all other estimators considered in this study. Otherwise, fixed (d = 0.01 or 0.1) improves the SRRE's performance. Furthermore, the RSJ estimator outperforms any biased estimator when the parameter d is smaller than 0.6 with fixed (k = 0.01 or 0.1). The RRRE performs better on the other hand.

References:

[1] Wooldridge, J. M. 2020. *Introductory Econometrics: A Modern Approach*. 7th Edition, Cengage Learning, Boston, USA.

[2] Sarkar, N., 1992. A new estimator combining the ridge regression and the restricted least squares

methods of estimation. Commun. Stat. Theory Methods21: 1987–2000.

[3] Özkale, M. R., and Kaciranlar. S. (2007). The restricted and unrestricted two-parameter estimators. Communications in Statistics—Theory and Methods, **36**: 2707-2725.

[4] Yang, H. and Xu, J.W. (2009) An alternative stochastic restricted Liu estimator in linear regression, Statist. Papers 50, 639 – 647

[5] Xu, G.X. and Yang, J.Z. (2011) Building and Application of PCA-GA-SVM Model-Empirical Analysis of Prediction Accuracy of Shanghai and Shenzhen Index. Quantitative & Technical Economics, 2, 135 – 147.

[6] Huang, H., Wu, J & Yi, W. (2016). On the restricted almost unbiased two-parameter estimator in linear regression model, Communications in Statistics-Theory and Methods, 46(4): 1668 – 1678.

[7] Mohammed, B .A and ALheety, M .I. (2023). New shrinkage restricted estimator for restricted linear regression model. AIP Conference Proceedings 2414, 040044.

[8] Gülesen Üstündağ Şiray, Selma Toker, Nimet Özbay. (2021) Defining a two-parameter estimator: a mathematical programming evidence. Journal of Statistical Computation and Simulation 91(11): 2133 – 2152.

[9] Batah F, Ramanathan TK, and Gore S. D. 2008. The efficiency of modified jackknife and ridge type regression estimators: A comparison. Surveys in Mathematics and its Applications, 3:111 – 122.

[10] Batah, F. Sh., Gore, S.D., and Verma, M. R. 2008, Effect of Jackknifing on Various Ridge Type Estimators, Model Assisted Statistics and Applications, 3(3): 201 – 210.

[11] Trenkler, G. and Toutenburg, H. (1990) Mean Square Error Matrix Comparisons between Biased Estimators: An Overview of Recent Results. Statistical Papers, 31: 165-179.

[12] Farebrother R. W. 1976, Further results on the mean square error of ridge regression. J R Stat Soc B. 38 : 248 – 250.

[13] Najarian, S., Arashi, M. and Kibria, B. M. G, (2013). A Simulation Study on Some Restricted Ridge Regression Estimators. Comm. Statist. Sim.Comp, 42:871-879.

[14] Batah, F. Sh. 2013, Recovering Jackknife Ridge Regression Estimates from OLS Results, Journal of University of Anbar for Pure Science, 7(2): 1 – 8.

حول مقدر جكنايف المتحيز المنكمش المقيد لنموذج الانحدار الخطي المقيد

احمد عبد الوهاب محمد علي، فراس شاكر محمود

المديرية العامة لتربية الانبار.

E-mail: ahm21u2008@uoanbar.edu.iq

E-mail: ferashaker2001@uoanbar.edu.iq

الخلاصة:

في نموذج الانحدار الخطي المقيد، هناك طرق كثيرة مقترحة لمعالجة مشكلة التعدد الخطي والتباين العالي، على سبيل المثال التقدير المتحيز المنكمش والتحسين (دالة لاكرانج). في هذا البحث قمنا باقتراح مقدرا متحيزا جديدا يعتمد على فلسفة جكنايف مع مقدر المربعات الصغرى المقيد يسمى مقدر جكنايف المنكمش المقيد (RSJ). كما بينا اعتمادا على دراسة المحاكاة الخصائص الاحصائية للمقدر الجديد مع بعض النظريات لمقارنة اداء (RSJ) مع بعض المقدرات المقيدة السابقة. تبين ان المقدر المقترح الجديد يمتلك اداء أفضل من المقدرات المقيدة السابقة. اخيرا تم النضر في مثال عددي لتوضيح اداء المقدرات.

الكلمات المفتاحية: نموذج الانحدار الخطي المقيد، مقدر جكنايف، مشكلة التعدد الخطي، التداخل الخطي، دراسة المحاكاة.