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Active Vibration Control of Cantilever Beam by Using Optimal LQR Controller

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ABSTRACT

Many of mechanical systems are exposed to undesired vibrations, so designing an active vibration control (AVC) system is important in engineering decisions to reduce this vibration. Smart structure technology is used for vibration reduction. Therefore, the cantilever beam is embedded by a piezoelectric (PZT) as an actuator. The optimal LQR controller is designed that reduce the vibration of the smart beam by using a PZT element.

In this study the main part is to change the length of the aluminum cantilever beam, so keep the control gains, the excitation, the actuation voltage, and mechanical properties of the aluminum beam for each length of the smart cantilever beam and observe the behavior and effect of changing the length of the smart cantilever beam. A cantilever beam with piezoelectric is modeled in Mechanical APDL ANSYS version 15.0 and verified this by using experimental work. The AVC was tested on a smart beam under different control gains in experimental work and chose the best control gain depending on FEM results for each length of the smart beam. The response of the smart beam is noticed to be different for every length and the reduction percentage for settling time was different for every length.

Keywords: active vibration control, LQR controller, piezoelectric, smart cantilever beam.

السيطرة الفعالة على تخميد الأهتزاز لعتبة حيدية بواسطة أستخدام طريقة السيطرة LQR

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الخلاصة

تتعرض العديد من الانظمة الميكانيكية للأهتزازات الغير مرغوب بها، للذلك تصميم طريقة سيطرة فعالة للانظمة مهمة في القرارات الهندسية لتقليل هذه الأهتزازات. لذلك فأن عتبة حيدية مثبتة من طرف واحد يدعم بمواد ذكية (PZT) كمشغل ميكانيكي. تم تصميم وحده تحكم LQR ألامثل التي من شأنها أن تخفف من اهتزاز العتبة الذكية بأستخدام العنصر الكهروضغطي.

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الهدف الرئيسي في هذه الدراسة هو تغيير طول عتبة حيدية مصنوعة من الالمنيوم، مع ابقاء ربح المسيطر، مقدار ألاثارة، مقدار فولطية المشغل الميكانيكي والخواص الميكانيكية لعتبة حيدية ذكية ونلاحظ تصرف وتأثير تغيير الطول لعتبة ذكية. تم نمذجته العتبة الحيدية مع المواد الكهروضغطية في برنامج ANSYS . والتحقيق منها بأستخدام الجانب العملي. السيطرة الفعالة للأهتزازات تم اختبارها على عتبة حيدية ذكية تحت مختلف قيم من ربح المسيطر في الجانب العملي واختيار أفضل قيمة للربح المسيطر أعتماداً على نتأئج الجانب النظري لكل طول من العتبة الذكية. نلاحظ ان استجابة العتبة الذكية مختلفة لكل طول ونسبة التقليل لزمن الاستقرار العائد الى الأهتزاز الحر هو مختلف لكل طول.

الكلمات الرئيسية: طريقة السيطرة الفعالة، طريقة سيطرة LQR، المواد الكهروضغطية، عتبة حيدية ذكية

1. INTRODUCTION

The finite element analysis (FEA) is a technique of a numerical solution and its one of the most popular method, which can be easily recognized as assemblages of elements, is connected together by nodes. The finite element analysis has been successfully applied to many engineering analysis of solid structures, heat transfer, electromagnetism, computational fluid dynamic, tribology…. etc. The term vibration points to the limited reciprocation motion of a particle or an object. In mechanical systems, vibrations are dangerous problems that are causing a lot of damage to structures and it is undesirable if the structure vibratory motion becomes excessive. The active vibration control is used to control the undesired vibration. There are several types of controllers of a smart structure such as PID, H∞, LQR, LQG, PPE, and state feedback, etc. The difference between these types are in the type of application and the range of natural frequencies, so collected control is very important for the stability of the mechanical system. An optimal LQR (Liner Quadratic Regulator) controller is designed to reduce the vibration of variable length smart cantilever beam by using a piezoelectric element.

Deepak, et al., 2011, investigated the active vibration control of cantilever beam that attached with piezoelectric as actuator and sensor at the top and bottom surfaces of the cantilever beam. Used Euler-Bernoulli theory as finite element model. The designing of optimal LQR and the technique of the state/output feedback control by pole placement control scheme is very effective in controlling the vibration. **Isabela, et al., 2016,** introduced two types of active vibration controller: Fractional Order Proportional Derivative (FO PD) and Linear Quadratic Regulator (LQR) controller, used to control the vibration of smart beam, the smart beam is widely used as a means of studying the dynamics and active vibration suppression possibilities in aircraft wings. **Shibly, et al., 2016,** studied vibration suppression of the smart structure and performed the smart structure by using a piezoelectric patch structure. The smart structures is represented by cantilever beam attached with two piezoelectric transducers as sensor and actuator, and design the controller that based on reducing the modal order of the large-scale system. Optimal LQR controller was used as a design control for the smart structure that attenuates the vibration of a smart cantilever beam with the attached piezoelectric transducer.

2. FINITE ELEMENT FORMULATION OF BEAM ELEMENT

The element of the beam is assumed with two nodes at its ends. Each of these nodes has two degrees of freedom (DOF). The element shape functions are derived by applying boundary

conditions and by taking into considerations an approximate solution. The stiffness and mass matrices are derived by using shape functions for the element of the beam. The final two row's two elements of the first matrix are appended with the first two row's two elements of next matrix. To generate the global stiffness and mass matrices is formative. The boundary conditions are applied to the global matrices for the cantilever beam. The first two columns two rows should be removed at one end of the cantilever beam is fixed. The real response of the system, i.e. the displacement $u(x,t)$ is attaining for all the diverse models of the cantilever beam with and without the controller by assuming the first prevailing vibratory modes. **Ravi1, et al., 2016**

The displacement u is given by $(x) = [N]^T[P]$ (1)

$$X = [N1(x)N2(x)N3(x)N4(x)]\begin{bmatrix}u_1\\ \theta_1\\ u_2\\ \theta_2\end{bmatrix} \quad (2)$$

Where $N1(x) = 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3}$ (3)

$$N2(x) = x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2} \quad (4)$$

$$N3(X) = \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3} \quad (5)$$

$$N4(x) = \frac{-x^2}{l_b} + \frac{x^3}{l_b^2} \quad (6)$$

The governing differential equation of motion for the beam is:

$$M\ddot{P} + C\dot{P} + KP = q \quad (7)$$

The mass matrix and stiffness matrix are obtained as:-

$$[M] = \rho A \int_0^l [\ddot{N}]^T [N]\, dx \quad (8)$$

$$[M] = \frac{\rho A l_b}{420}\begin{bmatrix}156 & 22l_b & 54 & -13l_b\\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2\\ 54 & 13l_b & 156 & -22l_b\\ -13l_b & -3l_b^2 & -22l_b & 4l_b^2\end{bmatrix} \quad (9)$$

$$[K] = EI \int_0^l [N^{\cdot}]^T [\dot{N}]\, dx \quad (10)$$

$$[K] = \frac{E_b I_b}{l_b^3}\begin{bmatrix}12 & 6l_b & -12 & 6l_b\\ 6l_b & 4l_b^2 & -6l_b & 2l_b^2\\ -12 & -6l_b & 12 & -6l_b\\ 6l_b & 2l_b^2 & -6l_b & 4l_b^2\end{bmatrix} \quad (11)$$

$$C = \alpha M + \beta K \quad (12)$$

The parameter α and β are calculated from eq. (13)

$$\xi i = \frac{\alpha}{2w_i} + \frac{\beta w_i}{2} \quad (13)$$

3. FINITE ELEMENT FORMULATION OF SMART BEAM ELEMENT

The stiffness and mass matrices for the element of the smart beam with piezoelectric patches are locating at the top of the surface of the beam as a construct pair is given by. **Deepak, et al., 2011**

$$M\ddot{P} + \Delta\dot{P} + KP = \{f_a\}\{\emptyset_a(t)\} \quad (14)$$

The mass matrix of the smart beam element of the following form:

$$[M] = \frac{\rho_b A_b l_b}{420}\begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix} + \frac{\rho_P A_P l_P}{420}\begin{bmatrix} 156 & 22l_P & 54 & -13l_P \\ 22l_P & 4l_P^2 & 13l_P & -3l_P^2 \\ 54 & 13l_P & 156 & -22l_P \\ -13l_P & -3l_P^2 & -22l_P & 4l_P^2 \end{bmatrix} \quad (15)$$

The stiffness matrix of the smart cantilever beam element of the following form:

$$[K] = \frac{EI_{eq}}{l_P^3}\begin{bmatrix} 12 & 6l_b & -12 & 6l_b \\ 6l_b & 4l_b^2 & -6l_b & 2l_b^2 \\ -12 & -6l_b & 12 & -6l_b \\ 6l_b & 2l_b^2 & -6l_b & 4l_b^2 \end{bmatrix} \quad (16)$$

$$EI_{eq} = E_b I_b + E_P I_P \quad (17)$$

$$Ip = \frac{1}{12} * b * t_a^3 + b * t_a \left(\frac{t_a + t_b}{2}\right)^2 \quad (18)$$

4. EQUATION OF THE VOLTAGE OF SENSOR AND CONTROL FORCE FROM ACTUATOR

The piezoelectric voltage of the sensor element is given by **Shibly, et al., 2016**

$$Vs(t) = Gs * Z * e_{31} * b * [0\ -1\ 0\ 1]\begin{bmatrix} \dot{\omega}_1 \\ \dot{\theta}_1 \\ \dot{\omega}_2 \\ \dot{\theta}_2 \end{bmatrix} \quad (19)$$

$$Vs(t) = gT\ [\dot{P}\ ] \quad (20)$$

Where $gT = Gs * Z * e_{31} * b\ [0 - 1\ 0\ 1]$ (21)

$$gT = Sc * [0\ -1\ 0\ 1] \quad (22)$$

The equation of actuator derived from the reverse the equation of the piezoelectric, the bending moment of the actuator in small cross-section area is given by **Shibly, et al., 2016**

$$dMa = E_P * I_P * \frac{d^2w}{dt^2} \quad (23)$$

To determine the resultant moment from Eq. (23) by integrating the stress in eq. (23)

$$Ma = E_P * d_{31} * Z * Va(t) \quad (24)$$

The controlling force produced by the actuator to the beam element is given by:

$$f_{ctrul} = E_P * d_{31} * b * Z * [-1\ 0\ 1\ 0]^T \quad (25)$$

$$f_{ctrul} = \mathrm{h} * \mathrm{Va(t)} \quad (26)$$

Which h is dependent on the type of the actuator, its characteristic properties, and its location on beam where

$$\mathrm{h} = \mathrm{E}_P * \mathrm{d}_{31} * \mathrm{b} * \mathrm{Z} * [-1\ 0\ 1\ 0] \quad (27)$$

5. THE STATE SPACE MODEL OF SMART BEAM

Many design instruments and model reduction in modernistic control theory notice a state space form the mathematical model of a plant. Consequently, the smart cantilever beam mathematical model can be written in form as follows, **Shibly, et al., 2016**

$$Let\ q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x,$$

$$\dot{q} = \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix} = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \dot{x}$$

$$and\ \ \ddot{q} = \begin{bmatrix} \dot{x_3} \\ \dot{x_4} \end{bmatrix}$$

Then the 4-element smart cantilever beam state space model is:-

$$M \begin{bmatrix} \dot{\mathrm{X}}_3 \\ \dot{\mathrm{X}}_4 \end{bmatrix} + \Delta \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + K \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = f_{ctral} \quad (28)$$

Which yield to

$$\begin{bmatrix} \dot{\mathrm{X}}_3 \\ \dot{\mathrm{X}}_4 \end{bmatrix} = -M^{-1}\Delta \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} - M^{-1}K \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + M^{-1}h\, V_a(t) \quad (29)$$

Or

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & I\\ -M^{-1}K & -M^{-1}\Delta\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ M^{-1}h\end{bmatrix} u(t) \quad (30)$$

And in a matrix form:-

$$\dot{\mathrm{X}}(t) = Ax(t) + Bu(t) \quad (31)$$

Where $u(t) = Va(t)$

With suitable zero and identity matrices dimensions. The voltage of the sensor is taken as the output of the system and the output equation is given as:-

$$Y(t) = Va(t) = g^T\,[\dot{\mathrm{P}}] = g^T\begin{bmatrix}x_3\\ x_4\end{bmatrix} \quad (32)$$

Thus, the output equation of sensor in state space form is given by:

$$or\ Y(t) = C\,x(t) \quad (33)$$

Where$C = [0 \quad gT]$. The single input single output state space model of smart structure progressing for the system is given by:

$$\left.\begin{aligned}\dot{\mathrm{X}}(t) &= Ax(t) + Bu(t)\\ Y(t) &= Cx(t)\end{aligned}\right\} \quad (34)$$

$$\left.\begin{aligned}With\ A &= \begin{bmatrix}0 & I\\ -M^{-1}K & -M^{-1}\Delta\end{bmatrix}\\ B &= \begin{bmatrix}0\\ M^{-1}h\end{bmatrix}\\ C &= [0 \quad gT]\\ D &= [0]\end{aligned}\right\} \quad (35)$$

6. LINEAR QUADRATIC REGULATOR (LQR)

The optimal controller (LQR) is used to compute the active control gain. The main idea of this type of control design is to minimize the quadratic cost function of the following formal**:** **Gergely, and Bori, 2012**

$$J_{LQR} = \int_0^\infty [X^T QX + u^T R\,u]d(t) \quad (36)$$

The integral from equation (36) is written as the cumulative deviousness of the states with respect to the equilibrium point and the cumulative control effect. By duly choosing the parameters of Q and R, the importance of control rendering and input energy are weighted access favorable intercession, **DAVID, 2014**

The state space format of the system is given by **Gergely, and Boris, 2012**

$$\left.\begin{aligned}\dot{X}(t) &= Ax(t) + Bu(t)\\ Y(t) &= Cx(t) + Du(t)\end{aligned}\right\} \quad (37)$$

The doctrinaire input $u(t)$ generated by the LQR controller is

$$u = -K_g\, x(t) \quad (38)$$

Where $K_g$is the feedback gain matrix calculate as

$$K_g = R^{-1}.B^T.X \quad (39)$$

X is a symmetric matrix determined from the Ricatti equation as a non-negative definite solution: -

$$A^T X + XA + C^T QC - XBR^{-1}B^T X = 0 \quad (40)$$

The solution of the closed-loop system from this eq. is

$$\dot{\mathrm{X}}(t) = (A - B * K_g) * X(t) \quad (41)$$

7. FINITE ELEMENT MODAL OF SMART CANTILEVER BEAM

The volume of beam and piezoelectric were created by using [BLOCK] command in ANSYS as shown in fig.1a. The type of element that used to mesh beam is SOLID45 and SOLID5 to mesh the piezoelectric. SOLID45 has 8 nodes for each node there is 3 degree of freedom (DOF). It is used to represent 3D solid structures. It has the plasticity, creep, and stress stiffness swelling, large deflection, and large strain capabilities. SOLID5 has 8 nodes for each node there is 6 DOF. It has 3D magnetic, piezoelectric, and structural field capacity with limited coupling between fields. The DOF of the beam has been chosen the displacement only and for piezoelectric chose electrical. After assigning appropriate the element type and material properties of beam and piezo have been input to the program by using [ET] and [MP] command in ANSYS.

The modeling of the material of the piezoelectric is given by the equations of constitutive:

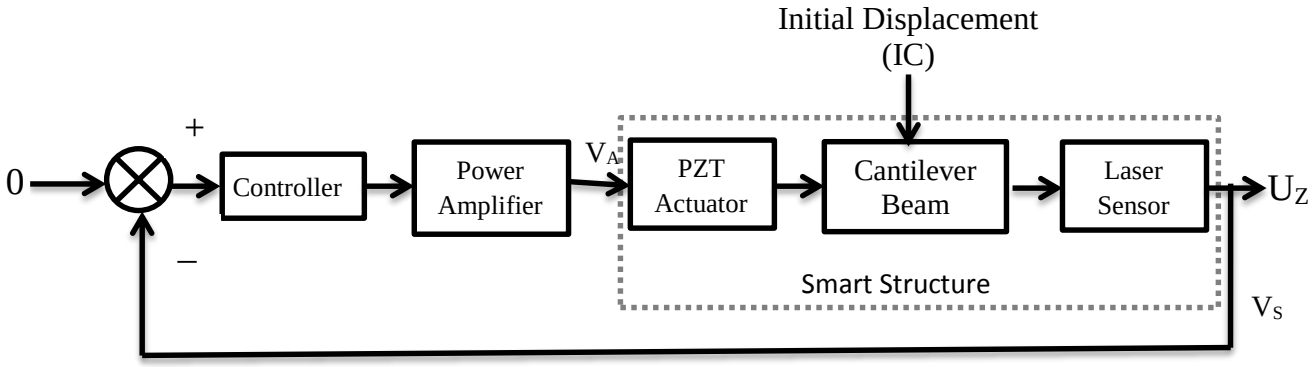
$$\{T\} = [c]\{S\} - [e]\{E\} \quad (42)$$

$$\{D\} = [e^T]\{S\} - [\varepsilon]\{E\} \quad (43)$$

Fig.1b shows the details of boundary conditions and the model of piezoelectric. In this study one piezoelectric patch was embedded on the top surface of the tested beam as an actuator, PPA-2014, PZT-5H, the dimensions of the piezoelectric is [**55 x 20.8 x 0.83]** mm. The material properties of aluminum and piezoelectric are listed in **Table 1**.

8. FREE VIBRATION ANALYSIS

Firstly initial displacement 0.0015m was applied to the tip of the smart cantilever beam to obtain free vibration, the static analysis was performed in order to create initial conditions by using [IC] command in the ANSYS program. Then these displacements were disposed in a matrix. The extracted displacement will lead the structure to freely oscillate.
The important thing is to note that the total number of nodes must equal to the number nodes in [IC] command. The block diagram of this analysis is shown in the frame below.



Frame. The Block diagram of free vibration analysis.

9. CLOSS LOOP SIMULATION

LQR controller is simulated in the ANSYS program by using [*USD] command to run the program of the controller that was created by micro file. Modal analysis for aluminum beam was performed to determine the time step of the ANSYS solution by using the equation $dt = 1/20/w_i$, where w_i is the natural frequency of aluminum beam. The finite element model of smart cantilever beam for free vibration is shown in **Fig. 3**. To determine the displacement of the system at each time step for LQR controller using the command [*DO ------ *ENDDO] in ANSYS solution. Active vibration control of a smart cantilever beam under free vibration is analyzed by using optimal LQR controller.

10. EXPERIMENTAL WORK OF ACTIVE VIBRATION CONTROL

The experimental work of active vibration control for the smart beam is shown in **Fig. 2**. It consists of smart aluminum beam embedded with one piezoelectric as an actuator. The smart aluminum beam is fixed from one end and free from the other. The piezoelectric is mounted close to the fixed end of the aluminum beam. The aluminum beam is of variable length (ranging from 200mm to 400mm), 40mm width, and 1.4mm thickness. The details view of the smart aluminum beam is shown in **Fig. 3**. The piezoelectric was produced by MIDE technology and the bearing the product designation PPA-2014. The piezoelectric (actuator) bonded onto beam using a special piezoelectric adhesive. The Laser Sensor LK-085 is used to measure the

displacement from the tip of the cantilever beam. The NI DAQPad-6353 is used as analog input to the received signal from Laser Sensor LK-085, also used as an analog output to applying the voltage to the piezo (actuator) patches used to mitigate vibration of the beam. The E-507 is used to amplify the control signal to the piezoelectric (actuator). Used LABVIEW Program is to implement the control algorithm.

11. RESULTS AND DISCUSSIONS

Increasing length of cantilever beam led to decrease the stiffness and increase the mass, hence the natural frequency will decrease (the natural frequency change directly with stiffness and indirectly with mass), and this led to the increasing the settling time of cantilever beam to reach the steady state. There is good agreement between the results of experimental and numerical.
From the experimental results of 200mm length as shown in **Fig. 4**, due to best control gain. It was [Q=20, R=30] due to the settling time of response to reach a steady state with active control at t=1 sec, while without active control settling time was 2sec. The reduction percentage for settling time-related to free vibration of the smart beam (200mm) is 50%.
Fig. 5 shows the results of 250mm length by using different control gains, the best control gain was [Q=10, R=10] due to the response take 1.5 sec to reach the steady state with active control, while without active control take 6 sec to reach steady state. The reduction percentage for settling time-related to free vibration of the smart beam (250mm) is 75%. The amplitude of the tip displacement without active control was $\pm 3\ cm$ while in active control was $\pm 1.5\ mm$, that mean all the undesired vibration are removed and the beam become safe from the unwanted vibration that caused fatigue, buckling ……..etc.
Fig. 6 shows the results of 350mm length by using different control gains, the best control gain was [Q=20, R=20] due to the response take 1 sec to reach the steady state with active control, while without active control take 7 sec to reach steady state. The reduction percentage for settling time-related to free vibration of the smart beam (350mm) is 80%.
Fig. 7 shows the results of 400mm length by using different control gains, the best control gain was [Q=30, R=30] due to the response take 2 sec to reach the steady state with active control, while without active control take 10 sec to reach steady state. The reduction percentage for settling time-related to free vibration of the smart beam (400mm) is 75%.

12. CONCLUSIONS

The simulations for the active vibration control of the cantilever beam for free vibration show that the settling time for the vibration reduced effectively and vibration suppression is obtained for various gains, the maximum reduced settling time was from 10 seconds to 2 seconds. Optimal LQR control (Active Vibration Control) is very affected for vibration reduction when compare it with another type of control such as passive control. The results of the experimental work are matching with the result obtained by APDL ANSYS, although the amplitude difference between the experimental and simulation amplitudes was got, the results are still good in matching and validity.

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NOMENCLATURE

A = the state matrix

B = the control matrix.

b = the width of the piezoelectric, mm

β = the stiffness (structural) damping constant.

C = the output matrix,

[c]= the matric of the elasticity,

C = the damping of the beam element.

D = the feedthrough matrix.

d_{31} = the piezoelectric strain constant, m/V x 10-12

D = the electrical vectors of displacement

E = the electric field vectors.

E_P = the young modulus of piezoelectric, GPa

e_{31} = the piezoelectric stress constant,$e_{31} = E_P * d_{31}$, NV/m *e-12.

$[\varepsilon]$= the matrix of the piezoelectric

$[e^T]$ = the matrix of the dielectric

$\{f_a\}$ = the force co-efficient vectors which maps the applied actuator voltage to the bread displacement of smart beam element.

Gs = the sensor gain, dimensionless.

h = the constant vector of the size (4 x 1), dimensionless.

Ip = the moment of inertia of cross-section of the piezoelectric, mm^4.

K = the stiffness of beam element.

M = the mass of beam element. Kg

$N1(x)$, $N2(x)$, $N3(x)$, and $N4(x)$ = the shape functions.

q = the force co-ordinate vectors.

Q = the positive semi-definite matrix having dimension n*n, where n is the number of states of the state space system, dimensionless.

$\{\emptyset_a(t)\}$ = the voltage that applied to the actuator, develops effect control force and moments.

R = a scalar quantity, dimensionless.

T = the mechanical vector of stress,

t_p = the thickness of piezoelectric, mm

t_a= the thickness of beam, mm

S = the mechanical vector of strain.

u = control force.

u_1, θ_1, and u_2, θ_2 = the DOF's at each node.

$V_a(t)$ = the voltage applied to actuator

α = the mass (frictional) damping constant

ξi = damping ratio of i^{th} mode.

wi = the natural frequency of i^{th} mode.

X = the state vector,

Y = the output vector.

$Z = \frac{t_b}{2} + t_a$ = distance between the neutral axis of the beam and the piezoelectric.

Table 1. Material Properties and Geometric Parameter.

Material property of piezoelectric	**Material property of beam**	**Geometric parameter of beam**	
$E = 26X10^9 N/M^2$	$E = 70x10^9 N/M^2$	$L = from\ 200\ to\ 400\ mm$	
$\rho = 7350\ kg/M^3$	$\rho = 2700\ kg/M^3$	$B = 40\ mm$	
Elastic stiffness matrix (N/ m^2) *E9 C11 = 126 C12 = 79.5 C13 = 84.1 C33 = 117 C44 = 23.3 Piezoelectric strain matrix (C/m^2) E31 = 6.5 E33 = 23.3 E15 = 17 Dielectric matrix (F/m)*E-9 e11 = 15.03 e22 = 15.03 e33 = 13	$v = 0.34$	$H = 1.4\ mm$	

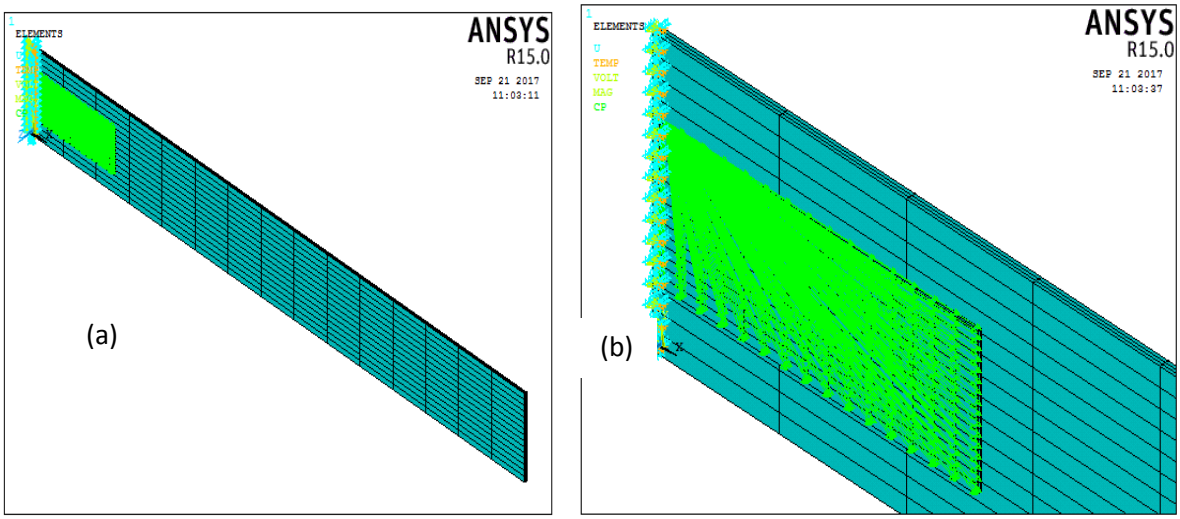


Figure 1. The finite element model of the cantilever beam embedded with piezoelectric. (a) Showing elements of the cantilever beam (b) Detail of boundary conditions and the model.

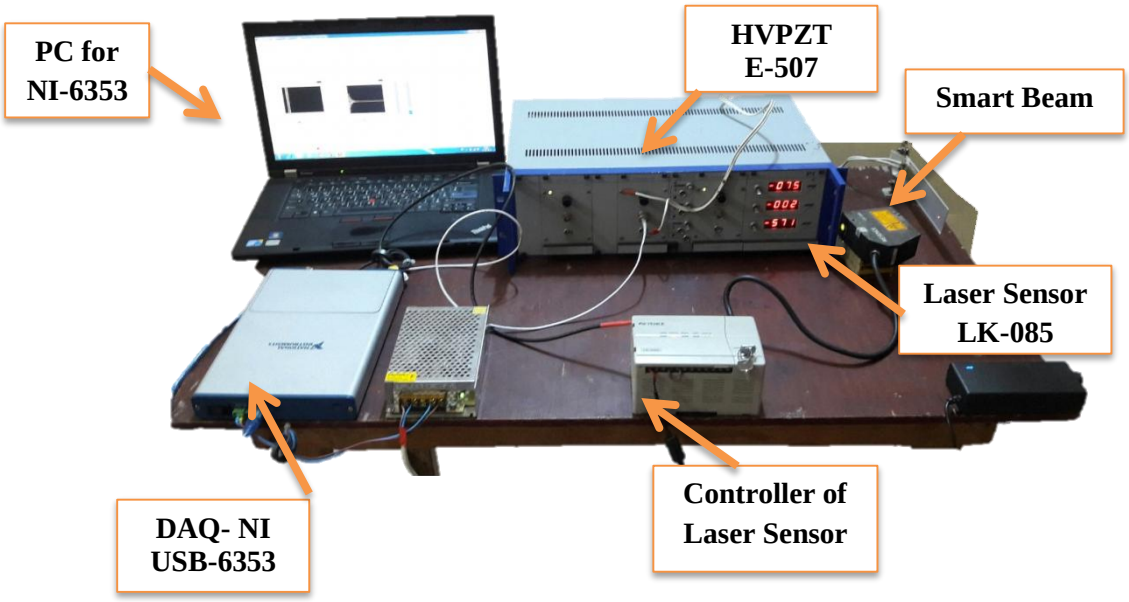


Figure 2. Experimental system.



Figure 3. Smart Cantilever beam.

The result of 200 mm length is shown below.

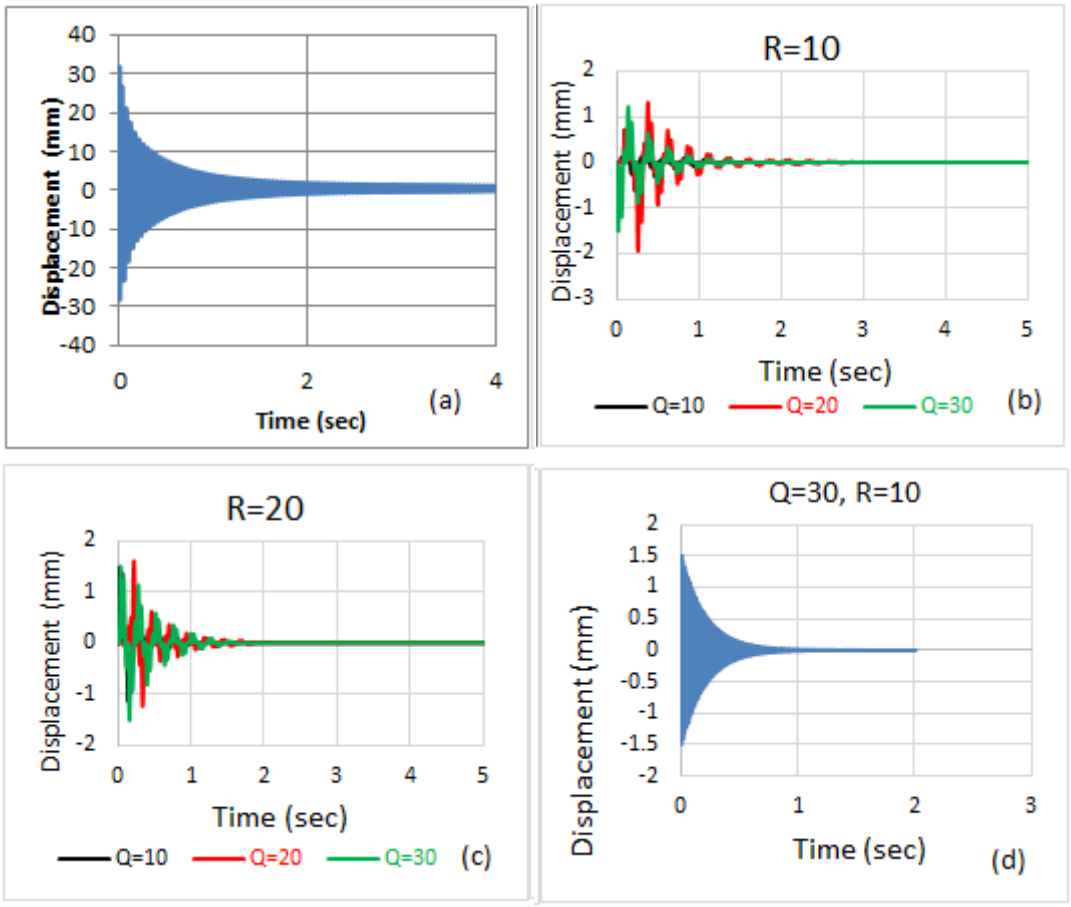


Figure 4. (a) Free vibration without active control. Results of active vibration control of experimental results shown in (b, c) while (d) show the numerical result.

result of 250 mm length is shown below.

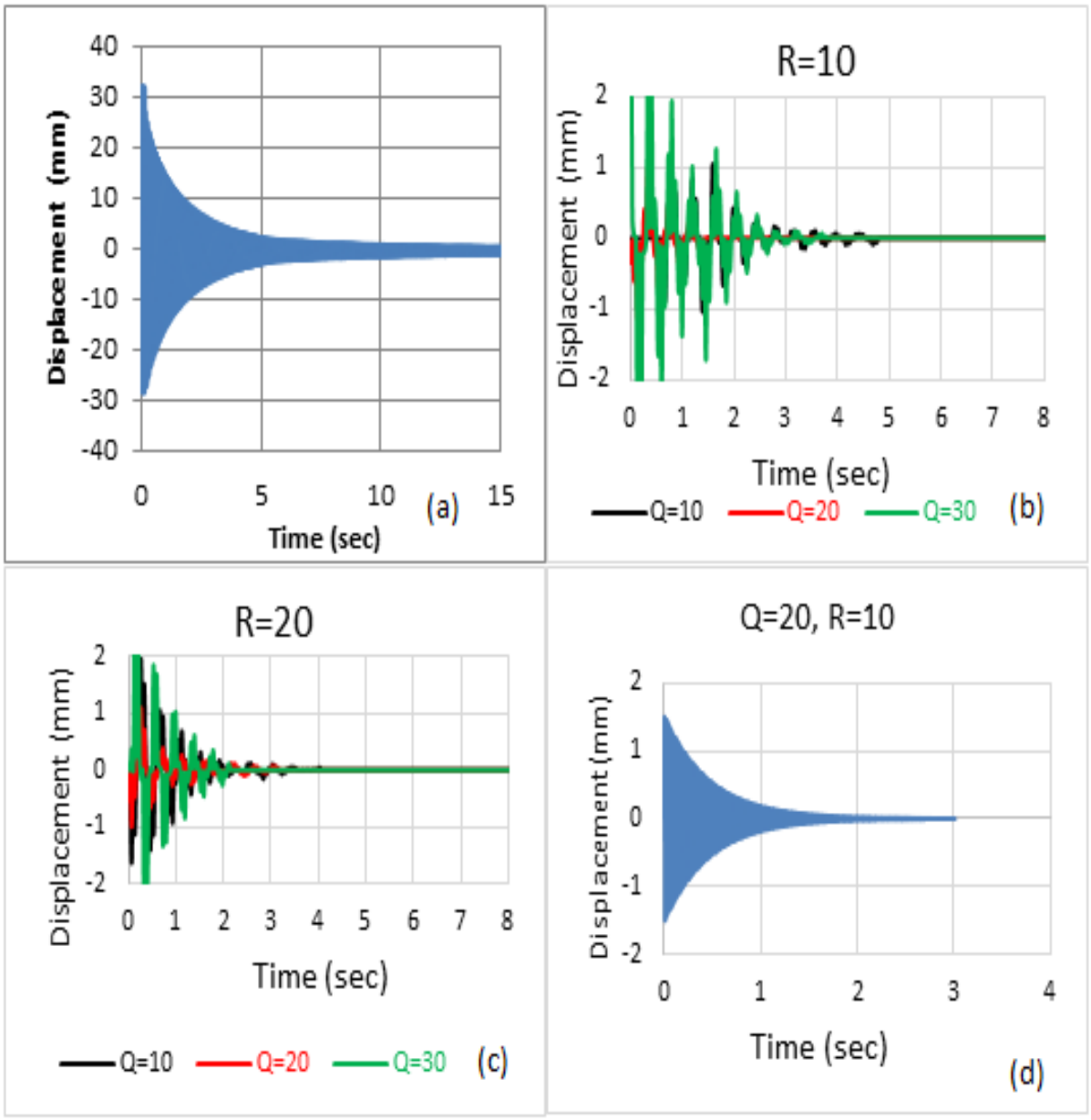


Figure 5. (a) Free vibration without active control. Results of active vibration control of experimental results shown in (b, c) while (d) show the numerical result.

The result of 350 mm length is shown below.

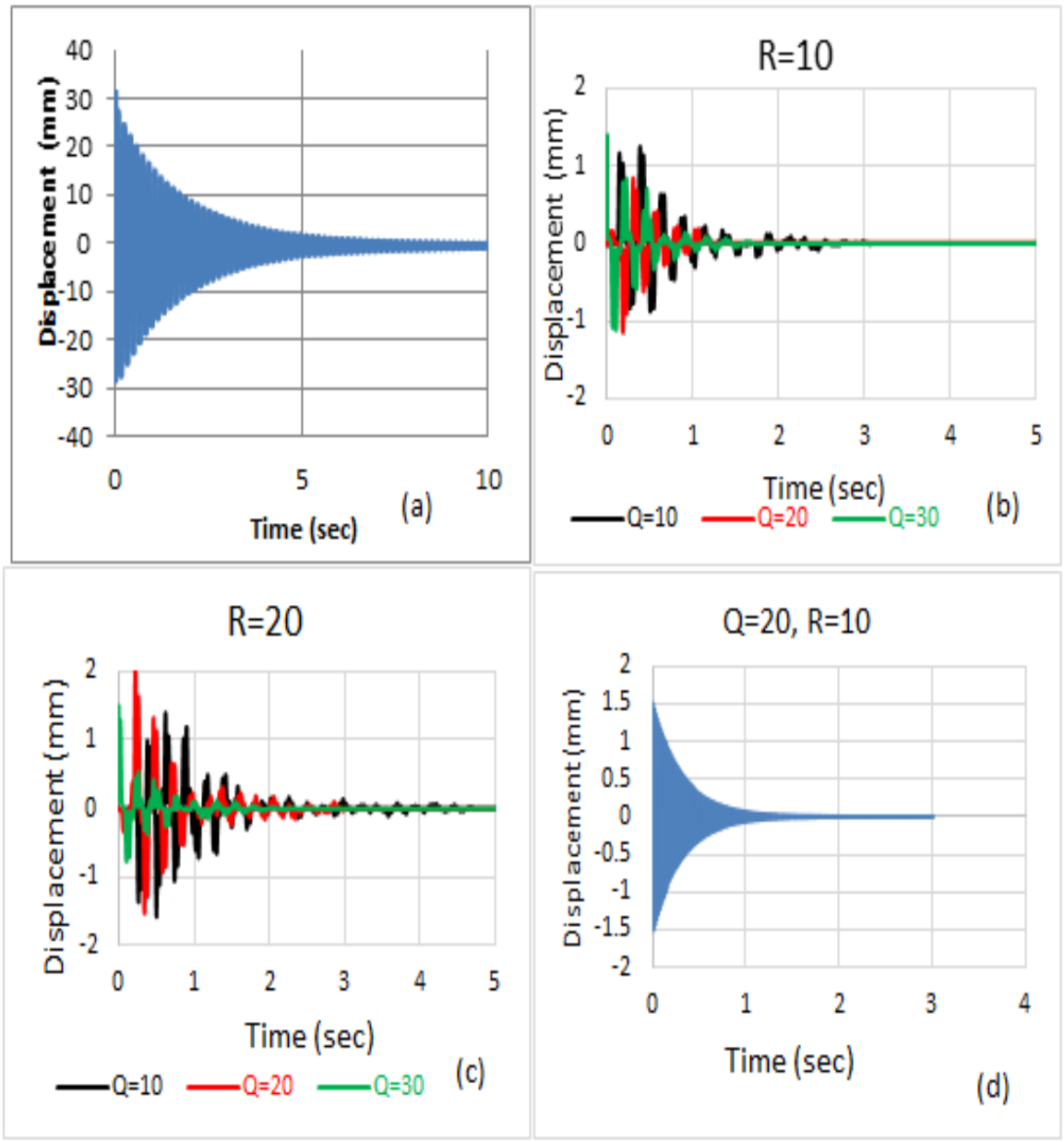


Figure 6. (a) Free vibration without active control. Results of active vibration control of experimental results shown in (b, c) while (d) show the numerical result.

The result of 400 mm length is shown below.

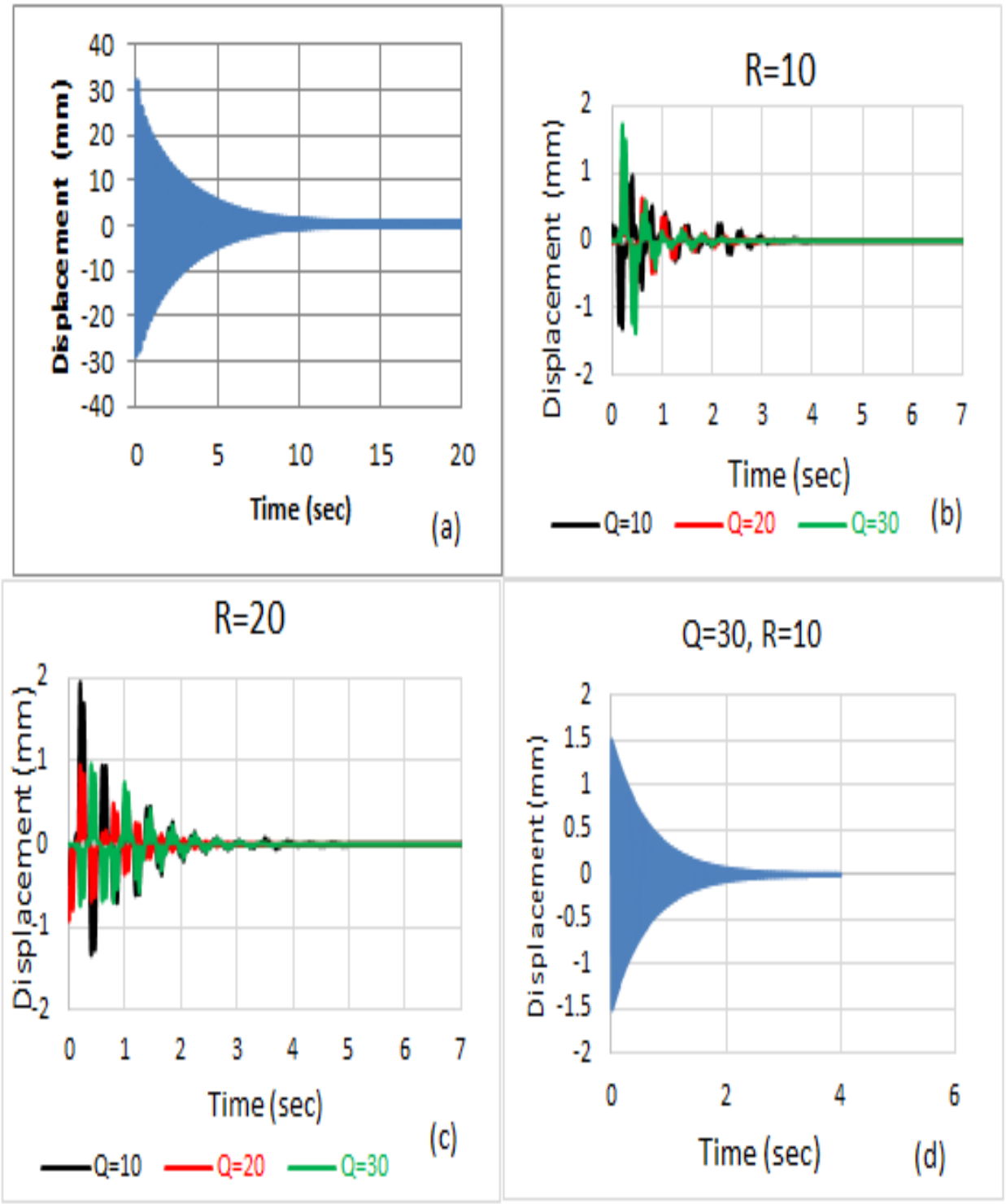


Figure 7. (a) Free vibration without active control. Results of active vibration control of experimental results shown in (b, c) while (d) show the numerical result.