# A UNIFIED METHOD FOR B-SPLINE'S SURFACE MODELING (INNOVATIVE METHOD FOR DATA TRANSFORMATION FROM CAD TO CAM PROGRAMS) 

Abdul Kareem Jaleel ${ }^{1} \quad$ Ahmed Abdul Samee ${ }^{2} \quad$ Raneen Sami Abid Ali ${ }^{3}$<br>Department of Mechanical Engineering, Babylon University, Babylon , Iraq<br>( drkareem959@yahoo.com , ahmed_abdulsamii7@yahoo.co., raneen_eng@yahoo.com)


#### Abstract

: In present work, a unified method for geometric transfer from CAD to CAM program has been investigated using iterative process. The B-spline objects (curves and surfaces) of $3^{\text {rd }}$ degree one patch matrix had been derived and formulated using MATLAB program to construct a computer program for the geometric transfer as a case study. The procedure of converting the sculpture surfaces from CAD program to CAM program without any geometrical distortion had been presented. The implementation of transfer data and the simulation using UGS program during transformation of any complex profile shape from CAD to CAM program is done without any distortion in final profile in CAM program. It was found that the results obtained by applying the present method which utilizes a less time as compared with the other methods are in a good agreement with the experimental results.


Keywords: cad/cam, b-spline, sculptured surfaces, tool path generation



في العمل الحالي تم أجراء طريقة هندسية للنقل من برنامج CAD إلى CAM باستخذام طريقة معدلة .تم اشتقاق
وصياغة مصفوفة لرقعة واحدة من الارجة الثالثة لأجسام B-spline (المنحنيات والأسطح) باستخذام برنامج لإنشاء برنامج حاسوبي لنقل اليانات الهناسية كدر اسة للحالة. تم تمثيل إجراءات تحويل الاسطح النحتية من برنامج CAD إلى برنامج CAM بدون أي تشوه هندسي .تم تنفيذ نقل البيانات والمحاكاة باستخدام برنامج UGS ألثناء تحويل لأي شكل معقد من برنامج CAD إلى CAM بدون أي تشوه في الثشكل النهائي في برنامج CAM ـ CAM وجد ان النتائج التي تم الحصول عليها مطابقة مع الننائج العملية بتطبيق الطريقة الحالية والتي تستغرق وقت اقل مقارنة مع الطرق الاخرى

## 1. INTRODUCTION:

The development of a new product is an iterative process. This includes: product design, analysis of performance, safety and reliability, product prototyping for experimental evaluation and design modification. Computer aided design is usually associated with interactive computer graphics. The designer can conceptualize the object to be designed more easily on the graphics screen and to consider alternative designs or modify a particular design quickly to meet the necessary design requirements or changes. Freeform surfaces, also called sculptured surfaces, have been widely used in CAD system to describe the surfaces of parts, such as aerospace, automotive, and die/mould manufacturing industries (Lin and Koren, 1996).
In pervious works when the researchers make their design of any profile using MATLAB program they save the extension of the design file as m-file or dxf-file. And then open it in CAM program such as SURFCAM or UGS programs to get the simulation of machining operation and get G-code (Zezhong et al,2003.Mansour,2002.Alan et al,1997. Hu et al,2000). This method can succeed for simple surfaces, but it's useless if a complex profile was designed and transferred using this basic method. While other researchers write the control points of any complex surface in CAM program manually(Akeel,2006.Mukdam,2007).
There are many executive programs that can convert the data from CAD program to CAM program. But if these programs are used for complex surfaces, the results may be distorted. So in the present work the matrix of $3^{\text {rd }}$ degree of the B -spline curve and surface were derived and formulated. These matrices were used with MATLAB to construct a program of data transfer from CAD to CAM.

## 2. B-SPLINE CURVES :

A B-spline is constructed from a string of curve segments whose geometry is determined by a group of local control points. These curves are known as piecewise polynomials. A curve segment does not have to pass through a control point, although this may be desirable at the two end points (John, 2010. Eric, 2004).
The general form of B-Spline curves can be represented in terms of their blending function is (Duncan, 2005):
$p(u)=\sum_{i=0}^{n} N_{i, k}(u) p_{i}$

Where $P_{i}$ is the set of control points and $N_{i, k}(u)$ represents the weight functions are defined recursively by the following expressions:

$$
\left.\begin{array}{rlrl}
N_{i, 1}(u) & =1 \quad & & \text { for } \quad t_{i} \leq u \leq t_{i+1}  \tag{2}\\
& =0 & & \text { otherwise }
\end{array}\right\}
$$

And:
$N_{i, k}(u)=\frac{u-t_{i}}{t_{i+k-1}-t_{i}} N_{i, k-1}(u)+\frac{t_{i+k}-u}{t_{i+k}-t_{i+1}} N_{i+1, k-1}$ (u)

So a 3rd degree of non-uniform B-Spline representation will be adopted for design of the curves and surfaces in this study, 3rd degree of non-uniform representation which is manually derived in this research. Appendix -A- shown the non-uniform derivative of B-spline curve.

## 3. B-SPLINE SURFACES REPRESENTATION:

B-spline surfaces are an extension of B-spline curves. The most common kind of a B-spline surface is the tensor product surface. The surface basis functions are products of two univariate (curve) bases. The surface is a weighted sum of surface (two dimensional) basis functions. The weights are a rectangular array of control points. The following Equation (4) shows a mathematical description of the tensor product B-spline surface (Xijun,2001).

$$
\begin{align*}
p(u, w) & =\sum_{i=0}^{m} \sum_{j=0}^{n} p_{i j} N_{i . k}(u) N_{j . l}(w)  \tag{4}\\
u, w & \in[0,1]
\end{align*}
$$

The $P_{i, j}$ is control points located at the vertices of the control polyhedron. The $N_{i, k}(u)$ and $N_{j, 1}(w)$ are the basis functions and they are same as those of B-spline curves. The degree of each of the basis function polynomials is controlled by k and $l$, respectively.
By the mathematical derivation, the general equation to represent B-spline surfaces can be shown as follow:
$\mathrm{P}(\mathrm{u}, \mathrm{w})=[\mathrm{U}][\mathrm{MB}][\mathrm{P}][\mathrm{MB}]^{\mathrm{T}}[\mathrm{W}]$
So, according to these equations and by selecting a different control point across $\mathrm{x}, \mathrm{y}$ and z -axis using MATLAB program. The B-spline surface is obtained as shown in Fig.(1).

And the procedure of the Matlab program that must be build up to obtain the surface can be shown in Fig.(2).

## 4. IMPLEMENTATION AND DISCUSSION:

The B-spline surface that described the sculpture surface was saved in the form of dat-extension. The CAM program (UGS-NX3) was used. To open this file of the surface which represent all points of the surface in $X, Y$ and $Z$ direction. The procedure followed to open the dat-extension file in UGS program can be divided into sub stages as follow:

- Import the dat-extension file from MATLAB files and open it in UGS program using Spline order.
- Draw spline curve pass through all points using the Chain with rectangle order.
- Draw splines surface that pass through all curves using through curve order.

Fig.(3) shows the representation of the B-spline surface produced by the derived and formulated equations of B-spline.

The surface were opened and represented successfully without any distortion in the original shape. So, the first step is to find the tool path and implement it on CNC milling machine can be taken, as follow:

- Plot a blank that contain all the dimensions of the proposed surface, and then the primitive operation can be used to get the final shape of the workpiece before making a program to get the tool-path. Fig. (4).
- Built a program for this workpiece to generate the tool path in 3-axis directions, and then make simulation for the machining. Fig.(5).
- Get the G-code in five-axis using the postprocessor order.

Finally, it can be noticed that the shape of parametric surface that was created using MATLAB program is the same shape when opened in UGS program. Also, it can be noticed that the proposed shape include concave and convex surfaces and not just a plane. So, this means that the program and the procedure followed in the proposed research is suitable for complex surfaces.

To prove this method, an experimental work was done to compare the final shape and dimensions with the profile proposed using MATLAB program. The profile of the final surface was obtained by the 3-axis machine in the Workshops and Training Center in the University of Technology /IraqBaghdad, is shown in Fig.(6).
The experimental results prove conformity with the original shape using less time as compared with other methods.

## 5. CONCLUSION:

There are many programs and many methods that can be used to transfer the sculpture surface from CAD to CAM program. But these programs may distort the surface after transferring to CAM program. The transfer method may need some modifications to get the original shape that was built in CAD program. In the present research a new method was proposed and implemented to transfer any complex plot without any distortion using $3^{\text {rd }}$ degree B-spline surface modeling. The present method matched the experimental results well, and used less time as compared to other methods.

Ahmed A.
Raneen S.


Fig.(1):Third degree B-spline surface using MATLAB software.


Fig.(2):Flowchart of the proposed program for B-spline surface presentation.

(a)

(b)

(c)

Fig.(3): (a) Importing data points for the case study to UGS program.(b)Case study as a set of curves. (c) Case study as a single surface.


Fig.(4).The blank in light color with the workpiece.

(a)

(b)

Fig.(5):(a)Contour tool path generation for the parametric surface.(b)The simulation of machining process of 3-axis milling using ball-end mill cutter.


Fig.(6): The specimen implemented in the Workshops and Training Center in the University of Technology /Iraq- Baghdad using C-TEK CNC milling machine.

## 6. REFERENCES:

Alan C. Lin, Shou-Yee Lin, Tse-HaoFang,"Automated sequence arrangement of 3D point data for surface fitting in reverse engineering", Computers in Industry, vol. 35, pp. 149-173, 1997.

Akeel Sabree Bedan, "Automatic surface generation from wireframe data in CAD applications", MSc. Thesis, University of Technology, Iraq, Baghdad, 2006.

Duncan Marsh,"APPLIEDGEOMETRY FOR COMPUTER GRAPHICS AND CAD ", $2^{\text {nd }}$ edition, Springer-Verlag London, (2005).

Eric Lengyel, "MATHEMATICS FOR 3D GAME PROGRAMMING AND COMPUTER GRAPGICS ", $2^{\text {nd }}$ edition, Charles river media, INC,Hingham ,Massachusetts (2004).

Hu Shi-Min, Tong Rou-Feng, Tao Ju, and Jia-Guang Sun "Approximate merging of a Pair of Bezier Curves", Computer-Aided Design, vol. 33, pp.125-136, 2000.

John Vince,"MATHEMATICS FOR COMPUTER GRAPGICS ", 3 rd edition, Springer-Verlag London Dordrecht Heidelberg New York,(2010).

Lin R.-S. and Koren Y., "Efficient Tool-Path Planning for Machining Free-Form Surfaces",ASME Journal of Engineering for Industry, vol. 118,pp.20-28 ,(1996).

Mansour.S, "Automatic generation of part programs for milling sculptured surfaces",Journal of Materials Processing Technology, vol.127,pp.31-39, 2002.

Mukdam Habib Shamoon, "Analysis and application of subdivision surfaces", MSc. Thesis, University of Technology, Iraq, Baghdad, 2007.

Xijun Wang, "Geometric Trimming and Curvature Continuous Surface Blending for Aircraft Fuselage and Wing Shapes ", M.Sc. thesis, Faculty of Virginia Polytechnic Institute and State University,(2001).

## APPENDI X A

## Derivation of the non-uniform B-Spline Basis Functions with ( $k=4, n=4$ ):

The B-spline basis function is determined by the general equation (3).

- Number of control points per segments $=\mathrm{k}=4$ points.
- Number of knots $(\mathrm{T})=\mathrm{n}+\mathrm{k}+1=9$ knots.

$$
\begin{equation*}
t_{i}=i-(k-1) \tag{A.1}
\end{equation*}
$$

By using the equation (A.1) it is easy to calculate the knot vector and to calculate the values from ( $\mathrm{t}_{\mathrm{i}}$ ) to $\left(\mathrm{t}_{\mathrm{i}+\mathrm{k}+\mathrm{n}}\right)$ as follow:
$\mathrm{T}=[\mathrm{i}-3, \mathrm{i}-2, \mathrm{i}-1, \mathrm{i}, \mathrm{i}+1, \mathrm{i}+2, \mathrm{i}+3, \mathrm{i}+4, \mathrm{i}+5]$
When $\mathrm{k}=1$

| $\begin{aligned} \mathrm{N}_{\mathrm{i}, 1}(\mathrm{u}) & =1 \\ & =0 \end{aligned}$ | for | $\begin{aligned} & \qquad \mathrm{i}-3 \leq \mathrm{u}<\mathrm{i}-2 \\ & \text { otherwise } \end{aligned}$ |
| :---: | :---: | :---: |
| $\begin{aligned} \mathrm{N}_{\mathrm{i}+1,1}(\mathrm{u}) & =1 \\ & =0 \end{aligned}$ | for | $\mathrm{i}-2 \leq \mathrm{u}<\mathrm{i}-1$ <br> otherwise |
| $\begin{aligned} & \mathrm{N}_{\mathrm{i}+2,1}(\mathrm{u})=1 \\ &=0 \end{aligned}$ | for | $\begin{aligned} & \mathrm{i}-1 \leq \mathrm{u}<\mathrm{i} \\ & \text { otherwise } \end{aligned}$ |
| $\begin{aligned} \mathrm{N}_{\mathrm{i}+3,1}(\mathrm{u}) & =1 \\ = & \end{aligned}$ | for | $\mathrm{i} \leq \mathrm{u}<\mathrm{i}+1$ <br> otherwise |
| $\begin{aligned} \mathrm{N}_{\mathrm{i}+4,1}(\mathrm{u}) & =1 \\ & =0 \end{aligned}$ | for | $\mathrm{i}+1 \leq \mathrm{u}<\mathrm{i}+2$ <br> otherwise |
| $\begin{array}{r} \mathrm{N}_{\mathrm{i}+5,1}(\mathrm{u})=1 \\ =0 \end{array}$ | for | $\mathrm{i}+2 \leq \mathrm{u}<\mathrm{i}+3$ <br> otherwise |
| $\begin{array}{r} \mathrm{N}_{\mathrm{i}+6,1}(\mathrm{u})=1 \\ =0 \end{array}$ | for | $\mathrm{i}+3 \leq \mathrm{u}<\mathrm{i}+4$ <br> otherwise |
| $\begin{array}{r} \mathrm{N}_{\mathrm{i}+7,1}(\mathrm{u})=1 \\ =0 \end{array}$ | for | $\mathrm{i}+4 \leq \mathrm{u}<\mathrm{i}+5$ <br> otherwise |

When $\mathrm{K}=2$
$N_{i, 2}(\mathrm{u})=\frac{(u-i+3) N_{i, 1}}{i-2-i+3}+\frac{(i-1-u) N_{i+1,2}(u)}{i-1-i+2}$
$\mathrm{N}_{\mathrm{i}, 2}(\mathrm{u})=(\mathrm{u}-\mathrm{i}+3) \mathrm{N}_{\mathrm{i}, 1}(\mathrm{u})+(\mathrm{i}-1-\mathrm{u}) \mathrm{N}_{\mathrm{i}+1,1}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+1,2}(\mathrm{u})=(\mathrm{u}-\mathrm{i}+2) \mathrm{N}_{\mathrm{i}+1,1}(\mathrm{u})+(\mathrm{i}-\mathrm{u}) \mathrm{N}_{\mathrm{i}+2,1}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+2,2}(\mathrm{u})=(\mathrm{u}-\mathrm{i}+1) \mathrm{N}_{\mathrm{i}+2,1}(\mathrm{u})+(\mathrm{i}+1-\mathrm{u}) \mathrm{N}_{\mathrm{i}+3,1}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+3,2}(\mathrm{u})=(\mathrm{u}-\mathrm{i}) \mathrm{N}_{\mathrm{i}+3,1}(\mathrm{u})+(\mathrm{i}+2-\mathrm{u}) \mathrm{N}_{\mathrm{i}+4,1}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+4,2}(\mathrm{u})=(\mathrm{u}-\mathrm{i}-1) \mathrm{N}_{\mathrm{i}+4,1}(\mathrm{u})+(\mathrm{i}+3-\mathrm{u}) \mathrm{N}_{\mathrm{i}+5,1}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+5,2}(\mathrm{u})=(\mathrm{u}-\mathrm{i}-2) \mathrm{N}_{\mathrm{i}+5,1}(\mathrm{u})+(\mathrm{i}+4-\mathrm{u}) \mathrm{N}_{\mathrm{i}+6,1}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+6,2}(\mathrm{u})=(\mathrm{u}-\mathrm{i}-3) \mathrm{N}_{\mathrm{i}+6,1}(\mathrm{u})+(\mathrm{i}+5-\mathrm{u}) \mathrm{N}_{\mathrm{i}+7,1}(\mathrm{u})$
When $\mathrm{K}=3$
$N_{i, 3}(u)=\frac{(u-i+3)}{i-1-i+3} N_{i, 2}(u)+\frac{(i-w)}{i-i+2} N_{i+1,2}(u)$
$\mathrm{N}_{\mathrm{i}, 3}(\mathrm{u})=$
$\frac{1}{2}\left[(u-i+3)^{2} N_{i, 1}(u)+(u-i+3)(i-1-u) N_{i+1,1}(u)+(i-u)(u-i+2) N_{i+1,1}(u)+\right.$ $\left.(i-u)^{2} N_{i+2,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+1,3}(\mathrm{u})=\frac{1}{2}$
$\left[(u-i+2)^{2} N_{i+1,1}(u)+(u-i+2)(i-u) N_{i+2,1}(u)+(i+1-u)(u-i+1) N_{i+2,1}(u)+\right.$ $\left.(i+1-u)^{2} N_{i+3,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+2,3}(\mathrm{u})=$
$\frac{1}{2}\left[(u-i+1)^{2} N_{i+2,1}(u)+(u-i+1)(i+1-u) N_{i+3,1}(u)+(i+2-u)(u-\right.$
i) $\left.N_{i+3,1}(u)+(i+2-u)^{2} N_{i+4,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+3,3}(\mathrm{u})=$
$\frac{1}{2}\left[(u-i)^{2} N_{i+3,1}(u)+(u-i)(i+2-u) N_{i+4,1}(u)+(i+3-u)(u-i-1) N_{i+4,1}(u)+\right.$ $\left.(i+3-u)^{2} N_{i+5,1}(u)\right]$

In the same way we can compute the $N_{i+4,3}(u)$, but this function not need in final solution.
Now to compute the $N_{i, 4}(u), N_{i+1,4}(u), N_{i+2,4}(u)$ and $N_{i+3,4}(u)$ functions for $3^{\text {rd }}$ degree of BSpline curve we will take the central duration when $t_{i+3} \leq u<t_{i+4}$ and propose $N_{i+3,1}(u)=1$ only and neglected any boundary don't have these function.

When $\mathrm{K}=4$
$\mathrm{N}_{\mathrm{i}, 4}(\mathrm{u})=\frac{(u-i+3)}{i-i+3} \mathrm{~N}_{\mathrm{i}, 3}(\mathrm{u})+\frac{(i+1-w)}{i+1-i+2} \mathrm{~N}_{\mathrm{i}+1,3}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}, 4}(\mathrm{u})=\frac{1}{6}\left[(i+1-u)^{3} N_{i+3,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}, 4}(\mathrm{u})=\frac{1}{6}\left[(1-u)^{3} N_{i+3,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+1,4}(\mathrm{u})=\frac{(u-i+2)}{i+1-i+2} \mathrm{~N}_{\mathrm{i}+1,3}(\mathrm{u})+\frac{(i+2-u)}{i+2-i+1} \mathrm{~N}_{\mathrm{i}+2,3}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+1,4}(\mathrm{u})=\frac{1}{6}\left[(u-i+2)(i+1-u)^{2} N_{i+3,1}(u)+(i+2-u)(u-i+1)\right.$
$\left.(i+1-u) N_{i+3,1}(u)+(i+2-u)^{2}(u-i) N_{i+3,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+1,4}(\mathrm{u})=\frac{1}{6}$
$\left[(u+2)(1-u)^{2} N_{i+3,1}(u)+(2-u)(u+1)(1-u) N_{i+3,1}(u)+(2-u)^{2}(u) N_{i+3,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+2,4}(\mathrm{u})=\frac{(u-i+1)}{i+2-i+1} \mathrm{~N}_{\mathrm{i}+2,3}(\mathrm{u})+\frac{(i+3-u)}{i+3-i} \mathrm{~N}_{\mathrm{i}+3,3}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+2,4}(\mathrm{u})=$
$\frac{1}{6}\left[(u-i+1)^{2}(i+1-u) N_{i+3,1}(u)+(u-i+1)(i+2-u)(u-i) N_{i+3,1}(u)+\right.$ $\left.(u-i)^{2}(i+3-u) N_{i+3,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+2,4}(\mathrm{u})=\frac{1}{6}$
$\left[(u+1)^{2}(1-u) N_{i+3,1}(u)+(u+1)(2-u)(u) N_{i+3,1}(u)+(u)^{2}(3-u) N_{i+3,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+3,4}(\mathrm{u})=\frac{(u-i)}{i+3-i} \mathrm{~N}_{\mathrm{i}+3,3}(\mathrm{u})+\frac{(i+4-u)}{i+4-i-1} \mathrm{~N}_{\mathrm{i}+4,3}(\mathrm{u})$
$\mathrm{N}_{\mathrm{i}+3,4}(\mathrm{u})=\frac{1}{6}\left[(u-i)^{3} N_{i+3,1}(u)\right]$
$\mathrm{N}_{\mathrm{i}+3,4}(\mathrm{u})=\frac{1}{6}\left[(u)^{3} N_{i+3,1}(u)\right]$
After finding the equations for different orders, these equations are compensated with each other to get the final equation.
$P(u)=N_{i, 4}(u) p_{i}+N_{i+1,4}(u) p_{i+1}+N_{i+2,4}(u) p_{i+2}+N_{i+3,4}(u) p_{i+3}$
$\mathrm{P}(\mathrm{u})=\frac{1}{6}\left\{\left[(1-u)^{3} N_{i+3,1}(u)\right] \mathrm{p}_{\mathrm{i}}+\left[(u+2)(1-u)^{2} N_{i+3,1}(u)+(2-u)(u+1)\right.\right.$
$\left.(1-u) N_{i+3,1}(u)+(2-u)^{2}(u) N_{i+3,1}(u)\right] p_{i+1}$
$+\left[(u+1)^{2}(1-u) N_{i+3,1}(u)+(u+1)(2-u)(u) N_{i+3,1}(u)+(u)^{2}(3-u) N_{i+3,1}(u)\right] p_{i+2}$ $\left.+(u)^{3} N_{i+3,1}(u) \mathrm{p}_{\mathrm{i}+3}\right\}$

$$
P(u)=\frac{1}{6}\left[\left(-u^{3}+3 u^{2}-3 u+1\right) p_{i}+\left(3 u^{3}-6 u^{2}+4\right) p_{i+1}+\left(-3 u^{3}+3 u^{2}+3 u+1\right) p_{i+2}+(u)^{3} p_{i+3}\right]
$$

As a result of compensation these equations, the matrix forms of cubic B-spline, when $\mathbf{k}=4$ is:

$$
P(u)=\frac{1}{6}\left[\begin{array}{llll}
u^{3} & u^{2} & u & 1
\end{array}\right]
$$

$$
\left[\begin{array}{cccc}
-1 & 3 & -3 & 1  \tag{A.2}\\
3 & -6 & 3 & 0 \\
-3 & 0 & 3 & 0 \\
1 & 4 & 1 & 0
\end{array}\right]\left[\begin{array}{c}
p_{i} \\
p_{i+1} \\
p_{i+2} \\
p_{i+3}
\end{array}\right]
$$

