

# Exact Solutions of Cardiovascular Models by using Upadhyaya Transform

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**Abstract**— In many practical fields, such as engineering, physics, chemistry, biology, psychology, economics, and finance, processes are simulated using differential equations. These models solutions, in contrast to algebraic equations, may be more intricate. In order to get at the solutions to these models, it is easy to employ integral transformations. In this paper, we use the Upadhyaya transform to obtain accurate solutions to two cardiovascular models. It is obvious that the Upadhyaya transform is an effective, dependable, and simple technique for solving differential equations.

**Keywords**— Differential equations, Upadhyaya transform (UT), Inverse Upadhyaya transform, cardiovascular models.

## I. INTRODUCTION

Due to the importance of the heart and blood vessels in humans and animals, models have been developed for them by specialists. In turn, as mathematician, we study finding exact solutions for these models in terms of identifying the problem and treating it. These difficult differential models can be converted into easy, simple algebraic equations using integral transformations, including the Upadhyaya transform.

Ordinary differential equations enable researchers to study the dynamics of biological systems, anticipate their behavior under various conditions, and obtain insights into underlying biological principles [1]. They are useful in investigating population dynamics, disease propagation, and many other biological processes. The mathematical solution of differential equations might be more complex than the solution of algebraic equations. As a result, scholars have investigated methods for translating differential equations into algebraic equations. Integral transforms, which convert differential equations to algebraic equations, are one of the solutions

developed as a result of this research. These transformations result in very successful solutions in a wide range of industries. The Upadhyaya transform (UT), which is the most significant generalization of virtually all conventional forms of the Laplace transform in mathematics, was developed by Upadhyaya [2]. In order to estimate the blood glucose concentration at the moment of an intravenous injection, Higazy *et al.* [3] established the Sawi decomposition approach and identified the primitive (solution) of Volterra integral equation. Mousa [4] employed the Upadhyaya transform to solve the V.I.E's (Volterra integral equations) of the first kind. In order to monitor a patient's blood glucose level while receiving continuous intravenous injections, Kumar *et al.* [5] suggested Anuj transformation. First-kind Volterra integral equations were solved by Kumar *et al.* [6] using the integral transform known as the Rishi Transform. Mansour *et al.* [7] solved partial differential equations using the Double Complex SEE Integral transform. To solve systems of ordinary differential equations, Zamil and Kuffi [8] applied Sadiq and

the complex Sadiq transforms. Peker *et al.* (9–14) utilized the Kashuri Fundo transform to solve various models, including steady heat transfer, decay, chemical reactions, Bratu's problem, Michaelis-Menten's biochemical reaction model, population growth, and mixing problems. Through these applications, they effectively showcased the capability of the transform in obtaining solutions for ordinary differential equations. Dinesh and Prakash [15] solved the linear second-kind Volterra integral equation using the Upadhyaya transform. Zamil and Kuffi [16] presented a novel integral transform known as the INEM-Transform, which is used to solve various ordinary differential equations. In their latest study, Peker *et al.* [17] used the Kashuri Fundo transform technique to find the precise solution of differential equations using two separate cardiovascular models. The purpose of this work is to show that Upadhyaya transform may be used to solve differential equations using two separate cardiovascular models: blood glucose concentration during continuous intravenous glucose injection and aortic pressure.

## II. NOMENCLATURE OF SYMBOLS

- $k$  - Constant velocity of elimination,
- $\alpha$  - Rate of infusion,
- $V$  - Volume in which glucose is distributed,
- $k, c, A, p_0$  - Constants,
- $U$  - Upadhyaya transform operator,
- $U^{-1}$  - Inverse Upadhyaya transform operator,
- $C(t)$  - Concentration of glucose in the blood,
- $p(t)$  - Pressure in the aorta.

## III. DEFINITION AND PROPERTIES

**Definition:** Upadhyaya transformation of the function  $f(t)$  is provided by [2]:

$$U\{f(t)\} = \lambda_1 \int_0^{\infty} \exp(-\lambda_2 t) f(\lambda_3 t) dt \quad \dots (1)$$

$$= u(\lambda_1, \lambda_2, \lambda_3), \quad (\lambda_1, \lambda_2, \lambda_3) > 0, \quad t \geq 0$$

The inverse Upadhyaya transform is represented as

$$f(t) = U^{-1}[u(\lambda_1, \lambda_2, \lambda_3)], \quad t \geq 0.$$

In the general formulation of the Upadhyaya transform,  $\lambda_1, \lambda_2$  and  $\lambda_3$  are complex parameters; however, for the sake of this research, we will assume that all of these parameters are positive real numbers (see Upadhyaya [2] and Upadhyaya *et al.* [18]).

### Property. (Linearity property) 1:

If  $f_1(t)$  and  $f_2(t)$  be two functions with UT's  $u_1(\lambda_1, \lambda_2, \lambda_3)$  and  $u_2(\lambda_1, \lambda_2, \lambda_3)$  in terms of the parameters  $\lambda_1, \lambda_2, \lambda_3$ , then [2]

$$U[b_1 f_1(t) + b_2 f_2(t); \lambda_1, \lambda_2, \lambda_3]$$

$$= b_1 U[f_1(t); \lambda_1, \lambda_2, \lambda_3] + b_2 U[f_2(t); \lambda_1, \lambda_2, \lambda_3]$$

$$= b_1 u_1(\lambda_1, \lambda_2, \lambda_3) + b_2 u_2(\lambda_1, \lambda_2, \lambda_3).$$

Here  $b_1, b_2$  are any constants.

### Property. (Convolution property) 2:

If the Upadhyaya transform of the functions  $f_1(t)$  and  $f_2(t)$  with respect to the parameters  $\lambda_1, \lambda_2, \lambda_3$  are  $u_1(\lambda_1, \lambda_2, \lambda_3)$  and  $u_2(\lambda_1, \lambda_2, \lambda_3)$ , then convolution of Upadhyaya transform of the functions  $f_1(t) * f_2(t)$  is given by [2]

$$U[f_1(t) * f_2(t); \lambda_1, \lambda_2, \lambda_3] = \frac{\lambda_3}{\lambda_1} u_1(\lambda_1, \lambda_2, \lambda_3) \cdot u_2(\lambda_1, \lambda_2, \lambda_3)$$

where  $f_1(t) * f_2(t)$  is given by

$$f_1(t) * f_2(t) = \int_0^t f_1(t-x) f_2(x) dx = \int_0^t f_1(x) f_2(t-x) dx$$

**III-I Upadhyaya transform of some functions** [18]:

$$U(1) = \frac{\lambda_1}{\lambda_2}, U(t^m) = \frac{m! \lambda_1 \lambda_3^m}{\lambda_2^{m+1}} (m \in N), U(e^{at}) = \frac{\lambda_1}{\lambda_2 - a\lambda_3}$$

$$U[\sin(at)] = \frac{a\lambda_1\lambda_3}{\lambda_2^2 + a^2\lambda_3^2}, U[\cos(at)] = \frac{\lambda_1\lambda_2}{\lambda_2^2 + a^2\lambda_3^2}$$

**III-II Upadhyaya transform of derivatives**

If  $U[f(t)] = u(\lambda_1, \lambda_2, \lambda_3)$  then from [18]

$$U[f'(t); \lambda_1, \lambda_2, \lambda_3] = \left(\frac{\lambda_2}{\lambda_3}\right) U[f(t); \lambda_1, \lambda_2, \lambda_3] - \frac{\lambda_1}{\lambda_3} f(0),$$

$$U[f''(t); \lambda_1, \lambda_2, \lambda_3] = \left(\frac{\lambda_2}{\lambda_3}\right)^2 U[f(t); \lambda_1, \lambda_2, \lambda_3] - \frac{\lambda_1\lambda_2}{\lambda_3^2} f(0) - \frac{\lambda_1}{\lambda_3} f'(0),$$

$$U[f^n(t); \lambda_1, \lambda_2, \lambda_3] = \left(\frac{\lambda_2}{\lambda_3}\right)^n U[f(t); \lambda_1, \lambda_2, \lambda_3] - \frac{\lambda_1\lambda_2^{n-1}}{\lambda_3^n} f(0) - \frac{\lambda_1\lambda_2^{n-2}}{\lambda_3^{n-1}} f'(0) - \frac{\lambda_1\lambda_2^{n-3}}{\lambda_3^{n-2}} f''(0) - \dots - \frac{\lambda_1}{\lambda_3} f^{n-1}(0).$$

**IV. APPLICATIONS (UPADHYAYA TRANSFORM TO CARDIOVASCULAR MODELS)**

This section provides Upadhyaya transform applications to two Cardiovascular Models.

**Application I. (Glucose concentration in the blood)**

During Continuous intravenous glucose concentration, the concentration of glucose in the blood is  $C(t)$  exceeding the baseline value at the start of the infusion. The function  $C(t)$  satisfies the initial value problem [16, 17]

$$\frac{dC(t)}{dt} + kC(t) = \frac{\alpha}{V}, t > 0, C(0) = 0 \quad \dots (2)$$

Using the Upadhyaya transform technique, we will determine the level of glucose in the blood.

Performing the Upadhyaya transform bilaterally to equation (2), we get

$$U\left[\frac{dC(t)}{dt}\right] + kU[C(t)] = U\left[\frac{\alpha}{V}\right]$$

$$U\left[\frac{dC(t)}{dt}\right] + kU[C(t)] = \frac{\alpha}{V} \frac{\lambda_1}{\lambda_2} \quad \dots (3)$$

Let  $U[C(t)] = u(\lambda_1, \lambda_2, \lambda_3)$  and using the derivative property of Upadhyaya transform, we obtain

$$\frac{\lambda_2}{\lambda_3} u(\lambda_1, \lambda_2, \lambda_3) - \frac{\lambda_1}{\lambda_3} C(0) + k u(\lambda_1, \lambda_2, \lambda_3) = \frac{\alpha}{V} \frac{\lambda_1}{\lambda_2} \quad \dots (4)$$

Equation (4) can be expressed as

follows by using the initial condition  $C(0) = 0$ .

$$\frac{\lambda_2}{\lambda_3} u(\lambda_1, \lambda_2, \lambda_3) + k u(\lambda_1, \lambda_2, \lambda_3) = \frac{\alpha}{V} \frac{\lambda_1}{\lambda_2} \quad \dots (5)$$

After simple computation:

$$\left[\frac{\lambda_2}{\lambda_3} + k\right] u(\lambda_1, \lambda_2, \lambda_3) = \frac{\alpha}{V} \frac{\lambda_1}{\lambda_2}$$

$$\left[\frac{\lambda_2 + \lambda_3 k}{\lambda_3}\right] u(\lambda_1, \lambda_2, \lambda_3) = \frac{\alpha}{V} \frac{\lambda_1}{\lambda_2}$$

$$u(\lambda_1, \lambda_2, \lambda_3) = \frac{\alpha}{V} \left[\frac{\lambda_1 \lambda_3}{\lambda_2 (\lambda_2 + k \lambda_3)}\right] \quad \dots (6)$$

Rearranging the equation (6)

$$u(\lambda_1, \lambda_2, \lambda_3) = \frac{\alpha}{kV} \left[\frac{\lambda_1}{\lambda_2} - \frac{\lambda_1}{\lambda_2 + k \lambda_3}\right] \quad \dots (7)$$

After applying bilaterally the inverse Upadhyaya transform to equation (7), we obtain

$$U^{-1}[u(\lambda_1, \lambda_2, \lambda_3)] = \frac{\alpha}{kV} U^{-1}\left[\frac{\lambda_1}{\lambda_2} - \frac{\lambda_1}{\lambda_2 + k \lambda_3}\right] \quad \dots (8)$$

Using the inverse Upadhyaya transform's linearity characteristic, we may rearrange equation (8) as follows:

$$U^{-1}[u(\lambda_1, \lambda_2, \lambda_3)] = \frac{\alpha}{kV} \left[U^{-1}\left(\frac{\lambda_1}{\lambda_2}\right) - U^{-1}\left(\frac{\lambda_1}{\lambda_2 + k \lambda_3}\right)\right] \quad \dots (9)$$

The equivalents of the terms in equation (9) are listed in table III-I.

$$U^{-1}[u(\lambda_1, \lambda_2, \lambda_3)] = C(t), U^{-1}\left(\frac{\lambda_1}{\lambda_2}\right) = 1,$$

$$U^{-1}\left(\frac{\lambda_1}{\lambda_2 + k\lambda_3}\right) = e^{-kt}.$$

The blood glucose concentration is finally determined by substituting these expressions in equation (9) as follows:

$$C(t) = \frac{\alpha}{kV} [1 - e^{-kt}] \quad \dots (10)$$

**Remark:** The solution to equation (10) corresponds exactly with the Kashuri Fundo Transform (Peker *et al.* [17]) and INEM-Transform approach (Zamil and Kuffi [16]). Unlike both of these transforms (Kashuri Fundo transform [17] and INEM-transform [16]), the U.T. (Upadhyaya transform) approach provides a precise (analytical) solution to the aforementioned problem without requiring considerable computing work.

**Application II: (Pressure in the Aorta)**

The blood is pumped into the aorta by the concentration of the heart. The pressure  $p(t)$  in the aorta satisfies the initial value problem [16, 17]

$$\frac{dp(t)}{dt} + \frac{c}{k} p(t) = c A \sin \omega t, t > 0 ; p(0) = p_0 \dots (11)$$

The pressure in the aorta will be calculated using the Upadhyaya transform method.

Performing the Upadhyaya transform bilaterally to equation (11), we get

$$U\left[\frac{dp(t)}{dt}\right] + \frac{c}{k} U[p(t)] = c A U[\sin \omega t]$$

$$U\left[\frac{dp(t)}{dt}\right] + \frac{c}{k} U[p(t)] = c A \frac{\omega \lambda_1 \lambda_3}{(\lambda_2^2 + \omega^2 \lambda_3^2)} \quad \dots (12)$$

Let  $U[p(t)] = u(\lambda_1, \lambda_2, \lambda_3)$  and utilizing the derivative property of Upadhyaya transform, we obtain

$$\frac{\lambda_2}{\lambda_3} u(\lambda_1, \lambda_2, \lambda_3) - \frac{\lambda_1}{\lambda_3} p(0) + \frac{c}{k} u(\lambda_1, \lambda_2, \lambda_3) = c A \frac{\omega \lambda_1 \lambda_3}{(\lambda_2^2 + \omega^2 \lambda_3^2)} \quad \dots (13)$$

Equation (13) can be expressed as

follows by using the initial condition  $p(0) = p_0$ .

$$\left[\frac{\lambda_2}{\lambda_3} + \frac{c}{k}\right] u(\lambda_1, \lambda_2, \lambda_3) = \frac{\lambda_1}{\lambda_3} p_0 + c A \frac{\omega \lambda_1 \lambda_3}{(\lambda_2^2 + \omega^2 \lambda_3^2)} \quad \dots (14)$$

After simple computation, equation (14) can be written as

$$u(\lambda_1, \lambda_2, \lambda_3) = \frac{\lambda_1}{\left(\lambda_2 + \frac{c}{k} \lambda_3\right)} p_0 + c A \frac{\omega \lambda_1 \lambda_3}{(\lambda_2^2 + \omega^2 \lambda_3^2)} \times \frac{\lambda_3 k}{(\lambda_2 k + c \lambda_3)} \quad \dots (15)$$

Rearranging the equation (15), we get

$$u(\lambda_1, \lambda_2, \lambda_3) = \frac{\lambda_1}{\left(\lambda_2 + \frac{c}{k} \lambda_3\right)} p_0 + \frac{Ac\omega}{\left(\omega^2 + \frac{c^2}{k^2}\right)} \left[ \frac{c\lambda_1\lambda_3}{k(\lambda_2^2 + \omega^2\lambda_3^2)} - \frac{\lambda_1\lambda_2}{(\lambda_2^2 + \omega^2\lambda_3^2)} + \frac{\lambda_1 k}{(\lambda_2 k + c\lambda_3)} \right] \quad \dots (16)$$

After applying bilaterally the inverse Upadhyaya transform to equation (16), we obtain

$$U^{-1}[u(\lambda_1, \lambda_2, \lambda_3)] = p_0 U^{-1}\left[\frac{\lambda_1}{\left(\lambda_2 + \frac{c}{k} \lambda_3\right)}\right] + \frac{Ac\omega}{\left(\omega^2 + \frac{c^2}{k^2}\right)} U^{-1}\left[\frac{c\lambda_1\lambda_3}{k(\lambda_2^2 + \omega^2\lambda_3^2)} - \frac{\lambda_1\lambda_2}{(\lambda_2^2 + \omega^2\lambda_3^2)} + \frac{\lambda_1 k}{(\lambda_2 k + c\lambda_3)}\right] \quad \dots (17)$$

Using the inverse Upadhyaya transform's linearity characteristic, we may rearrange equation (17) as follows

$$U^{-1}[u(\lambda_1, \lambda_2, \lambda_3)] = p_0 U^{-1} \left[ \frac{\lambda_1}{\left( \lambda_2 + \frac{c}{k} \lambda_3 \right)} \right] + \frac{Ac\omega}{\left( \omega^2 + \frac{c^2}{k^2} \right)} \left[ U^{-1} \left( \frac{c\lambda_1\lambda_3}{k(\lambda_2^2 + \omega^2\lambda_3^2)} \right) - U^{-1} \left( \frac{\lambda_1\lambda_2}{(\lambda_2^2 + \omega^2\lambda_3^2)} \right) + U^{-1} \left( \frac{\lambda_1}{\left( \lambda_2 + \frac{c\lambda_3}{k} \right)} \right) \right] \dots (18)$$

The equivalents of the terms in equation (18) are listed in III-I.

$$U^{-1}[u(\lambda_1, \lambda_2, \lambda_3)] = p(t), U^{-1} \left( \frac{\lambda_1}{\lambda_2 + k\lambda_3} \right) = e^{-kt}$$

$$U^{-1} \left( \frac{\omega\lambda_1\lambda_3}{(\lambda_2^2 + \omega^2\lambda_3^2)} \right) = \sin \omega t$$

$$U^{-1} \left( \frac{\lambda_1\lambda_2}{(\lambda_2^2 + \omega^2\lambda_3^2)} \right) = \cos \omega t ,$$

$$U^{-1} \left( \frac{\lambda_1}{\left( \lambda_2 + \frac{c\lambda_3}{k} \right)} \right) = e^{-\frac{c}{k}t} .$$

Finally, when we plug these formulas into Equation (18), we get the pressure in the aorta as

$$p(t) = p_0 e^{-\frac{c}{k}t} + \frac{Ac\omega}{\left( \omega^2 + \frac{c^2}{k^2} \right)} \left[ \frac{c}{\omega k} \sin \omega t - \cos \omega t + e^{-\frac{c}{k}t} \right] \dots (19)$$

**Remark:** In contrast to the Kashuri Fundo Transform (Peker et al. [17]) and INEM-Transform approach (Zamil and Kuffi [16]), the U.T. (Upadhyaya Transform) approach provides a precise (analytical) solution to the aforementioned problem without the need for extensive computational work.

## V. CONCLUSION

In this paper, the exact solution for these models of importance in the medical and epistemological fields was found through the above-mentioned integral transform, noting that the transform parameters are nothing but experimental medical parameters taken by the advanced medical staff in the hospital after verification through biological laboratories. As mathematicians, we take the model ready. This integral transformation gave precise accuracy in the solution and precise scalability to these models.

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