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Comparing Different Fuzzy Reliability Function of Exponentiated Maxwell

د. أنعام عبد الوهاب عبد الجبار

Dr. Anaam Abdel Wahab Abdul Jabbar

Abstract :

This paper deals with comparing different estimators of fuzzy Reliability function for new probability distribution called expontiated Maxwell the methods of estimations are maximum likelihood and L- Moments and proposed method One (probability weighted moments), The comparison is done by simulation as well as application on time to failure of machines , at Dura refinery Baghdad All results of comparison are explained in tables , and compared by statistical Measures Mean square error(MSE). The comparison is done using simulation procedure , and all the results are explained in tables.

Key words: Maximum Likelihood Estimator(MLE), probability Weighted Moments (PWM), Mean square error (MSE), Proposed Method (PROP).

Introduction:

The concepts of fuzzy groups , were introduced by (Zadeh) in 1965 , were $(\tilde{A})$ denote the fuzzy group which is defined on universal set of element (X) as a set of groups with its Membership function as follows $\tilde{A}=\{ M_{\tilde{A}}\ (x_i)\ \ (x_i) \}$ $i = 1, 2, \ldots, n$

Were $M_{\tilde{A}}\ (x_i) \in [0, 1]$

and when $X \notin \tilde{A}$ then its membership function $\mu\, \tilde{A}(x) = 0$ and the element belong to fuzzy set

then $X \in \tilde{A}$ and $M_{\tilde{A}}\ (x_i) = 1$

Due to the expansion in the field of probability distribution in all type , were it is unique or mixed or compound , the research er's introduce many expellants research in Reliability and hazard Rate function , were Shanker R and Mishra A (2013) gives a studying of estimation of Reliability function for quasi lindley distribution to Journal of Mathematics and Computer Science , while Sankaran M. In (1970) introduce discrete distribution called Poisson- Lindley Distribution , In (2010) (Mohamoudi and Zakerzaadeh) introduce generalized Poisson- Lindley Distribution and introduced different methods of estimation and comparing results by MSE, with it's properties and estimation method .

The mixture distributions have vital role in practical applications for research that deal with economics, medicine, agriculture, life testing, and reliability for classical reliability theory .There are several methods and models in which the parameters assume precise, but in real world application ,due to vague and randomness affect the life times distribution , also when the parameters of life time distribution are fuzzy , then there is difficulty for handling reliability and hazard functions Many researches work on fuzzy reliability and introduce development for this field , as this in dictated . In (2017) two researchers M.A. Hussian and Esssam A. discussed fuzzy exponential distribution and discussed how to compute reliability in case of stress- strength model and ranked set sampling .Also in (2013) two researchers Elbatal and M.elgarhy introduced Transmuted Quasi Lindley Distribution and worked on deriving (r_{th}) moment and moment generating function., while in 2016 Nedjar and Zeghdoudi worked on deriving gamma Lindley Distribution and studied its properties by Simulation.

A Generalization of Lindley Distribution was introduced by Zakerzadeh and Dolati in (2009). Also Zeghdoudi and Nedjar in 2016 introduced properties and application on Poisson Gamma Lindley Distribution. In May –Jun. (2015) Dutta and Borah introduced a study about Poisson –Quasi Lindley Distribution, and derived its mathematical properties of coefficient of skewness and kurtosis coefficient of variation . In (2017) Rama Shanker and etl. discuss three parameters Lindley ,and studied their different estimators of parameter and reliability function ,all mathematical and statistical properties were discussed. The aim of this research is to build mixed failure to time model from exponential and Gamma distribution where this model is necessary when the observation of time to failure cannot be represented by single probability distribution , which is recognized by scatter diagram , so two types of distributions need to be mixed which are exponential and Gamma using certain proportion as weights function , and the sum of this weight equals one.

In this researcher we continue the work in the field of fuzzy Reliability and work on comparing three deferent's estimators of fuzzy Reliability function , and explain the results of comparison by simulation.

Theoretical Parts:

The p.d.f of one scale parameter Maxwell is

$$f(t,\theta)=\frac{4}{\sqrt{\pi}\,\theta^3}\ t^2\ \mathrm{e}^{-(\frac{t}{\theta})^{2}} \qquad \ldots..(1)$$

And:

$$E(t)=\frac{2\theta}{\sqrt{\pi}}$$

$$E(t^2)=\frac{33}{22}\theta^2$$

$$V(t)=\frac{5\theta^2}{22}$$

$$F(t,\theta) = \frac{4}{3\sqrt{\pi}}\left(1 - \mathrm{e}^{-\left(\frac{t}{\theta}\right)^{3}}\right) \qquad \ldots..(2)$$

While exponetiated Maxwell is obtained from

$G\ (t) = F_{(t,\theta)}$

$$G\ (t) = (\frac{4}{3\sqrt{\pi}})^{\beta}\left(1 - \mathrm{e}^{-\left(\frac{t}{\theta}\right)^{3}}\right)^{\boldsymbol{\beta}} \qquad \ldots\ldots(3)$$

New exponetiated Maxwell is

$$g_T(t,\theta,\beta) = (\frac{4}{3\sqrt{\pi}})^{\beta}\left(1 - \mathrm{e}^{-\left(\frac{t}{\theta}\right)^{3}}\right)^{\boldsymbol{\beta}} (-)\ \mathrm{e}^{-\left(\frac{t}{\theta}\right)^{3}} \quad \frac{(-)3}{\theta}\left(\frac{t}{\theta}\right)^{2}$$

$$g_T(t,\theta,\beta) = \frac{3\beta}{\theta}(\frac{4}{3\sqrt{\pi}})^{\beta} \quad t^2 \quad \mathrm{e}^{-\left(\frac{t}{\theta}\right)^{3}}\left(1 - \mathrm{e}^{-\left(\frac{t}{\theta}\right)^{3}}\right)^{\boldsymbol{\beta - 1}} \qquad \ldots\ldots(4)$$

And Its C.D. F

$$G_T(t) = (\frac{4}{3\sqrt{\pi}})^{\beta} \quad \left(1 - \mathrm{e}^{-\left(\frac{t}{\theta}\right)^{3}}\right)^{\boldsymbol{\beta}} \qquad \ldots\ldots.(5)$$

And $\quad R_T(t) \ = 1 - \ G_T(t) \qquad \ldots\ldots.(6)$

Estimation Methods:

The two parameters (θ,β) are estimated by Maximum Likelihood and frequency Ratio , and proposed one .

1- Estimation by Maximum Likelihood (MLE):

Let $t_1\ , t_2\ , t_3, \ldots\ldots.., t_n$ be ar.s from p.d.f in equation (4) , then

$$L = \prod_{i=1}^{n} g_T(ti,\theta,\beta)$$

$$= \ (\frac{3\beta}{\theta^3})^n \quad (\frac{4}{3\sqrt{\pi}})^{n\beta} \quad \prod_{i=1}^{n} t^2 \quad \mathrm{e}^{-\sum_{i=1}^{n}\left(\frac{ti}{\theta}\right)^{3}} \quad \prod_{\boldsymbol{i=1}}^{\boldsymbol{n}} \quad \left(1 - \mathrm{e}^{-\left(\frac{ti}{\theta}\right)^{3}}\right)^{\boldsymbol{\beta - 1}} \qquad \ldots\ldots..(7)$$

$$\log L = \quad \mathrm{n} \log 3\,\beta - n\log(\theta^3) + (n\beta)\log 4 - 3\sqrt{\pi}\ \log n\beta + \sum_{i=}^{n} \log {t_i}^2 - \frac{\sum_{i=1}^{n} {t_i}^3}{\theta^3} + (\beta - 1)$$

$$\sum_{i=}^{n} \log(1 - \mathrm{e}^{-\left(\frac{ti}{\theta}\right)^{3}})$$

Then

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{3\beta}(3) - nlog\, 4 - 3\sqrt{\pi}\ \left(\frac{1}{n}\right) \sum_{i=1}^{n} \log\ \left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)$$

$$from\ \frac{\partial \log L}{\partial \beta} = 0 \ \rightarrow$$

$$\frac{n}{\hat{\beta}} = n \log 4 + 3\ \frac{\sqrt{\pi}}{n}\ - \sum_{i=1}^{n} \log\ \left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)$$

$$\hat{\beta}_{MLE} = \frac{n}{n \log 4 + 3\ \frac{\sqrt{\pi}}{n}\ - \sum_{i=1}^{n} \log\ \left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)} \quad \ldots \ldots (8)$$

And from

$$\frac{\partial \log L}{\partial \theta} = \frac{-3n}{\theta} + \frac{3\sum ti^{3}}{\theta^{4}} + (\beta - 1)\frac{\sum_{i=1}^{n}(-1)e^{-\left(\frac{ti}{\theta}\right)^{3}}\ \frac{(-3)ti^{3}}{\theta^{4}}}{\left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)}$$

$$\rightarrow\ -3n\theta^{3} + 3\sum ti^{3} + (\beta - 1)\frac{\sum_{i=1}^{n} 3\, ti^{3}\ e^{-\frac{ti^{3}}{\theta^{3}}}}{\left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)} \qquad \div 3$$

$$\text{From}\ \frac{\partial \log L}{\partial \theta} = 0\ \rightarrow$$

$$\hat{\theta}_{MLE} = \sqrt[3]{\frac{\sum ti^{3} + (\hat{\beta} - 1)\ \frac{\sum_{i=1}^{n}\ ti^{3}\ e^{-\frac{ti^{3}}{\theta^{3}}}}{\left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)}}{n}}$$

2- Estimation by Proposed Methods (prop):

The exponentiated distribution have one scale parameter (θ) and one shape parameter (β) ,so according to the generated observations we can estimate shape parameter (β) , depending on non parametric estimator of scale parameter (θ),

Were $\hat{\theta} = Y_{(1)}$ which is min (observation) .

(i. e) the smallest observation and the shape parameter (β)

$$\hat{\beta} = \frac{Ln\ 2}{Ln\ (\frac{Median}{\hat{\theta}})}$$

Also the two parameters (β, θ) can be estimated by probability weighted Moments (PWM) as:

$$\hat{\beta} = \frac{\hat{\mu}_{1,0,1} - \hat{\mu}_{1,0,0}}{(\ 2\hat{\mu}_{1,0,1} - \hat{\mu}_{1,0,0}\ )}$$

Were: $\hat{\mu}_{1,0,0} = \bar{t}\ \ or\ \ \bar{Y}$

And

$$\hat{\mu}_{1,0,0} = \frac{1}{n(n-1)} \sum_{i=1}^{n} (\mathrm{i} - 1)\, Y_{(i)}$$

Were $Y_{(1)} < Y_{(2)} < Y_{(3)} \ \ldots\ldots < Y_{(n)}$

And the scale parameter

$$\hat{\theta} = \frac{\bar{t}\ (\hat{\beta} - 1)}{\hat{\beta}}$$

3-L- Moment Estimation Method (LM):

The shape parameter (β)

$$\hat{\beta} = \frac{SLM_2 - \bar{Y}}{2\ SLM_2}$$

Were :

$$SLM_2 = \frac{2}{n} \sum_{j=1}^{n} \left(\frac{\mathrm{j}-1}{\mathrm{n}-1}\right) Y_{(j)} \ - \bar{Y}$$

or $$SLM_2 = \frac{2}{n(n-1)} \sum_{i=2}^{n} (\mathrm{j} - 1\ ) \ \left(Y_{(j)} \ - \bar{Y}\right)$$

While scale parameter (θ) is estimated depending on $\hat{\beta}_{LM}$ as:

$$\hat{\theta}_{LM} = \frac{\bar{Y} - (\ \hat{\beta}_{LM} - 1)}{\hat{\beta}_{LM}}$$

Simulation Procedures:

According to given values of (θ,β), first of all the values of (ti) are generated according to pre-determined values of (θ,β) and inverse trans formation as:

$$G(ti) = Ui$$

$$ui = (\frac{4}{3\sqrt{\pi}})^{\beta} \left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)^{\boldsymbol{\beta}}$$

Let

$$K = \frac{4}{3\sqrt{\pi}} = \frac{4}{1.103752} \qquad \pi = 2.718$$

$$ui = (K)^{\beta} \left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)^{\boldsymbol{\beta}}$$

$$ui = (0.955408)^{\beta} \left(1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right)^{\boldsymbol{\beta}}$$

$$\text{Log } ui = \beta \log(0.955408) + \beta \log\left[1 - e^{-\left(\frac{ti}{\theta}\right)^{3}}\right]$$

According to the given values of (β , θ) and observation (ti), we obtain $Zi = log\, ui$ and then the observation of exponentiated Maxwell , to be used in estimation.

$$(\frac{ui}{K})^{\frac{1}{\beta}} = (1 - e^{-\left(\frac{ti}{\theta}\right)^{3}})$$

$$e^{-\left(\frac{ti}{\theta}\right)^{3}} = 1 - (ui^{*})^{\frac{1}{k}}$$

Were

$$ui^{*} = \frac{ui}{K} \qquad , K = \frac{4}{3\sqrt{\pi}}$$

$$-\left(\frac{\text{ti}}{\theta}\right)^{3} = \log 1 - (ui^{*})^{\frac{1}{k}}$$

$$ti = \left[-\theta^{3}\log(1 - ui^{*})^{\frac{1}{k}}\right]^{\frac{1}{3}}$$

Were :

$$ui^{*} = \frac{ui}{k} \qquad 0 < ui \leq 1$$

Since the aim of research is to compare fuzzy Reliability function of new exponentiated Maxwell distribution .

Were:

$$\breve{R}_T(ki\ ti) = \quad 1 - (\frac{4}{3\sqrt{\pi}})^{\hat{\beta}} \left[1 - e^{-\left(\frac{ki\ ti}{\theta}\right)^3}\right]^{\hat{\hat{\beta}}}$$

Ki: is vague factor

And β: is shape parameter , θ: is scale parameter

And ($\hat{\theta}$, $\hat{\beta}$) are estimated first by Maximum Likelihood and second by Proposed Method and third by probability Weighted Moment.

The sample size taken n=(25, 50, 75) and the results are compared by mean square error .

There are different values for ($\theta = 1.5$,3) , ($\beta = 2$,3.5) and ($\tilde{k} = 0.4$,0.6)

And each experiment is repeated $R = 300$

All the results of comparisons of fuzzy reliability functions, are explained in different Tables(1 ,2 ,3 ,4 ,5 ,6 ,7 ,8) and according to different sets of initial values . The comparison is done using statistical Measures Mean Square Error .

Table 1. Estimator Fuzzy Reliability when $\beta = 2$, $\theta = 1.5$, $\tilde{k} = 0.4$

n	t_i	Rreal Ri	$\hat{R}_{mle}$	$\hat{R}_{prop}$	$\hat{R}_{pwm}$	Best
25	1.5	0.4886	0.3862	0.4188	0.3996	Prop
	2.5	0.4637	0.4566	0.4886	0.4722	Prop
	3.5	0.5072	0.5029	0.5304	0.5169	Prop
	4.5	0.5614	0.5319	0.5592	0.5472	Prop
	5.5	0.5783	0.5536	0.5804	0.5891	Pwm
	6.5	0.5915	0.5702	0.5964	0.5986	Pwm
	7.5	0.6021	0.5832	0.6092	0.6089	Prop
	8.5	0.6172	0.6029	0.6179	0.6175	prop
50	1.5	0.4886	0.3874	0.3952	0.3916	prop
	2.5	0.4637	0.4597	0.4681	0.4647	MLE
	3.5	0.5072	0.5243	0.5132	0.5122	MLE
	4.5	0.5614	0.5346	0.5434	0.5406	Pwm
	5.5	0.5783	0.5565	0.5654	0.5628	MLE
	6.5	0.5915	0.5732	0.5822	0.5794	Prop
	7.5	0.6021	0.5862	0.5952	0.6032	Pwm

	8.5	0.6172	0.5966	0.6065	0.6118	pwm
75	1.5	0.4886	0.4613	0.4619	0.4582	Prop
	2.5	0.4637	0.5062	0.5072	0.5034	Prop
	3.5	0.5072	0.5367	0.5377	0.5342	Prop
	4.5	0.5614	0.5578	0.5588	0.5562	MLE
	5.5	0.5783	0.5759	0.5766	0.5732	Prop
	6.5	0.5915	0.5884	0.5898	0.5862	Prop
	7.5	0.6021	0.6092	0.6224	0.5967	Prop
	8.5	0.6172	0.6152	0.6162	0.6055	Prop

Table 2. Estimator Fuzzy Reliability when $\beta = 2 , \theta = 3 , \tilde{k} = 0.4$

n	t_i	Rreal Ri	$\widehat{R}_{mle}$	$\widehat{R}_{prop}$	$\widehat{R}_{pwm}$	Best
25	1.5	0.3287	0.3056	0.3396	0.3187	Prop
	2.5	0.3754	0.3689	0.3977	0.3832	Prop
	3.5	0.4166	0.4087	0.4352	0.4235	Prop
	4.5	0.4658	0.4373	0.4612	0.4513	Prop
	5.5	0.4817	0.4573	0.4803	0.4616	Prop
	6.5	0.4943	0.4852	0.4952	0.4772	Prop
	7.5	0.5043	0.4948	0.5068	0.4883	Prop
	8.5	0.5127	0.5030	0.5163	0.4993	prop
50	1.5	0.3287	0.3103	0.3261	0.3142	Prop
	2.5	0.3754	0.3742	0.3875	0.3778	Prop
	3.5	0.4166	0.4147	0.4266	0.4169	Prop
	4.5	0.4658	0.4427	0.4538	0.4578	Pwm
	5.5	0.4817	0.4632	0.4626	0.4684	Pwm
	6.5	0.4943	0.4787	0.4882	0.4942	Pwm
	7.5	0.5043	0.4912	0.5011	0.5065	Pwm
	8.5	0.5127	0.5013	0.5108	0.5216	pwm
75	1.5	0.3287	0.3116	0.3226	0.3229	Pwm
	2.5	0.3754	0.3762	0.3752	0.3768	Pwm
	3.5	0.4166	0.4168	0.4237	0.4272	Pwm
	4.5	0.4658	0.4452	0.4511	0.4455	Pwm
	5.5	0.4817	0.4656	0.4713	0.4662	Pwm
	6.5	0.4943	0.4873	0.4867	0.4832	MLE
	7.5	0.5043	0.5936	0.4987	0.4945	MLE
	8.5	0.5127	0.5637	0.5077	0.5129	MLE

Table 3. Estimator Fuzzy Reliability when $\beta = 3.5$, $\theta = 1.5, \tilde{k} = 0.4$

n	t_i	Rreal Ri	$\hat{R}_{mle}$	$\hat{R}_{prop}$	$\hat{R}_{pwm}$	Best
25	1.5	0.55	0.5821	0.5716	0.5836	Pwm
	2.5	0.63	0.6336	0.6216	0.6355	MLE
	3.5	0.643	0.6676	0.6543	0.6687	Pwm
	4.5	0.6572	0.6917	0.6672	0.6844	MLE
	5.5	0.6653	0.7097	0.6946	0.6932	MLE
	6.5	0.6750	0.7236	0.7082	0.7027	MLE
	7.5	0.6889	0.7348	0.7274	0.7272	MLE
	8.5	0.7091	0.7438	0.7352	0.7466	pwm
50	1.5	0.55	0.5618	0.6418	0.6084	Prop
	2.5	0.63	0.6234	0.6429	0.6421	Prop
	3.5	0.643	0.6568	0.6388	0.6633	MLE
	4.5	0.6572	0.6828	0.6684	0.6824	MLE
	5.5	0.6653	0.6994	0.6855	0.6957	MLE
	6.5	0.6750	0.7232	0.7006	0.7044	MLE
	7.5	0.6889	0.7245	0.7122	0.7056	MLE
	8.5	0.7091	0.7332	0.7266	0.7236	MLE
75	1.5	0.55	o.5626	0.5332	0.5536	MLE
	2.5	0.63	0.6133	0.6214	0.6024	Prop
	3.5	0.643	0.6468	0.6362	0.6342	MLE
	4.5	0.6572	0.6708	0.6612	0.6568	MLE
	5.5	0.6653	0.6888	0.6778	0.6672	MLE
	6.5	0.6750	0.7028	0.6949	0.6889	MLE
	7.5	0.6889	0.7142	0.7056	0.7009	MLE
	8.5	0.7091	0.7307	0.7152	0.7173	MLE

Table 4. Estimator Fuzzy Reliability when $\beta = 3.5$, $\theta = 1.2, \tilde{k} = 0.4$

n	t_i	Rreal Ri	$\hat{R}_{mle}$	$\hat{R}_{prop}$	$\hat{R}_{pwm}$	Best
25	1.5	0.5520	0.5832	0.5726	0.5825	MLE
	2.5	0.6021	0.6335	0.6214	0.6351	Pwm
	3.5	0.6333	0.6676	0.6531	0.6687	Pwm
	4.5	0.6472	0.6927	0.6672	0.6944	Pwm
	5.5	0.6888	0.7097	0.6846	0.7127	Pwm
	6.5	0.7202	0.7234	0.7672	0.7372	Prop
	7.5	0.7092	0.7346	0.7588	0.7474	Prop
	8.5	0.7167	0.7515	0.7276	0.7555	prop
50	1.5	0.5520	0.5624	0.5703	0.5726	Prop
	2.5	0.6021	0.6122	0.6083	0.6232	Prop
	3.5	0.6333	0.6468	0.6812	0.6573	Prop
	4.5	0.6472	0.6708	0.6544	0.6816	Prop
	5.5	0.6888	0.8844	0.6720	0.6997	Prop
	6.5	0.7202	0.7228	0.6857	0.7138	MLE
	7.5	0.7092	0.7142	0.7977	0.7141	Prop
	8.5	0.7167	0.7232	0.7166	0.7343	pwm
75	1.5	0.5520	0.5593	0.6546	0.5626	Prop
	2.5	0.6021	0.6088	0.6035	0.6132	Pwm
	3.5	0.6333	0.6426	0.6352	0.6469	Pwm
	4.5	0.6472	0.6652	0.6586	0.6721	Pwm
	5.5	0.6888	0.6828	0.6762	0.6892	Pwm
	6.5	0.7202	0.6965	0.6888	0.7028	Pwm
	7.5	0.7092	0.7075	0.7092	0.7142	Pwm
	8.5	0.7167	0.7243	0.7173	0.7233	pwm

Table 5. Estimator Fuzzy Reliability when $\beta = 2, \theta = 1.5, \tilde{k} = 0.6$

n	t_i	Rreal Ri	$\hat{R}_{mle}$	$\hat{R}_{prop}$	$\hat{R}_{pwm}$	Best
25	1.5	0.3102	0.3068	0.3386	0.3188	Prop
	2.5	0.3764	0.3698	0.3974	0.3820	Prop
	3.5	0.4166	0.4069	0.4345	0.4234	Prop
	4.5	0.4450	0.4371	0.4611	0.4416	Prop
	5.5	0.4658	0.4564	0.4802	0.4615	Prop
	6.5	0.4817	0.4728	0.4952	0.4782	Prop
	7.5	0.4942	0.4852	0.5066	0.4886	Prop
	8.5	0.5043	0.4948	0.5163	0.5082	Prop
50	1.5	0.3102	0.3104	0.3102	0.3186	Pwm
	2.5	0.3764	0.3742	0.3618	0.3822	Pwm
	3.5	0.4166	0.4146	0.4221	0.4241	Pwm
	4.5	0.4450	0.4426	0.4581	0.4423	Prop
	5.5	0.4658	0.4634	0.4596	0.4629	MLE
	6.5	0.4817	0.4966	0.4934	0.4766	MLE
	7.5	0.4942	0.5022	0.5054	0.5012	Prop
	8.5	0.5043	0.5093	0.5152	0.5193	Pwm
75	1.5	0.3102	0.3116	0.3216	0.3142	Pwm
	2.5	0.3764	0.3762	0.3847	0.3787	Prop
	3.5	0.4166	0.4168	0.4248	0.4196	Prop
	4.5	0.4450	0.4463	0.4525	0.4468	Pwm
	5.5	0.4658	0.4856	0.4728	0.4841	MLE
	6.5	0.4817	0.4823	0.4883	0.4755	Prop
	7.5	0.4942	0.4937	0.5004	0.5065	Pwm
	8.5	0.5043	0.5036	0.5186	0.5217	pwm

Table 6. Estimator Fuzzy Reliability when $\beta = 2, \theta = 3, \tilde{k} = 0.6$

n	t_i	Rreal Ri	$\hat{R}_{mle}$	$\hat{R}_{prop}$	$\hat{R}_{pwm}$	Best
25	1.5	0.2777	0.2967	0.3136	0.2848	Prop
	2.5	0.3172	0.3374	0.3506	0.3367	Prop
	3.5	0.3462	0.3663	0.3679	0.3677	Prop
	4.5	0.3667	0.3872	0.3962	0.3884	Prop
	5.5	0.3824	0.4031	0.4107	0.4250	Pwm
	6.5	0.4046	0.4166	0.4226	0.478	Pwm
	7.5	0.4132	0.4255	0.4397	0.4484	Pwm
	8.5	0.424	0.4338	0.4522	0.4369	prop
50	1.5	0.2777	0.2868	0.2869	0.4442	Pwm
	2.5	0.3172	0.3282	0.3284	0.4506	Pwm
	3.5	0.3462	0.3563	0.3563	0.4157	Pwm
	4.5	0.3667	0.3769	0.3769	0.4243	Pwm
	5.5	0.3824	0.3927	0.3927	0.4376	Pwm
	6.5	0.4046	0.4061	0.4051	0.4009	MLE
	7.5	0.4132	0.4162	0.4235	0.4102	MLE
	8.5	0.424	0.4305	0.4365	0.4154	prop
75	1.5	0.2777	0.2866	0.2718	0.2794	MLE
	2.5	0.3172	0.3265	0.3172	0.3198	MLE
	3.5	0.3462	0.3542	0.3474	0.3479	MLE
	4.5	0.3667	0.3743	0.3695	0.3684	MLE
	5.5	0.3824	0.3889	0.3858	0.3852	MLE
	6.5	0.4046	0.4018	0.4094	0.3965	Prop
	7.5	0.4132	0.4116	0.4179	0.4066	Prop
	8.5	0.424	0.4199	0.4256	0.4147	prop

Table 7. Estimator Fuzzy Reliability when $\beta = 3.5$, $\theta = 3$, $\tilde{k} = 0.6$

n	t_i	Rreal Ri	$\widehat{R}_{mle}$	$\widehat{R}_{prop}$	$\widehat{R}_{pwm}$	Best
25	1.5	0.5521	0.5722	0.5726	0.5825	Pwm
	2.5	0.5802	0.6235	0.6016	0.6350	Pwm
	3.5	0.5333	0.6576	0.6441	0.6697	Pwm
	4.5	0.5617	0.6827	0.6772	0.6944	Pwm
	5.5	0.6650	0.6087	0.6943	0.7128	Pwm
	6.5	0.6782	0.7236	0.7082	0.7274	Pwm
	7.5	0.7082	0.7347	0.7188	0.7383	Pwm
	8.5	0.7167	0.7525	0.7352	0.7466	MLE
50	1.5	0.5521	0.5663	0.5425	0.5556	MLE
	2.5	0.5802	0.6630	0.5704	0.6620	MLE
	3.5	0.5333	0.6664	0.6186	0.6623	MLE
	4.5	0.5617	0.6810	0.6524	0.7021	Pwm
	5.5	0.6650	0.6882	0.6758	0.7232	Pwm
	6.5	0.6782	0.7132	0.6788	0.7233	Pwm
	7.5	0.7082	0.7243	0.7032	0.7309	Pwm
	8.5	0.7167	0.7408	0.7161	0.7426	Pwm
75	1.5	0.5521	0.5625	0.7042	0.5506	Prop
	2.5	0.5802	0.6133	0.7066	0.6062	Prop
	3.5	0.5333	0.6468	0.7156	0.6396	Prop
	4.5	0.5617	0.4708	0.7231	0.6635	Prop
	5.5	0.6650	0.6888	0.7284	0.6814	Prop
	6.5	0.6782	0.7143	0.7372	0.6953	Prop
	7.5	0.7082	0.7232	0.7309	0.7064	Prop
	8.5	0.7167	0.7307	0.7233	0.7155	MLE

Table 8. Estimator Fuzzy Reliability when $\beta = 3.5 \quad , \theta = 1.5 \quad , \tilde{k} = 0.6$

n	t_i	Rreal Ri	$\hat{R}_{mle}$	$\hat{R}_{prop}$	$\hat{R}_{pwm}$	Best
25	1.5	0.5562	0.5832	0.5726	0.5825	MLE
	2.5	0.6034	0.6336	0.6216	0.6350	Pwm
	3.5	0.6333	0.6676	0.6543	0.6672	MLE
	4.5	0.6571	0.6908	0.6862	0.6933	Pwm
	5.5	0.6752	0.7097	0.6945	0.7028	MLE
	6.5	0.6889	0.7236	0.7081	0.7374	Pwm
	7.5	0.7372	0.7348	0.7176	0.7476	Pwm
	8.5	0.7167	0.7515	0.7352	0.7556	pwm
50	1.5	0.5562	0.5728	0.5725	0.7620	Pwm
	2.5	0.6034	0.6230	0.6703	0.6573	Prop
	3.5	0.6333	0.6568	0.6932	0.6814	Prop
	4.5	0.6571	0.6810	0.7088	0.6889	Prop
	5.5	0.6752	0.69990	0.7211	0.7138	Prop
	6.5	0.6889	0.7132	0.7311	0.7255	MLE
	7.5	0.7372	0.7242	0.7393	0.7343	Prop
	8.5	0.7167	0.7333	0.7463	0.7420	prop
75	1.5	0.5562	0.5635	0.6418	0.5592	MLE
	2.5	0.6034	0.6144	0.6029	0.6082	MLE
	3.5	0.6333	0.6468	0.6388	0.6412	MLE
	4.5	0.6571	0.6708	0.6653	0.6644	MLE
	5.5	0.6752	0.6889	0.6857	0.6820	MLE
	6.5	0.6889	0.7028	0.7005	0.6957	MLE
	7.5	0.7372	0.7140	0.7122	0.7066	MLE
	8.5	0.7167	0.7231	0.7218	0.7156	MLE

Conclusion:

From the above results for comparing fuzzy Reliability estimators we find that the first best one is the proposed estimator which depend on non parametric estimator for shape parameter , and also the probability Weighted Moment (pwm).

Also we apply the expontiated Maxwell , rathan ordinary Maxwell , for comparing fuzzy Reliability , since the exponentiated is necessary , when the values are small , so the powering to certain exponent , make the data , flexible and feasible for estimation of parameters , and then estimating Reliability or Risk function .

The results of comparison indicates that the first best fuzzy Reliability is the proposed one , and the second is the probability Weighted Moments , and the third one is maximum Likelihood estimator .

Here are the results of preference of fuzzy Reliability estimators for all tables percentage of preference of three different methods estimators:

Tables	$\hat{R}_{MLE}$	$\hat{R}_{prop}$	$\hat{R}_{pwm}$
Table (1)	$\frac{4}{24} = 0.1666$	$\frac{16}{24} = 0.6666$	$\frac{4}{24} = 0.1666$
Table(2)	$\frac{3}{24} = 0.125$	$\frac{11}{24} = 0.4583$	$\frac{10}{24} = 0.4166$
Table(3)	$\frac{18}{24} = 0.75$	$\frac{3}{24} = 0.125$	$\frac{3}{24} = 0.125$
Table(4)	$\frac{2}{24} = 0.0833$	$\frac{10}{24} = 0.4166$	$\frac{12}{24} = 0.5$
Table(5)	$\frac{3}{24} = 0.125$	$\frac{13}{24} = 0.5416$	$\frac{8}{24} = 0.3333$
Table(6)	$\frac{7}{24} = 0.2916$	$\frac{9}{24} = 0.375$	$\frac{8}{24} = 0.3333$
Table(7)	$\frac{5}{24} = 0.2083$	$\frac{7}{24} = 0.2916$	$\frac{12}{24} = 0.5$
Table(8)	$\frac{11}{24} = 0.4583$	$\frac{7}{24} = 0.2916$	$\frac{6}{24} = 0.25$

From the summary of results in this Table, it is concluded that MLE is best with percentage $\frac{53}{192} = 0.27604$ %, and then proposed is best with $\frac{86}{192} = 0.4479166\%$, And also Weighted Moments is best with $\frac{63}{192} = 0.328125\%$.

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