

Design And Simulation Of Programmable Finite Impulse Response (PFIR) Digital Filter Using MATLAB And MAC Filter

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Abstract :

In this paper, the fundamental and philosophy of FIR digital filter design with MATLAB program and multiply accumulate (MAC) Filter is present. The principle of low pass filters is concentrated here which will apply to other response type as well, and the degree of freedom available for finite impulse response (FIR) filter design is discussed in details. The importance of Equiripple and Least square design to implement the FIR filter in hardware is evaluated. The realistic aspects of FIR filter design and fixed point completion beside algorithms in the filter design toolbox and signal processing toolbox is illustrated. Different design algorithms and theory behind is wanted to contribute them. Result shows that the least square methods give less pass band ripple compare with the Equiripple methods but with Gibbs phenomena in the filter response. The implementation results with MAC filter from Xilinx shows an important reduction in multiplication process, consequently low power consumption.

Keyword: Equiripple, Least Square, FIR, MATLAB

تصميم ومحاكاة مرشح رقمي مبرمج ذو الاستجابة المحدودة باستخدام برنامج ماتلاب

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الخلاصة :

هذا البحث يتناول فلسفة ومبادئ تصميم المرشح الرقمي ذو الاستجابة المحدودة في برنامج ماتلاب. يركز هذا العمل على مبادئ ترشيح الحزم المنخفضة من الترددات ومقدار الحرية المتاحة في تصميم مرشح الاستجابة المحدودة. أن أهمية تصميم المرشحات المتوازنة التموج في التنفيذ تم تقييمها ومقارنة نتائجها. الجوانب الواقعية في تصميم هذا النوع من المرشحات بالقيم الثابتة بالاعتماد على لوغاريتمات صممت لهذا الغرض استخدمت فيها مختلف

برامج معالجات الإشارة. النتائج التي تم الحصول عليها تبين أن طريقة التربيع الأدنى تعطي تموج اقل في حزمة المرور بالمقارنة مع طريقة التمرير المتعادل ولكن مع ظاهرة جيبس الغير مرغوبة في استجابة الفلتر.

1. Introduction

The finite impulse response (FIR) filters and infinite impulse response (IIR) filter is the most important filters in the modern communication system and user daily life. It has been long recognized these filter design nowadays with MATLAB engineer. The optimization problems could be constrained in the design of FIR filter in some applications. The optimization-based methods used present a design structure in which a diversity of design criteria and qualifications can be readily included. Equivalent to the growth of these methods, the past more than a few years have witnessed the development of new optimization algorithms that have proven extremely capable for the design of digital filters. These powerful optimization methods in combination with the ever-increasing power of computers have made optimization-based filter design an increasingly important field of research ^[1]. In this paper, several recently proposed optimization based algorithms for the design of FIR and IIR filters and filter banks are reviewed. The essential function of a decimation or interpolation filter is to decrease or increase the sampling rate and to keep the pass band aliasing or imaging error within prescribed bounds. The filters require no multipliers and use limited storage thereby leading to more economical hardware implementations ^[2]. In most digital filter applications, they prefer to use FIR filter better than IIR filter due to phase linearity could be achieved easily which provide more stability in the filter response ^[3]. However, this paper will focused on FIR filter design in MATLAB with its most important applications. The basic design of FIR filter is shown in **Figure(1-a)**. The difficulty operation of FIR filter with coefficients values are the sum of two power and the multiplications can be replaced by bit serial shifters. However, the coefficients values could be represented by fixed point for this type of filter.

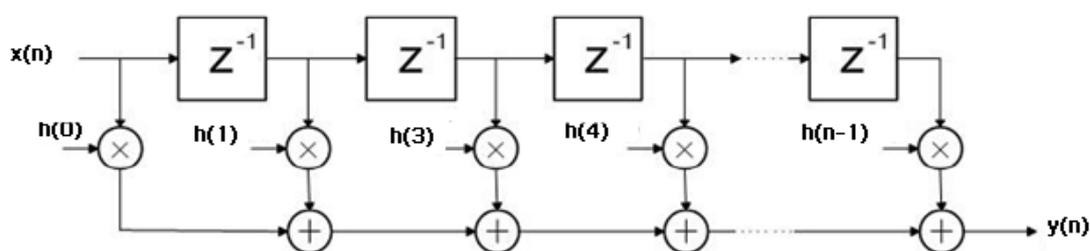


Fig.(1-a). Basic design of FIR filter.

The multiplication and addition achieved during multiplier and accumulator which is the major computational in digital signal processing design. Many approach to high speed implementation of FIR filter has been proposed ^[4-5-6]. In order to achieve high presentation, parallel development approach such as binary tree technique have been applied. The architecture of new DSP processor is mainly based on modified Harvard design with multi buses system. Mixture of higher design and high speed memory integrated on chip and optimized education set is the reason why DSP processor are appropriate for digital processing .

2. Linear Phase FIR Filter Design

The exact linear phase of FIR filters design are mostly used a different methods like, window, frequency sampling, and approximation methods. All these methods are focused on the error reduction to improve the performance of FIR filter by addressing the following issues:

- a. The power consumption of the filter to be minimum, while maintaining high efficiency.
- b. Minimizing the pass band ripple to avoid the distortion within the received signal.
- c. Compensating the adjacent band rejection and blocker requirement.

The filter that allows through all frequency components of signal below a design cutoff frequency and rejects all frequency components of a signal above cutoff frequency is called low pass filter. However, the frequency response of ideal low pass filter could be express as:

$$H_{LP}(e^{j\omega}) = \begin{cases} 1, & 0 \leq \omega \leq \omega_c \\ 0, & \omega_c \leq \omega \leq \pi \end{cases} \quad (1)$$

FIR filter design essentially consists of two problems, approximation problems and realization problems. The realization problem deals with choosing the structure to implement the transfer function in the form of a program. The most important specifications of FIR filter are the filter order N, Transition width and pass/stop band ripple. This specification could be illustrated in **Figure(1-b)**. The unwanted specifications in low pass filter response are pass/stop band ripple and transition width, but these specifications are unavoidable. However, an approximation should be achieved. Typically, the FIR filter design consists of filter that meets exact design specifications and not exceeds the acceptable values. In same time one must choose the truncated filter response in term of filter order or consistently the filter length. A practical symbol for the FIR filter design is to imagine of every specification as one of the angles in the triangle as illustrated in **Figure(2)**. A practical idea illustrated in Figure 2 could be used to know and understand the freedom degree in digital filter design. Imagine that every filter parameter represents one angle in the triangle, so the sum of all three angles in the triangle is fixed, and then any change in one triangle angle will effect on other two angles. The same thing happens in the case of selecting the filter parameters. Therefore, after selecting the two parameters, the third parameter must be designed with special algorithm. At the same time, as in angles of triangle, if a larger or smaller parameter is selected, it will affect other parameter.

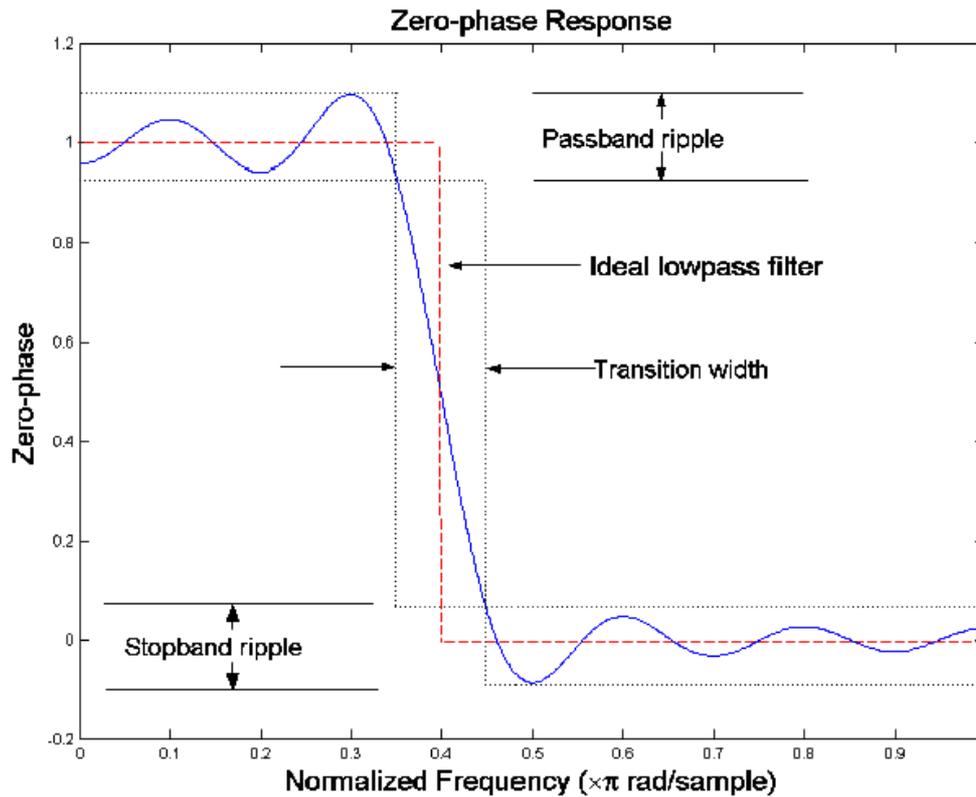


Fig.(1-b) The ideal low pass filter when approximating with an FIR filter

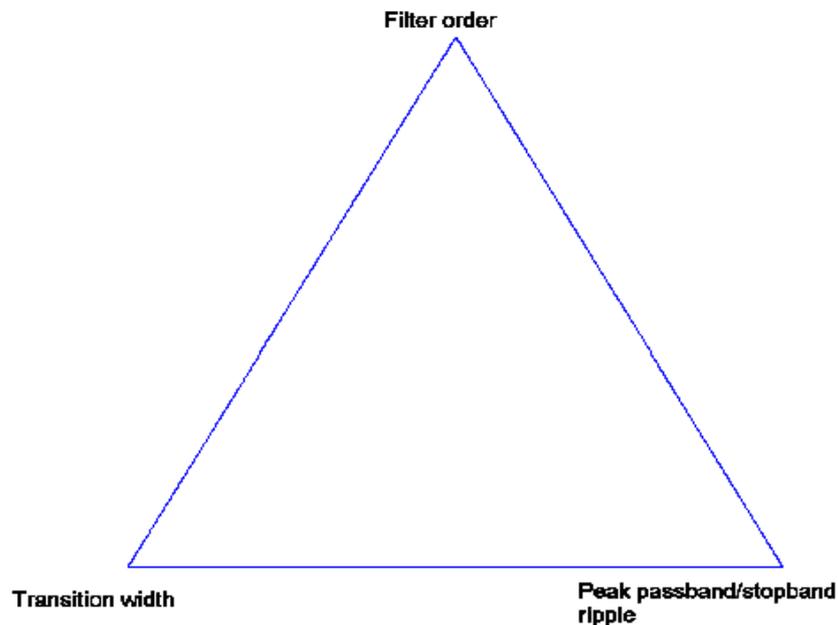


Fig.(2) FIR filter design condition correspond to triangle

In most applications, the linear phase design of FIR filter is attractive, especially in image processing and data transmission. The linear phase FIR filter has symmetrical or unsymmetrical impulse response and the zero phase response could be determined logically. However, the filter design problems become mathematical approximation problems.

Therefore, the polynomial of FIR filter could be used as best approximation to a given ideal low passes filter response with certain filter order which minimize the difference between the desired and ideal response. To provide this condition, Remez algorithms should be used to compute a solution of design problems for linear phase FIR filter which minimize the maximum error between ideal and actual filters. This kind of filter guide to so call Equiripple filters in which the peak deviation from the ideal response are equal.

3. Equiripple FIR filter Design

Under this type of FIR filter design, many methods have been presented and the basic idea in each method is to design the filter coefficients again and again until a particular error is minimized. One of these methods is the idea by [7] or mini-max algorithms. In the mini-max algorithms [8], if the desired or real frequency response of the filter is $H_d(\omega)$ and the actual or designed frequency response of the filter is $H(\omega)$, hence, the frequency response of the weighted function is present as $W(\omega)$. However, the error function could be defined as $E(\omega)$ which represent the difference between the desired response $H_d(\omega)$ and actual response $H(\omega)$ of the filter. This error requires reducing to lowest level by mean of difference between desired response and actual response to optimize the filtering process. At this time the filter coefficients play an important role in the limitation of this error, which is a trade-off between the filter coefficients, and the error should be achieved to optimize the filter design [8]. The error criteria between the desired response and actual response in pass band and stop band of LPF could be explained in **Figure .(3)** .

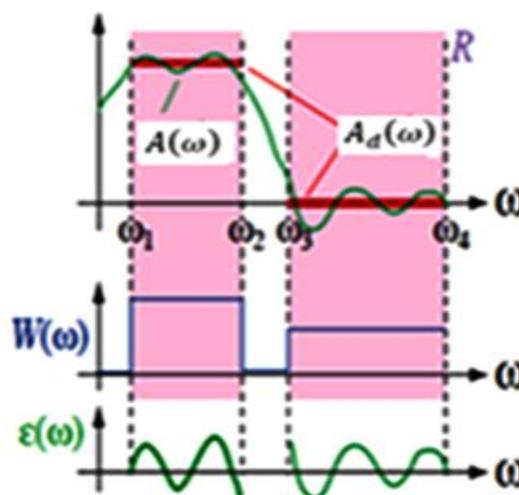


Fig.(3) Error function of Pass-band edge in LPF response

The attractive properties of linear phase Equiripple filters is the smallest maximum deviation from the ideal filter when compared to all other linear phase FIR filters in the same filter order. The Equiripple feature in the filter response is perfectly matched for applications

in which exact tolerance should be met. If it is required to design a filter with specific minimum stop band attenuation or given maximum pass band ripple. For example if one interested in LP filter satisfying the following requirements:

$$1 - \delta_p \leq H_z(e^{j\omega}) \leq 1 + \delta_p; \quad \text{pass band} \quad (2)$$

$$-\delta_s \leq H_z(e^{j\omega}) \leq \delta_s; \quad \text{stop band} \quad (3)$$

Considering type (I) FIR filter and symmetric impulse response, the real valued frequency response is:

$$H(Z) = \sum_{n=0}^{N-1} h(n)Z^{-n} \quad (4)$$

$$H(\omega) = \sum_{k=0}^M h[k] \cos(k\omega) \quad (5)$$

The first step in designing a poly-phase filter is to analyze the coefficients $h[k]$ into M dissimilar poly-phase filters and, $\cos(k\omega)$ could be express as a polynomial in $\cos(\omega)^k$ with lower powers [e.g. $\cos(2\omega) = 2(\cos\omega)^2 - 1$] thus, one can found $h[k]$ as

$$H(\omega) = \sum_{k=0}^M h[k] (\cos\omega)^k \quad (6)$$

and

$$h[k] = \begin{cases} h[M] & k = 0 \\ 2h[M - K] & 1 \leq K \leq M \end{cases} \quad (7)$$

An M^{th} order the polynomial has at most M-1 maxima and minima, so, $H(\omega)$ has at most M-1 min-max ripple. For example, if 5th order polynomial in $\cos \omega$, the response could be shown in **Figure 4**.

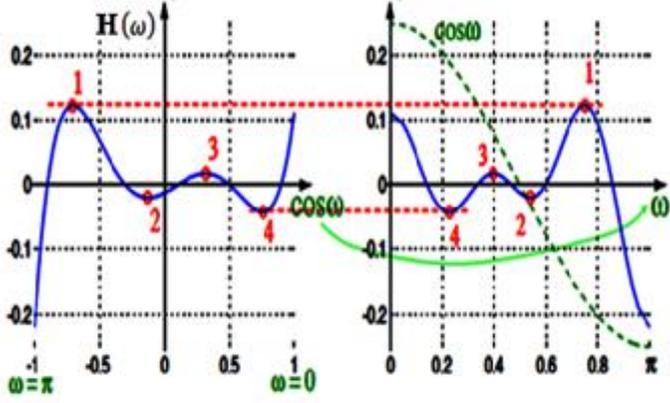


Fig.(4) Mini-max FIR filter response

The error function $E(\omega)$ and a weight function $W(\omega)$ which define the relative significance of the error at any certain frequency ω . Then, the error function can be described as in [7-8-9-10-11].

$$E(\omega) = W(\omega)[A_d(\omega) - A(\omega)] \tag{8}$$

Where $A_d(\omega)$ is the desired amplitude response and $A(\omega)$ is the designed amplitude response. A simple weight function $W(\omega)$, can be defined in [8] as:

$$W(\omega) = \begin{cases} 1, \omega \in (\text{passband}) \\ 0, \omega \in (\text{stopband}) \end{cases} \tag{9}$$

And the actual amplitude response of a linear-phase FIR filter could be represented as a product of two filters functions as given by [8].

$$A(\omega) = F(\omega) G(\omega) \tag{10}$$

and

$$G(\omega) = \sum_{k=0}^M g[k](\cos\omega)^k, \quad M = \frac{N}{2}, \quad K = 0,1,2 \dots \tag{11}$$

where $F(\omega)$ and M are obtained from **Table (1)**.

Table 1: Parameters of the four FIR filters [5],[6]

Type	I	II	III	IV
Order	Even	Odd	Even	Odd
$F(\omega)$	1	$\cos(\omega/2)$	$\sin(\omega)$	$\sin(\omega/2)$
M	$N/2$	$(N-1)/2$	$(N-2)/2$	$(N-1)/2$
ω_0	0	0	$\pi/2$	$\pi/2$

The filter coefficients $g[k]$ should be obtained to reduce the maximum absolute weighted error $|E(\omega)|$. A specific algorithm should be used to reduce the magnitude of the error $|E(\omega)|$ across the specified pass-band and stop-band. Thus, to optimize the error in pass-band response, the mini-max algorithm could be used as mentioned in [9],[10]:

$$\|E(\omega)\|_{\infty} = \max_{\omega \in [0, \pi]} |W(\omega)[A_d(\omega) - A(\omega)]| \quad (12)$$

where ω is in the desired operating frequency range of the filter

The Kaiser Window with 40 filter order could be designed using M-file program and fvtool environments in MATLAB. Thus, the Equiripple algorithms (Remez) with filter order of N=40, and the weight in pass band region is duplicated comparing with stop band as [1100]. The maximum coefficient is chosen here as 0.37, 0.43 and 1. According to these data the following M-code is used to give the filter response as illustrated in Figure 5.

```
fr = remez(40,[0 0.37 0.43 1],[1 1 0 0]); (fr mean Equiripple design)
fvtool(fr)
```

If one needs to minimize the energy of the error between the ideal and actual filter in the pass/stop band, the least squares design is desirable in this case as show in Figure 6 with the following M-code program in MATLAB for the same transition width and filter order as in Equiripple filter.

```
fls = firls(40,[0 0.37 0.43 1],[1 1 0 0]); ( fls mean least squares design)
fvtool(fl)
```

As illustrated from **Figure(5)** and **Figure(6)**, the ripple in the pass band rejoin is reduced by using least squares methods and the slope in the stop band rejoin is better compare with Equiripple which will used to suppressed the unwanted signal rejected from following stages. To see more clear difference between two methods, one could zoom in the pass band area as show in **Figure(7 and 8)**. The peak-to-peak ripple with Equiripple FIR filter is approximately -0.7 dB, while in the case of least square FIR filter is around of -0.4 dB but one can see the Gibbs phenomena is appear.

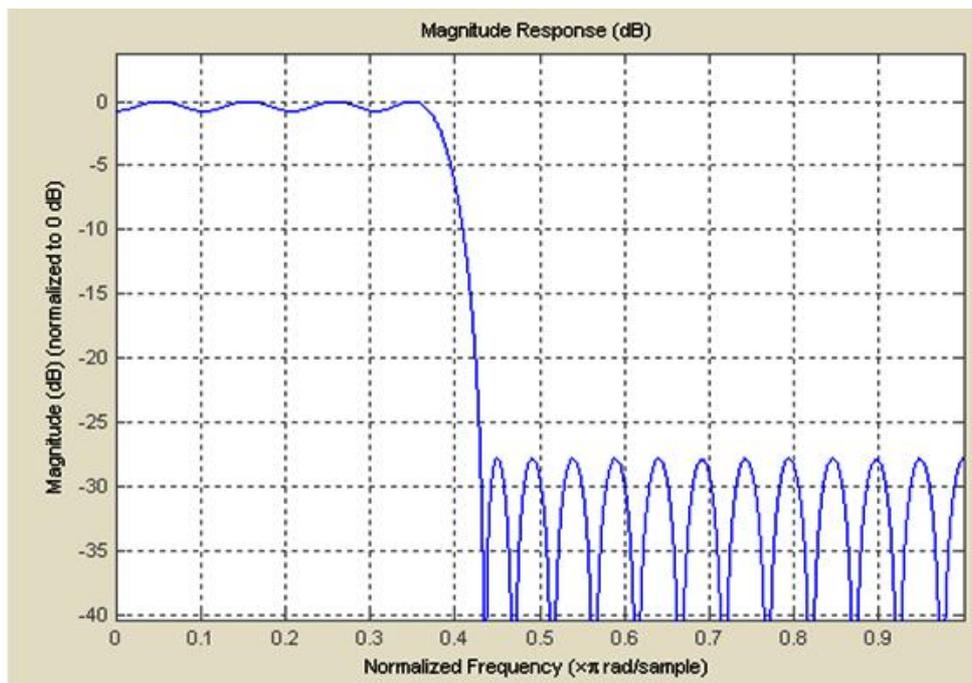


Fig.(5) Equiripple FIR filter response

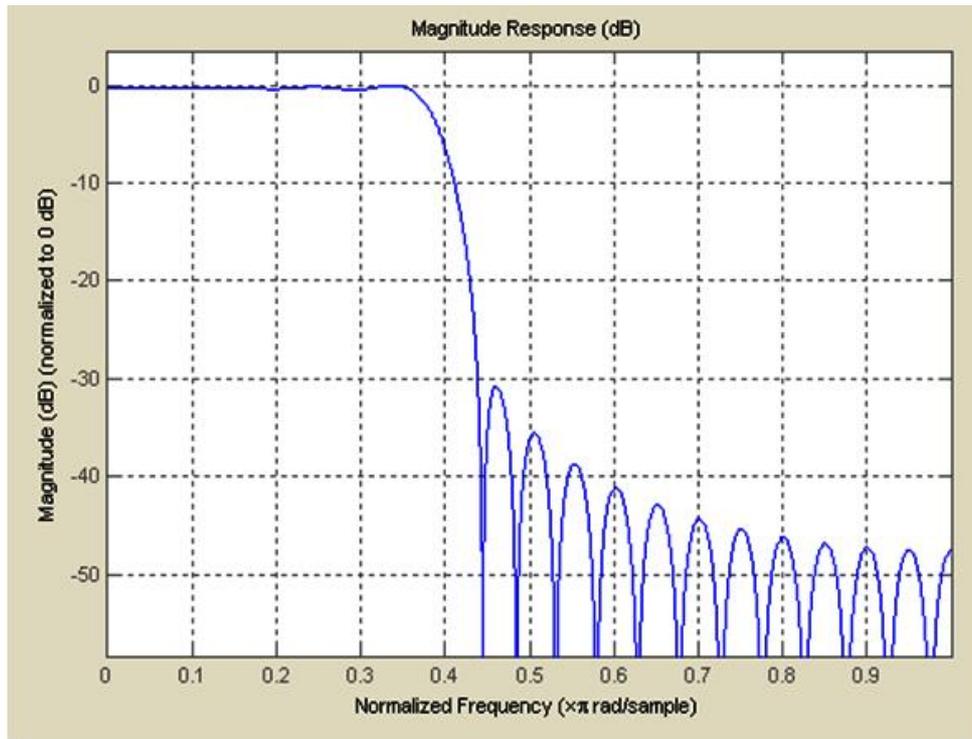


Fig.(6) Least square FIR filter response

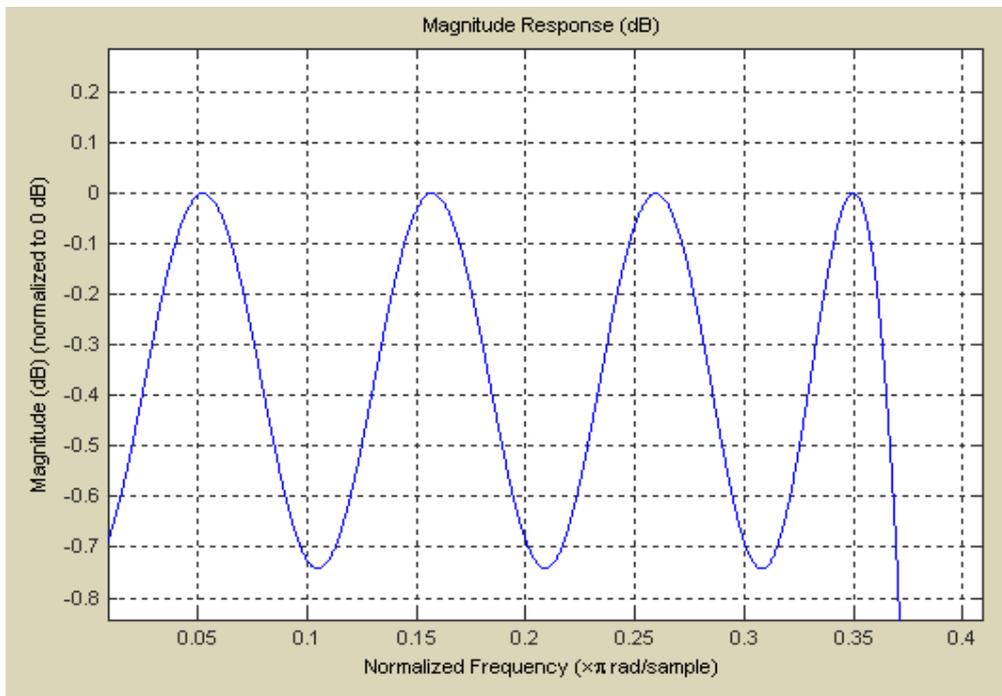


Fig.(7) Equiripple FIR filter response after zoom in the pass band area

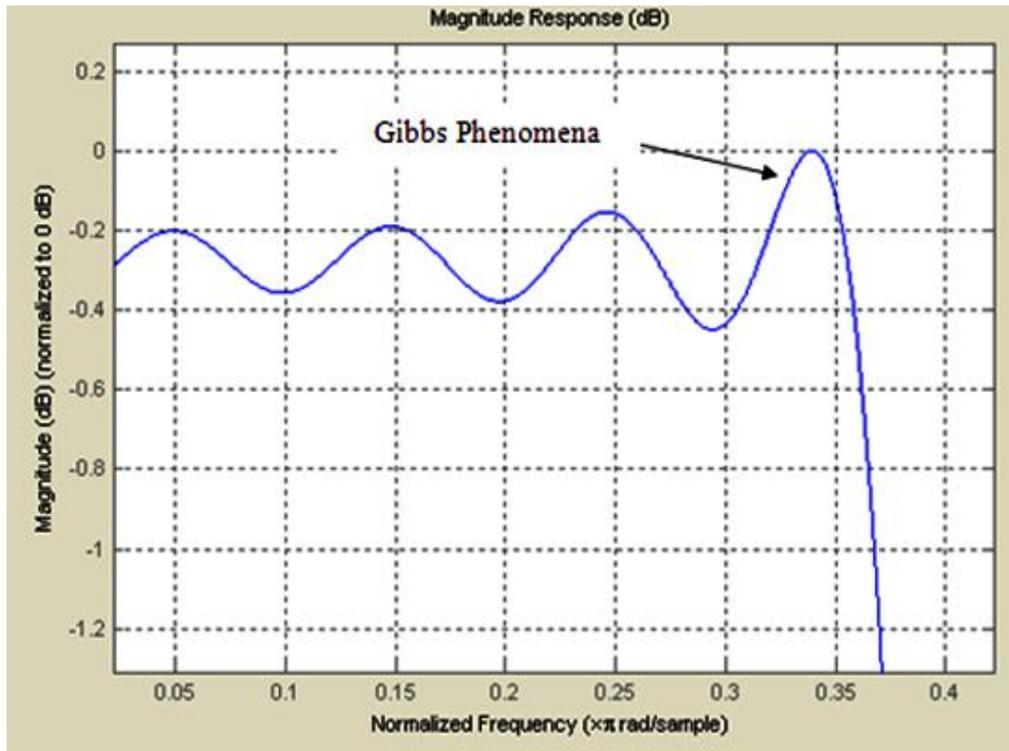


Fig.(8) Least square FIR filter response after zoom in the pass band area

4. PFIR Filter Implementation Using MAC Filter

The multiply accumulate (MAC) FIR ^[12] core uses one or more time-shared (MAC) functional units to serve the calculations in the filter. The core automatically determines the minimum number of MAC engines required to meet the user specified throughput. Figure 9 shows a basic view of a MAC-based FIR utilizing. The block diagram of a single MAC engine storage for the filter coefficients ($a(0), a(1), \dots, a(39)$) and the control circuit that sequences the appropriate coefficients and data to the multiply-accumulator for the specified integration period.

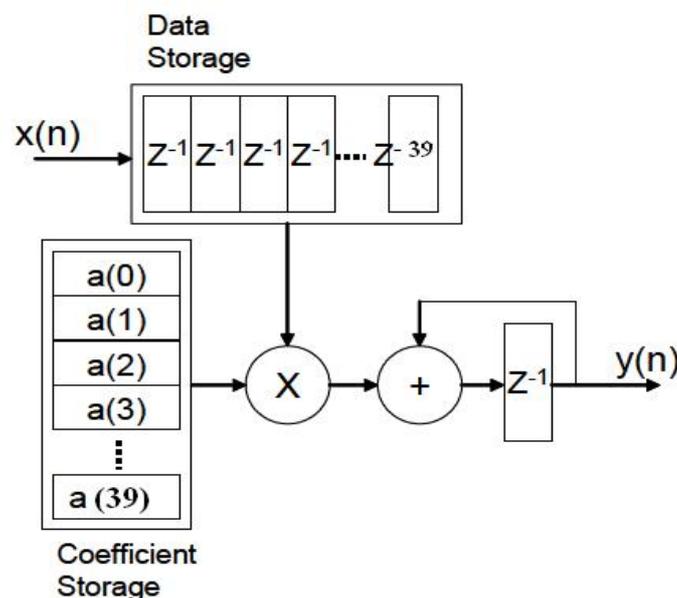


Fig.(9) Multiple MAC Engine Implementation ^[12]

The single implementation is expanded to multi-MAC implementations for use in realizing advanced presentation filter condition like larger numbers of coefficients, top sample rates and extra channels. The number of multipliers important to implement a filter is hard by calculating the number of multiplies required to carry out the calculation and then dividing by the number of clocks available to process every input sample. The available clock cycles value is for all time rounded down and the number of multipliers rounded up to the nearest integer. But there is a non-zero remainder, a number of of the MAC engines calculate less coefficients than others, and the coefficients are filling with zeros to provide somewhere to stay the excess cycles. The output samples replicate the padding of the coefficient vector. Hence, the response to an applied impulse surround a certain number of zero outputs before the first coefficient of the specified impulse response emerge at the output. The core mechanically generates an implementation that meets the user-defined performance requirements based on the system clock rate, the sample rate, the number of taps and channels, and the rate change. The core inserts one or more multipliers to meet the overall throughput requirements. The proposed PFIR need 39 multiplication process because the filter order is $N=40$, hence with MAC filter will need only one multiplication process.

5. Conclusion

This paper deal with the different techniques of designing an efficient FIR filter such as Equiripple and least square methods and the comparison between them is presented. As clear from the results obtained by MATLAB, the ripple in the pass band is changed according to filter order and design methods. Linear phase Equiripple filter design could be done by solving the frequency response equation with different frequency components. Iterative algorithms like Remez or so called Equiripple algorithm could be used to solve these problems. The least square algorithm also can be used with the same filter order to get the desired frequency response but with different pass band ripple. The least square is suffered from Gibbs phenomena but the slop in stop band is better. The results obtained shows that the ripple also depend on the transition width of the filter, such that the ripple is increased as the bandwidth is decrease and vice versa. The implementation of the proposed filter with optimize ripple could be achieved by using the MAC filter from Xilinx in order to offers lowest multiplication process as well as low power consumption.

6. References

1. **A. Abedelgwad, "High speed and area efficient multiply Accumulate (MAC) Unit for Digital signal Processing applications," *IEEE International Symposium on Circuits and Systems, ISCAS 2007.***
2. **Xiaoping, and Ruijie, "On Chebyshev Design of Linear-Phase FIR FiltersWith Frequency Inequality Constraints", *IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—II: EXPRESS BRIEFS, VOL. 53, NO. 2, PP. 120-124,2006.***

3. Tianqi and Cheng, “Sample Rate Conversion Technology in Software Defined Radio” Student Number, IEEE, Memorial University of Newfoundland St. John’s, Newfoundland and Labrador, Canada, 1-4244-0038-4 2006, IEEE CCECE/CCGEI, Ottawa, pp.1355-1358, 2006.
4. Lingwen, Xiaojun and Xia, “Design of Optimum Filters for WCDMA Base Station Based on SDR”, IEEE journal in Communications, ICSP2006 Proceedings: pp. 1-4, 2006.
5. Shahana, Babita, Poulose and Sreela, “Decimation Filter Design Toolbox for Multi- Standard Wireless Transceivers using MATLAB” International Journal of Signal Processing, 5;2 PP. 154-163, 2009.
6. Tony, “RF and Digital Signal Processing for Software-Defined Radio”, Elsevier, ISBN 978-0-7506-8210-7, UK, pp. 319, 2009.
7. Proakis, and Salehi. “Digital Communications”, 5th ed., New York, NY: McGraw-Hill, Ch. 3, pp. 95-148, 2008.
8. Ricardo, “practical FIR filter design In matlab”, The Math Works inc. Revision 1.1 , USA: PP. 5, 26-27, 2008.
9. Faheem and Shahid, “Sample rate conversion filter design for multi-standard software radios”, Elsevier Inc. All rights reserved, Digital Signal Processing Vol.20, pp.3–12, 2010.
10. Dan, “Digital Signal Processing Topic-9 filter design FIR”, pp.1-30, 2008
Available: <http://www.ee.columbia.edu/~dpwe/e4810/lectures/L09-firdesign-2up.pdf> .
11. Chen, “Design of Equiripple Linear-Phase FIR Filters Using MATLAB”, IEEE transaction, 2011
available at: <http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5997704>.
12. Xilinx Inc. “ MAC FIR v 5.1 Product specifications”, 2005
Available at: www.xilinx.com pp.1-19.