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Some Types of Mappings in Bitopological Spaces

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Abstract:

This work, introduces some concepts in bitopological spaces, which are *nm-j-ω*-converges to a subset, *nm-j-ω*-directed toward a set, *nm-j-ω*-closed mappings, *nm-j-ω*-rigid set, and *nm-j-ω*-continuous mappings. The mainline idea in this paper is *nm-j-ω*-perfect mappings in bitopological spaces such that n = 1,2 and m =1,2 $n \neq m$. Characterizations concerning these concepts and several theorems are studied, where j = θ, δ, α, *pre*, *b*, β.

Key words: Filter base, *nm-j-ω*-converges, *nm-j-ω*-closed mappings, *j-ω*-rigid a set, *nm-j-ω*-perfect mappings.

Introduction and Preliminaries:

In 1963 Kelly J. C. (1) introduced the definition, a set G with two topologies σ_1 and σ_2 is said to be bitopological space and denoted by (G, σ_1, σ_2) and a subset $K \subseteq G$. The closure and interior of K in (G, σ_n) is denoted by σ_n–cl(K) and σ_n–int(K), where n = 1, 2. A topological space (G, σ) and a point g in G is said to be condensation point of $K \subseteq G$ if every open neighborhood S in σ with $g \in S$, the set $K \cap S$ is uncountable (2). In 1982 the ω-closed set was first exhibited by H. Z. Hdeib in (3) defined it as a subset $K \subseteq G$ is called ω-closed if it incorporates each its condensation points, and the ω-open set is the complement of the ω-closed set and the ω-closed of the set $K \subseteq G$ denoted by $\mathrm{cl}^{\omega}(K)$. The ω-interior of the set $K \subseteq G$ is defined as the union of all ω-open sets content in K and is denoted by $\mathrm{int}^{\omega}(K)$. In (4) a point $g \in G$ is said to θ-cluster points of $K \subseteq G$ if $\mathrm{cl}(S) \cap K \neq \varphi$ for each open set S of G contained g.Al so in (4) the set of each θ-cluster points of K is called the θ-closure of K and is denoted by $\mathrm{cl}_{\theta}(K)$. A subset $K \subseteq G$ is called θ-closed (4) if $K = \mathrm{cl}_{\theta}(K)$. The complement of θ-closed set is said to be θ-open. A point $g \in G$ is said to θ-ω-cluster points of $K \subseteq G$ if $\mathrm{cl}^{\omega}(S) \cap K \neq \varphi$ for each ω-open set S of G containing g. The set of each θ-ω-cluster points of K is called the θ-ω-closure of K and is denoted by $\mathrm{cl}_{\theta}^{\omega}(K)$. A subset $K \subseteq G$ is called θ-ω-closed (4) if $K = \mathrm{cl}_{\theta}^{\omega}(K)$. The complement of θ-ω-closed set is said to be θ-ω-open. A subset $K \subseteq G$ is said to be δ-closed (5) if $K = \mathrm{cl}_{\delta}(K) = \{g \in G : \mathrm{int}(\mathrm{cl}(S)) \cap K \neq \varphi, S \in \tau \text{ and } g \in S\}$. The complement of δ-closed is called δ-open set, and K is δ-ω-closed if $K = \mathrm{cl}_{\delta}^{\omega}(K) = \{g \in G : \mathrm{int}^{\omega}(\mathrm{cl}(S)) \cap K \neq \varphi, S \in \tau \text{ and } g \in S\}$. For other notions or notations not defined here, R. Englking (6) should be followed closely. Several characterizations of ω-closed sets were provided in (4, 5, 8, 9, and 10). Some of the results in (11), (12), (13), (14) and (15) will be bult.

Definition 1. (1) A nonempty family $\Im$ of nonempty subsets of G is called filter base if $M_1, M_2 \in \Im$ then $M_3 \subseteq M_1 \cap M_2$ for some $M_3 \in \Im$.

The filter generated by a filter base $\Im$ consists of all supersets of elements of $\Im$. An open filter base on a space G is a filter base with open members.

The set $\aleph_g$ of all neighborhoods (nbds) of $g \in G$ is a filter on G, and any nbd base at g is a filter base for $\aleph_g$. This filter called the nbd filter at g.

Definition 2. (1) Let $\Im$ and $\wp$ be filter bases on G. Then $\wp$ is called finer than $\Im$ (written as $\Im < \wp$) if for all $M \in \Im$, there is $\mathcal{G} \in \wp$, $\mathcal{G} \subseteq M$ also, that $\Im$ meets $\mathcal{G}$ if $M \cap \mathcal{G} \neq \phi$ for all $M \in \Im$ also, $\mathcal{G} \in \wp$. Notice, $\Im \to g$ iff $\aleph_g < \Im$.

Definition 3. (7) A subset K of a space G is called:

(a) α-ω-open if $K \subseteq \text{int}^{\omega}(\text{cl}(\text{int}^{\omega}(K)))$.
(b) *pre*-ω-open if $K \subseteq \text{int}^{\omega}(\text{cl}(K))$.
(c) *b*-ω-open if $K \subseteq \text{cl}(\text{int}^{\omega}(K)) \cup \text{int}^{\omega}(\text{cl}(K))$.
(d) β-ω-open if $K \subseteq \text{cl}(\text{int}^{\omega}(\text{cl}(K)))$.
The complement of an α-ω-open (resp., *pre*-ω-open, *b*-ω-open, β-ω-open) is called (resp. α-ω-closed (resp., *pre*-ω-closed, *b*-ω-closed, β-ω-closed).
The *j*-ω-closure of $K \subseteq G$ is denoted by $\text{cl}_j^{\omega}(K)$ and defined by $\text{cl}_j^{\omega}(K) = \cap\{M \subseteq G;\ M$ is *j*-ω-closed and $K \subseteq M\}$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Filter Bases and *nm-j-ω*-Perfect Mappings in Bitopological Spaces

This section, defines filter bases and *nm-j-ω*-converges to a subset, *nm-j-ω*-directed toward a set, *nm-j-ω*-closed mapping, *j-ω*-continuous mappings, *j-ω*-rigid a set, and used to obtain characterization theorem for an *nm-j-ω*-perfect mappings in bitopological spaces.

Definition 4. A point *g* in bitopological space (G, σ_1, σ_2) is said to be *nm-j-ω*-condensation point of a subset K of G iff for any σ_n–open nbd S of g, $(\text{cl}_j^{\omega}(S)) \cap K \neq \varphi$. The set of all *nm-j-ω*-condensation point of K is called *nm-j-ω*-closure of K and means by $nm\ \omega\text{-}\text{cl}_j^{\omega}(K)$. A set $K \subseteq G$ is said to be *nm-j-ω*-closed if $K = nm\text{-}\omega\text{-}\text{cl}_j^{\omega}(K)$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Definition 5. A point *g* in a bitopological space (G, σ_1, σ_2) is said to be *nm-j-ω*-condensation point of a filter base $\Im$ on K if it is an *nm-j-ω*-condensation point of every number of $\Im$. The set of all *nm-j-ω*-condensation point of $\Im$ is called *nm-j-ω*-condensed of $\Im$ and means by $nm\text{-}j\text{-}\omega\text{-}cod\Im$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Definition 6. A filter base $\Im$ on a bitopological space (G, σ_1, σ_2) is called *nm-j-ω*-converges to a subset $K \subseteq G$ (written as $\Im nm\text{-}j\text{-}\omega \rightarrow K$) if for each σ_n–open cover $\mathcal{K}$ of K, yond is a finite subfamily $\mathcal{L} \subseteq \mathcal{K}$ and $M \in \Im$ such that $M \subseteq \cup\{\sigma_n\text{–}\text{cl}_j^{\omega}(L) : L \in \mathcal{L}\}$. $\Im$ *nm-j-ω*-converges to a point $g \in G$ (written as $\Im nm\text{-}j\text{-}\omega \rightarrow g$) iff $\Im nm\text{-}_j\text{-}\omega \rightarrow \{g\}$, or equivalently, $\sigma_n\text{-}\text{cl}_j^{\omega}(S)$ of every σ_m–open nbd S of g contains some member of $\Im$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 1. In a bitopological space (G, σ_1, σ_2) a point *g* is an *nm-j-ω*-condensation of a filter base $\Im$ on G if there subsistent a filter base $\Im^*$ finer than $\Im$ such that $\Im^* nm\text{-}_j\text{-}\omega \rightarrow g$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Proof: ($\Rightarrow$) Let *g* be an *nm-j-ω*-condensation point of a filter base $\Im$ on G, then every σ_n–open nbd S of g, the *j-ω*-closure of S contains a member of $\Im$ and thus contains a member of any filter base $\Im^*$ minutes than $\Im$, so that $\Im^*\ nm\text{-}_j\text{-}\omega \rightarrow g$.
($\Leftarrow$) Assume that *g* is not an *nm-j-ω*-condensation point of a filter base $\Im$ on G, then there subsistent an σ_n–open nbd S of g, such that *j-ω*-closure of S contains no member of $\Im$, denote by $\Im^*$ the family of sets $M^* = M \cap (G - (\text{cl}_j^{\omega}(S))$ for $M \in \Im$, then the sets M^* are nonempty. And $\Im^*$ is a filter base and indeed it is minute than $\Im$, since $M_1^* = M_1 \cap (G - \text{cl}_j^{\omega}(S))$ and $M_2^* = M_2 \cap (G - \text{cl}_j^{\omega}(S))$, so there is an $M_3 \subseteq M_1 \cap M_2$ and this lead to:
$M_3^* = M_3 \cap (G - (\text{cl}_j^{\omega}(S)) \subseteq M_1 \cap M_2 \cap (G - (\text{cl}_j^{\omega}(S))$
$= M_1 \cap (G - (\text{cl}_j^{\omega}(S)) \cap M_2 \cap (G - (\text{cl}_j^{\omega}(S))$.
By construction $\Im^*$ not *nm-j-ω*-convergent to *g*.
This contradiction, and thus *g* is an *nm-j-ω*-condensation point of a filter base $\Im$ on G.

Definition 7. A filter base $\Im$ on a bitopological space (G, σ_1, σ_2) is said to be *nm-j-ω*- directed toward to a set $K \subseteq G$ (written as $\Im nm\text{-}_{j\text{-}\omega}\text{-}dir\text{-}tow \rightarrow K$) if for each filter base $\wp$ finer $\Im$ has an *nm-j-ω*-condensation point in *K*. i.e $(nm\text{-}\ j\text{-}\omega\text{-}cod\,\wp) \cap K \neq \varphi$. $\Im nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \rightarrow g$ used to mean $\Im nm\text{-}_{j\text{-}\omega}\text{-}dir\text{-}tow \rightarrow \{g\}$, where $g \in G$, and $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 2. Let $\Im$ be a filter base on a bitopological space (G, σ_1, σ_2) and point $g \in G$, then $\Im\ nm\text{-}_j\text{-}\omega \rightarrow g$ if and only if $\Im nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \rightarrow g$, where $j = \theta, \delta, \alpha, pre, b, \beta$.
Proof: ($\Rightarrow$) Clear.
($\Leftarrow$) Assume that $\Im$ is not an *nm-j-ω*-converge to *g*, there exists an σ_n–open nbd S of g, such that $M \not\subset \text{cl}_j^{\omega}(S)$, for all $M \in \Im$. Then $\wp = \{(M \cap (G - (\sigma_n - \text{cl}_j^{\omega}(S)) : M \in \Im\}$ is a filter base on G finer than $\Im$, and conspicuously $g \notin nm\text{-}j\text{-}\omega\text{-}cod\,\wp$. So $\Im$ cannot be *nm- j-ω*- directed towards *g*.

Definition 8. A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is said to be *nm-j-ω*-perfect if for every filter base $\Im$ on $\lambda(G)$, *nm-j-ω*-directed towards some subset L of $\lambda(G)$, the filter base $\lambda^{-1}(\Im)$ is *nm-j-ω*-directed towards $\lambda^{-1}(L)$ in G, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 4. Let $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be a mapping. Then the following are equivalent:
(a) λ is *nm-j-ω*-perfect.
(b) For every filter base $\Im$ on $\lambda(G)$, which is *nm- j-ω*-convergent to a point *h* in *H*, $\lambda^{-1}(\Im) nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to \lambda^{-1}(h)$.
(c) For any filter base $\Im$ on *G*, $nm\text{-}j\text{-}\omega\text{-}cod\ \lambda(\Im) \subset \lambda(nm\text{-}j\text{-}\omega\text{-}cod\ \Im)$, where $j = \theta, \delta, \alpha, pre, b, \beta$.
Proof: (a) ⇒ (b) Proof by Theorem (2).
(b) ⇒ (c) Let $h \in nm\text{-}j\text{-}\omega\text{-}cod\ \lambda(\Im)$. By Theorem (1), there is a filter base $\wp$ in $\lambda(G)$ finer than $\lambda(\Im)$, $\wp\ nm\text{-}_j\text{-}\omega \to h$. Let $\upsilon = \{\lambda^{-1}(\mathcal{G}) \cap M : \mathcal{G} \in \wp$ and $M \in \Im\}$. Then υ is a filter base on *G* finer than $\lambda^{-1}(\wp)$. Since $\wp\ nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to h$, and by Theorem (2) and λ is *nm- j-ω*-perfect, $\lambda^{-1}(\wp)\ nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to \lambda^{-1}(h)$. υ Being finer than $\lambda^{-1}(\wp)$, then $\lambda^{-1}(h) \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \upsilon) \neq \phi$. It is then clear that $\lambda^{-1}(h) \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \Im) \neq \phi$. Then, $h \in \lambda(nm\text{-}j\text{-}\omega\text{-}cod\ \Im)$.
(c) ⇒ (a) Suppose $\Im$ be a filter base on $\lambda(G)$, it is *nm-j-ω*-directed towards some subset *L* of $\lambda(G)$. Let $\wp$ be a filter base on *G* finer than $\lambda^{-1}(\Im)$. Hence, $\lambda(\wp)$ is a filter base on $\lambda(G)$ finer than $\Im$ and so $L \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \lambda(\wp)) \neq \phi$. Then by (c) $L \cap \lambda(nm\text{-}j\text{-}\omega\text{-}cod\ \wp)) \neq \phi$, so that $\lambda^{-1}(L) \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \wp) \neq \phi$. Then, $\lambda^{-1}(\Im)$ is *nm-j-ω*-directed towards $\lambda^{-1}(L)$. Thus, λ is *nm-j-ω*-perfect.

Definition 9. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be *nm-j-ω*-closed if the image of every *nm-j-ω*-closed set in *G* is *nm-j-ω*-closed in *H*, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 5. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-closed if $nm\text{-}\omega\text{-}\,\mathrm{cl}_j^{\omega}\ \lambda(K) \subset \lambda(nm\text{-}\omega\text{-}\,\mathrm{cl}_j^{\omega}(K))$, for *n, m*= 1and 2 such that ($n \neq m$), and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.
Proof: Straightforward.

Theorem 6. The *nm-j-ω*-perfect mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-closed, where $j = \theta, \delta, \alpha, pre, b, \beta$.
Proof: Follow from Theorem (5) and Theorem (3) (a) ⇒ (c) taking $\Im = \{K\}$.

Definition 10. A subset *K* of bitopological space (G, σ_1, σ_2) is said to be *nm*-Supra-*ω*-rigid (written as *nm-j-ω*-rigid) in *G* if for every filter base $\Im$ on *G* with $(nm\text{-}j\text{-}\omega\text{-}cod\ \Im) \cap K = \phi$, there is $S \in \sigma_n$ and $M \in \Im$, such that $K \subset S$ and $\mathrm{cl}_j^{\omega}(S) \cap M = \phi$. or equivalent, if for every filter base $\Im$ on *G* whenever, $K \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \Im) = \phi$, then for some $M \in \Im$, $K \cap (nm\text{-}\omega\text{-}\,\mathrm{cl}_j^{\omega}(M)) = \phi$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 7. If a mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-closed such that for every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in *G*, then λ is *nm- j-ω*-perfect, where $j = \theta, \delta, \alpha, pre, b, \beta$.
Proof: Assume that $\Im$ be a filter base on $\lambda(G)$ such that $\Im\ nm\text{-}_j\text{-}\omega \to h$ in *H*, for some $h \in H$. If $\wp$ is a filter base on *G* finer than the filter base on $\lambda^{-1}(\Im)$. Thus $\lambda(\wp)$ is a filter base *H*, finer than $\Im$. Since $\Im nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to g$, by Theorem (1), $h \in nm\text{-}j\text{-}\omega\text{-}cod\lambda(\wp)$, i.e., $h \in \cap\{nm\text{-}\omega\text{-}\,\mathrm{cl}_j^{\omega}\lambda(\mathcal{G}) : \mathcal{G} \in \wp\}$ and $h \in \cap\{\lambda(nm\text{-}\omega\text{-}\,\mathrm{cl}_j^{\omega}(\mathcal{G})) : \mathcal{G} \in \wp\}$ by Theorem (5), since λ is *nm- j-ω*-closed. Then $\lambda^{-1}(h) \cap nm\text{-}\omega\text{-}\,\mathrm{cl}_j^{\omega}(\mathcal{G}) \neq \phi$, for all $\mathcal{G} \in \wp$. Hence for all $S \in \sigma_n$ with $\lambda^{-1}(h) \subset S$, $\mathrm{cl}_j^{\omega}(S) \cap \mathcal{G} \neq \phi$, for all $\mathcal{G} \in \wp$. Since $\lambda^{-1}(h)$ is *nm- j-ω*-rigid, it then that $\lambda^{-1}(h) \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \wp) \neq \phi$. Then $\lambda^{-1}(\Im) nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to \lambda^{-1}(h)$, and by Theorem (4 (b) ⇒ (a)). Thus λ is *nm-j-ω*-perfect.

Definition 11. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be *nm*-Supra-*ω*-continuous (written as *nm-j-ω*-continuous) if for any ς_n-open nbd *T* of $\lambda(g)$, there exists a σ_n- open nbd *S* of *g*, $\lambda(\mathrm{cl}_j^{\omega}(S)) \subset \varsigma_{m}\text{-}\,\mathrm{cl}_j^{\omega}(T)$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Definition 12. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be weakly *nm- j-ω*-continuous if for any ς_n-open nbd *T* of $\lambda(g)$, there exists a σ_n-open nbd *S* of *g* such that $\lambda(S) \subset \varsigma_{m}\text{-}\,\mathrm{cl}_j^{\omega}(T)$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Definition 13. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be strongly *nm-j-ω*-continuous if for any ς_n-open nbd *T* of $\lambda(g)$, there exists a σ_n- open nbd *S* of *g*, $\lambda(\mathrm{cl}_j^{\omega}(S)) \subset T$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Definition 14. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be super *nm-j-ω*-continuous if for any ς_n-open nbd *T* of $\lambda(g)$, there exists a σ_n-open nbd *S* of *g*, $\lambda(\mathrm{int}_j^{\omega}(\mathrm{cl}_j^{\omega}(S)) \subset \varsigma_{m}\text{-}\,\mathrm{cl}_j^{\omega}(T)$, for *n, m* = 1and 2 such that ($n \neq m$), where $j = \theta, \delta, \alpha, pre, b, \beta$.

Definition 15. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be almost *nm-j-ω*-continuous if for any ς_n-open nbd *T* of $\lambda(g)$, there exists a σ_n-open nbd *S* of *g*, $\lambda(S) \subset (\varsigma_{m}\text{-}\mathrm{int}_j^{\omega}(\mathrm{cl}_j^{\omega}(T))$, for *n, m* =

1and 2 such that ($n \neq m$), where $j = \theta, \delta, \alpha, pre, b, \beta$.

The relation between weakly and strongly *nm-j-ω*-continuous mappings are given by the following

Strongly *nm- j-ω*-continuous mapping	$\Rightarrow$	*nm- j-ω*-continuous mapping	$\Rightarrow$	Weakly *nm-j-ω*-continuous mapping
$\Downarrow$		$\Downarrow$		
Super *nm- j-ω*-continuous mapping		almost *nm- j-ω*- continuous mapping		

Figure 1. The relation between weakly and strongly *nm-j-ω*-continuous mappings, where $j = \theta, \delta, \alpha, pre, b, \beta$.

In the higher figure the converses not be true such that the demonstrated by the following examples:

Example 1. Let A be the upper half of the plane and B be the x-axis. Let $G = A \cup B$. If τ_{hdis} be the half disc topology on G and τ_r be the relative topology that G inherits by virtue of being a subspace of $\Re^2$. The identity mapping $\lambda : (G, \tau_r) \rightarrow (G, \tau_{hdis})$. Then, λ is weakly *nm- j-ω*-continuous mapping but it is not *nm- j-ω*-continuous mapping.

Example 2. Let $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (G, \varsigma_1, \varsigma_2)$ be a mapping such that $G = \{u, v, w\}$, and $\sigma_1 = \{G, \varphi\}$, $\sigma_2 = \{G, \varphi, \{u, v\}\}$ and $\varsigma_1 = \{G, \varphi\}$, $\varsigma_2 = \{G, \varphi, \{w\}\}$. Such that $\lambda(u) = \lambda(v) = \lambda(w) = u$. Then λ is almost *nm-j-ω*-continuous mapping but it is not *nm- j-ω*-continuous mapping.

Example 3. Let $\lambda : (\mathscr{R}, \tau) \rightarrow (\mathscr{R}, \tau)$ be a mapping. Define by $\lambda(g) = g$, and let $(\mathscr{R}, \tau)$ where τ is the topology with basis whose members are of the form (a, b) and $(a, b) - N$ such that $N = \{1/n;\ n \in Z^+\}$. Then $(\mathscr{R}, \tau)$ is Hausdorff but is not *ω*-regular. Then λ is *nm-j-ω*-continuous mapping but is not strongly *nm- j-ω*-continuous mapping.

Example 4. Let $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (G, \sigma_1, \sigma_2)$ be identity mapping, such that $G = \{u, v, w\}$ and $\sigma_1 = \{G, \varphi, \{u, v\}\}$, $\sigma_2 = \{\varphi, G, \{u\}, \{v\}, \{u, v\}\}$. Then λ is super *nm- j-ω*-continuous mapping but it is not strongly *nm- j-ω*-continuous mapping.

Theorem 8. If an *nm- j-ω*-continuous mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is *nm- j-ω*- perfect , then:
(a) λ is *nm-j-ω*- closed.
(b) For every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in G, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Proof: (a) By Theorem (6) λ an *nm- j-ω-* perfect mapping is *nm- j-ω*- closed.
(b) To prove $\lambda^{-1}(h)$ is *nm- j-ω*-rigid, let $h \in H$, and assume that $\Im$ be a filter base on G such that $(nm\text{-}j\text{-}\omega\text{-}cod\ \Im) \cap \lambda^{-1}(h) = \phi$. Then $h \notin \lambda\ (nm\text{-}j\text{-}\omega\text{-}cod\ \Im)$, since λ is *nm- j-ω*-perfect, by Theorem (3 (a) $\Rightarrow$ (c)). Then, $h \notin (nm\text{-}j\text{-}\omega\text{-}cod\ \lambda(\Im))$, so there exists an $M \in \Im$ such that $h \notin nm\text{-}\omega\text{-}cl_j^{\omega}\lambda(M)$, yond exists an ς_n-open nbd T of h also, $\varsigma_{m}\text{-}cl_j^{\omega}(T) \cap \lambda(M) = \phi$, since λ is *nm- j-ω*-continuous, for every $g \in \lambda^{-1}(h)$, then σ_n-open nbd S_g of g such that $\lambda(cl_j^{\omega}(S_g)) \subset \varsigma_{m}\text{-}cl_j^{\omega}(T) \subset H\text{-}\lambda(M)$. Then $\lambda(cl_j^{\omega}(S_g)) \cap \lambda(M) = \phi$, so that $cl_j^{\omega}(S_g) \cap M = \phi$, then $g \notin nm\text{-}\omega\text{-}cl_j^{\omega}(M)$, for every $g \in \lambda^{-1}(h)$, then $\lambda^{-1}(h) \cap (nm\text{-}\omega\text{-}cl_j^{\omega}(M)) = \phi$, so $\lambda^{-1}(h)$ is *nm- j-ω*-rigid in G, where $j = \theta, \delta, \alpha, pre, b$.

Corollary 1. An *nm-j-ω*-continuous mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-perfect if λ is *nm- j-ω*- closed and for every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in G, where $j = \theta, \delta, \alpha, pre, b, \beta$.

The results show that thereupon the higher theorem remainders aright if *nm- j-ω*-closeness of λ is replaced by a stringently enfeeble condition which will be called as a weak *nm- j-ω*-closeness and strong *nm- j-ω*-closeness of λ. Thus, these will be predefined as follows:

Definition 16. A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is called weakly *nm- j-ω*-closed if for every $h \in \lambda(G)$, and each σ_n-open set S containing $\lambda^{-1}(h)$ in G, there exists a ς_m-open nbd T of h, $\lambda^{-1}(\varsigma_{m}\text{-}cl_j^{\omega}(T)) \subset cl_j^{\omega}(S)$, for n, m = 1and 2 such that ($n \neq m$), where $j = \theta, \delta, \alpha, pre, b, \beta$.

Definition 17. A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is said to be strongly *nm- j-ω*-closed if for each $h \in \lambda(G)$, and each σ_n-open set S containing $\lambda^{-1}(h)$ in G, there exists a ς_m-open nbd T of h, $\lambda^{-1}(\varsigma_{m}\text{-}cl_j^{\omega}(T)) \subset (S)$, for n, m = 1and 2 such that ($n \neq m$), where $j = \theta, \delta, \alpha, pre, b, \beta$.

The relation between weakly and strongly *nm-j-ω*-closed mappings are given by the following figure:

Strongly *nm- j-ω*-closed $\Rightarrow$ *nm- j-ω*-closed $\Rightarrow$ weakly *nm- j-ω*-closed

Figure 2. The relation between weakly and strongly *nm-j-ω*-continuous mappings, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 9. An *nm-j-ω*-closed mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is weakly *nm- j-ω*-closed, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Proof: Assume that $h \in \lambda(G)$ also, let S be a σ_n-open set containing $\lambda^{-1}(h)$ in G , by Theorem (5) and λ is *nm- j-ω*-closed mapping, then $nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(G - \mathrm{cl}_j^{\omega}(S)) \subset \lambda[(\sigma_n - \mathrm{cl}_j^{\omega}(G - \mathrm{cl}_j^{\omega}(S))]$. Since $h \notin \lambda[(\sigma_n - \mathrm{cl}_j^{\omega}(G - \mathrm{cl}_j^{\omega}(S))]$, and $h \notin nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(G - \mathrm{cl}_j^{\omega}(S))$. Thus, there exists an ς_n-open nbd T of h in H, $\varsigma_n\text{-}\mathrm{cl}_j^{\omega}(T) \cap \lambda(G - \mathrm{cl}_j^{\omega}(S)) = \phi$, then $\lambda^{-1}(\varsigma_m\text{-}\mathrm{cl}_j^{\omega}(T)) \cap \lambda(G\text{-} \mathrm{cl}_j^{\omega}(S)) = \phi$, i.e $\lambda^{-1}(\varsigma_m\text{-}\mathrm{cl}_j^{\omega}(T)) \subset \mathrm{cl}_j^{\omega}(S)$, then λ is weakly *nm- j-ω*-closed.

The inversion of the Theorem (9) is not be right, it will be shown by next example:

Example 5. Let $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be a constant mapping and σ_1, σ_2 and ς_1, ς_2 be any topology, then λ is weakly *nm- j-ω*-closed for n, m= 1and 2 such that ($n \neq m$), let $G = H = \Re$. If ς_1 or ς_2 is discrete topology on H, then $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ given by $\lambda(g) = 0$, for every $g \in G$, is neither *12- j-ω*-closed nor *21- j-ω*-closed, regardless of the topologies σ_1 , σ_2 also, ς_2 (or ς_1), where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 10. An strongly *nm-j-ω*-closed mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm- j-ω*-closed, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 11. If an *nm-j-ω*-continuous mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-perfect, then:
(a) λ is strongly *nm-j-ω*-closed.
(b) for every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in G, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 12. Let $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be *nm-j-ω*-continuous mapping. Then λ is *nm-j-ω*-perfect, if:
(a) λ is weakly *nm- j-ω*- closed.
(b) for every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in G, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Proof: Assume that λ is *nm-j-ω*-continuous mapping then satisfying the condition for (a) and (b). To show that λ is *nm-j-ω*-perfect, Theorem (7) show that λ is *nm-j-ω*-closed, let $h \in nm\text{-}j\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for some non- null subset K of G. However $h \notin \lambda(nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K))$, so $\mathcal{L} = \{K\}$ is a filter base on G, also $(nm\text{-}j\text{-}\omega\text{-}cod\ \mathcal{L}) \cap \lambda^{-1}(h) = \phi$, by *nm-j-ω*-rigidity of $\lambda^{-1}(h)$. There is σ_n-open set S containing $\lambda^{-1}(h)$ such that $\mathrm{cl}_j^{\omega}(S) \cap K = \phi$, and by a mapping λ is weakly *nm- j-ω*-closed, there exists an ς_n-open nbd T of h, such that $\lambda^{-1}(\varsigma_m\text{-}\mathrm{cl}_j^{\omega}(T)) \subset \mathrm{cl}_j^{\omega}(S)$. Then $\lambda^{-1}(\varsigma_m\text{-}\mathrm{cl}_j^{\omega}(T)) \cap K = \phi$, i.e $(\varsigma_m\text{-}\mathrm{cl}_j^{\omega}(T)) \cap \lambda(K) = \phi$, this is impossible because that $h \in nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$. So $h \in \lambda(nm\text{-}j\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K))$. Then λ is *nm-j-ω*-closed.

Study on some Types of *j*-ω-perfect Mappings in Bitopological Spaces.

In this section, *nm-j-ω*-perfect mappings are given and used the definitions of characterizations theorems for an *nm-j-ω*-continuous mapping and weakly *nm-j-ω*-continuous mapping and strongly *nm-j-ω*-continuous mapping and super *nm-j-ω*-continuous mapping and almost *nm-j-ω*-continuous mapping are indicated to this end, and $n, m = 1, 2$ where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 13. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-continuous if $\lambda(nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)) \subset nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for n, m= 1and 2 such that ($n \neq m$), and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Proof: ($\Rightarrow$) Assume that $h \in nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)$ and T is ς_n- open nbd of $\lambda(g)$. Because of λ is *nm-j-ω*-continuous, there exists a σ_n-open nbd S of g such that $\lambda(\mathrm{cl}_j^{\omega}(S)) \subset \varsigma_m\text{-}\mathrm{cl}_j^{\omega}(T)$. Since, $\mathrm{cl}_j^{\omega}(S) \cap K \neq \phi$, then $\varsigma_m\text{-}\mathrm{cl}_j^{\omega}(T) \cap \lambda(K) \neq \phi$. Thus, $\lambda(g) \in nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$. This shows that $\lambda(nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)) \subset nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$ for n, m = 1and 2 such that ($n \neq m$)
($\Leftarrow$) Clear.

Theorem 14. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is weakly *nm-j-ω*-continuous if $\lambda(nm\text{-}\omega\text{-}(K)) \subset nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for n, m = 1and 2 such that ($n \neq m$), and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 15. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is strongly *nm-j-ω*- continuous if $\lambda(nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)) \subset nm\text{-}\omega\text{-}\lambda(K)$, for n, m = 1and 2 such that ($n \neq m$), and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 16. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is super *nm-j-ω*-continuous if $\lambda(nm\text{-}\omega\text{-}\mathrm{int}\text{-}\mathrm{cl}_j^{\omega}(K)) \subset nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for n, m = 1and 2 such that ($n \neq m$), for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 17. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is almost *nm-ω*-continuous if $\lambda(nm\text{-}\omega\text{-}(K)) \subset nm\text{-}\omega\text{-}\mathrm{int}\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for n, m = 1and 2 such that ($n \neq m$), and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Theorem 18. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be *nm-j-ω*-continuous and *nm- j-ω*-perfect, Then λ^{-1} preserves *nm-j-ω*-rigidity, where $j = \theta, \delta, \alpha$, *pre, b*, β.

Proof: Assume that L be an *nm- j-ω*-rigid set in H and suppose $\Im$ be a filter base on G, then $\lambda^{-1}(L) \cap$ (*nm- j-ω-cod* $\Im$) = ϕ, since λ is *nm- j-ω*-perfect and $L \cap \lambda($ *nm- j-ω-cod* $\Im)$ = ϕ. By Theorem (3 (a) ⇒ (c)) then $L \cap$ (*nm- j-ω-cod*$\lambda(\Im)$) = ϕ, now L being an *nm- j-ω*-rigid set in H , there exists an $M \in \Im$ such that $L \cap$ (*nm-ω*-$\mathrm{cl}_j^{\omega}\lambda(M)$) = ϕ, since λ is *nm- j-ω*-continuous, by Theorem (14) it follows that $L \cap \lambda$ (*nm-ω*-$\mathrm{cl}_j^{\omega}(M)$) = ϕ. Then $\lambda^{-1}(L) \cap$ (*nm-ω*-$\mathrm{cl}_j^{\omega}(M)$) = ϕ. This proves that $\lambda^{-1}(L)$ is *nm- j-ω*-rigid.

Definition 18. A subset K of a bitopological space (G, σ_1, σ_2) is said to be *nm-j-ω*-set in G if for every σ_n-open cover $\mathcal{K}$ of K, there is a finite sub collection $\mathcal{L}$ of $\mathcal{K}$ such that $K \subset \cup$ { $\mathrm{cl}_j^{\omega}(S)$: $L \in \mathcal{L}$}, where $j = \theta, \delta, \alpha$, *pre, b*, β.

Theorem 19. Let (G, σ_1, σ_2) be a bitopological space, and a subset K of space for every filter base $\Im$ on K such that (*nm-j-ω-cod* $\Im$) ∩ $K \neq$ ϕ, is an *nm-j-ω*-set, where $j = \theta, \delta, \alpha$, *pre, b*, β.

Proof: Let $\mathcal{K}$ be an σ_n-open cover of K, σ_m- *j-ω*-closed of union of any finite subcollection of $\mathcal{K}$ is not cover K. So $\Im$ = { K / cl_j^{ω} $_g(\cup_{\mathcal{L}}(S_{\mathcal{L}}))$: $\mathcal{L}$ is finite subcollection of $\mathcal{K}$} is a filter base on K and (*nm-j-ω-cod* $\Im$) ∩ K = ϕ, this contradiction yield that K is an *nm- j-ω*-set.

Theorem 20. If $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-perfect, and $L \subset H$ is *nm- j-ω*-set in H, then $\lambda^{-1}(L)$ is an *nm- j-ω*-set in G, for n, m = 1and 2 such that $(n \neq m)$, and where $j = \theta, \delta, \alpha$, *pre, b*, β.

Proof: Assume that $\Im$ be a filter base on $\lambda^{-1}(L)$, then $\lambda(\Im)$ is a filter base on L. Because L is an *nm- j-ω*-set in H, such that $L \cap$ *nm- j-ω-cod* $\lambda(\Im) \neq$ ϕ, by Theorem (12). By Theorem (3 (a) ⇒ (c)), $L \cap \lambda$(*nm- j-ω-cod* $\Im$) ≠ ϕ, so $\lambda^{-1}(L) \cap$ *nm- j-ω-cod* ($\Im$) ≠ ϕ. Therefore by Theorem (12), $\lambda^{-1}(L)$ is an *nm- j-ω*-set in G.

The inversion of the Theorem (20) is not right, as shown by the example following:

Example 6. Let $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be an identity mapping and σ_1, σ_2 be the cofinite and discrete topologies respectively on G, and ς_1 , ς_2 respectively denote the indiscrete and usual topologies on H such that $G = H = \Re$,then every subset of either of (G, σ_1, σ_2) and $(H, \varsigma_1, \varsigma_2)$ is a *12-j-ω*-set. Now , any nonvoid finite set $K \subset G$ is *12- j-ω*-closed in G, but $\lambda(K)$ (i.e K) is not *12-j-ω*-closed in H, (in fact, the only *12- j-ω*-closed subset of H are H and ϕ), where $j = \theta, \delta, \alpha$, *pre, b*, β.

The Theorem (20) and the above Example (6) allude the definition of a strictly weaker transcription of *nm-j-ω*- perfect mapping as given below.

Definition 19. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to almost *nm-j-ω*-perfect if for every *nm-j-ω*-set K in H, $\lambda^{-1}(K)$ is *nm-j-ω*-set in G, where $j = \theta, \delta, \alpha$, *pre, b*, β.

By analogy to Theorem (20), amplest condition for a mapping to be almost *nm-j-ω*-perfect, is prove as follows.

Theorem 21. Let $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be any mapping such that

(a) $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in G, such that for every $h \in H$

(b) λ is weakly *nm- j-ω*- closed.

Then λ is almost *nm- j-ω*-perfect, where $j = \theta, \delta, \alpha$, *pre, b*, β.

Proof: Assume that L be an *nm-j-ω*-set in H and let that $\Im$ be a filter base on $\lambda^{-1}(L)$, then $\lambda(\Im)$ is a filter base on L. Also, by Theorem (20), (*nm-j-ω-cod* $\Im$) ∩ $L \neq$ ϕ, let $h \in$ [(*nm- j-ω-cod* $\Im$)] ∩ L. Assume that $\Im$ has no *nm- j-ω*-condensation point in $\lambda^{-1}(L)$, then (*nm- j-ω-cod* $\Im$) ∩ $\lambda^{-1}(h)$ = ϕ. Because of $\lambda^{-1}(h)$ is *nm- j-ω*-rigid in G, there exists an $M \in \Im$ and a σ_n-open S containing $\lambda^{-1}(h)$, such that $M \cap \sigma_n$- $\mathrm{cl}_j^{\omega}(S)$ = ϕ. By λ is weakly *nm- j-ω*- closed, then there is a ς_m –open nbd T of h, $\lambda^{-1}(\varsigma_m - \mathrm{cl}_j^{\omega}(T)) \subset \sigma_n$- $\mathrm{cl}_j^{\omega}(S)$. Therefore which implies that $\lambda^{-1}(\varsigma_m - \mathrm{cl}_j^{\omega}(T)) \cap M$ = ϕ, i.e., ς_m - $\mathrm{cl}_j^{\omega}(T) \cap \lambda(M)$ = ϕ, which is a contradiction . Therefore by Theorem (20), $\lambda^{-1}(L)$ is an *nm- j-ω*-set in G. So λ is almost *nm-j-ω*-perfect.

Conclusion.

The main purpose of the present work is the starting point for some application of pairwise supra-*ω*-perfect mappings of abstract topological structures in filter base by using bitopological spaces. Definitions of characterizations theorems are used for an *nm- j-ω*-continuous mapping and weakly *nm- j-ω*-continuous mapping and strongly *nm- j-ω*-continuous mapping and super *nm- j-ω*-continuous mapping and almost *nm- j-ω*-continuous mapping.

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بعض انواع التطبيقات في الفضاءات التبولوجية الثنائية

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الخلاصة:

قدمنا بعض المفاهيم في الفضاءات التبولوجية الثنائية وهي الاقتراب من المجموعة الجزئية من النمط $nm\text{-}j\text{-}\omega$ ، الاتجاه المباشر لمجموعة من النمط $nm\text{-}j\text{-}\omega$ ، التطبيقات المغلقة من النمط $nm\text{-}j\text{-}\omega$ ، صلابة المجموعة من النمط $nm\text{-}j\text{-}\omega$ ، التطبيقات المستمرة من النمط $nm\text{-}j\text{-}\omega$ ، والخط الرئيسي لهذا البحث هو التطبيقات التامة من النمط $nm\text{-}j\text{-}\omega$ في الفضاءات التبولوجية الثنائية. المميزات المتعلقة بهذه المفاهيم والعديد من المبرهنات قد درسنا حيث $j = \theta, \delta, \alpha, pre, b, \beta$.

الكلمات المفتاحية : المرشحات الاساسية ، التقارب من النمط $nm\text{-}j\text{-}\omega$ ، التطبيقات المغلقة من النمط $nm\text{-}j\text{-}\omega$ ، مجموعة صلبة من النمط $j\text{-}\omega$ ، التطبيقات التامة من النمط $nm\text{-}j\text{-}\omega$.