The Iraqi Journal For Mechanical And Material Engineering, Vol.14, No3, 2014

MATHEMATICAL MODEL AND NUMERICAL SIMULATION FOR THE EXTRUSION OF POLYGONAL SECTIONS FROM ROUND BILLETS THROUGH POLYNOMIAL STREAMLINED DIES

Ahmed Waleed Hussein [1] and Abdulkareem Jalil Kadhim [2]

Department of Mechanical Engineering, University of Babylon, Babylon, Iraq

[1] E-mail: ahmed_wleed_1988@yahoo.com , [2]

ABSTRACT:

Despite increasing demand for the application of three – dimensional extrusion of various shaped sections through continuous dies, so far little work has been done by general analytical and numerical analyses to predict the total extrusion pressure for the extrusion of polygonal sections from round billets through polynomial streamlined dies. For effective die design, efficient design method and the related method of theoretical analysis are required for extrusion of complicated sections. A systematic method for the die surface representation using blending function and trigonometric relationships is proposed in which smooth transitions of the die contour from the die entrance to the die exit are obtained. The upper bound extrusion pressure is obtained by derived a general velocity and strain rate felids. The effects of area reduction, the optimum die length, the shape of streamline function and frictional conditions are also discussed in relation to the relative extrusion pressure. Another advantage of the present work is that it could easily be applied to the extrusion of many different shapes just by defining the entry and exit sections functions and putting them into the general formulations. The results obtained in this work were compared with the theoretical results of other workers and found to be in highly compatible. The extrusion process is also simulated using the finite element code, ANSYS (V 14.0) in order to assist the mathematical solution and to show the stress and strain distributions for the products when the strain hardening effect taking into the account.

KEYWORDS: Upper bound method; finite element; streamlined dies; extrusion process.

النموذج الرياضي والمحاكاة العددية لبثق المقاطع المضلعة من الخامات الدائرية خلال قوالب انسيابية متعددة الحدود

احمد وليد حسين عطية ، عبد الكريم جليل كاظم /العراق- جامعة بابل-كلية الهندسة-قسم الميكانيك

الخلاصة

على الرغم من زيادة الطلب على بثق المقاطع الثلاثية الأبعاد خلال القوالب الانسيابية، الا ان القليل من البحوث تناولت تحليلا نظريا وعدديا عاما لإيجاد الضغط الكلي لعملية بثق الأشكال المضلعة من الخامات الدائرية خلال قوالب انسيابية متعددة الحدود. من اجل تصميم فعال لقوالب بثق الاشكال المعقدة يجب ان تكون هنالك طريقة كفؤة للتصميم بما ترتبط فيها من نظريات. في هذا البحث استخدمت طريقة منهجيه لتمثيل سطح القالب للأشكال المعقدة استعمله فيها دالة استنزاف (blending function) مع الدوال المثلثية من اجل تحول كفؤ وسلس لسطح القالب من منطقة دخول الخامة وحتى منطقة خروجها . تم ايجاد الحد الاعلى لضغط البثق عن طريق اشتقاق عام لحقول السرعة ومعدل الانفعال. الميزة الاضافية لهذا البحث هي امكانية تطبيق التحليل الرياضي على مقاطع اخرى فقط بتغيير الدوال التي تعبر عن منطقة الدخول والخروج من القالب ووضعها بالصيغة العامة. تم مناقشة تأثير مقدار النقصان بالمساحة ، الطول المثالي للقالب، شكل دالة انسياب القالب، حالات الاحتكاك مع ضغط البثق النسبي. ايضا في هذا البحث تم استخدام طريقة العناصر المحددة من خلال برنامج ANSYS (V14) من اجل اثبات صحة الموديل الرياضي وكذلك للحصول على توزيع الاجهادات والانفعالات اللدنة خلال الخامة المبثوقة.

1. NOMENCLATURE:

$f(z)$: Streamline function.

J : Total powers consume.
L : Die length.
m : Friction factor.
r, θ, z : Cylindrical co-ordinates systems.

R_o : Billet radius.

$R_S(\theta,z)$: Die surface contour.
V_o : Billet velocity.
V_r, V_θ, V_z: Velocity components
$W(\theta,z)$: Angular velocity.
ΔV : Relative velocity slip.

σ_o : Yield stress of the billet material.
θ_s : Angle of symmetry.

2. INTRODUCTION :

Extrusion of shaped sections through continuous dies has been increasingly used to improve the quality of products and raise the productivity. However, the designing of optimal die shapes depends on the predictions of actual metal flow characteristics within the die under the given conditions, Blazynski (1989). Recently, analytical methods for predicting the metal flow during the three-dimensional extrusion of sections have been proposed by the following main researches that enable us to design extrusion dies, the most other works coming next built on these main researches. Nagpal and Altan (1975) proposed dual stream functions to obtain a kinematically admissible velocity field that demonstrated a three-dimensional metal flow. Their model, however, was limited to the extrusion of ellipse bar or rod. Nagpal et al. (1979) presented a computer-aided technique for extrusion of T- section from round billets. The die was designed by dividing the initial and final cross-sections into a number of sectors in which the extrusion ratio of these sectors equal to the overall extrusion ratio. Yang and Lee (1978) used conformal transformation techniques for the extrusion of generalized sections from similar billets in which the intermediate sections was transferred into a unit circle. This formulation becomes very complex for the extrusion of complex sections, also includes redundant work relating to the velocity discontinuities at the entrance and the exit sections of the die respectively. Kiuchi et al. (1981) introduced a new concept in the upper-bound analysis of extrusion and drawing of sections through straight converging dies. They allowed to a velocity discontinuities at entrance and exit sections of the die respectively. Gunasekara and Hoshino (1985) proposed a purely analytical work based on an upper-bound theorem for extrusion of polygonal sections from round billets through curved dies. The geometry of the die was constructed by an envelope of streamlines between the entry and the exit sections. Yang and Han (1986) proposed their work for extrusion of a rectangular section from round billet through curved die. The die surface was constructed analytically by using Fourier series expansion. The Fourier series was expanded for many terms until the rectangular shape generated. A general kinematically velocity field was proposed by Abrinia and Bloobar (2000) which becomes automatically admissible whatever the die geometry is used. A formulation was derived based upon the upper bound theorem. The die surface was constructed by the same concept that used by Nagpal et al. (1979) but the entrance and exit sectors were connected using third degree Bezier curves. The effects of area reductions and friction factors on the extrusion pressure were discussed. Ismail (2002) adopted a rational tool design based on the CRHS and CMSR concept in the designing of the extrusion dies for the extrusion of the polygonal sections. FEM simulator for the material behavior in the plastic zone was conducted. A criterion based on total plastic effective strain was proposed by the author to determine the plastic zone and consequently pressure at which steady-state extrusion occurs. Sahoo and Kar (2003) focused their research on attempting to find an upper-bound solution for the problems of steady-state extrusion of polygonal section bars from polygonal billet through the square dies using the same SERR technique.

It became clear that the streamlined extrusion dies were designed by the following methods: (i) polynomial equation based dies (ii) area and line mapping technique (iii) analytical method. The main aims of this work: (1) obtaining an easily and systematic representation of the dies surfaces, (2) obtaining a 3-D upper bound solution based on continuous velocity filed to evaluate the extrusion pressure of polygonal sections from round billets, and (3) Obtaining 3-D elasto-plastic finite element simulations to assist the mathematical models.

3. KINEMATICALLY ADMISSIBLE VELOCITY AND STRAIN RATE FIELDS:

It is necessary to build the proposed kinematically admissible velocity field using streamlines concept. **Figure (1)**, shows the general die shape in cylindrical co-ordinate systems in which the z- direction is coincident with the extrusion axis. The boundary limits for the die surfaces are given by:

$$0 \le r \le R_S(\theta, z)$$

$$0 \le \theta \le \theta_f(z)$$

$$0 \le z \le L \qquad (1)$$

Throughout this analysis, the following assumptions are employing (Kiuchi et al. (1981), Gunasekara and Hoshino (1985)):

- The billet materials are isotropic and homogeneous.

- The billet materials are incompressible and undergo plastic deformation predominantly and its elastic strain is neglected.
- The die is assumed rigid body and the effect of temperature between the billet and the die is neglected i.e. the process is assumed isothermal because practically the extrusion die is kept cool by circulating water around it.
- The longitudinal velocity V_z, is uniform at each cross-section of the material in the die.
- The Von-Misses yield criterion is assumed applicable.
- The rotational velocity component $V_\theta(r,\theta,z)$, is expressed as product of two functions as follows:

$$V_\theta(r,\theta,z) = r.w(\theta,z) \quad (2)$$

- Velocity components normal to the die surface should vanish hence,

$$V_r|_{r=R_s(\theta,z)} = 0 \quad (3)$$

The volume constancy in cylindrical co-ordinate system is expressed by the next equation,

$$\frac{\partial V_r(r,\theta,z)}{\partial r} + \frac{1}{r}\,V_r(r,\theta,z) + \frac{\partial V_z(r,\theta,z)}{\partial z} + \frac{1}{r}\,\frac{\partial V_\theta(r,\theta,z)}{\partial \theta} = 0 \quad (4)$$

Using Eqs.(2-4) , the following velocity field can be derived:

$$V_z(z) = \frac{Vo\int_0^{\theta_s} R_s^2(\theta,0)\,d\theta}{\int_0^{\theta_s} R_s^2(\theta,z)\,d\theta} \quad (5)$$

$$V_\theta(r,\theta,z) = \frac{-r}{R_s^2(\theta,z)}\int_0^{\theta}\frac{\partial}{\partial z}\{\,V_z(z)\;R_s^2(\theta,z)\}\,d\theta \quad (6)$$

$$V_r(r,\theta,z) = -\frac{r}{2}\left\{\frac{\partial V_z(z)}{\partial z} + \frac{\partial W(\theta,z)}{\partial \theta}\right\} \quad (7)$$

Where V_o is the billet velocity and θ_s is the angle of symmetry for the die.

According to cylindrical co-ordinate systems, the strain rate components are defined as follows referring to the kinematically admissible velocity field (Kiuchi et al. (1981)):

$$\dot{\varepsilon}_{rr} = \frac{\partial V_r}{\partial r} = -\frac{1}{2}\left\{\frac{\partial V_z(z)}{\partial z} + \frac{\partial W(\theta,z)}{\partial \theta}\right\},\ \dot{\varepsilon}_{\theta\theta} = \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r} = \frac{1}{2}\left\{\frac{\partial W(\theta,z)}{\partial \theta} - \frac{\partial V_z(z)}{\partial z}\right\},$$

$$\dot{\varepsilon}_{zz} = \frac{\partial V_z}{\partial z},\ \dot{\varepsilon}_{r\theta} = \frac{1}{2}\left\{\frac{1}{r}\frac{\partial V_r}{\partial \theta} + \frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r}\right\} = -\frac{1}{4}\frac{\partial^2 W(\theta,z)}{\partial \theta^2},\ \dot{\varepsilon}_{\theta z} = \frac{1}{2}\left\{\frac{\partial V_\theta}{\partial z} + \frac{1}{r}\frac{\partial V_z}{\partial \theta}\right\} = \frac{r}{2}\frac{\partial V_\theta}{\partial z},\ \text{and}$$

$$\dot{\varepsilon}_{rz} = \frac{1}{2}\left\{\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}\right\} = -\frac{r}{4}\left\{\frac{\partial^2 W(\theta,z)}{\partial z \partial \theta} + \frac{\partial^2 V(z)}{\partial z^2}\right\}. \quad (8)$$

4. UPPER BOUND SOLUTION:

The upper bound theorem is based on an energy balance such that the internal rate of dissipation of energy must be less than or equal to the rate at which external forces do work. According to this theorem, the total power required J is given by (Dieter and Bacon (1988)):

$$J = \frac{2\sigma_o}{\sqrt{3}} \int_v \sqrt{\frac{1}{2}\dot{\varepsilon}_{ij}\,\dot{\varepsilon}_{ij}}\; dv + \frac{\sigma_o}{\sqrt{3}} \int_{s_i} |\Delta V_i|\; ds_i + \frac{m\,\sigma_o}{\sqrt{3}} \int_{s_j} |\Delta V_j| ds_j \quad (9)$$

$$|\Delta V| = \sqrt{V_z^2 + V_\theta^2 + V_r^2}$$

$$\mathrm{dS_j} = \sqrt{1 + \frac{1}{R_s^2(\theta,z)}\left[\frac{\partial R_s(\theta,z)}{\partial \theta}\right]^2 + \left[\frac{\partial R_s(\theta,z)}{\partial z}\right]^2}\; R_s(\theta,z)\; d\theta dz$$

Where, $s_{i,}$ is the surface at the velocity discontinuity, s_j is the surface of friction and σ_o is the yield stress. The first term of Eq.(9) represents the internal power of deformation and the second and third terms represent the energy dissipated due to the velocity discontinuities and frictional resistance respectively.

5. GENERAL DIE SHAPE FOR THE EXTRUSION THROUGH STREAMLINED DIES:

The difficulty of the analysis of three- dimensional extrusion of sections is due to the fact that it is not simple to find analytic expression for a complicated die surface and develop a corresponding theoretical formulation for the complicated material flow bounded by a three-dimensional surface. In this work, a new approach was used in which a blending function and trigonometric relationships is attempted. This method has some advantages in CAD/CAM application of the die manufacturing (Abrinia and Bloobar (2000)). When the cross sectional shapes of the both entrance and exit sections of the die are given by analytical functions $R(\theta,0)$ and $R(\theta,L)$, then the intermediate sectional contour $R_s(\theta,z)$ that satisfies the die surface can be obtained by:

$$R_s(\theta,z) = f(z)\big(R(\theta,0) - R(\theta,L)\big) + R(\theta,L) \quad (10)$$

Where $f(z)$ is the streamline function.

In order to minimize the total power dissipation i.e. Eq.(9), zero slopes and curvature boundary conditions were applied at the entrance and exit sections of the die respectively. In this case, the energy dissipated due to the velocity discontinuities will vanish.

The streamline function f(z) can be taken as a polynomial of any order. In this work, third and five-order polynomial were utilized in order to make a comparison between them. The boundary conditions using to find the constants of these functions were:

$$f(0) = 1 , \qquad \left.\frac{\partial f(z)}{\partial z}\right|_{z=0} = 0 , \qquad \left.\frac{\partial^2 f(z)}{\partial z^2}\right|_{z=0} = 0 ,$$

$$\left.\frac{\partial f(z)}{\partial z}\right|_{z=L} = 0 , \quad \left.\frac{\partial^2 f(z)}{\partial z^2}\right|_{z=L} = 0 \quad \text{and} \quad f(L) = 0 \qquad (11)$$

Hence, the third and fifth order streamline functions are:

$$f(z) = 1 - 3\left[\frac{y}{L}\right]^2 + 2\left[\frac{y}{L}\right]^3 \qquad (12\text{-a})$$

$$f(z) = 1 - 10\left[\frac{z}{L}\right]^3 + 15\left[\frac{z}{L}\right]^4 - 6\left[\frac{z}{L}\right]^5 \qquad (12\text{-b})$$

The similarity of the polygonal section was taken into the account in this investigation. So, first of all, let (a) be the side length of polygon and, θ_s be the angle of symmetry of the polygon, therefore,

$$\theta_s = \frac{\pi}{N_S} \qquad (13)$$

Where N_S is the number of sides of the polygon ($N_s \geq 4$).

Figure (2, A) takes a special case when N_S equals six (hexagonal section) to evaluate the governing equations. Assuming that the sector AB is gradually transfers to the vertical straight line CD when θ change from zero to θ_s . Hence, the equation of the exit straight line in cylindrical co-ordinate systems is,

$$r = k \sec(\theta) \qquad (14)$$

Where $= \frac{a}{2\tan(\theta_s)}$.

Hence, the sectional contour that satisfies the die surface for the extrusion of any polygonal section have N_s sides from round billets i.e. Eq. (3-41), becomes,

$$R_s(\theta, z) = f(z)(R_o - k\sec(\theta)) + k\sec(\theta) \quad (15)$$

Figure (2, B) shows the overall surface contour for the extrusion of polygonal section from round billet produced the numerical application of Eq. (15) in MATLAB 7.11 when the die dimensions were L=19 mm ,Ro=12.5 mm, RA%= 52% , N_s=6, and f(z) from Eq.(12, a).

By substituting of Eq.(15) into the Eqs. (5-7), the following velocity components for any polygonal section were obtained:

$$V_z(z) = \frac{V_0 R_0^2\, \theta_s}{D(z)}, \quad (16)$$

$$V_\theta(r, \theta, z) = \frac{-r}{R_s^2(\theta, z)}\{V_z(z) I_1 + F_1 I_2\}, \text{ and}$$

$$V_r(r, \theta, z) = -\frac{r}{2}\{F_1 + F_2\}$$

Where,

$$D(z) = R_o^2 f(z)^2 \theta_s + 2k R_o f(z)\big(1 - f(z)\big)[\mathrm{B}_1]_0^{\theta_s} + k^2(1 - f(z))^2 [\mathrm{B}_2]_0^{\theta_s}$$

$$I_1 = \int_0^\theta \frac{\partial R_s^2(\theta, z)}{\partial z}\, d\theta = 2R_O^2 f(z) f'(z)\theta - 2R_o k\, f(z) f'(z) B_1 + 2R_o k\, f'(z)\big(1 - f(z)\big)\mathrm{B}_1 - 2\,k^2\big(1 - f(z)\big) f'(z)\, B_2$$

,

$$I_2 = \int_0^\theta R_s^2(\theta, z)\, d\theta = R_O^2 f(z)^2\theta + 2R_o k f(z)(1 - f(z))\mathrm{B}_1 + k^2\big(1 - f(z)\big)^2\, \mathrm{B}_2$$

$$\mathrm{B}_1 = \ln|\tan(\theta) + \sec(\theta)|,$$

$$\mathrm{B}_2 = \tan(\theta),$$

$$F_1 = \frac{1}{\{D(z)\}^2}\Big[-2VoR_o^2\, \theta_s * \Big[R_o f(z) f'(z)\theta_s + R_o k\, f'(z)\big(1 - f(z)\big)[\mathrm{B}_1]_0^{\theta_s} - R_o k\, f(z) f'(z)[\mathrm{B}_1]_0^{\theta_s} - k^2 f'(z)(1 - f(z))[\mathrm{B}_2]_0^{\theta_s}\Big]\Big]$$

, and

$$F_2 = \frac{-2}{R_s(\theta, z)} k\big(1 - f(z)\big)\sec(\theta)\tan(\theta)\frac{V_\theta(r, \theta, z)}{r} - F_1 - \frac{2\, V_z(z)}{R_s(\theta, z)} f'(z)(R - k\, sec(\theta)).$$

Where $f'(z)$ represent the differentiation of the function $f(z)$ with respect to z

After the velocity components become known, it becomes easily to find the strain rates components from Eq.(8). The total power of extrusion becomes:

$$J = 2\, N_s \left[\frac{2\,\sigma_o}{\sqrt{3}} \int_v \sqrt{\frac{1}{2}\, \dot{\varepsilon}_{ij}\, \dot{\varepsilon}_{ij}}\; d + \frac{m\,\sigma_o}{\sqrt{3}} \int_{s_j} \left|\Delta V_j\right| ds_j \right] \tag{17}$$

The area and volume integrals of Eq. (17) were performed using 15-point Gaussian quadrature integration technique. According to this technique, the last equation becomes,

$$J = 2\, N_s (J_1 + J_3) \tag{18}$$

Where,

$$J_1 = \iiint P(r,\theta,z)\; dr\; d\theta\; dz = \frac{L}{2}\,\frac{\theta_S}{2} \sum_{i=1}^{i=n} \sum_{j=1}^{j} \sum_{k=1}^{k=h}$$

$$W_i W_j W_k \left[\frac{R_s(\eta_j,\zeta_i)}{2}\, P\left(\frac{(1+\psi_k) R_s(\eta_j,\zeta_i)}{2}, \frac{(1+\eta_j)\theta_S}{2}, \frac{(1+\zeta_i)L}{2} \right) \right]$$

$$J_3 = \iint F(\theta,z)\; d\theta\; dz = \frac{L}{2}\,\frac{\theta_S}{2} \sum_{i=1}^{i=n} \sum_{j=1}^{j=m} W_i W_j\; P\left(\frac{(1+\eta_j)\theta_S}{2}, \frac{(1+\zeta_i)L}{2} \right)$$

Where n,m and h are the number of sample points and (ψ,η,ζ) and $(W_i W_j W_k)$ are the sample points and its weights. The non-dimensional average extrusion pressure can be written as:

$$\frac{P_{avg}}{\sigma_o} = \frac{J}{\pi\, R_O^2 V_O\, \sigma_o} \tag{19}$$

6. FINITE ELEMENT SUMULATION:

The dimensional accuracy of the cold forged products is strongly dependent on the elastic characteristics of the die. Most of the finite element models studied by the previous researchers were simple (rigid- plastic material) and neglected the elastic behavior of the working material and the strain hardening. So in this study, FEM code, ANSYS (V 14) was used to simulate the nonlinear extrusion of the considered sections by taking elasto-plastic material model.

The original billet radius = 12.5 mm, reduction in area = 60 %, the relative die length (L/Ro) =1 and the friction factor $m = 0.1$. The solid billet, die, punch and container were modeled using Solid186. It can tolerate irregular shapes without as much loss of accuracy also; it has plasticity, creep, stress stiffening, large deflection, and large strain capabilities (ANSYS guide release 14). Contact elements, named CONTA174 and TARGE170, havebeen created between billet-die, billet-container and billet - punch interfaces. The die, the container and the punch were made from tool steel, which has isotropic properties of $E = 210$ GPa and $\upsilon = 0.28$ .The billets material used in this simulation were aluminum alloy which have bilinear isotropic hardening, modulus of elasticity E= 68 GPa, tangent modulus E_t= 0.1 GPa, yield stress σ_y= 70.2 MPa and Poisson's ratio $\nu = 0.3$. Free meshing was applied to the entire models. The loading were conducted in the form of a prescribed displacement in which the punch displacement was 20 mm in the z- direction. The entire meshed models are shown in **Fig. (3).**

7. RESULTS AND DISCUSSION :

Upper bound and the FE solutions were successfully applied for predication of plastic deformation work (J) for the extrusion of polygonal sections from round billets through streamlined dies. Through, in this paper, priority is given to the analysis of the sections through streamlined die which produces no shear energy at inlet and outlet of the velocity boundaries, the present method is applicable to analyze other die profiles simply by changing the profile function f(z). Gunasekera and Hoshino (1985) carried out a purely analytical work for extrusion of polygonal sections from round billets. polynomial equation based dies was used in this work in which the die surface constructed by an envelope of curve lines drawn from points on the perimeter of the circular entry section to the corresponding points on the square exiting shape. Also, different formulation was used to find the velocity field in this work. The results given by these authors were compared with the theoretical results of this paper in order to verify the upper bound solution.

It can be observed from **Fig.(4)** that the relative extrusion pressure (P_{avg}/σ_o) decreased with increasing relative die length up to optimal relative die length and then it increased. The frictional load has always an increasing tend with relative die length, while the deformation load gradually decreased with increasing relative die length. When f(z) from 3^{rd} degree, the optimum relative die lengths under the conditions of this figure was 1.5 and the corresponding optimum relative extrusion pressure equals 2.1. In addition, it can be observed from this figure that as the degree of streamline function f(z) increased up to 5^{th} degree, the relative extrusion pressure increased due to the increasing in fluctuations of the die surface which caused increasing in the deformation power (J_1).

The comparison between the author's results and that obtained by Gunasekera and Hoshino (1985) at zero hardening condition (E_t=0) also shown in **Fig.(4).** It can be toted that the average error between the authors solution and that obtained by Gunasekera and Hoshino (1985) were 2%. This error probably due to the difference in the methods used to solve the area and volume integrations. This suggests that the analytical method developed by this study can give quite reasonable results.

The comparison between the upper bound solution FE solution at zero hardening condition (E_t=0) is shown in **Fig.(5).** It can be noted that there is a good compatible between the upper bound and FE solutions with average error of about 10%. This error is acceptable because the FE solution always lower the upper bound solution.

Figure (6) shows the effect of the number of polygon sides (Ns) on the total relative extrusion pressure. It can be observed that, when the number of sides increased, the relative extrusion pressure decreased this because as the number of sides increases the product shape becomes closer to the circular shape in which the rotational velocity value becomes closer to the zero.

At optimum die length, the effects of area reduction and friction factor on the relative extrusion pressure are shown in **Fig.(7).** It can be noted that, when the reduction in area increased the relative extrusion pressure increased due to the increasing of internal power of deformation. While, increasing in friction factor causes increasing in relative extrusion pressure due to the increasing in frictional power.

The Von Misses stress and corresponding plastic strain contours at the end of 110 substep at hardening condition is shown in **Fig.(8)**. In general, the maximum stress and corresponding plastic strain exist on the billet at the exit region at the billet-die interfaces, and tend to be a minimum along the centerline of the billet in which the rotational and radial velocities become zero. Another point can be noted in these figures are smoothly variation in the stress values from the entrance of the die to its exit; this is the main advantage of the streamlined die which reduces the cracks of the products and improved its fatigue life.

8. CONCLUSIONS:

The following were the main conclusions that noted throughout this study:

1- The method of analysis presented here not only was capable of computing the upper bound on load but also was used to determine analytically both the velocity and the strain rate distributions for the extrusion of polygonal sections.
2- Three-dimensional material flows were obtained analytically for the extrusion of a polygonal section. The results were improved by the present method due to the elimination of the internal discontinuities provided.
3- It is concluded that the streamlined extrusion dies designed based on polynomial streamline function from third degree is superior to the dies designed based on polynomial equation from fifth degree.
It was saw that when the number of sides of polygon increased, the extrusion pressure decreased.

4- Comparison with FEM results has shown that the present method as a fast and precise analytical tool is comparable with the present day FEM commercial software.
5- By using the elasto-plastic material model, it was concluded that the maximum damage occurs on the billet surface in the exit region of the die.
6-The extrusion through the streamlined die gives little variation in the stresses values from the die entrance to the die exit; this reduces the cracks in products and improves its fatigue life.
7- The present method can be extended to obtain the solution of generalized problems of non-axisymmetric extrusion through converging dies.

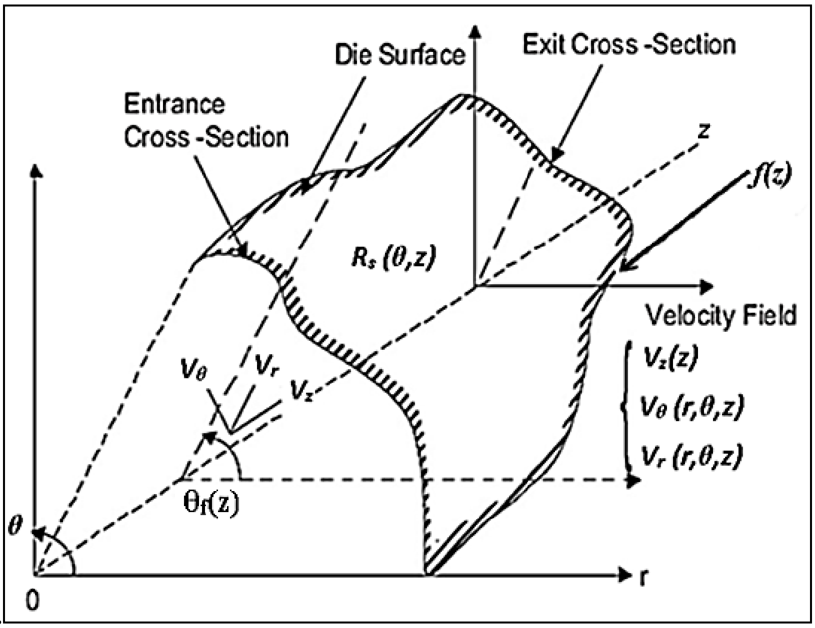

Figure (1) General die shape (Kiuchi et al. (1981)).

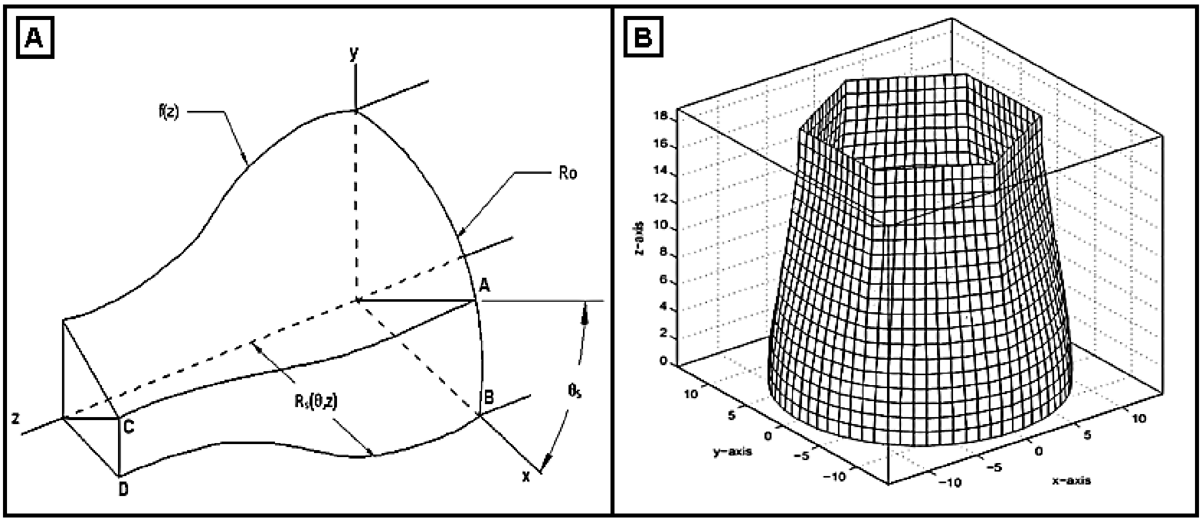

Figure (2) Die surface for extrusion of polygonal section from round billet,
A: Quarter model, B: Overall surface.

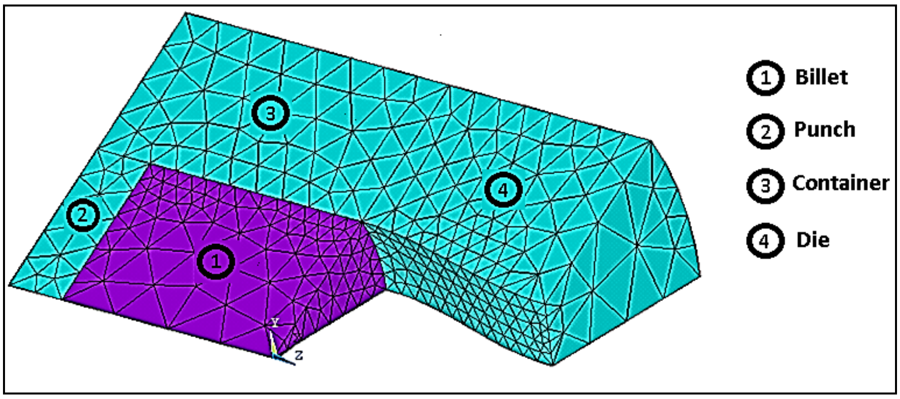

Figure (3) Meshed model.

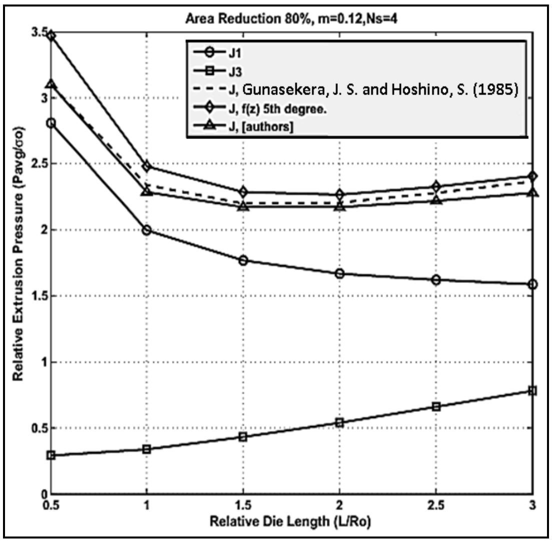

Figure (4) The relative extrusion pressure against the relative die length.

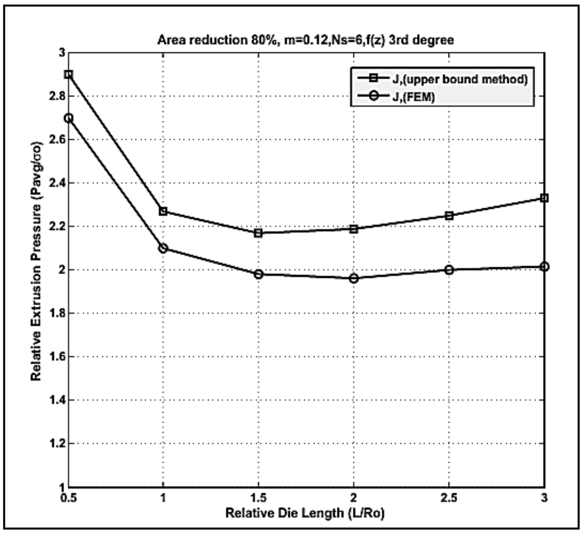

Figure (5) Comparison between the upper bound and FE solutions.

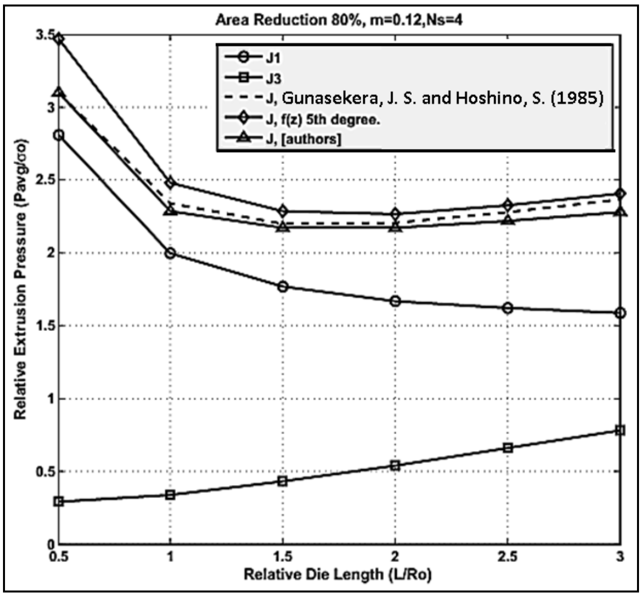

Figure (6) The effect of the number of polygon sides on relative extrusion pressure.

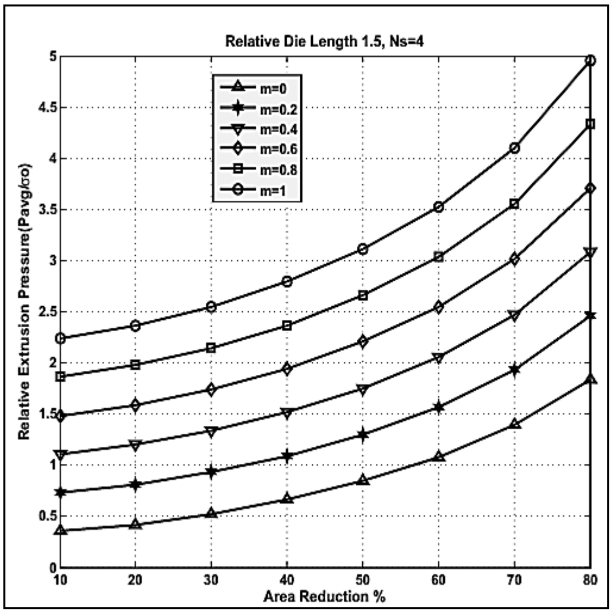

Figure (7) the effects of the area reduction and friction factor on relative extrusion pressure.

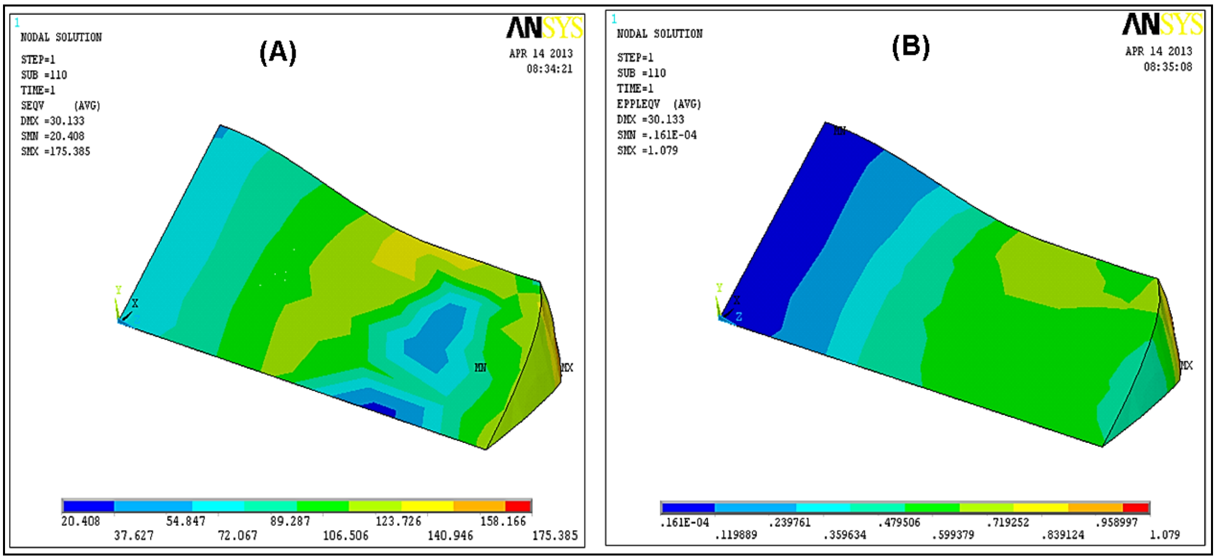

Figure (8) The von Mises: (A) Stress contour (MPa), (B) plastic strain

9. REFRENCES:

Abrinia, K, and Bloorbar, H. , 2000," A New Upper Bound Solution for the Extrusion of Shaped Sections using CAD Techniques", European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS) Barcelona, Spain.

Blazynski, T. Z., 1989," *Plasticity and Modern Metal-Forming Technology*", Published by ElsevierApplied Science, London.

G. E. Dieter and D. Bacon, 1988, "*Mechanical Metallurgy*", McGraw-Hill Book Company. Guide to the ANSYS Documentation Release 14.0.

Gunasekera, J. S., and Hoshino, S., 1985, "Analysis of Extrusion of Polygonal Sections through Streamlined Dies", Transactions of the ASME Journal of Engineering for Industry, 107:229.

Ismail, A. R., 2002, "Numerical Simulation of 3-D Regular Polygonal Sections in Forward Extrusion By Elastic- Plastic Finite Element Method", Ph.D. thesis, Department of Production Engineering and Metallurgy, University of Technology, Baghdad.

Kiuchi, M., Kishi, H., and Ishikawa, M., 1981, "Study on Non-Symmetric Extrusion and Drawing", Proceedings of the 22nd International Machine Tool Design Research Conference, pp. 523- 532.

Nagpal, V., and Altan, T., 1975," Analysis of the Three Dimensional Metal Flow in Extrusion of Shapes with the Use of Dual Stream Functions", In: Proceedings of the 3rd NAMRC Conference, PA, Pittsburg.

Nagpal, V., Billhart, C. F. and Altan, T. , 1979, "Lubricated Extrusion of T Sections From Aluminum, Titanium and Steel Using Computer-Aided Techniques", Trans. ASME J. Eng. Ind., Vol. 101, pp. 319–325.

Sahoo, S. K. and Kar, P. K., 2003, "Polygonal Section Extrusion from Square Billet through Flat Die - An Application of Upper-Bound Method", IE (I) Journal-PR, Vol. 84, pp. 12-18.

Yang, D. Y., and Lee, C.H., 1978," Analysis of Three-Dimensional Extrusion of Sections through Curved Dies by Conformal Transformation", Int. J. Mech. Sci, 20 (9), pp. 541–552.

Yang, D. Y., Han, C. H., and Kim, M. U., 1986, "A Generalized Method for Analysis of Three- dimensional Extrusion of Arbitrarily Shaped Sections", Int. J. Mech. Sci., Vol. 28, No. 8, pp. ' 517-534.