A Technique To Solve The Multi Index Transportation Problems

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المستخلص :

تعتبر تطبيقات مسائل البرمجة الخطية متعدده المؤشر Multi index linear programming problems ذات اهمية كبيره في مجالات متعدده، أمثلة على ذلك مشاكل تخصيص الموارد في النظم الهرمية: معالجة مشاكل مكثفات الغاز ، مشاكل نقل البيانات في الوصلات ذات السعة المحدودوغيرها من المشاكل .في هذا البحث تم تناول اصناف البرمجة الخطية متعدده المؤشر ، كذلك العروج الى مسائل النقل عندما تكون هناك محطات وسطية بين الوحدات المنتجة والوحدات المستهلكه وقد تم بناء خريطة التي توضح عمل خوارزمية orlin المستخدمة لحل النموذج النقل المتكون من ثلاث مؤشرات، اضافة الى ما تقدم فقد تناول البحث مثال يبيناسلوب الحل.

Abstract

In this paper, the problem multi-index transportation with intermediate resource centers (warehouses) was taken. The mathematical model of this problem and discussed possible approaches using known algorithm (orlin,1993, 338) to solve it. We applied this algorithm for finding optimal Plan distribution goods that corresponds to criteria optimality in the volume of production, consumption, and the amount of storage stocks.

Keywords Multi index linear programming problems, transportation problem, and the problem of resource allocation.

1.Introduction

There is a wide class of applications to formalize in the form of multi-index linear programming transport type problems. Examples of such problems are the problems of resource allocation in hierarchical systems: task processing of gas condensate (Kostukov, 2010,5), the task of distribution facilities data channels, the transportation problem with intermediate points (Afraimovich, 2010, 148, 2006, 194).Multi index linear programming problem transportation type belongs to a class of linear programming problems, which is in accordance with polynomial solvable (Khachiyan,1979,1093,Yule,2013,25). In the case of multi-index (index number at least three) most attention on two classes of problems: problems and index axial multi-index planar problem. Research questions and build consistency algorithms for solving these problems are discussed (Queyranne,1997,239,Vlach,1986,61).Geometric properties of the set of feasible solutions multi-index transportation problems are discussed in (De Loera, 2009,1306).

2.Formulation and classification of multi-index linear programming

Let $f = \{1,2,...,s\}$, where f represented s of natural numbers.

Each number $\ell \in f$ put into correspondence to parameter j_ℓ named index which can take one value from the set $\{1,2,...,n_\ell\} = J_\ell$, set values of the indices $(j_1, j_2,..., j_s)$ is called s-index denoted by F .

$$F=(j_1,j_2,...,j_s)\ ,\ j_1\in\{1,2,...,n_1\}=J_1$$
$$j_2\in\{1,2,...,n_2\}=J_2$$
$$\dots\dots\dots\dots\dots\dots$$
$$j_s\in\{1,2,...,n_s\}=J_s$$

It is clear that there $N=\prod_{\ell=1}^{s}$

Different sets of values of the indices $(j_1,j_2,...,j_s)$ these sets combine to form a set E s -index F , which is the direct product of sets $J_1,J_2,...,J_s$.

$E=\{F_1,F_2,...,F_N\}=J_1\times J_2\times...\times J_s$

Each s – index element $F\in E$ put in line with the real number $x_{j_1j_2...j_s}=x_F$ .The set of numbers for all possible values of the indices $j_1,j_2,...,j_s$ called s – index matrix and denoted: $\{x_{j_1j_2...j_s}\}=\{x_F\}$

Obviously, s – index array $\{x_F\}$ contain N element.

Let $f_i=\{k_1^i,k_2^i,...,k_{t_i}^i\}\subset f$ some random non-empty proper (own) subset of F , let us assume that the order of element in f_i not important, then $M=\sum_{\ell}^{s-1}C_s^{\ell}=2^s-2$

Options formulate subsets f_i . Each subset f_i corresponds to a subset of indices $\{j_{k_1^i},j_{k_2^i},...,j_{k_n^i}\}\subset F.$

Set in indexes $\{j_{k_1^i},j_{k_2^i},...,j_{k_{t_i}^i}\}$ that is t_i index denoted F_i. so $F_i=(j_{k_1^i},j_{k_2^i},...,j_{k_{t_i}^i})\subset F$.

Where $j_{k_1^i}\in(1,2,...,n_{k_1^i})=j_{k_1^i},j_{k_2^i}\in(1,2,...,n_{k_2^i})=j_{k_2^i},...,j_{k_{t_i}^i}\in(1,2,...,n_{k_{t_i}^i})=j_{k_{t_i}^i}$.

The set F_i for all possible values of indices $j_{k_1^i},j_{k_2^i},...,j_{k_{t_i}^i}$ forms a set of $E_i=J_{k_1^i}\times J_{k_2^i}\times...\times J_{k_{t_i}^i}$ .which obviously contains $N_i=\prod_{\ell\in f_i}n_\ell$ elements.

Addition :

$\bar{f}_i=f\setminus f_i$ subsets f_i to the set of f is called the set of elements f , are not include in f_i , addition f_i corresponds to subset $\overline{F}_i=F\setminus F_i$ and $\overline{E}_i=\prod_{\ell\subseteq f_i}J_\ell$.

Note that E_i and $\overline{E}_i$ interconnected so direct formation forms a set of $E=E_i\times\overline{E}_i$

The set of all component s – index $\{x_F\}$ with fixed indices $F_i=(j_{k_1^i},j_{k_2^i},...,j_{k_{t_i}^i})$ called $(s-t_i)$ dimensional cross-section orientation $\overline{F}_i$ and denoted $X_{Fi}^{\overline{F}_i}$.

The cross section is denoted by corresponding letters on the right side which written $(s-t_i)$ index orientation $\overline{F_i}$, and below the value of fixed t_i index F_i.

Cross section $X_{Fi}^{\overline{F_i}}$ forms $(s-t_i)$ index-sub matrices $\{x_{\overline{F_i}}\}$matrix $\{x_F\}$. The number of section of the orientation F_i equal N_i -number of elements of the set E_i.

Sometimes it will be more convenient s – index matrix $\{x_F\}$denoted $\{x_{F_i\overline{F_i}}\}$.

Note that the equality $\{x_{j_1,j_2,\dots j_s}\}=\{x_F\}=\{x_{F_i\overline{F_I}}\}$ for all f_i.

In order to reduce and simplify the results , we introduce the following notation:

$$\sum_{j_1\in J_1}\sum_{j_2\in J_2}\dots\sum_{j_s\in J_S} x_{j_1 j_2\cdots j_s} = \sum_{J_1\times J_2\times\dots\times J_s} x_{j_1 j_2\cdots j_s} = \sum_E x_F \tag{2.1}$$

The equation above called full amount .partial sums in the set E_i called the sum type

$$\sum_{j_{k_1^i}\in J_{k_1^i}} \sum_{j_{k_2^i}\in J_{k_2^i}} \dots \sum_{j_{k_{t_i}^i}\in J_{k_{t_i}^i}} x_{j_1 j_2\cdots j_s} = \sum_{J_{K_1^I}\times J_{K_2^I}\times\dots\times J_{k_{t_i}^i}} x_{j_1 j_2\cdots j_s} = \sum_{E_i} x_F$$

We introduce s – index matrix $\{c_F\},\{a_F^1\},\{a_F^2\},\dots,\{a_F^{(m)}\}$ and m subset f_i, $i\in\{1,2,\dots,m\}=I$.

Using the notation introduction formulates the most general form linear programming problems multi-index:

Finding a set of $X=\{x_F\}$, that minimizing the linear function

$$L(X)=c_F x_F \tag{2.2}$$

And that satisfy the constraints

$$\begin{aligned}
&\sum_{E_1} a_F^{(1)} x_F = b_{\overline{F_1}}^{(1)}\ ,\overline{F_1}\in\overline{E}_1,\\
&\sum_{E2} a_F^{(2)} x_F = b_{\overline{F_2}}^{(2)}\ ,\overline{F_2}\in\overline{E}_2,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
&\sum_{E_m} a_F^{(m)} x_F = b_{\overline{F_m}}^{(m)}\ ,\overline{F}_m\in\overline{E}_m,\\
&x_F\ge 0, F\in E
\end{aligned} \tag{2.3}$$

3. Multi – index transportation problem of distribution resources

In general, multi – index transportation problem of distribution resources,it is possible to submit a collection of elements of the three sets:

1. production points;
2. intermediate points (warehouses);
3. points of consumption,

Functional relationship which is represented as a graph, multi-index problem with constraints of transport is the problem of determining the optimal transportation plan, which provides the effective functioning of the system that is finding the optimal volumes:

1. Production T_{i_1} resource $i_1 - M$ productionpoints $\left(i_1 = \overline{1, N_1}\right)$, where N_1 -the total amount of production points;

2. Consumption t_{i_3} resource $i_3 - M$ consumption points $\left(i_3 = \overline{1, N_3}\right)$, where N_3 -the total amount of consumption points.

Subject to the following constraints:

1. Maximum capacity allowable of production T'_{i_1} resource $i_1 - M$ production points;
2. Minimum and maximum capacity allowable of consumption t''_{i_3} and t'_{i_3} resource $i_3 - M$ consumption points;
3. Throughput capacity C_{i_2} during transport through the resource $i_2 - e$ intermediate points $\left(i_2 = \overline{1, N_2}\right)$, where N_2 -the total amount of intermediate points.

The formal statement of this problem is the following:

$$\max t_1; \max t_2; ...; \max t_{N_3}, \tag{3.1}$$

Subject to the following constraints:

$$\sum_{i_3=1}^{N_3} t_{i_3} = \sum_{i_1=1}^{N_1} T_{i_1}; \; T_{i_1} \le T'_{i_1}, \; i_1 = \overline{1, N_1}, \tau_{i_2} \le C_{i_2}, \quad i_2 = \overline{1, N_2}, t''_{i_3} \le t_{i_3} \le t'_{i_3},$$

$$i_3 = \overline{1, N_3}$$

$$, \sum_{i_3=1}^{N_3} t''_{i_3} \le \sum_{i_1=1}^{N_1} T'_{i_1} \le \sum_{i_3=1}^{N_3} t'_{i_3}; \tag{3.2}$$

Where τ_{i_2} amount of resource that is available to the input i_2 intermediate point.

4.Geometric graph of the set of feasible solutions of multi-index transportation problems

we represent the distribution of multi-index transportation problem with intermediate points in the form of a directed graph $G(V, E)$, without loops and parallel arcs are represented by a set of non-empty set V of vertices and set E arcs of the set of two-element subsets of the plural V :

$$E \subset \{v_i, v_j\} \text{ and } \forall e \subset E, \; v_i, v_j \in V, i \ne j \; G(V, E) \overset{'def'}{=} \langle V, E \rangle, V \ne \varnothing, (|e| = 2), |E = |V| + 1|$$

Where $V = \{v_1, v_2, ..., v_N\}$, $E = \{e_1, e_2, ..., e_M\}$, N and M -thetotal number of vertices and arcs, respectively.And, in the formulated problem, the set V vertex graph $G(V, E)$ represented by a set of subsets that do not intersect:

1. V_s The subset of production points (initial nodes of the graph);
2. V_p The subset of intermediate points (intermediate nodes of a directed graph);
3. V_e The subset of the points of consumption (ends directed graph), that is $V = V_s \cup V_p \cup V_e$, $\left(V_s \cup V_p\right) \cap V_e = \varnothing, \left|V_s\right| = N_1, \left|V_p\right| = N_2, \left|V_e\right| = N_3, N = N_1 + N_2 + N_3$

Under the a set of (union) of subsets of vertices - production points V_s and intermediate points V_P will be understood in a subset of distributors $V_d = V_s \cup V_p$,then the weight of the arc from set E ,that come out of a subset of vertices V_d ,determined the values according to distribution coefficients $E' = \{e'_1, e'_2, ..., e'_M\}$ from subset of V_d .we denoted by $F(i) \subset E$ -subset of set arc graph $G(V, E)$,which out of i-vertex, and $E = \bigcup_{i=1}^{N} F(i)$, $\bigcap_{i=1}^{N} F(i) = \varnothing$,view of the above, the problem of optimal distribution of a homogeneous resource - is the problem of determining the weights of arcs that come out of a subset of vertices V_d taking into account the criteria

$$f\left(T_1,...,T_{N_1}, e'_1,...,e'_M\right) = (-1)\sum_{i_3=1}^{N_3} t\left(T_1,...,T_{N_1}, e'_1,...,e'_M\right) \rightarrow \min_{\substack{e'_1,...,e'_M \in E \\ T_1,...,T_{N_1} \in T}} \tag{4.1}$$

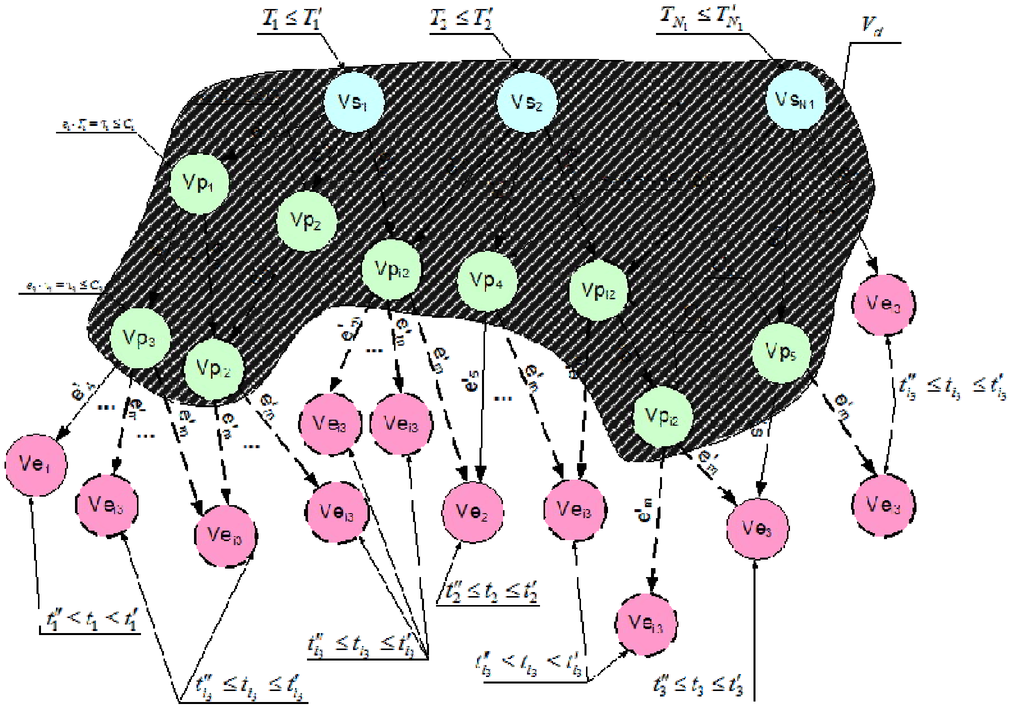
Subject to

$$\sum_{i_3=1}^{N_3} t_{i_3} = \sum_{i_1=1}^{N_1} T_{i_1};\ T_{i_1} \le T'_{i_1};\ \tau_{i_2} \le C_{i_2};\ t''_{i_3} \le t_{i_3} \le t'_{i_3} \tag{4.2}$$

And additional constraints imposed on the distribution coefficients $k{-}x$ $k = \overline{1, K}$,where K- the total number of vertices that belong to the subset V_d ,i.e. $\left|V_d\right| = K = \left|V_s\right| + \left|V_p\right| = N_1 + N_2$, divisors (weights of arcs), which are defined by the following equation:

$\sum_{j=1}^{J_k} e'^k_j = 1$, where $e'^k_j < 1 \in F(k), F(k) = J_k$, $F(k) \subset E$, $k = \overline{1, K}$.

Geometry of solving the problem is presented in the following Figure :



To determine the vector of function $t(T_1,\ldots,T_{N_1},e'_1,\ldots,e'_M)$,which characterizes the amount distributed in the i_3 - th element of uniform resource consumption, we introduce the following notation. Directed graph $G(V,E)$ we specify matrices incidents for foreword and reverse, respectively, and the elements of which are defined by the following expressions:

$$H^{in}_{i,m} = \begin{cases} 1, \text{if the node } v_i \; is\ incident\ arc\ e_m\ and\ represented\ its\ end; \\ 0, otherwise \end{cases} \tag{4.3}$$

$$H^{out}_{i,m} = \begin{cases} -1, if \text{ the node } v_i \; is\ incident\ arc\ e_m\ and\ represented\ its\ start \\ 0, otherwise \end{cases} \tag{4.4}$$

We introduce the vector function $v(T_1,\ldots,T_{N_1},e'_1,\ldots,e'_M), i-th$ element $(i=\overline{1,N})$ which determined all $i-th$ vertexes, and in the formulation of the problem,characterizes the amount of the resource, which is transmitted in the $i-th$ vertex of the graph $G(V,E)$. It is defined by the following relation:

$$v(T_1,\ldots,T_{N_1},e'_1,\ldots,e'_M) =$$

$$= \left[\sum_{i=1}^{N}\left[H^{in} * diag\left(\gamma_r\left(T_1,\ldots,T_{N_1},e'_1,\ldots,e'_M\right)\right) * \left(-1 * H^{out}\right)^T\right]^{(i)} + v'\left(T_1,\ldots,T_{N_1}\right)\right] \tag{4.5}$$

Where $v'(T_1,\ldots,T_{N_1})$-vector function of the $i-th$ element $(i=\overline{1,N})$ which determines the amount of resource that is made of the $i-th$ vertex of the graph

$G(V,E)$; $\gamma_r\left(T_1,...,T_{N_1},e'_1,...,e'_M\right)$ $R-th$ vector function is calculated according to the recurrence formula

$$\gamma_R\left(T_1,...,T_{N_1},e'_1,...,e'_M\right)=$$

$$=\left[\sum_{m=1}^{M}\left[\beta\left(e'_1,...,e'_M\right)H^{in}diag\left(\gamma_{R-1}\left(T_1,...,T_{N_1},e'_1,...,e'_M\right)\right)\right]^{(m)}+\gamma_1\left(T_1,...,T_{N_1},e'_1,...,e'_M\right)\right] \quad (4.6)$$

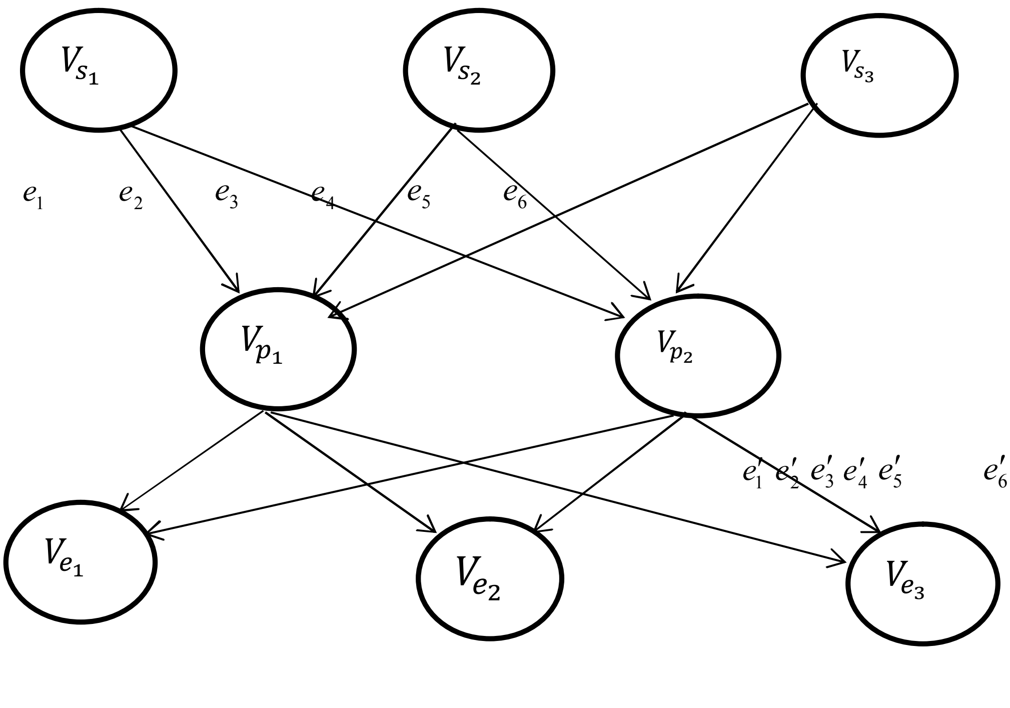
In the expression above the initial value of the function $\gamma_1\left(T_1,...,T_{N_1},e'_1,...,e'_M\right)$ and matrix function $\beta\left(e'_1,...,e'_M\right)$, determined in the following equation:

$$\gamma_1(T_1,...,T_{N_1},e'_1,...,e'_M)=\sum_{i=1}^{N}\left[\beta(e'_1,...,e'_M)*diag(v'(T_1,...,T_{N_1}))\right]^{(i)};$$

$$\beta\left(e'_1,...,e'_M\right)=\left[\left(-1*H^{out}\right)*diag\left(E'\left(e'_1,...,e'_M\right)\right)\right]^T \quad (4.7)$$

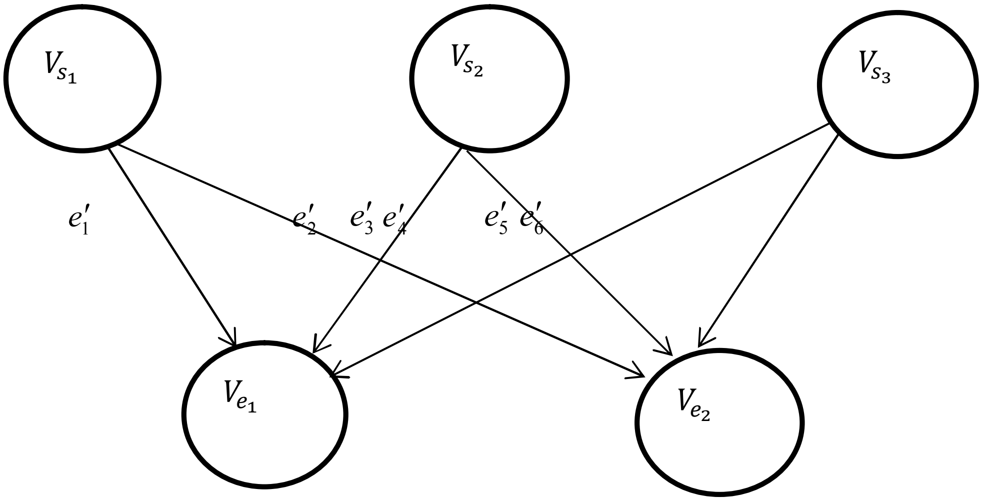
5.Example to clarify the procedure of the algorithm

Construct a radial map our graph such that the center had all the elements of subsets, and placed all vertices, respectively, which corresponding to intermediate and consumption points.



<u>Stage 1</u>

Calculate the amount that transformed from each production points to intermediate points



$$H^{in} = \begin{pmatrix} 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 1\,0\,1\,0\,1\,0 \\ 0\,1\,0\,1\,0\,1 \end{pmatrix} \; ; \quad H^{out} = \begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\beta(e'_1,\ldots,e'_6) = \left[\left(-1 * H^{out}\right) * diag(E'(e'_1,\ldots,e'_6)\right]^T$$

$$\beta(e'_1,\ldots,e'_6) = \left[\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e'_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & e'_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & e'_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & e'_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & e'_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & e'_6 \end{pmatrix} \right]^T \; ; \gamma_1 = \begin{pmatrix} e'_1 & 0 & 0 & 0 & 0 \\ e'_2 & 0 & 0 & 0 & 0 \\ 0 & e'_3 & 0 & 0 & 0 \\ 0 & e'_4 & 0 & 0 & 0 \\ 0 & 0 & e'_5 & 0 & 0 \\ 0 & 0 & e'_6 & 0 & 0 \end{pmatrix} \begin{pmatrix} T_1 & 0 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 & 0 \\ 0 & 0 & T_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma_1 = \begin{pmatrix} e'_1.T_1 & 0 & 0 & 0 & 0 \\ e'_2.T_1 & 0 & 0 & 0 & 0 \\ 0 & e'_3T_2 & 0 & 0 & 0 \\ 0 & e'_4T_2 & 0 & 0 & 0 \\ 0 & 0 & e'_5T_3 & 0 & 0 \\ 0 & 0 & e'_6T_3 & 0 & 0 \end{pmatrix}; \quad v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{pmatrix} = H^{in} * \begin{pmatrix} \gamma_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_1^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_1^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_1^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_1^5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_1^6 \end{pmatrix} * \left(-1 * H^{out}\right)^T$$

$$\gamma_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ e'_1T_1 & e'_3T_2 & e'_5T_3 & 0 & 0 \\ e'_2T_1 & e'_4T_2 & e'_6T_3 & 0 & 0 \end{pmatrix}; \quad \begin{pmatrix} v_1^{optimal} \\ v_2^{optimal} \\ v_3^{optimal} \\ v_4^{optimal} \\ v_5^{optimal} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ e'_1T_1 + e'_3T_2 + e'_5T_3 \\ e'_2T_1 + e'_4T_2 + e'_6T_3 \end{pmatrix}$$

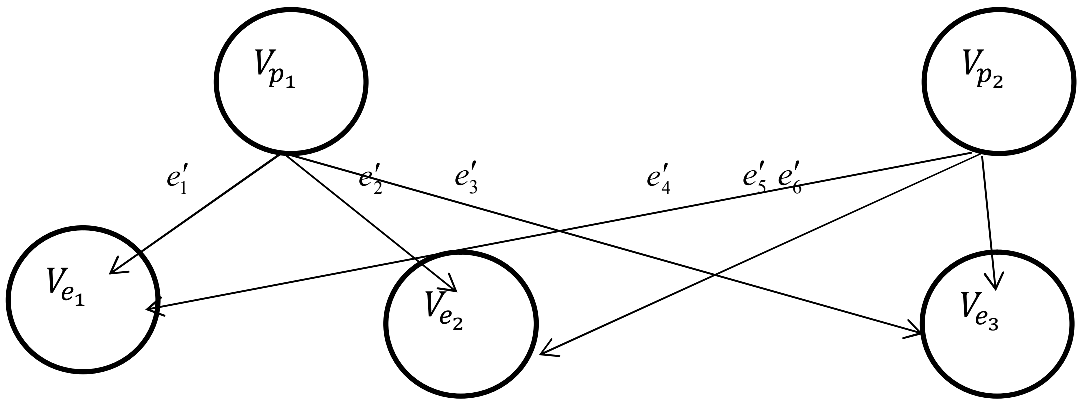
When T_1=100, T_2=80, T_3=90, e'_1=0.4, e'_2=0.6, e'_3=0.25, e'_4=0.75, e'_5=0.5, e'_6=0.5

$v_4^{optimal}$ =105, which represented the amounts that income to the first intermediate point.

$v_5^{optimal}$ =165, which represented the amounts that income to the second intermediate point.

Stage2

Calculate the amount that transformed from each intermediate point to consumptions points.



$$H^{in}=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\1&0&0&1&0&0\\0&1&0&1&0&1\\0&0&1&0&0&1\end{pmatrix};\quad H^{out}=\begin{pmatrix}-1&-1&-1&0&0&0\\0&0&-1&-1&-1&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix};\beta(e'_1,\dots,e'_6)=\begin{pmatrix}e'_1&e'_2&e'_3&0&0&0\\0&0&0&e'_4&e'_5&e'_6\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}^T$$

$$\gamma_1=\begin{pmatrix}e'_1&0&0&0&0&0\\e'_2&0&0&0&0&0\\e'_3&0&0&0&0&0\\0&e'_4&0&0&0&0\\0&e'_5&0&0&0&0\\0&e'_6&0&0&0&0\end{pmatrix}\begin{pmatrix}T_1(v_4^{op})&0&0&0&0\\0&T_2(v_5^{op})&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}$$

$$\begin{pmatrix}v_1^{optimal}\\v_2^{optimal}\\v_3^{optimal}\\v_4^{optimal}\\v_5^{optimal}\end{pmatrix}=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\1&0&0&1&0&0\\0&1&0&1&0&1\\0&0&1&0&0&1\end{pmatrix}\begin{pmatrix}\gamma_1^1&0&0&0&0&0\\0&\gamma_1^2&0&0&0&0\\0&0&\gamma_1^3&0&0&0\\0&0&0&\gamma_1^4&0&0\\0&0&0&0&\gamma_1^5&0\\0&0&0&0&0&\gamma_1^6\end{pmatrix}\begin{pmatrix}1&0&0&0&0\\1&0&0&0&0\\1&0&0&0&0\\0&1&0&0&0\\0&1&0&0&0\\0&1&0&0&0\end{pmatrix}$$

$$\begin{pmatrix}v_1^{optimal}\\v_2^{optimal}\\v_3^{optimal}\\v_4^{optimal}\\v_5^{optimal}\end{pmatrix}=\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\e'_1T_1(v_4^{op})&e'_4T_2(v_5^{op})&0&0&0\\e'_2T_1(v_4^{op})&e'_5T_2(v_5^{op})&0&0&0\\e'_3T_1(v_4^{op})&e'_6T_2(v_5^{op})&0&0&0\end{pmatrix},$$

$$\begin{pmatrix}v_1^{optimal}\\v_2^{optimal}\\v_3^{optimal}\\v_4^{optimal}\\v_5^{optimal}\end{pmatrix}=\begin{pmatrix}0\\0\\e'_1T_1(v_4^{op})+e'_4T_2(v_5^{op})\\e'_2T_1(v_4^{op})+e'_5T_2(v_5^{op})\\e'_3T_1(v_4^{op})+e'_6T_2(v_5^{op})\end{pmatrix}$$

When T_1=105, T_2=165, e_1'=0.3, e_2'=0.3, e_3'=0.4, e_4'=0.4, e_5'=0.4, e_6'=0.2

$v_3^{optimal}$=97.5, which represented the amounts that income to the first consumption point.
$v_4^{optimal}$=97.5, which represented the amounts that income to the second consumption point.
$v_5^{optimal}$=75, which represented the amounts that income to the third consumption point.

6-Conclusion

Of particular interest is attracted multi index problemslinear programming transportation type, since there is a wide class of applied problems of resource allocation, formalized as multi index tasks transportation problems .In this paper, the problem multi index, including problem uniform distribution with intermediate resource centers (warehouses). The mathematical model of this problem using known methods to solve it. The proposed algorithm for finding optimal plan distribution goods that responsible criteria optimality in the volume of production, consumption, and the amount of storage stocks.

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