Smarandache Completely Semi Prime Ideal With Respect To An Element Of A Near Ring

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Abstract

In this paper ,we introduce the notions of smarandache completely semi prime ideal (S.C.S.P.I),and smarandache completely semi prime ideal with respect to an element x of a near ring N denoted by (x-S.C.S.P.I) , and smarandache completely semi prime ideal with respect to an element x near ring .Also we give some properties of these notions .

Key Words

Near ring ,ideal of near ring , near-ring homomorphism, the direct product of the near-rings, completely semi prime ideal, completely semi prime ideal with respect to an element of a near ring, near field, smarandache near ring, smarandache ideal, Smarandache direct product, Smarandache near-ring homomorphism.

Introduction

Throughout this paper N will be a left near ring . In 1989 the notion of completely semi prime ideal of a near ring (C.S.P.I) was introduced by P.DHeena [6] . In 2011 H.Hadi and Showq M. the notions of completely semi prime ideal with respect to an element x of a near ring and the completely semi prime ideals with respect to an element near ring (x-C.S.P.I near ring) [4] . They established many results and obtained many correspondents between (C.S.P.I) and (x-C.S.P.I) of a near ring . The purpose of this paper is as mention in the abstract .

1. Preliminaries

In this section we give some basic concepts that we need in the second section.

Definition (1.1) [2]

A left near ring is a set N together with two binary operations "+" and "." such that

a. (N,+) is a group (not necessarily abelian)

b. (N, .) is a semigroup.

c. $(n_1 + n_2) . n_3 = n_1 . n_3 + n_2 . n_3$

For all $n_1, n_2, n_3, \in N$;

Definition (1.2) [3]:

Let N be a near-ring. A normal subgroup I of (N,+) is called a left ideal of N if

i. $IN \in I$.

ii. $\forall\ n, n_1 \in N$ and for all $i \in I$,

$n.(n_1 + i) - n.n_1 \in I$.

Remark (1.3) [7]

We will refer that all near rings and ideals in this paper are left .

Definition (1.4)([8]

Let $\{Nj\}_{j\in J}$ be a family of near rings , J is an index set and $\prod_{j\in J} N_j = \{(x_j) : x_j \in N_j, \textit{for}$ all $j \in J$ $\}$ be the directed product of N_j with the component wise defined operations '+' and '.', is called the direct product near ring of the near rings N_j .

Definition (1.5) [1]

If I_1 and I_2 are ideals of a near ring N then $I_1 \cdot I_2 = \{i_1 \cdot i_2 : i_1 \in I_1, i_2 \in I_2\}$.

Definition (1.6) [8]

A near ring *N is called an integral domain if N has non -zero divisors*

Definition (1.7) [8]

Let N_1 and N_2 be two near-rings. The mapping $f : N_1 \rightarrow N_2$ is called a near-ring homomorphism if for all m, n $\in N_1$

f(m + n) = f(m) + f(n) and f(m. n) = f(m) f(n).

Theorem (1.8) [8]

Let $f : N_1 \rightarrow N_2$ is homomorphism

(1) If I is ideal of a near ring N_1 then f(I) is ideal of a near ring N_2.

(2) If J is ideal of a near ring N_2then $f^{-1}(J)$ is ideal of a near ring N_1.

Definition (1.9) [6]

An ideal I of N is called completely semi prime ideal(C.S.P.I) of a near ring .if $x^2 \in I$ imples $x \in I$ for any $x \in N$.

Definition (1.10)[7]

Let I be an ideal of a near ring N. Then I its called completely prime ideal of N if $\forall x, y \in N$, $x.y \in I$ implies $x \in I$ or $y \in I$, denoted by C.P.I of N .

Definition (1.11)[4]

let N be a near ring and $x \in N$, I is called completely semi prime ideal with respect to an element x denoted by (x-C.S.P.I) or(x- completely semi prime ideal)of N if for all $y \in N$,if $x \cdot y^2 \in I$ implies $y \in I$.

Definition (1.12) [8]

Anon- empty set N is said to be a near field if on N is defined by two binary operations "+","." such that

(1) (N,+) is a group ,

(2) (N\{0}, .) is a group ,

(3) a.(b+c)=a.b+a.c for all a,b,c belong to N .

Definition (1.13) [8]

The near ring (N,+,.) is said to be a smarandache near ring denoted by (S-near ring) if it has a proper subset M such that (M,+,.) is a near field .

Definition(1.14) [8]

Let N be S-near ring ,a normal subgroup I of N is called a smarandache ideal (S-ideal) of N related to M if ,

(1) $\forall$ y,z $\in$ M and $\forall$ i $\in$ I, y.(z+i)-y.z $\in$ I , where M is the near field contained in N.

(2) I.M $\subseteq$ I

Definition (1.15) [8]

Let $\{N_j\}_{j\in J}$ be a family of near-rings which has at least one S-near ring. Then this direct product $\prod_{j\in J} N_j$ with component wise defined operations

'+' and '.' is called Smarandache direct product (S-direct product) of near-rings.

Theorem (1.16) [8]

The S-direct product of family of near rings is a S-near-ring.

Definition (1.17) [8]

Let(N_1,+,.) and (N_2 ,+′,.′) be two S-near-rings, a function f : $N_1 \rightarrow N_2$ is called a Smarandache near-ring homomorphism (S-near-ring homomorphism) if for all m, n $\in M_1$ (M_1 is a proper subset of N_1 which is a near-field) we have f(m + n) = f(m) +′ f(n) and f(m. n) = f(m) .′ f(n) , where f (m) and f (n) $\in M_2$ (M_2 is a proper subset of N_2 which is a near-field)

Definition (1.18)[8]

Let(N_1,+,.) and (N_2 ,+′,.′) be two S-near-rings, a function f : $N_1 \rightarrow N_2$ is called a Smarandache near-ring homomorphism (S-near-ring homomorphism) if for all m, n $\in M_1$ (M_1 is a proper subset of N_1 which is a near-field) we have f(m + n) = f(m) +′ f(n) and f(m. n) = f(m) .′ f(n) , where f (m) and f (n) $\in M_2$ (M_2 is a proper subset of N_2 which is a near-field)

2. The main Results

This section is devoted to study Smarandache as completely semi prime ideal with respect to an element of a near ring .

Definition (2.1)

A S- ideal I of the S- near ring N related to the near field M is called a smarandache completely semi prime ideal with respect to an element x of N denoted by x- S. C.S.P.I of N if $x.y^2 \in I$ implies $y \in I$ for all $y \in M$.

Example (2.2)

Consider the S- near ring N $=z_{12}$,the ideal I ={0,3,6,9} is S-ideal related to the near field M={0,8,4}, I is 2-S.C.S.P.I of N since

$$2.y^2 \in I \text{ implies } y \in I \text{ for all } y \in M.$$

Remark (2.3)

In general not all x-S.C.S.P.I related to the near field M of a near ring N are x- C.S.P.I of N

Example (2.4)

Consider the S- near ring N in example (2.2) The S-ideal {0 } is 7-S.C.S.P.I related to the near field M of N but is not 7-C.S.P.I of N .

Remark (2.5)

Let N_1 and N_2 be two S- near rings, $f:N_1 \rightarrow N_2$ be an epimomorphism and N_1 has M_1 as near field then $M_2=f(M_1)$ is a near field of N_2 .

Proposition (2.6)

Let N_1 and N_2 be two S- near rings and $f:(N_1,+,.) \rightarrow (N_2,+',.')$ be an epimomorphism and I be S-ideal of N_1 .Then f(I) is S- ideal of N_2 .

Proof

Let M_2 be a near field of N_2 , M_1 be a near field of N_1 and $M_2 = f(M_1)$

(1) $Let\ y, z \in M_2 = f(M_1)$

$and\ i' \in f(I)$

$\exists r, s \in M_1, i \in I$ such that

$y = f(r)$, $z = f(s)$, $i' = f(i)$

$\Rightarrow f(r).'(f(s)+'f(i)) - f(r).'f(s)$

$= f(r\,(s+i) - r.s) \in f(I)$

$since\ \ r.(s+i) - r.s \in I$

(2) $I.M_1 \subseteq I$

$f(I.M_1) \subseteq f(I)$

$f(I).'f(M_1) \subseteq f(I)$

form (1),(2) we have f(I) is S-ideal of N_2. ▪

Theorem (2.7)

Let N_1 and N_2 be two S- near rings , $f : (N_1, +, .) \to (N_2, +', .')$ be an epimomorphism .If I be x- S. C.S.P.I of N_1 related to the near field M , then f(I) is f(x)- S . C.S.P.I related to the near field f(M) of N_2 .

Proof

Let I be x-S- C.S.P.I related to the near field M of N_1 $\Rightarrow$ f(I) is S- ideal related to the near field f(M) of N_2 .To prove f(I) is f(x)-S.C.P.I related to the near field f(M) of N_2 .

Let $c = f(y) \in f(M)$, $y \in M$ such that

$f(x).'c^2 = f(x).'(f(y))^2 \in f(I)$

$\Rightarrow f(x).'f(y^2) = f(x.y^2) \in f(I)$

[since f is an epimomorphism]

$\Rightarrow x.y^2 \in I \Rightarrow y \in I$ [since I is x-S.C.S.P.I related to the near field M of N_1]

$\Rightarrow c = f(y) \in f(I) \Rightarrow f(I)$ is a f(x)-S.C.S.P.I of N_2 . ▪

Proposition (2.8)

Let N_1 and N_2 be two S- near rings , $f : (N_1, +, .) \to (N_2, +', .')$ be epimomorphism and J be S-ideal related to the near field M_2 when $f(M_1)=M_2$ of N_2 .Then $f^{-1}(I)$ is S-ideal related to the near field M_1 of N_1 where y=f(x), $\ker f \subseteq f^{-1}(I)$.

Proof

To proof $f^{-1}(J)$ is S-ideal of N_1 , since J is S-ideal of N_2 $\Rightarrow$

(1) $Let\ r, s \in M_1 = f^{-1}(M_2)$

$and\ j \in f^{-1}(J)$

To prove

$r.(s+j) - r.s \in f^{-1}(J)$

$f(r.(s+j) - r.s) \in J$ $\quad since$ [J is S-ideal N_2].

$= f(r).'(f(s)+'f(j)) - f(r).'f(s) \in J$

$r.(s+j) - r.s \in f^{-1}(J)$

(2) $J.M_2 \subseteq J$

$f^{-1}(J.M_2) \subseteq f^{-1}(J)$

$f^{-1}(J).f^{-1}(M_2) \subseteq f^{-1}(J)$

form (1),(2) we have $f^{-1}(J)$ is S-ideal related to the near field M_1 of N_1. ▪

Theorem (2.9)

Let N_1 and N_2 be two S- near rings , $f : N_1 \to N_2$ be an epimomorphism and J be y-S. C.S.P.I related to the near field M_2 of N_2 .Then $f^{-1}(I)$ is x-S. C.S.P.I related to the near field M_1 of N_1 where y=f(x) , $\ker f \subseteq f^{-1}(I)$ and $M_2 = f(M_1)$.

Proof

By using proposition (2.8) we have $f^{-1}(J)$ is S- ideal related to the near field M_1 , Now to proof $f^{-1}(J)$ is a x-S.C.S.P.I related to the near field M_2 of N_1 .

Let $z \in N_1$ such that

$x.z^2 \in f^{-1}(J)$

$\Rightarrow f(x.z^2) \in J$

$= f(x).f(z)^2 = f(x).(f(z))^2$

$f(x).(f(z))^2 = y.(f(z))^2$

$\Rightarrow f(z) \in J$ [since J is y-S.C.S.P.I related to the near field M_2 of N_2]

$\Rightarrow z \in f^{-1}(J)$

$\Rightarrow f^{-1}(J)$ is x-S.C.S.P.I related to the near field M_1 of N_1 , y = f(x).

Definition (2.10)

Let N be S- near ring we call the S-ideal related to the near field M as a completely prime related to the near field M of N if $y.z \in I$ implies $y \in I$ or $z \in I$ for any $z,y \in M$. denoted by S.C.P.I of N .

Example (2.11)

Consider the near ring $N=Z_6$ with addition and multiplication as defined by the following tables .

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$\cdot_6$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

N has M={0,2,4} as a near field then N is a S-near ring the S- ideal I={0,3} related to the near field M of N is S.C.P.I related to the near field M of N since $y.z \in I$ *implies* $y \in I$ or $z \in I$ for any $z,y \in M$.

Theorem (2.12)

Let N_1 and N_2 be two S- near rings , $f : N_1 \to N_2$ be epimomorphism and I be S. C.P.I related to the near field M of N of N_1 . Then f(I) is S. C.P.I related to the near field f(M) of N of N_2 .

Proof

By Proposition (2.6) we have f(I) is S- ideal related to the near field M of N_2.

To proof f(I) is a S.C.P.I of N_2 .

Let $f(y), f(z) \in f(M)$ such that

$f(y).f(z) \in f(I)$

$\Rightarrow f(y.z) \in f(I)$

$\Rightarrow y.z \in I$

$\Rightarrow y \in I$ or $z \in I$ [since I is S.C.P.I related to the near field M of N_1]

$\Rightarrow f(y) \in f(I)$ or $f(z) \in f(I)$

$\Rightarrow$ f(I) is S.C.P.I related to the near field f(M) of N_2 .▪

Theorem (2.13)

Let N_1 and N_2 be two S- near rings , $f : N_1 \to N_2$ be an epimomorphism and J be S. C.P.I related to the near field M_2 of N_2 .Then $f^{-1}(J)$ is S. C.P.I related to the near field M_1 of N_1 ,where $f(M_1)=M_2$, $\ker f \subseteq f^{-1}(I)$.

Proof

Let $y, z \in M_1$ such that

$y.z \in f^{-1}(J)$

$\Rightarrow f(y.z) \in J$

$\Rightarrow f(y.z) = f(y).f(z) \in J$

$\Rightarrow f(y) \in J$ or $f(z) \in J$ [since J is S.C.P.I related to the near field M_2 =f(M_1) of N_2]

$\Rightarrow y \in f^{-1}(J)$ or $z \in f^{-1}(J)$

$\Rightarrow f^{-1}(J)$ is S.C.P.I related to the near field M_1 of N_1 .▪

Definition (2.14)

A S- ideal I of the near ring N is called smarandache completely semi prime ideal related to the near field M denoted by S.C.S.P.I of N if $y^2 \in I$ *implies* $y \in I$ for any $y \in M$.

Example (2.15)

Consider N =Z_{12} in example (2.2)

See that ..

N is S- near ring since N is near ring has proper subset a near field M= {0,4,8}.

Let I={0,2,4,6,8,10}is S- ideal related to the near field M ,Since

1. ∀ y,z ∈ M and for all i ∈ I ,y.(z+i)-y.z ∈ I , where M is the near field contained in N .
2. I.M ⊆ I .

I is S. C.S.P.I related to the near field M of N since $y^2 \in I$ *implies* $y \in I$ for any $y \in M$.

Remark (2.16)

Not all S-C.S.P.I related to a near field M of S-near ring N is x-S. C.S.P.I related to a near field M of N .

Example (2.17)

Consider the S- near ring N in example (2.11) ,the ideal I={0,3}is S-C.S.P.I related to a near field M={0,2,4} of N but is not is 3-S. C.S.P.I related to a near field M of N since $3.2^2 = 0 \in I$ but $2 \notin$ I , $2 \in$ M..

proposition (2.18)

If N is non zero S-near ring and I={0} then I is not 0-S.C.S.P.I of N .

Proof

Suppose I is 0-S.C.S.P.I related to a near field M of N and y∈N .

$\Rightarrow 0.y^2 \in I$

$\Rightarrow y \in I$ [since I is 0-S.C.S.P.I related to the near field M]

$\Rightarrow y = 0$

$\Rightarrow M = \{0\}$

and this contradiction since $M \neq \{0\}$.

Then I is not 0-S.C.S.P.I related to the near field M of N .

Proposition (2.19)

Let I be S- ideal related to a near field M of S-near ring N such that $M \not\subset I$, then I is not 0-S.C.S.P.I related to a near field M of N .

Proof

Suppose that I is 0-S.C.S.P.I related to a near field M of N *and* $y \in M$

$\Rightarrow o.y^2 = 0 \in I$

$\Rightarrow y \in I$ [since I is 0-S.C.S.P.I related the near field M of N]

$\Rightarrow$ M $\subseteq$ I this contradiction since M $\not\subset$ I]

$\Rightarrow I$ is not 0-S.C.S.P.I of N. ▪

Definition (2.20)

The S- near ring N is called Smarandache completely semi prime ideals related to a near field M with respect to an element x of N it denoted by (x-S. C.S.P.I near ring), if every S-ideal related to a near field M of N is x-S. C.S.P.I of N .

Example (2.21)

Consider the S- near ring N in example (2.11) let M={0,2,4} is a near field I_1={0}, I_2 ={0,3}and I_3 ={0,2,4} are the only 5-S.C.S.P.I of N that mean N is 5-S-C.S.P.I related to a near field M near ring .

Theorem (2.22)

Let $\{N_j\}_{j \in J}$ be a family of S- near rings $x_j \in N_j$ and I_j be x_j –S. C.S.P.I related to a near field M_j of N_j for all $j \in J$ .Then $\prod_{j \in J} I_j$ is (x_j)–S. C.S.P.I related to each near field M'_j of $\prod_{i \in J} N_j$ where M'_j is the near field of $\prod_{i \in J} N_j$ whos elements (m_j) is equal to m_j and all other components are zeros.

Proof

By theorem (1.12), we have $\prod_{i\in J} N_j$ is S- near ring

$\Rightarrow \prod_{j\in J} I_j$ be S-ideal of $\prod_{i\in J} N_j$ since

(1) let $(y_j),(z_j) \in M'_j$ and $i_k \in \prod_{j\in J} I_j$

$(y_j).((z_j)+(i_k))-(y_j).(z_j) = (y_j.z_j + y_j.i_k - y_j.z_j) \in \prod_{j\in J} I_j$

(2) $\prod_{j\in J} I_j . M'_j \subseteq \prod_{j\in J} I_j \ \forall j \in J$

$\Rightarrow \prod_{j\in J} I_j$ is S-ideal related to the near field M'_j

To proof $\prod_{j\in J} I_j$ is (x_j)-S. C.S.P.I related to each near field M'_j of $\prod_{j\in J} N_j$

Let $(y_j) \in \prod_{j\in J} M'_j$ such that

$(x_j)(y_j)^2 \in \prod_{j\in J} I_j$

$\Rightarrow (x_j.y_j^2) \in \prod_{j\in J} I_j$

$\Rightarrow x_j.y_j^2 \in I_j$ [since I_j is $x_j - S.C.S.P.I$]

$\Rightarrow y_j \in I_j$

$\Rightarrow (y_j) \in \prod_{j\in J} I_j$

$\prod_{j\in J} I_j$ is (x_j)- S.C.S.P.I related to each near field M'_j of $\prod_{j\in J} N_j$ ▪

Remark (2.23)

Not all S- ideal related to a near field M is an ideal of the S-near ring N.

Example (2.24)

Consider the S-near ring N in example (2.2) , Let I={0,3,9} is S- ideal related to a near field M but it is not an ideal of the S-near ring N.

Proposition (2.25)

Every trivial S- ideal related to a near field M is an ideal of the S-near ring N.

Proof

Let $r,s \in M$

If I={0}

$\Rightarrow$ r.(s+0)-r.s= $0 \in I$

$\forall$r,s$\in$M

{0}.X={0}$\subseteq${0}.

{0} is S- ideal related to near field M

if I=N and r,s$\in$M

$\Rightarrow$r.(s+i)-r.s $\in N$ $\forall$r,s$\in$X,i$\in$N,

$\Rightarrow$N.X$\subseteq$N

$\Rightarrow$ I is an ideal of N. ▪

Remark (2.26)

Not all S- C.S.P.I related to a near field M of the S-near ring N is a prime ideal .

Example (2.27)

Consider the S-near ring N in example (2.11)

I={0} is S- C.S.P.I related to a near field M={0,2,4} of N when I_1 ={0,3} and I_2 ={0,2,4}

$I_1.I_2 = \{0\} \subset I$ but $\{0,3\} \not\subset I$ and $\{0,2,4\} \not\subset I$

$\Rightarrow$ I is not a prime ideal .

Remark (2.28)

Not all S- C.S.P.I related to a near field M of the S-near ring N is C.P.I of N .

Example (2.29)

Consider the S-near ring N in example (2.11)

Let I={0} is S- C.S.P.I of N

4.3=0 ,but $4 \notin I$ and $3 \notin I$

$\Rightarrow$ I is not C.P.I ideal of N .

Remark (2.30)

Not all x-S. C.S.P.I of the S-near ring N is C.P.I of N .

Example (2.31)

Consider the S-near ring N in example (2.11)

Let I={0} is 5-S. C.S.P.I related to a near field M={0,2,4} of N

2.3=0 , but $2 \notin I$ and $3 \notin I$

$\Rightarrow$ I is not C.P.I ideal of N .

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الخلاصة

قدمنا في هذا البحث مفاهيم وهي مثالية سمرندش الشبه الأولية التامة ومثالية سمرندش الاولية الشبه الاولية التامة بالنسبة لعنصر x في الحلقة القريبة والذي يرمز لها (x-S-C.S.P.I) كما درسنا الحلقة القريبة للمثاليات سمرندش الاولية الشبه تامة بالنسبة لعنصر x. كما أعطينا بعض خواص هذه المفاهيم .