Compare Bayes estimators under Different Priors with the Classical estimators for Maxwell-Boltzmann distribution

Dr. Jinan Abbas Naser Al-obedy -Assistant professor
Technical College of Management-Baghdad,

Abstract

In this study, different estimators were used for estimating scale parameter for the Maxwell–Boltzmann distribution, such as maximum likelihood estimator, moment estimator and the Bayes estimator, in three types when the prior distribution for the scale parameter is (SRIG) distribution and ,the non-informative prior distribution and, the natural conjugate family of priors when the Bayesian estimation based on Squared Loss Function. Several cases from Maxwell–Boltzmann distribution for data generating , for different sample sizes (small, medium, and large).The results were obtained by using simulation technique, Programs written using MATLAB-R2008a program were used.

Simulation results shown that bayes estimation when the prior distribution is (SRIG) distribution with (a=b=3) gives the smallest value of MSE and MAE for all (n).And bayes estimation when the prior distribution is the non-informative prior distribution with (c=6) gives the smallest value of MSE and MAE for all (n).

Key words: The Maxwell–Boltzmann, Maximum likelihood method, Moment estimation method, Bayes method, the SRIG prior distribution, the non-informative prior distribution, the natural conjugate family of priors, mean squared errors (MSE), Mean Absolute Errors (MAE).

1. Introduction

The Maxwell–Boltzmann is the distribution for molecular speeds, and it can also refer to the distribution for velocities, moments, and magnitude of the moments of the molecules, each of which will have different probability distribution function. Many of studies discussed Maxwell distribution in different fields; we mention some of them in a brief manner: In (1989a, b) Tyagi and Bhattacharya [1], [2] considered Maxwell distribution as a lifetime model for the first time. They obtained Bayes estimates and minimum variance unbiased estimators of the parameter ad reliability function for the Maxwell distribution. In (1998) Chaturvedi and Rani [3] generalized Maxwell distribution by introducing one more parameter. They obtained classical and bayesian estimation procedures for this generalized distribution. In (2005) Bekker and Roux [4] studied empirical bayes estimation for Maxwell distribution. These studies give mathematical handling to Maxwell distribution but ignore the application aspect of the Maxwell distribution, and they have assumed that complete sample information is available. In (2006) Pasha , Muhammad & Muhammad [5] derived the variances of classical estimators and negative integer moment estimator from minimum variance bound with reference to Maxwell Distribution. In (2009) Krishna and Malik [6] studied reliability estimation for Maxwell distribution with type-II censored data. In (2010) Sanku and Maiti [7] derived bayes estimators of parameter of Maxwell distribution by considering non-informative as well as conjugate priors under different scale invariant loss functions, namely, quadratic Loss function, squared-Log. In (2011) Lu [8] used the Maxwell distribution as the acceptance sampling plans based on truncated life tests. In (2011) Sanku [9] studied bayes estimators of the parameter of a Maxwell distribution and obtained associated based on conjugate prior under scale invariant symmetric and a symmetric loss function. In (2012) Syed Mohsin , Muhammad and Sajid [10] studied the properties of bayes estimators of the parameter under different loss functions via simulated and real life data. They used the Maxwell distribution as a particular case, and they used simulation scheme based on non-informative and informative priors. Then they compared the loss functions through posterior risk. Also they derived closed form expressions for the complete sample, and they made some of other interesting comparison like credible interval (CI) and highest posterior density (HPD) intervals. In (2013) Sanku and Maiti [11] studied bayesian estimation and prediction for Maxwell distribution under conjugate prior. In (2013) Radha & Vekatesan [12] obtained the posterior distribution for the unknown parameter of Maxwell distribution by using general uniform and inverse gamma distribution. They developed the posterior predictive distribution using these informative priors.

So in this paper, we try to find best method to estimate parameter of Maxwell distribution .According to the smallest value of Mean Square Errors (MSE) and Mean Absolute Errors (MAE) were calculated to compare the methods of estimation. We used the maximum likelihood estimator, the moment estimator and the bayes estimator in three types, when the prior distribution is (SRIG) distribution and is the non-informative prior distribution and is the natural conjugate family of priors when the Bayesian estimation based on Squared Loss Function. Several cases from Maxwell–Boltzmann distribution for data generating ,for different sample sizes (small, medium, and large).The results were obtained by using simulation technique, Programs written using MATLAB-R2008a program were used.

2. Model Description

The Maxwell–Boltzmann is the distribution for molecular speeds, and it can also refer to the distribution for velocities, moments, and magnitude of the moments of the molecules, each of which will have different probability distribution function [13] ,[14]:

Defining, $\theta=\sqrt{\frac{KT}{M}}$ where K is the Maxwell constant, T is temperature, m is the mass of a molecule. The probability density function and the cumulative distribution function of Maxwell–Boltzmann distribution over the range for the random variable $x\in[0,\infty)$ are given by:

$$f(x;\theta)=\sqrt{\frac{2}{\pi}}\ \frac{x^2}{\theta^3}\ e^{-\frac{x^2}{2\theta^2}} \quad \text{for} \quad x,\theta>0 \qquad ...(1)$$

Where θ is a scale parameter, and $\pi=3.1416$.And the cumulative distribution function (cdf) of Maxwell distribution defined as follow:

$$F(x)=2\varphi(\frac{x}{\theta})-1-\sqrt{\frac{2}{\pi}}\ \frac{x}{\theta}\ e^{-\frac{x^2}{2\theta^2}}$$

Where $\varphi(x)$ is the unit normal function. We can define the r^{th} moment of Maxwell distribution as follow [15], [16]:

$$M_r=E(x^r)=\theta^r 2^{\frac{2+r}{2}}\ \frac{1}{\sqrt{\pi}}\ \Gamma(\frac{3+r}{2}) \quad , \quad r>-3$$

So that, for r=1 we get the mean of Maxwell distribution as follows:

$$M_1=E(x)=2\theta\sqrt{\frac{2}{\pi}} \qquad ...(4)$$

And the Variance (σ^2) as follows

$$\sigma^2=\theta^2\ \frac{(3\pi-8)}{\pi}\ ,$$

3. Parameter Estimation Methods

In this section, we can use several methods to estimation parameter θ.

3.1 Maximum likelihood method

We introduce the concept of maximum likelihood estimation with Maxwell–Boltzmann distribution. Let $\underline{x}=(x_1,x_2,\ldots,x_n)$ be a random sample of size (n) have an independent and identically distributed from Maxwell distribution, then the likelihood of the sample from Maxwell distribution with parameter θ is given by [17],[5]:

$$L(\underline{x}\setminus\theta)=\prod_{i=1}^{n} f(x;\theta)=\left(\frac{2}{\pi}\right)^{\frac{n}{2}}\prod_{i=1}^{n}x_i^2\ \theta^{-3n}\ e^{-\frac{\sum_{i=1}^{n}x_i^2}{2\theta^2}} \quad ...(6)$$

Then by taking the log-likelihood function for the equation (6), we get

$$\log L(\underline{x}\setminus\theta)=\frac{n}{2}\log\left(\frac{2}{\pi}\right)+\log\prod_{i=1}^{n}x_i^2-3n\log\theta-\frac{\sum_{i=1}^{n}x_i^2}{2\theta^2} \quad ...(7)$$

Now, differentiating partially equation (7) with respect to θ:

$$\frac{\partial}{\partial\theta}\log L(\underline{x}\setminus\theta)=-\frac{3n}{\theta}+\frac{\sum_{i=1}^{n}x_i^2}{\theta^3} \quad ...(8)$$

The MLE of θ is the solution of the likelihood equation (8) equal to zero, then the maximum likelihood estimator :

$$\frac{\partial}{\partial\theta}\log L(\underline{x}\setminus\theta)=0 \quad \Rightarrow \hat{\theta}_{MLE}=\sqrt{\frac{\sum_{i=1}^{n}x_i^2}{3n}} \quad ...(9)$$

3.2 Moments method (MM)

The method of moments is another technique commonly used in the field of estimation of parameters. If $\underline{x}=(x_1,x_2,\dots,x_n)$ be a random sample of size (n) represent a set of data, then an unbiased estimator for the r^{th} origin moment is[17],[5]:

$$m_r=\frac{\sum_{i=1}^{n}x_i^r}{n}$$

Where m_r stands for the r^{th} sample moment. The first moment of Maxwell distribution as equation (4)

$$M_1=E(x)=2\theta\sqrt{\frac{2}{\pi}}$$

Therefore by equating sample and population moments we get

$$m_1=M_1=E(x)=2\theta\sqrt{\frac{2}{\pi}} \quad ...(12)$$

From (12) we get

$$\bar{x}=2\theta\sqrt{\frac{2}{\pi}} \quad \Rightarrow \quad \hat{\theta}_{MM}=\bar{x}\ \left(\frac{1}{2}\right)\sqrt{\frac{\pi}{2}} \quad ...(13)$$

3.3 Bayes Estimation Method

Let $\underline{x}=(x_1,x_2,\dots,x_n)$ be a random sample of size n with probability density function given in equation (1) and likelihood function given in equation (6).We consider the Bays estimation of the parameter under different prior distributions which is mentioned below, here we consider three types of priors [17]:

3.3.1 Bayes estimation using first type:

It is assumed that θ follows Square Root Inverted Gamma (SRIG) distribution with pdf as given below [18]:

$$P(\theta)\ \alpha\ \frac{2b^a}{\Gamma(a)}\theta^{-(2a+1)}\ e^{-\frac{b}{\theta^2}} \quad \text{for} \quad a, b,\ \theta \geq 0 \qquad ...(14)$$

Where $\Gamma(.)$ is a gamma function .Then the posterior distribution of given the data $\underline{x} = (x_1, x_2, ..., x_n)$ is:

$$P(\theta \setminus x) = \frac{L(\underline{x} \setminus \theta)\ P(\theta)}{\int_{\theta} L(\underline{x} \setminus \theta)\ P(\theta) d\theta} \qquad ...(15)$$

Substituting the equation (6) and the equation (14) in equation (15), we get:

$$P(\theta \setminus x) = \frac{(\frac{2}{\pi})^{\frac{n}{2}} \prod_{i=1}^{n} x_i^2\ \theta^{-3n}\ e^{-\frac{\sum_{i=1}^{n} x_i^2}{2\theta^2}} \left[\frac{2b^a}{\Gamma a}\theta^{-(2a+1)}\ e^{-\frac{b}{\theta^2}}\right]}{\int_0^{\infty} (\frac{2}{\pi})^{\frac{n}{2}} \prod_{i=1}^{n} x_i^2\ \theta^{-3n}\ e^{-\frac{\sum_{i=1}^{n} x_i^2}{2\theta^2}} \left[\frac{2b^a}{\Gamma a}\theta^{-(2a+1)}\ e^{-\frac{b}{\theta^2}}\right] d\theta}$$

$$P(\theta \setminus x) = \frac{\theta^{-(2a+3n+1)}\ e^{-\frac{1}{\theta^2}\left[\frac{\sum_{i=1}^{n} x_i^2}{2} + b\right]}}{\int_0^{\infty} \theta^{-(2a+3n+1)} e^{-\frac{1}{\theta^2}\left[\frac{\sum_{i=1}^{n} x_i^2}{2} + b\right]} d\theta} \qquad ...(17)$$

We can write $\theta^{-(2a+3n+1)}$ as $\theta^{-(2(\frac{2a+3n}{2})+1)}$, and by multiplying the integral in equation (17) by the quantity which equals to

$$\left(\frac{2\left(\frac{\sum_{i=1}^{n} x_i^2}{2} + b\right)^{(\frac{2a+3n}{2})}}{\Gamma(\frac{2a+3n}{2})}\right) \left(\frac{\Gamma(\frac{2a+3n}{2})}{2\left(\frac{\sum_{i=1}^{n} x_i^2}{2} + b\right)^{(\frac{2a+3n}{2})}}\right)$$

, where $\Gamma(.)$ is a gamma function .Then we get ,

$$P(\theta \backslash x) = \frac{\theta^{-(2(\frac{2a+3n}{2})+1)}\; e^{-\frac{1}{\theta^2}[\frac{\sum_{i=1}^{n} x_i^2}{2}+b]}}{\left(\frac{\Gamma(\frac{2a+3n}{2})}{2(\frac{\sum_{i=1}^{n} x_i^2}{2}+b)^{(\frac{2a+3n}{2})}}\right) A(x;\theta)}$$

Where $A(x;\theta)$ equals to

$A(x;\theta) = \int_0^\infty \frac{2(\frac{\sum_{i=1}^{n} x_i^2}{2}+b)^{(\frac{2a+3n}{2})}}{\Gamma(\frac{2a+3n}{2})} \theta^{-(2(\frac{2a+3n}{2})+1)} e^{-\frac{1}{\theta^2}(\frac{\sum_{i=1}^{n} x_i^2}{2}+b)} d\theta = 1$ Be the integral of the pdf of the SRIG distribution .Then we get the posterior distribution of θ given the data $\underline{x} = (x_1, x_2, \ldots, x_n)$ is:

$$P(\theta \backslash x) = \frac{2(\frac{\sum_{i=1}^{n} x_i^2}{2}+b)^{(\frac{2a+3n}{2})}}{\Gamma(\frac{2a+3n}{2})} \theta^{-(2(\frac{2a+3n}{2})+1)} e^{-\frac{1}{\theta^2}(\frac{\sum_{i=1}^{n} x_i^2}{2}+b)} \quad \text{for} \quad n, a, b, \ \theta \geq 0 \qquad ...(20)$$

It means that $P(\theta \backslash x) \sim \text{SRIG}$ distribution with new parameters $\left(a_{(new)} = (\frac{2a+3n}{2}), b_{(new)} = (\frac{\sum_{i=1}^{n} x_i^2}{2}+b) \right)$.

Bayes estimator of θ by using Squared Loss Function $\ell(\hat{\theta}-\theta) = (\hat{\theta}-\theta)^2$, the risk function is [19]:

$$R(\hat{\theta}-\theta) = E[\ell(\hat{\theta}-\theta)] \qquad ...(21)$$

$$R(\hat{\theta}-\theta) = \int_\theta \ell(\hat{\theta}-\theta)\; P(\theta \backslash x)\, d\theta$$

$$R(\hat{\theta}-\theta) = \int_\theta (\hat{\theta}-\theta)^2\; P(\theta \backslash x)\, d\theta \quad \Rightarrow R(\hat{\theta}-\theta) = \int_\theta (\hat{\theta}^{\,2} - 2\theta\hat{\theta} + \theta^2)\; P(\theta \backslash x)\, d\theta$$

$$R(\hat{\theta}-\theta) = \hat{\theta}^{\,2} \int_0^\infty P(\theta \backslash x)\, d\theta - 2\hat{\theta} \int_0^\infty \theta\, P(\theta \backslash x)\, d\theta + \int_0^\infty \theta^2\, P(\theta \backslash x)\, d\theta \Rightarrow$$

$$R(\hat{\theta}-\theta) = \hat{\theta}^{\,2} - 2\hat{\theta} E(\theta \backslash x) + E(\theta^2 \backslash x) \qquad ...(22)$$

Let $\frac{\partial}{\partial \hat{\theta}} R(\hat{\theta}-\theta)=0$, we get Bayes estimator of θ denoted by $\hat{\theta}_{Bayes}$ for the above prior as follows

$$\hat{\theta}_{Bayes} = E(\theta \setminus x) = \int_0^\infty \theta \, P(\theta \setminus x)\, d\theta \qquad ...(23)$$

Substituting the equation (20) in equation (23), we get:

$$\hat{\theta}_{Bayes} = \int_0^\infty \theta \; \frac{2\left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)^{\left(\frac{2a+3n}{2}\right)}}{\Gamma\left(\frac{2a+3n}{2}\right)} \; \theta^{-\left(2\left(\frac{2a+3n}{2}\right)+1\right)} \; e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)} \; d\theta \qquad ...(24)$$

By multiplying the integral in equation (24) by the quantity which equals to $A_1 = \left(\frac{\Gamma\left(\frac{2a+3n-1}{2}\right)}{\Gamma\left(\frac{2a+3n-1}{2}\right)}\right)$, where $\Gamma(.)$ is a gamma function.

Then, we have

$$\hat{\theta}_{Bayes} = A_1 \int_0^\infty \frac{2\left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)^{\left(\frac{2a+3n}{2}\right)-\frac{1}{2}+\frac{1}{2}}}{\Gamma\left(\frac{2a+3n}{2}\right)} \; \theta^{-\left(2\left(\frac{2a+3n-1}{2}\right)+1\right)} \; e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)} \; d\theta$$

Then we have

$$\hat{\theta}_{Bayes} = \frac{\Gamma\left(\frac{2a+3n-1}{2}\right)}{\Gamma\left(\frac{2a+3n}{2}\right)} \left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)^{\frac{1}{2}} \left(A_2(x;\theta)\right)$$

Where $A_2(x;\theta)$ equals to

$$A_2(x;\theta) = \int_0^\infty \frac{2\left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)^{\left(\frac{2a+3n-1}{2}\right)}}{\Gamma\left(\frac{2a+3n-1}{2}\right)} \; \theta^{-\left(2\left(\frac{2a+3n-1}{2}\right)+1\right)} \; e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)} \; d\theta = 1$$

Be the integral of the pdf of the SRIG distribution. Then we get the Bayes estimator of θ as the following formula:

$$\hat{\theta}_{Bayes1} = \frac{\Gamma\left(\frac{2a+3n-1}{2}\right)}{\Gamma\left(\frac{2a+3n}{2}\right)} \left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)^{\frac{1}{2}} \qquad ...(26)$$

3.3.2 <u>Bayes estimation using second type:</u>

In this section, we consider that the parameter θ has the non-informative prior distribution and is given by

$$P_1(\theta)\ \alpha\ \frac{1}{\theta^c} \qquad \text{for} \quad \theta, c > 0 \qquad ...(27)$$

Then the posterior distribution of given the data $\underline{x} = (x_1, x_2, \ldots, x_n)$ according to the equation (15), we get it by substituting the equation (6) and the equation (27) in equation (15), so we have

$$P(\theta \setminus x) = \frac{(\frac{2}{\pi})^{\frac{n}{2}} \prod_{i=1}^{n} x_i^2\ \theta^{-3n}\ e^{-\frac{\sum_{i=1}^{n} x_i^2}{2\theta^2}} [\theta^{-c}]}{\int_0^{\infty} (\frac{2}{\pi})^{\frac{n}{2}} \prod_{i=1}^{n} x_i^2\ \theta^{-3n}\ e^{-\frac{\sum_{i=1}^{n} x_i^2}{2\theta^2}} [\theta^{-c}]\, d\theta}$$

$$P(\theta \setminus x) = \frac{\theta^{-3n-c}\ e^{-\frac{1}{\theta^2}[\frac{\sum_{i=1}^{n} x_i^2}{2}]}}{\int_0^{\infty} \theta^{-3n-c} e^{-\frac{1}{\theta^2}[\frac{\sum_{i=1}^{n} x_i^2}{2}]}\, d\theta} \qquad ...(29)$$

We can write θ^{-3n-c} as $\theta^{-(2(\frac{3n+c}{2})+1-1)} = \theta^{-(2(\frac{3n+c-1}{2})+1)}$, and by multiplying the integral in equation (29) by the quantity which equals to

$$\left(\frac{2(\frac{\sum_{i=1}^{n} x_i^2}{2})^{(\frac{3n+c-1}{2})}}{\Gamma(\frac{3n+c-1}{2})}\right) \left(\frac{\Gamma(\frac{3n+c-1}{2})}{2(\frac{\sum_{i=1}^{n} x_i^2}{2})^{(\frac{3n+c-1}{2})}}\right)$$

, where $\Gamma(.)$ is a gamma function .Then we get,

$$P(\theta \setminus x) = \frac{\theta^{-(2(\frac{3n+c-1}{2})+1)}\ e^{-\frac{1}{\theta^2}[\frac{\sum_{i=1}^{n} x_i^2}{2}]}}{\left(\frac{\Gamma(\frac{3n+c-1}{2})}{2(\frac{\sum_{i=1}^{n} x_i^2}{2})^{(\frac{3n+c-1}{2})}}\right) B(x;\theta)}$$

Where $B(x;\theta)$ equals to

$$B(x;\theta)=\int_0^\infty \frac{2\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)^{\left(\frac{3n+c-1}{2}\right)}}{\Gamma\left(\frac{3n+c-1}{2}\right)}\,\theta^{-\left(2\left(\frac{3n+c-1}{2}\right)+1\right)}\,e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)}\,d\theta=1$$

Be the integral of the pdf of the SRIG distribution .Then we get the posterior distribution of θ given the data $\underline{x}=(x_1,x_2,\ldots,x_n)$ is:

$$P(\theta\setminus x)=\frac{2\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)^{\left(\frac{3n+c-1}{2}\right)}}{\Gamma\left(\frac{3n+c-1}{2}\right)}\,\theta^{-\left(2\left(\frac{3n+c-1}{2}\right)+1\right)}\,e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)}\quad \text{for}\quad n,c,\theta\ge 0 \quad \ldots(31)$$

It means that $P(\theta\setminus x)\sim$ SRIG distribution

with parameters $\left(a=\left(\frac{3n+c-1}{2}\right),\, b=\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)\right)$.

Bayes estimator of θ by using Squared Loss Function

$\ell(\hat{\theta}-\theta)=(\hat{\theta}-\theta)^2$, the risk function according equation (21) and using the equation (22) .Let $\frac{\partial}{\partial\hat{\theta}}R(\hat{\theta}-\theta)=0$ for equation (22) , we get Bayes estimator of θ denoted by $\hat{\theta}_{Bayes2}$.

Substituting the equation (31) in equation (23), we get:

$$\hat{\theta}_{Bayes2}=\int_0^\infty \theta\,\frac{2\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)^{\left(\frac{3n+c-1}{2}\right)}}{\Gamma\left(\frac{3n+c-1}{2}\right)}\,\theta^{-\left(2\left(\frac{3n+c-1}{2}\right)+1\right)}\,e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)}\,d\theta \qquad \ldots(32)$$

By multiplying the integral in equation (32) by the quantity which equals to $B1=\left(\frac{\Gamma\left(\frac{3n+c-2}{2}\right)}{\Gamma\left(\frac{3n+c-2}{2}\right)}\right)$, where $\Gamma(.)$ is a gamma function. Then, we have

$$\hat{\theta}_{Bayes2}=B1\int_0^\infty \frac{2\left(\frac{\sum_{i=1}^n x_i^2}{2}+b\right)^{\left(\frac{3n+c-1}{2}\right)-\frac{1}{2}+\frac{1}{2}}}{\Gamma\left(\frac{3n+c-1}{2}\right)}\,\theta^{-\left(2\left(\frac{3n+c-2}{2}\right)+1\right)}\,e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)}\,d\theta$$

Then we have

$$\hat{\theta}_{Bayes2}=\frac{\Gamma\left(\frac{3n+c-2}{2}\right)}{\Gamma\left(\frac{3n+c-1}{2}\right)}\left(\frac{\sum_{i=1}^n x_i^2}{2}\right)^{\frac{1}{2}}\,(B2(x;\theta))$$

Where $B2(x;\theta)$ equals to

$$B2(x;\theta)=\int_0^{\infty}\frac{2\left(\frac{\sum_{i=1}^{n}x_i^2}{2}\right)^{\left(\frac{3n+c-2}{2}\right)}}{\Gamma\left(\frac{3n+c-2}{2}\right)}\,\theta^{-\left(2\left(\frac{3n+c-2}{2}\right)+1\right)}\,e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^{n}x_i^2}{2}\right)}\,d\theta=1$$

Be the integral of the pdf of the SRIG distribution. Then we get the Bayes estimator of θ as the following formula:

$$\hat{\theta}_{Bayes2}=\frac{\Gamma\left(\frac{3n+c-2}{2}\right)}{\Gamma\left(\frac{3n+c-1}{2}\right)}\left(\frac{\sum_{i=1}^{n}x_i^2}{2}\right)^{\frac{1}{2}} \qquad ...(34)$$

3.3.3 Bayes estimation using third type:

Here, we consider that the parameter θ has the natural conjugate family of priors is given by

$$P_2(\theta)\ \alpha\ \frac{1}{\theta^{\alpha+1}}\,e^{-\frac{\beta}{2\theta^2}} \qquad \text{for} \quad \theta,\alpha,\ \beta>0 \qquad ...(35)$$

Then the posterior distribution of given the data $\underline{x}=(x_1,x_2,\dots,x_n)$ according to the equation (15), we get it by substituting the equation (6) and the equation (35) in equation (15), so we have

$$P(\theta\setminus x)=\frac{\left(\frac{2}{\pi}\right)^{\frac{n}{2}}\prod_{i=1}^{n}x_i^2\,\theta^{-3n}\,e^{-\frac{\sum_{i=1}^{n}x_i^2}{2\theta^2}}\left[\frac{1}{\theta^{\alpha+1}}\,e^{-\frac{\beta}{2\theta^2}}\right]}{\int_0^{\infty}\left(\frac{2}{\pi}\right)^{\frac{n}{2}}\prod_{i=1}^{n}x_i^2\,\theta^{-3n}\,e^{-\frac{\sum_{i=1}^{n}x_i^2}{2\theta^2}}\left[\frac{1}{\theta^{\alpha+1}}\,e^{-\frac{\beta}{2\theta^2}}\right]d\theta}$$

$$P(\theta\setminus x)=\frac{\theta^{-3n-\alpha-1}\,e^{-\frac{1}{\theta^2}\left[\frac{\sum_{i=1}^{n}x_i^2+\beta}{2}\right]}}{\int_0^{\infty}\theta^{-3n-\alpha-1}\,e^{-\frac{1}{\theta^2}\left[\frac{\sum_{i=1}^{n}x_i^2+\beta}{2}\right]}d\theta} \qquad ...(37)$$

We can write $\theta^{-3n-\alpha-1}$ as $\theta^{-(3n+\alpha+1)}=\theta^{-\left(2\left(\frac{3n+\alpha}{2}\right)+1\right)}$, and by multiplying the integral in equation (37) by the quantity which equals to

$$\left(\frac{2\left(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2}\right)^{(\frac{3n+\alpha}{2})}}{\Gamma(\frac{3n+\alpha}{2})}\right)\left(\frac{\Gamma(\frac{3n+\alpha}{2})}{2\left(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2}\right)^{(\frac{3n+\alpha}{2})}}\right)$$, where $\Gamma(.)$ is a gamma function .Then we get,

$$P(\theta \backslash x)=\frac{\theta^{-(2(\frac{3n+\alpha}{2})+1)}\, e^{-\frac{1}{\theta^2}[\frac{\sum_{i=1}^{n} x_i^2+\beta}{2}]}}{\left(\frac{\Gamma(\frac{3n+\alpha}{2})}{2\left(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2}\right)^{(\frac{3n+\alpha}{2})}}\right) C(x;\theta)}$$

Where $C(x;\theta)$ equals to

$$C(x;\theta)=\int_0^\infty \frac{2\left(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2}\right)^{(\frac{3n+\alpha}{2})}}{\Gamma(\frac{3n+\alpha}{2})}\,\theta^{-(2(\frac{3n+\alpha}{2})+1)}\, e^{-\frac{1}{\theta^2}(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2})}\, d\theta=1$$ Be the integral of the pdf of the SRIG distribution .Then we get the posterior distribution of θ given the data $\underline{x}=(x_1,x_2,\dots,x_n)$ is:

$$P(\theta \backslash x)=\frac{2\left(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2}\right)^{(\frac{3n+\alpha}{2})}}{\Gamma(\frac{3n+\alpha}{2})}\,\theta^{-(2(\frac{3n+\alpha}{2})+1)}\, e^{-\frac{1}{\theta^2}(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2})} \quad ...(39)$$

It means that $P(\theta \backslash x) \sim SRIG$ distribution with parameters $(a=(\frac{3n+\alpha}{2}), b=(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2}))$.

Bayes estimator of θ by using Squared Loss Function

$\ell(\hat{\theta}-\theta)=(\hat{\theta}-\theta)^2$, the risk function according equation (21) ,and using the equation (22).

Let $\frac{\partial}{\partial \hat{\theta}} R(\hat{\theta}-\theta)=0$ for equation (22) , we get Bayes estimator of θ denoted by $\hat{\theta}_{Bayes3}$ .Substituting the equation (39) in equation (23), we get:

$$\hat{\theta}_{Bayes3}=\int_0^\infty \theta\, \frac{2\left(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2}\right)^{(\frac{3n+\alpha}{2})}}{\Gamma(\frac{3n+\alpha}{2})}\,\theta^{-(2(\frac{3n+\alpha}{2})+1)}\, e^{-\frac{1}{\theta^2}(\frac{\sum_{i=1}^{n} x_i^2+\beta}{2})}\, d\theta$$

By multiplying the integral in equation (32) by the quantity which equals to $C_1 = \left(\frac{\Gamma(\frac{3n+\alpha-1}{2})}{\Gamma(\frac{3n+\alpha-1}{2})} \right)$, where $\Gamma(.)$ is a gamma function.

Then, we have

$$\hat{\theta}_{Bayes3} = C_1 \int_0^{\infty} \frac{2\left(\frac{\sum_{i=1}^{n} x_i^2 + \beta}{2}\right)^{(\frac{3n+\alpha}{2}) - \frac{1}{2} + \frac{1}{2}}}{\Gamma(\frac{3n+\alpha}{2})} \theta^{-(2(\frac{3n+\alpha}{2})+1)} e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^{n} x_i^2 + \beta}{2}\right)} d\theta$$

Then we have

$$\hat{\theta}_{Bayes3} = \frac{\Gamma(\frac{3n+\alpha-1}{2})}{\Gamma(\frac{3n+\alpha}{2})} \left(\frac{\sum_{i=1}^{n} x_i^2 + \beta}{2}\right)^{\frac{1}{2}} \left(C_2(x;\theta) \right)$$

Where $C_2(x;\theta)$ equals to

$$C_2(x;\theta) = \int_0^{\infty} \frac{2\left(\frac{\sum_{i=1}^{n} x_i^2 + \beta}{2}\right)^{(\frac{3n+\alpha-1}{2})}}{\Gamma(\frac{3n+\alpha-1}{2})} \theta^{-(2(\frac{3n+\alpha-1}{2})+1)} e^{-\frac{1}{\theta^2}\left(\frac{\sum_{i=1}^{n} x_i^2 + \beta}{2}\right)} d\theta = 1$$

Be the integral of the pdf of the SRIG distribution. Then we get the Bayes estimator of θ as the following formula:

$$\hat{\theta}_{Bayes3} = \frac{\Gamma(\frac{3n+\alpha-1}{2})}{\Gamma(\frac{3n+\alpha}{2})} \left(\frac{\sum_{i=1}^{n} x_i^2 + \beta}{2}\right)^{\frac{1}{2}} \qquad ...(42)$$

4. Simulation Study

In this study, we have generated random samples from Maxwell-Boltzmann distribution and compared the performance of MLE and MME and Bayes estimator based on them. So we have considered several steps to perform simulation study as follow:

1. We have chosen sample size n = 10, 25, 50 and 100 to represent small, moderate and large sample size.
2. we generated data from Maxwell-Boltzmann distribution according to the following equation $x_i = \sqrt{U_{1i}^2 + U_{2i}^2 + U_{3i}^2}$ where $U_{ji} = \theta z_{ji}$ for $j = 1,2,3$ & $i = 1,2,3,\ldots,n$. And $z_i \sim$ standard normal distribution $(\mu = 0, \sigma = 1)$. For the scale parameter, we have considered several values for the parameter of Maxwell-Boltzmann distribution $\theta = 1,3,5,7$.
3. We used three values for the parameters of the SRIG distribution ($a = 1,2,3$ and $b = 1,2,3$) as prior distribution for θ.

4. We used three values for the function of the non-informative prior distribution $(P_1(\theta))$ c = 2, 4, 6.
5. We used three values for the function of the natural conjugate family of priors $(P_2(\theta))$ $\alpha = 1,2,3$ and $\beta = 1,2,3$.
6. The number of replication used was ($r = 1000$) for each sample size (n).
7. We obtained estimators for scale parameter from equations (9), (13), (26), (34), (42).

The simulation program was written by using MATLAB-R2008a program. After the parameter θ was estimated, Mean Square Errors (MSE) and Mean Absolute Errors (MAE) were calculated to compare the methods of estimation, where:

- $MSE = \frac{1}{r}\sum_{r=1}^{1000} (\hat{\theta}_i(r) - \theta_i)^2$ ...(43)
- $MAE = \frac{1}{r}\sum_{r=1}^{1000} | \hat{\theta}_i(r) - \theta_i |$ ...(44)

The results of the simulation study are summarized and tabulated in Tables (1,2,3,4).In each row of Tables (1,2,3,4) ,we have four estimated values for θ $(\hat{\theta})$ with MSE and MAE for all sample size (n) and values (a, b, c, α, β) respectively .By using different estimation methods that is maximum likelihood estimator and the moment estimator .The Bayes estimators in three types of prior distribution The best method is the method that gives the smallest value of (MSE) and (MAE). We list the results in the following tables (1 ,2,3,4).

Table 1: Shows the values for $\hat{\theta}$ with MSE and MAE.

Parameters			Estimator	Empirical value for (θ)				MSE				MAE			
				Sample Size(n)				Sample Size(n)				Sample Size(n)			
θ	a	b		10	25	50	100	10	25	50	100	10	25	50	100
1	-	-	$\hat{\theta}_{mle}$	0.9819	0.9999	0.9998	0.9967	0.0161	0.0070	0.0033	0.0017	0.1017	0.0673	0.0464	0.0328
1	-	-	$\hat{\theta}_{mm}$	0.9931	1.0024	1.0023	0.9976	0.0169	0.0075	0.0035	0.0018	0.1039	0.0696	0.0474	0.0335
			$\hat{\theta}_{Bayes1}$	by using (SRIG) prior											
1	1	1		1.0075	1.0099	1.0048	0.9992	0.0145	0.0069	0.0033	0.0017	0.0966	0.0664	0.0461	0.0326
	1	2		1.0399	1.023	1.0114	1.0026	0.0152	0.0071	0.0033	0.0016	0.0984	0.0674	0.0465	0.0325
	1	3		1.0714	1.036	1.018	1.0059	0.0179	0.0077	0.0035	0.0017	0.1073	0.0703	0.0474	0.0327
1	2	1		0.9759	0.9968	0.9982	0.9959	0.0142	0.0066	0.0032	0.0017	0.0953	0.0654	0.0457	0.0327
	2	2		1.0074	1.0098	1.0048	0.9992	0.0128	0.0065	0.0032	0.0016	0.0906	0.0647	0.0455	0.0323
	2	3		1.0379	1.0225	1.0113	1.0025	0.0134	0.0068	0.0032	0.0016	0.0925	0.0657	0.0459	0.0323
1	3	1		0.9473	0.9842	0.9917	0.9926	0.0156	0.0067	0.0032	0.0017	0.0996	0.0660	0.0459	0.0330
	3	2		0.9778	0.9970	0.9982	0.9959	0.0125	0.0063	0.0031	0.0016	0.0896	0.0637	0.0451	0.0324
	3	3		1.0073	1.0096	1.0047	0.9992	0.0114	0.0062	0.0031	0.0016	0.0854	0.0630	0.0449	0.0321
θ		c	$\hat{\theta}_{Bayes2}$	by using ($P_1(\theta)$) prior											
1	-	2		0.9902	1.0033	1.0015	0.9975	0.0145	0.0069	0.0033	0.0017	0.0966	0.0664	0.0461	0.0326
	-	4		0.9583	0.9901	0.9948	0.994	0.0152	0.0071	0.0033	0.0016	0.0984	0.0674	0.0465	0.0325
	-	6		0.9292	0.9774	0.9883	0.9909	0.0179	0.0077	0.0035	0.0017	0.1073	0.0703	0.0474	0.0327
θ	α	β	$\hat{\theta}_{Bayes3}$	by using ($P_2(\theta)$) prior											
1	1	1		1.0074	1.01	1.0048	0.9992	0.0156	0.0071	0.0033	0.0017	0.0999	0.0673	0.0464	0.0327
	1	2		1.0244	1.0167	1.0081	1.0009	0.0156	0.0071	0.0033	0.0017	0.0997	0.0675	0.0465	0.0326
	1	3		1.041	1.0233	1.0115	1.0026	0.0162	0.0073	0.0034	0.0017	0.1016	0.0683	0.0468	0.0326
1	2	1		0.9908	1.0033	1.0015	0.9975	0.0151	0.0069	0.0033	0.0017	0.0985	0.0666	0.0462	0.0327
	2	2		1.0075	1.0099	1.0048	0.9992	0.0145	0.0068	0.0032	0.0017	0.0966	0.0664	0.0461	0.0326
	2	3		1.0238	1.0165	1.0081	1.0009	0.0146	0.0070	0.0032	0.0016	0.0965	0.0667	0.0462	0.0325
1	3	1		0.9749	0.9967	0.9982	0.9959	0.0151	0.0068	0.0032	0.0017	0.0984	0.0663	0.0460	0.0328
	3	2		0.9913	1.0033	1.0015	0.9975	0.0141	0.0067	0.0032	0.0017	0.0953	0.0657	0.0458	0.0326
	3	3		1.0074	1.0098	1.0048	0.9992	0.0136	0.0067	0.0032	0.0016	0.0935	0.0655	0.0458	0.0325

Table 2: Shows the values for $\hat{\theta}$ with MSE and MAE.

Parameters			Estimator	Empirical value for (θ)				MSE				MAE			
				Sample Size(n)				Sample Size(n)				Sample Size(n)			
θ	a	b		10	25	50	100	10	25	50	100	10	25	50	100
3	-	-	$\hat{\theta}_{mle}$	2.9824	2.9845	2.9855	3.0011	0.1435	0.0612	0.0305	0.0143	0.1021	0.0659	0.0459	0.0318
3	-	-	$\hat{\theta}_{mm}$	3.0077	2.9948	2.991	3.0052	0.1548	0.0658	0.0325	0.0155	0.1059	0.0685	0.0472	0.0331
			$\hat{\theta}_{Bayes1}$	by using (SRIG) prior											
3	1	1		2.9689	2.9791	2.9827	2.9997	0.1407	0.0608	0.0305	0.0142	0.1013	0.0657	0.0459	0.0318
	1	2		2.9801	2.9836	2.985	3.0008	0.1390	0.0604	0.0304	0.0142	0.1006	0.0655	0.0458	0.0318
	1	3		2.9913	2.988	2.9872	3.0019	0.1377	0.0601	0.0302	0.0142	0.1	0.0653	0.0457	0.0318
3	2	1		2.8761	2.9404	2.9631	2.9897	0.1465	0.0623	0.0311	0.0143	0.1040	0.0664	0.0465	0.0319
	2	2		2.887	2.9448	2.9653	2.9908	0.1429	0.0616	0.0309	0.0142	0.1027	0.0660	0.0463	0.0319
	2	3		2.8978	2.9492	2.9675	2.992	0.1396	0.0610	0.0307	0.0142	0.1015	0.0657	0.0462	0.0319
3	3	1		2.7915	2.9032	2.9439	2.9799	0.1670	0.0667	0.0325	0.0145	0.1112	0.0689	0.0478	0.0323
	3	2		2.8021	2.9075	2.9461	2.981	0.1617	0.0657	0.0323	0.0144	0.1094	0.0683	0.0475	0.0322
	3	3		2.8126	2.9119	2.9483	2.9821	0.1568	0.0647	0.0320	0.0144	0.1077	0.0678	0.0473	0.0322
θ		c	$\hat{\theta}_{Bayes2}$	by using ($P_1(\theta)$) prior											
3	-	2		3.0074	2.9945	2.9904	3.0036	0.1407	0.0608	0.0305	0.0142	0.1013	0.0657	0.0459	0.0318
	-	4		2.9103	2.9551	2.9706	2.9936	0.1390	0.0604	0.0304	0.0142	0.1006	0.0655	0.0458	0.0318
	-	6		2.8222	2.9172	2.9512	2.9837	0.1377	0.0601	0.0302	0.0142	0.1	0.0653	0.0457	0.0318
θ	α	β	$\hat{\theta}_{Bayes3}$	by using ($P_2(\theta)$) prior											
3	1	1		3.0131	2.9968	2.9916	3.0041	0.1452	0.0612	0.0305	0.0143	0.1023	0.0660	0.0458	0.0319
	1	2		3.0188	2.999	2.9927	3.0047	0.1448	0.0611	0.0305	0.0143	0.1020	0.0660	0.0458	0.0319
	1	3		3.0245	3.0013	2.9938	3.0052	0.1445	0.0610	0.0304	0.0144	0.1018	0.0659	0.0458	0.0319
3	2	1		2.9633	2.9768	2.9816	2.9991	0.1416	0.0610	0.0305	0.0142	0.1017	0.0658	0.0459	0.0318
	2	2		2.9689	2.9791	2.9827	2.9997	0.1407	0.0608	0.0305	0.0143	0.1013	0.0657	0.0459	0.0318
	2	3		2.9745	2.9813	2.9839	3.0002	0.1398	0.0606	0.0304	0.0143	0.1010	0.0656	0.0458	0.0318
3	3	1		2.9159	2.9573	2.9718	2.9941	0.1429	0.0615	0.0308	0.0142	0.1026	0.0660	0.0462	0.0319
	3	2		2.9214	2.9595	2.9729	2.9947	0.1415	0.0612	0.0307	0.0142	0.1020	0.0658	0.0461	0.0319
	3	3		2.9269	2.9618	2.974	2.9952	0.1401	0.0609	0.0306	0.0142	0.1015	0.0657	0.0460	0.0318

Table 3: Shows the values for $\hat{\theta}$ with MSE and MAE.

Parameters			Estimator	Empirical value for (θ)				MSE				MAE			
				Sample Size(n)				Sample Size(n)				Sample Size(n)			
θ	a	b		10	25	50	100	10	25	50	100	10	25	50	100
5	-	-	$\hat{\theta}_{mle}$	4.9711	4.9978	4.9943	4.9867	0.4477	0.1638	0.0843	0.0438	0.1071	0.0648	0.0470	0.0333
5	-	-	$\hat{\theta}_{mm}$	5.006	5.0088	5.0062	4.9922	0.4618	0.1754	0.0897	0.0462	0.1092	0.0667	0.0481	0.0339
			$\hat{\theta}_{Bayes1}$	by using (SRIG) prior											
5	1	1		4.9366	4.9838	4.9873	4.9833	0.4423	0.1628	0.0841	0.0439	0.1065	0.0646	0.0470	0.0333
	1	2		4.9434	4.9865	4.9887	4.9839	0.4403	0.1625	0.0840	0.0439	0.1062	0.0645	0.0469	0.0333
	1	3		4.9501	4.9892	4.99	4.9846	0.4384	0.1623	0.0839	0.0438	0.1060	0.0645	0.0469	0.0333
5	2	1		4.7824	4.9191	4.9545	4.9668	0.4587	0.1649	0.0849	0.0444	0.1095	0.0651	0.0473	0.0336
	2	2		4.7889	4.9217	4.9559	4.9674	0.4548	0.1643	0.0847	0.0443	0.1090	0.0650	0.0473	0.0336
	2	3		4.7954	4.9244	4.9572	4.9681	0.4509	0.1637	0.0845	0.0443	0.1085	0.0649	0.0472	0.0336
5	3	1		4.6417	4.8568	4.9224	4.9504	0.5159	0.1749	0.0878	0.0455	0.1175	0.0673	0.0482	0.0341
	3	2		4.6481	4.8594	4.9237	4.9511	0.5103	0.1740	0.0875	0.0454	0.1168	0.0671	0.0481	0.0340
	3	3		4.6544	4.862	4.925	4.9518	0.5048	0.1731	0.0873	0.0453	0.1161	0.0670	0.0481	0.0340
θ		c	$\hat{\theta}_{Bayes2}$	by using $(P_1(\theta))$ prior											
5	-	2		5.0127	5.0145	5.0027	4.9909	0.4423	0.1628	0.0841	0.0439	0.1065	0.0646	0.0470	0.0333
	-	4		4.851	4.9485	4.9695	4.9743	0.4403	0.1625	0.0840	0.0439	0.1062	0.0645	0.0469	0.0333
	-	6		4.704	4.885	4.937	4.9579	0.4384	0.1623	0.0839	0.0438	0.1060	0.0645	0.0469	0.0333
θ	α	β	$\hat{\theta}_{Bayes3}$	by using $(P_2(\theta))$ prior											
5	1	1		5.0161	5.0158	5.0033	4.9912	0.4540	0.1651	0.0845	0.0438	0.1076	0.0651	0.0471	0.0332
	1	2		5.0196	5.0171	5.004	4.9916	0.4535	0.1650	0.0845	0.0438	0.1075	0.0650	0.0470	0.0332
	1	3		5.023	5.0185	5.0047	4.9919	0.4531	0.1650	0.0845	0.0438	0.1074	0.0650	0.0470	0.0332
5	2	1		4.9332	4.9825	4.9867	4.9829	0.4434	0.1629	0.0841	0.0439	0.1066	0.0646	0.0470	0.0333
	2	2		4.9366	4.9838	4.9873	4.9833	0.4423	0.1628	0.0841	0.0439	0.1065	0.0646	0.0470	0.0333
	2	3		4.94	4.9851	4.988	4.9836	0.4413	0.1627	0.0840	0.0438	0.1064	0.0645	0.0470	0.0333
5	3	1		4.8543	4.9498	4.9702	4.9747	0.4462	0.1630	0.0843	0.0441	0.1073	0.0647	0.0471	0.0334
	3	2		4.8577	4.9511	4.9709	4.975	0.4446	0.1628	0.0842	0.0441	0.1071	0.0646	0.0471	0.0334
	3	3		4.861	4.9525	4.9715	4.9753	0.4431	0.1626	0.0841	0.0440	0.1069	0.0646	0.0471	0.0334

Table 4: Shows the values for $\hat{\theta}$ with MSE and MAE.

Parameters			Estimator	Empirical value for (θ)				MSE				MAE			
				Sample Size(n)				Sample Size(n)				Sample Size(n)			
θ	a	b		10	25	50	100	10	25	50	100	10	25	50	100
7	-	-	$\hat{\theta}_{mle}$	6.9595	6.9929	6.9784	6.9858	0.8776	0.308	0.1697	0.0850	0.1071	0.0625	0.0474	0.0331
7	-	-	$\hat{\theta}_{mm}$	7.0084	7.0192	6.9955	6.9918	0.9052	0.3388	0.1778	0.0903	0.1092	0.0653	0.0482	0.0338
			$\hat{\theta}_{Bayes1}$	by using (SRIG) prior											
7	1	1		6.9066	6.9716	6.9677	6.9804	0.8690	0.3065	0.1697	0.0851	0.1066	0.0623	0.0474	0.0331
	1	2		6.9115	6.9735	6.9687	6.9809	0.8669	0.3062	0.1695	0.0851	0.1065	0.0623	0.0474	0.0331
	1	3		6.9163	6.9754	6.9696	6.9814	0.8649	0.306	0.1694	0.0850	0.1064	0.0623	0.0474	0.0331
7	2	1		6.6908	6.881	6.9219	6.9573	0.9029	0.312	0.1725	0.0859	0.1098	0.0629	0.0479	0.0333
	2	2		6.6955	6.8829	6.9228	6.9578	0.8989	0.3113	0.1723	0.0859	0.1095	0.0629	0.0478	0.0333
	2	3		6.7002	6.8848	6.9238	6.9583	0.8950	0.3107	0.1721	0.0858	0.1092	0.0628	0.0478	0.0333
7	3	1		6.494	6.7939	6.8769	6.9344	1.0166	0.3328	0.1794	0.0879	0.1178	0.0653	0.0489	0.0338
	3	2		6.4986	6.7958	6.8779	6.9349	1.011	0.3319	0.1791	0.0878	0.1175	0.0652	0.0488	0.0337
	3	3		6.5031	6.7976	6.8788	6.9354	1.0054	0.3309	0.1788	0.0877	0.1171	0.0651	0.0488	0.0337
θ		c	$\hat{\theta}_{Bayes2}$	by using ($P_1(\theta)$) prior											
7	-	2		7.0178	7.0163	6.99	6.9916	0.8690	0.3065	0.1697	0.0851	0.1066	0.0623	0.0474	0.0331
	-	4		6.7914	6.9239	6.9437	6.9684	0.8669	0.3062	0.1695	0.0851	0.1065	0.0623	0.0474	0.0331
	-	6		6.5856	6.8352	6.8983	6.9454	0.8649	0.306	0.1694	0.0850	0.1064	0.0623	0.0474	0.0331
θ	α	β	$\hat{\theta}_{Bayes3}$	by using ($P_2(\theta)$) prior											
7	1	1		7.0202	7.0172	6.9905	6.9918	0.8905	0.3102	0.1699	0.0850	0.1076	0.0628	0.0475	0.0330
	1	2		7.0227	7.0182	6.991	6.9921	0.8899	0.3101	0.1698	0.0850	0.1076	0.0628	0.0475	0.0330
	1	3		7.0251	7.0191	6.9915	6.9923	0.8894	0.3101	0.1698	0.0851	0.1075	0.0628	0.0475	0.0331
7	2	1		6.9042	6.9706	6.9672	6.9802	0.8700	0.3066	0.1697	0.0851	0.1067	0.0624	0.0474	0.0331
	2	2		6.9066	6.9716	6.9677	6.9804	0.8690	0.3065	0.1697	0.0851	0.1066	0.0623	0.0475	0.0331
	2	3		6.9091	6.9725	6.9682	6.9807	0.8679	0.3064	0.1696	0.0850	0.1066	0.0623	0.0475	0.0331
7	3	1		6.7938	6.9249	6.9442	6.9686	0.8761	0.3074	0.1706	0.0854	0.1075	0.0624	0.0476	0.0332
	3	2		6.7962	6.9258	6.9447	6.9688	0.8745	0.3072	0.1706	0.0854	0.1073	0.0624	0.0476	0.0332
	3	3		6.7985	6.9268	6.9452	6.9691	0.8730	0.3070	0.1705	0.0853	0.1072	0.0624	0.0476	0.0332

5. Discussion

In general, as we see in the tables (1-4) by using different estimation methods ,we find the Mean Square Error (MSE) and the Mean Absolute Error (MAE) decreased when sample size increased in all cases.

In table (1), we see that the estimated values for θ $(\hat{\theta})$ be greater than the true value of $\theta\,(\theta=1)$ by using the following methods of :- Moment (MM) for the two samples sizes n = 25, 50. Bayes when the prior distribution for the scale parameter θ is (SRIG) distribution with the same values for parameters a and b (a=b) for all samples sizes except n = 100, and for a less than b (a < b) in all samples sizes (n).Bayes when the prior distribution for the scale parameter θ is the non-informative prior distribution with (c =2) for the two samples sizes n = 25, 50. Bayes when the prior distribution for the scale parameter θ is the natural conjugate family of priors with the same values for parameters α and β $(\alpha=\beta)$ for all samples sizes except n = 100, and for α less than β $(\alpha<\beta)$ in all samples sizes (n).

For the estimated values for MSE and MAE, we see in bayes estimation when the prior distribution is (SRIG) distribution with fixed value of (a) and increases for value of (b), we obtain increased value of MSE and MAE for all (n).And in bayes estimation when the prior distribution is the non-informative prior distribution $(P_1(\theta))$ for increase value of c, we obtain increased value of MSE and MAE for all (n).And in bayes estimation when the prior distribution is the natural conjugate family of priors $(P_2(\theta))$ with fixed value of (α) and increases for value of (β),we obtain increased values for MSE and we obtain decrease values for MAE for all (n).

The best method is the bayes estimation when the prior distribution is (SRIG) distribution with (a=b=3) that gives the smallest value of MSE and MAE for all (n).

In table (2), we see that the estimated values for θ $(\hat{\theta})$ be greater than the true value of $\theta\,(\theta=3)$ by using the following methods of :- Maximum likelihood (MLE) for the sample size n =100.Moment (MM) for the two samples sizes n = 10,100.Bayes when the prior distribution for the scale parameter θ is (SRIG) distribution with the value for parameter a=1 and b=2, 3 for the sample size n = 100.Bayes when the prior distribution for the scale parameter θ is the non-informative prior distribution with c =2 for the two samples sizes n = 25, 50.Bayes when the prior distribution for the scale parameter θ is the natural conjugate family of priors with the values for parameters $(\alpha<\beta)$,when $\alpha=1$ and $\beta=1,2$ for the two samples sizes n = 10, 100, and with the value for parameter $\alpha = 1$ and $\beta = 3$ for all the samples sizes except n = 50.Also with $\alpha = 2$ and $\beta = 3$ for n=100.

And in table (3), we see that the estimated values for θ ($\hat{\theta}$) be greater than the true value of θ ($\theta = 5$) by using the following methods of :-Moment (MM) for all the samples sizes except n = 100.Bayes when the prior distribution for the scale parameter θ is the non-informative prior distribution with c =2 for all the samples sizes except n = 100.Bayes when the prior distribution for the scale parameter θ is the natural conjugate family of priors with the values for parameters $\alpha < \beta$,when $\alpha = 1$ and $\beta = 1,2,3$ all the samples sizes except n = 100.

And in table (4), we see that the estimated values for θ ($\hat{\theta}$) be greater than the true value of θ ($\theta = 7$) by using the following methods of :-Moment (MM) for the two samples sizes n = 10, 25.Bayes when the prior distribution for the scale parameter θ is the non-informative prior distribution with c =2 for the two samples sizes n = 10, 25.Bayes when the prior distribution for the scale parameter θ is the natural conjugate family of priors with the values for parameters $\alpha < \beta$, when $\alpha = 1$ and $\beta = 1,2,3$ for the two samples sizes n = 10, 25.

For the estimated values for MSE and MAE in tables (2-4) ,we see in bayes estimation when the prior distribution is (SRIG) distribution with fixed value of (a) and increases for value of (b), we obtain decreases value of MSE and MAPE for all (n).And in bayes estimation when the prior distribution is the non-informative prior distribution $(P_1(\theta))$ for increase value of c , we obtain decreases value of MSE and MAE for all (n). And in bayes estimation when the prior distribution is the natural conjugate family of priors $(P_2(\theta))$ with fixed value of (α) and increases for value of (β),we obtain decreases value of MSE and MAE for all (n).

The bayes estimation when the prior distribution is (SRIG) distribution with (a=1,b=3) and the bayes estimation when the prior distribution is the non-informative prior distribution with (c=6) are the best methods that gives the smallest value of MSE and MAE for all (n).

6.Conclusion

When we compared the estimated values for θ ($\hat{\theta}$) for the scale parameter of the Maxwell-Boltzmann distribution by using the methods in this study .We find that the (MSE) and the (MAE) decreased when sample size increased in all cases. And the MSE increased in all samples sizes (n) when the true value of θ increased. The best method is the bayes estimation when the prior distribution is (SRIG) distribution with (a=b=3) that gives the smallest value of MSE and MAE for all (n) see table (1).The bayes estimation when the prior distribution is (SRIG) distribution with (a=1,b=3) and the bayes estimation when the prior distribution is the non-informative prior distribution with (c=6) are the best methods that give the smallest value of MSE and MAE for all (n) see tables (2,3,4).

References

1. Tyagi, R.K. and Bhattacharya, S.K. (1989a). Bayes estimation of the Maxwell's velocity distribution function, Statistica, XLIX, 29(4), 563-567.
2. Tyagi, R.K. and Bhattacharya, S.K. (1989b). A note on the MVU estimation of reliability for the Maxwell failure distribution, Estadistica, 41(137), 73-79.
3. Chaturvedi, A. and Rani, U. (1998). Classical and Bayesian reliability estimation of the generalized Maxwell failure distribution. Statist. Research, 32, 113-120.
4. Bekker, A., Roux, J.J., (2005),'Reliability characteristics of the Maxwell distribution': a Baye estimation study', Comm. Stat (Theory & Meth.). Vol.34, No.11, pp. 2169-2178.
5. Pasha, G. R. & Muhammad, A. & Muhammad, J., (2006). 'Deviation of the Variances of Classical Estimators and Negative Integer Moment Estimator from Minimum Variance Bound with Reference to Maxwell Distribution', Pak. j. stat. oper. res. Vol.2, No.2 2006 pp145-150.
6. Krishna, H and Malik, M. (2009). Reliability estimation in Maxwell distribution with Type-II censored data. Inter Quality and Reliability Management, 26(2), 184-195.
7. Sanku, D., and Maiti, S. S., (2010),'Bayesian estimation of the parameter of Maxwell distribution under different loss functions', Journal of Statistical Theory and Practice, 4(2), 279-287.

8. Lu, W., (2011). Acceptance sampling plans based on truncated life tests for Maxwell distribution. Pak. J. Statist., 27(2), 159-170.
9. Sanku, D., (2011),'Bayesian estimation and prediction for Maxwell distribution', St .Anthony's college, Shillong, Meghalaya, India.
10. Syed Mohsin, A. K. and Muhammad, A. and Sajid, A., (2012),' preference of prior for the class of life – time distributions under different loss functions', Pak. J. Statist. Vol. 28(4), 467-484.
11. Sanku, D., and Maiti, S. S. (2013),' Bayesian estimation and prediction for Maxwell distribution under conjugate prior', Model Assisted Statistics and Applications, 8(3), 193-203. DOI: 10.3233/MAS-130249.
12. Radha, R.K., &Vekatesan, P., (2013),'On the double prior selection for the parameter of Maxwell distribution', IJSER © 2013.
13. Papoulis, A., (1984). Probability, Random Variables, and Stochastic Processes, 2nd ed. New York: McGraw-Hill, pp. 104 and 148-149.
14. Maxwell–Boltzmann distribution. (2014). Available at: From Wikipedia, the free encyclopedia. This page was last modified on 19 April 2014 at 16:46. http://en.wikipedia.org/wiki.
15. The Maxwell Distribution. Available at: http:// www.math .uah.edu /stat/ special/ Maxwell.html.
16. The Maxwell Distribution (2014). Available at: This page was last modified on 19 April 2014 at 16:46. http:// math world. Wolfram. Com/ Maxwell Distribution.html.
17. Bickel, P.J. &Doksum, K. A., (1977), " Mathematical Statistics: Basic Ideas and Selected Topics ", Holden- Day, Inc., San Francisco.
18. Ohakwe, J. and Okolic, N. and Obij, C. and Ugwud, N., (2012),' Properties of a square root gamma distribution', J. Math. Comput. Sci. 2(2012), No. 6, 1588-1597.ISSN: 1927-5307.
19. Nadia , J. Al-obedy (By internet) ,'Comparison of the scale parameter estimators for Maxwell distribution using different prior distributions',

مقارنة مقدرات بيز تحت افتراض دوال اولية مختلفة مع المقدرات الكلاسيكية لتوزيع Maxwell–Boltzmann

د. جنان عباس ناصر العبيدي – استاذ مساعد
الكلية التقنية الادارية – بغداد

الخلاصة

في هذا البحث ،استخدمت طرائق مختلفة لتقدير معلمة القياس لتوزيع Maxwell–Boltzmann ، كمقدر الإمكان الأعظم ومقدر العزوم ومقدر بيز في ثلاثة انواع مختلفة عندما يكون التوزيع الاولي لمعلمة القياس توزيع جذر مربع معكوس كاما و عندما يكون التوزيع الاولي توزيع non–informative والتوزيع الاولي لعائلة الدالة المرافقة الطبيعية، حيث اعتمد تقدير بيزن على مربع دالة الخسارة.عدة حالات لمعلمة القياس للتوزيع Maxwell–Boltzmann استخدمت لتوليد البيانات ولاحجام مختلفة من العينات (صغيرة ، متوسطة ، كبيرة).استحصلت النتائج باستخدام أسلوب المحاكاة،بكتابة برامج باستخدام MATLAB–R2008a. تبين نتائج المحاكاة بان مقدر بيزعندما يكون التوزيع الاولي لمعلمة القياس توزيع SRIG بالمعلمتين (a=b=3) يعطي اصغر قيمة لـ MSE و MAE لكل قيم n.ومقدر بيزعندما يكون التوزيع الاولي توزيع non–informative بالمعلمة (c=6) يعطي اصغر قيمة لـ MSE و MAE لكل قيم n.

مفاتيح الكلمات : توزيع Maxwell–Boltzmann،طريقة الإمكان الأعظم،طريقة العزوم ،طريقة بيز ،التوزيع الأولي SRIG،التوزيع الأولي non–informative ،التوزيع الأولي لعائلة الدالة المرافقة الطبيعية، متوسط مربع الاخطاء (MSE) ،متوسط الخطاء المطلق(MAE).