Eng. & Tech. Journal,Vol.29, No.15, 2011

# The Geometrical Analysis Accuracy for Parallel Robotic Mechanisms

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Received on: 31/3/2011
Accepted on: 8/9/2011

**Abstract**

The geometrical analysis of the robotic mechanisms means the formulation of the position equation of the mechanism in terms of dependent and independent coordinates. This function should describe the position of its links and the end effecter (or moving platform center) in terms of dependent and independent coordinates.

In this paper will be derived a mathematical model, that is, a model in terms of dependent and independent generalized coordinates, for the position accuracy of the end effecter (platform) of the closed kinematics robotic mechanisms. In these types of mechanisms it is necessary to solve the inverse problem of the geometrical analysis. The solving of the inverse problem in the robotic systems means the definition of independent coordinates on preset values and evaluation of the dependent coordinates with several degrees of mobility.

The aim of this paper is to obtain a new mathematical formulation for evaluation of the deviation of the end effecter position due to the deviation of the independent coordinates. It will be evaluated only the deformations of the arms caused by the weights and the forces.

**Keywords**: Parallel robotics mechanisms, Geometrical analysis, Gough – Stewart manipulators.

# دقة التحليل الجيومتري لاليات الروبوتات المتوازية

**الخلاصة**

ان التحليل الجيومتري لاليات الروبوتات يمثل صياغة المعادلة التي تصف موقع الية الروبوت بالاحداثيات المستقلة (التي تسبب الحركة) والاحداثيات غير المستقلة (التي تنتج بسبب الحركة). ان هذه المعادلة يجب ان تصف موقع اذرع الروبوت وكذلك موقع مركز المنصة المتحركة.

يتناول البحث اشتقاق نموذج رياضي يبين دقة موقع مركز المنصة المتحركة للروبوتات "المغلقة". ومن المعلوم انه لغرض الوصول الى صياغة معادلة التحليل الجيومتري في هذا النوع من الروبوتات يجب صياغة وحل المشكلة العسكية للتحليل الجيومتري. ان حل المشكلة العكسية في منظومات الروبوتات يمثل تحديد قيم معينة للاحداثيات غير المستقلة عند درجات مختلفة من الحركة واحتساب الاحداثيات المستقلة التي تنتجها.

ان الدافع الرئيسي لهذا البحث هو لاشتقاق معادلة جديدة لاحتساب الانحرافات في موقع مركز المنصة المتحركة والذي ينتج بسبب الانحرافات المختلفة في الاحداثيات المستقلة وسيتم اعتماد الانحرافات الناتجة في اذرع الروبوت نتيجة الانحناء بسبب الاوزان والقوى المؤثرة على هذه الأذرع فقط.

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## 1. Introduction

In the field of robotics, there are two types of robotic systems mechanisms, the first type is named open kinematics chain mechanisms (as shown in fig 2), and the second is the closed kinematics chain mechanisms. In open kinematics mechanisms the links connected sequentially starting from a fixed base. The last link in the chain is connected from one end to a previous link but is free from the other end [10].

Closed kinematics chains are formed when several arms (links) coordinated to handle on object [3]. Gough – Stewart manipulator is the famous kind of the closed kinematics chain mechanisms. It consists of a set of serial links each connected to a fixed base from one end, and connected to a common moving platform (or end effecter) on the other end.

In general, Gough-Stewart platform manipulator is a six degree of freedom with two main bodies [3]. The fixed body is called the base, while another body is regarded as movable and is called the moving plate (platform). These two bodies are connected together by six extensible legs [6].

It will be assumed that every leg of the legs of the manipulator consists of two parts connected together with a prismatic kinematic joint (P). All the legs are connected with the base by spherical kinematic joints (S) in the points $A_i$ while they are connected with the moving plate by spherical joints with fingers in the points $B_i$ as shown in Figure1.

Gough – Stewart manipulator implement a certain task when the moving platform (center of the moving platform) transforms from point 1 to point 6. The coordinates and orientations of these points are known. The track of the moving platform center between these two points will be divided into six points.

Solving of the inverse problem of the geometrical analysis will lead to obtain the initial length of the manipulator arms (legs).

Forward geometrical analysis problem solution will lead to evaluate the programmable coordinates and the orientations of the moving platform center in each point of the track between point 1 and point 6.

In this paper, the deviation between the programmable values of coordinates and orientations and the actual values will be evaluated by obtaining a novel formulation and using MATH CAD program.

## 2. Problem formulation

The determination and solving of the position equation for the closed kinematics chain mechanisms (geometrical analysis) is complicated. The formulation of this problem for these types of mechanisms often leads to a nonlinear set of equations. The determination of the position equation is more complicated and has many solutions [4]. In this paper proposed a new method for establishing the position equation for these types of mechanisms. In this method the full mechanism will be divided into many kinematics chain mechanisms, arrange the position equation for each of them and then connect all of them into the main position equation (position equation of the platform or the end effecter).

It has been found that our methods can be applied to more complex problems, including the motion planning problem for planner closed chains and to spatial kinematics chains with revolute joints.

In this paper we proposed Gough – Stewart manipulator as a closed kinematics chain mechanism. Proposed Gough – Stewart characteristics will be defined as follow:

$R_{ai}$ : The coordinate vector of the spherical joints connected the fixed base with the legs of the manipulator (in the global coordinate system);

$R_{bi}$ : The coordinate vector of the spherical with finger joints connected the moving platform with the legs of the manipulator (in the local coordinate system);

$R_o$ : The coordinate vector of the moving platform center (in the global coordinate system);

$A_{ol}$ : The matrix of the coordinate transformation from the local coordinate system to the global coordinate system;

$s_i$ : Prismatic joints displacement (legs extension).

The whole mechanism will be divided into six configuration systems. Each configuration system will consist from the following parts (as shown in fig 3):

- Coordinate vector of one spherical joint between the leg (arm) and the fixed base of the manipulator;
- Prismatic joint of the arm of the manipulator;
- Coordinate vector of the symmetrical joint between the moving platform and the same leg (arm);
- The coordinate vector of the moving platform center.

There will be six configuration systems for the manipulator mechanism connected together to obtain the main equation of the coordinates and orientation for the moving platform center at any time in terms of independent coordinate.

## 3. Platform center position

The mechanism will be divided into six structures each leg of the Gough-Stewart platform manipulator is treated as an independent substructure. The platform center position or the structure configuration can be defined from each independent structure as follow:

$$s_i = R_o + A_{ol} R_{bi} - R_{ai} \quad \ldots\ldots \ (1)$$

Where:

$R_o = \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}$ Coordinates of platform center in the global coordinate system;

$R_{ai} = \begin{bmatrix} x_{ai} \\ y_{ai} \\ z_{ai} \end{bmatrix}$ Coordinates of the joints between the manipulator legs and the fixed base in the global coordinate system;

$R_{bi} = \begin{bmatrix} x_{bi} \\ y_{bi} \\ z_{bi} \end{bmatrix}$ Coordinates of the joints between the manipulator legs and the platform in the local coordinate system;

$$A_{ol} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Where:

$$a_{11} = \cos(y)\cos(j) - \sin(y)\cos(q)\sin(j)$$
$$a_{12} = -\cos(y)\sin(j) - \sin(y)\cos(q)\cos(j)$$
$$a_{13} = \sin(y)\sin(q);$$
$$a_{21} = \sin(y)\cos(j) + \cos(y)\cos(q)\sin(j)$$

$$a_{22} = -\sin(y)\sin(j) + \cos(y)\cos(j)\cos(q)$$
$$a_{23} = -\cos(y)\sin(q);$$
$$a_{31} = \sin(q)\sin(j);$$
$$a_{32} = \sin(q)\cos(j);$$
$$a_{33} = \cos(q)$$

where ψ, θ and φ are Euler angles.

Simply we can find that equation (1) presents the length of the manipulator leg in this structure configuration. By squaring the two sides of equation (1) and because of the matrix $A_{ol}$ orthogonally the following equation can be obtained:

$$s_i^2 = R_o^T R_o + R_{bi}^T R_{bi} + R_{ai}^T R_{ai} + 2R_o^T A_{ol} R_{bi} - 2R_{ai}^T A_{ol} R_{bi} - 2R_o^T R_{ai} \quad ..(2)$$

The formulation of the geometrical analysis of the Gough – Stewart manipulator when all the six configurations connect together can be written as follow:

$$G_i = R_o^T R_o + R_{bi}^T R_{bi} + R_{ai}^T R_{ai} + 2R_o^T A_{ol} R_{bi} - 2R_{ai}^T A_{ol} R_{bi} - 2R_o^T R_{ai} - s_i^2 \quad \ldots\ldots \ (3)$$

Or in the standard form

$$G_i(s,u) = 0 \quad \ldots\ldots\ldots\ldots\ldots \ (4)$$

The symbol $u$ is the vector of moving platform center. This vector presents the Cartesian coordinates and Euler angles of orientation of the moving plate center as follow:

$$u = [x \quad y \quad z \quad y \quad q \quad j]^T$$

The extension of the six legs of the manipulator:

$$s_i = [s_1 \quad s_2 \quad s_3 \quad s_4 \quad s_5 \quad s_6]^T$$

Equation (4) presents a nonlinear set of six equations. The forward solution of this equation will be obtained the position of the moving platform center (end effecter).

**4. Geometrical analysis accuracy**

Deviation between the theoretical and actual values of the coordinates vector of moving platform center will be occur due to manufacturing errors of the parts, deformation of the prismatic joints under the static and dynamic loads, backlash of the screws and the errors between the nominal and actual characteristics of the actuators.

The above reasons will be caused deviation in the coordinates of the connection joints of the manipulator legs with the base $R_{ai}$ and moving platform $R_{bi}$. Also they will be caused a deviation in the prismatic joint displacement $s_i$.

The actual values of them will be as follow:

$$R_{ai} = R_{ai} + \Delta R_{ai} \quad ;$$

$$R_{bi} = R_{bi} + \Delta R_{bi} \quad ;$$

$$s_i = s_i + \Delta s_i$$

Solution of the forward geometrical analysis, in case of evaluation the above deviation will be result an error in the coordinate and the orientation of the moving platform center. It will be written as follow:

$$u = u^N + du$$

Because of the changes listed above, the geometrical analysis equation (4) can be written in another form:

$$G_i(s_i + \Delta s_i, u + du, R_{ai} + \Delta R_{ai}, R_{bi} + \Delta R_{bi}) = 0 \quad (5)$$

The left side of the equation (5) can be implementing with the Taylor series and will be as follow:

$$G_i(s_i, u, R_{ai}, R_{bi}) + \frac{\partial G_i}{\partial s_i}\Delta s_i + \frac{\partial G_i}{\partial R_{ai}}\Delta R_{ai} + \frac{\partial G_i}{\partial R_{bi}}\Delta R_{bi} + \frac{\partial G_i}{\partial u} du = 0 \quad (6)$$

The error in the position coordinates and orientation of the moving platform center will be found from the following equation:

$$du = -\left[\frac{\partial G_i}{\partial u}\right]^{-1}\left[\frac{\partial G_i}{\partial s_i}\Delta s_i + \frac{\partial G_i}{\partial R_{ai}}\Delta R_{ai} + \frac{\partial G_i}{\partial R_{bi}}\Delta R_{bi}\right] \quad (7)$$

The error $\partial u$ will be as a vector of the deviation of the three coordinates and three orientation angles (Euler angles) as follow:

$$du = [\partial x \quad \partial y \quad \partial z \quad \partial y \quad \partial q \quad \partial j]^T$$

## 5. Example of analysis

In this example the given mechanism is a parallel robotic system type of Gough Stewart manipulator with the following parameters:

- The fixed plate diameter is 2.4m.
- The spherical joints of the robot legs with the fixed plate locate on the angles ($30^\circ$, $90^\circ$, $135^\circ$, $225^\circ$, $270^\circ$, $330^\circ$) in the Global coordinates.
- The moving platform diameter is 0.8m.
- The joints between the manipulator legs and the moving platform locate on the angles ($15^\circ$, $90^\circ$, $160^\circ$, $200^\circ$, $270^\circ$, $345^\circ$) in the local coordinates.
- The track of the moving platform center will be between point 1 and point 6 through the specified points 2,3,4,5. the coordinates and the Euler angles of these points in the global coordinates system are as in the following matrix:

$$M = \begin{pmatrix} 0.66 & -0.35 & 1.8 & -5 & 2 & 1.4 \\ 0.34 & -0.17 & 1.83 & -7 & 10 & 2.2 \\ 0.07 & -0.09 & 1.85 & -10 & 10 & 2.6 \\ -0.15 & 0.24 & 1.92 & -12 & 11.5 & 2.7 \\ -0.3 & 0.36 & 1.94 & -12 & 12.6 & 3.4 \\ -0.44 & 0.58 & 1.98 & -13 & 12.7 & 4.2 \end{pmatrix}$$

- From the solution of the inverse geometrical problem we can evaluate the actual length of the manipulator legs $S$ in the first location of the moving platform as in the following vector:

$$s = (2.003 \ \ 2.255 \ \ 2.375 \ \ 2.153 \ \ 1.948 \ \ 1.80)m$$

- When the moving platform center will arrive to the last point (point 6), the length of the manipulator legs are as in the following vector:

$$S = (2.318 \quad 2.155 \quad 2.02 \quad 2.281 \quad 2.38 \quad 2.541)m$$

- The deformations in the manipulator arms (legs) due to the forces and weights acting on the manipulator are as in the vector:

$$\Delta s_i = (0.12 \quad 0.45 \quad 0.26 \quad 0.32 \quad 0.28 \quad 0.41) \times 10^{-3} m$$

- The deviation of the moving platform center position can be evaluated by implementing of equation (7). The results arranged in tables 1 – 6 enclosed with this paper.

## 6. Conclusions

There is a deviation between the theoretical (programmable) position of the moving plate center and its actual position. This deviation occurs due to many reasons, for example deformation of the arms, errors in the parts manufacturing, backlash of the actuators and the errors between the theoretical and actual inputs of the motors.

The accuracy of the moving plate center position and orientation occurred due to the effect of the deformation of the manipulator arms under the applied forces and their weights has been evaluated in this paper. In case of the deviation value is not so little, the suitable solution is to change the design or the dimensions of the arms to reduce the deformation and buckling of the arms or to reduce the forces (weights) transforming by the manipulator.

These criteria can be used to examine the singularity of the manipulator configuration because it has been found that, the value of the deviation between the theoretical (programmable) position of the moving plate center and its actual position near the singularity configuration of the manipulator was very large and the manipulator can not accurately implement the programmable moving.

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**Table 1 Accuracy of moving plate center coordinates “X – coordinate”**

| Point | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Programmable value of X in m | 0.66 | 0.34 | 0.07 | -0.15 | -0.3 | -0.44 |
| Deviation $\delta x$ in $10^{-3}$ m | 0.213 | 0.146 | 0.129 | 0.201 | 0.222 | 0.256 |

**Table 2 Accuracy of moving plate center coordinates “Y – coordinate”**

| Point | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Programmable value of Y in m | -0.35 | -0.17 | -0.09 | 0.24 | 0.36 | 0.58 |
| Deviation $\delta y$ in $10^{-3}$ m | 0.135 | 0.211 | 0.183 | 0.186 | 0.177 | 0.232 |

**Table 3 Accuracy of moving plate center coordinates “Z – coordinate”**

| Point | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Programmable value of Z in m | 1.8 | 1.83 | 1.85 | 1.92 | 1.94 | 1.98 |
| Deviation $\delta z$ in $10^{-3}$ m | 0.996 | 0.946 | 0.987 | 0.849 | 1.064 | 1.213 |

**Table 4 Accuracy of moving plate center orientation "ψ - angle"**

| Point | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Programmable value of ψ in degrees | -5 | -7 | -10 | -12 | -12 | -13 |
| Deviation δψ in Degrees | 0.059 | 0.066 | 0.057 | 0.074 | 0.063 | 0.054 |

**Table 5 Accuracy of moving plate center orientation "θ - angle"**

| Point | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Programmable value of θ in degrees | 2 | 10 | 10 | 11.5 | 12.6 | 12.7 |
| Deviation δθ in Degrees | 0.0145 | 0.0132 | 0.015 | 0.0174 | 0.0201 | 0.0167 |

**Table 6 Accuracy of moving plate center orientation "ϕ - angle"**

| Point | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Programmable value of ϕ in degrees | 1.4 | 2.2 | 2.6 | 2.7 | 3.4 | 4.2 |
| Deviation δϕ in Degrees | 0.0121 | 0.0265 | 0.0214 | 0.0272 | 0.0197 | 0.0188 |

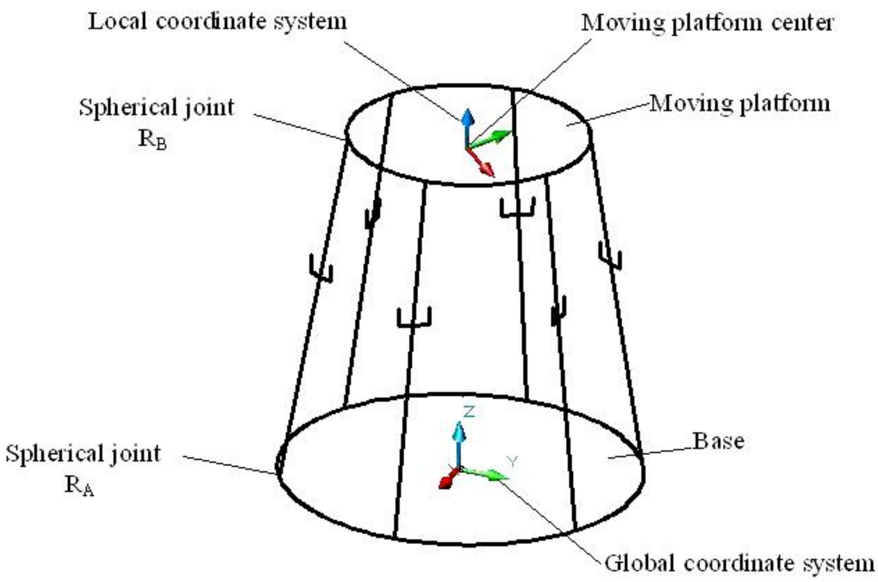


**Figure 1 Gough – Stewart Parallel Robotic Mechanism**

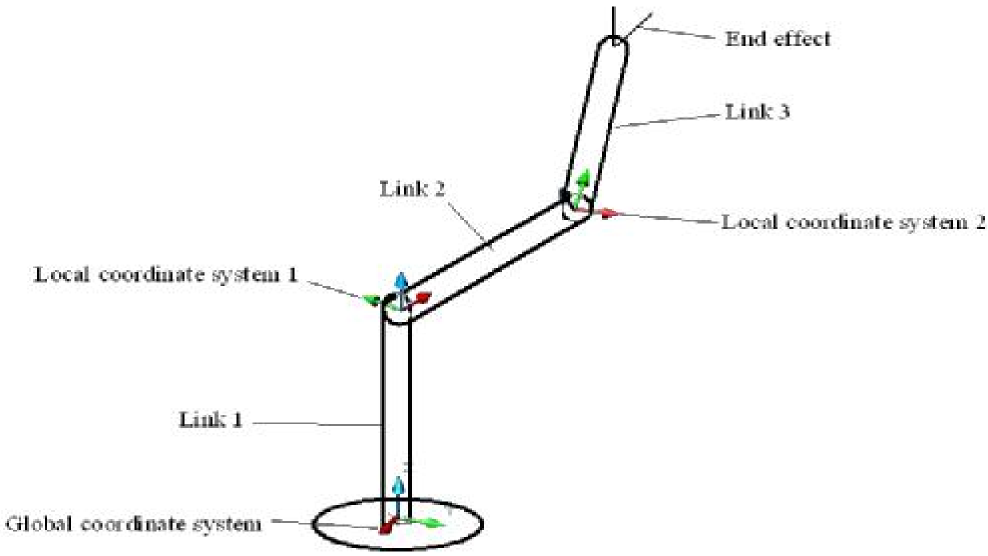


**Figure 2 Open Kinematics Chain Mechanism**

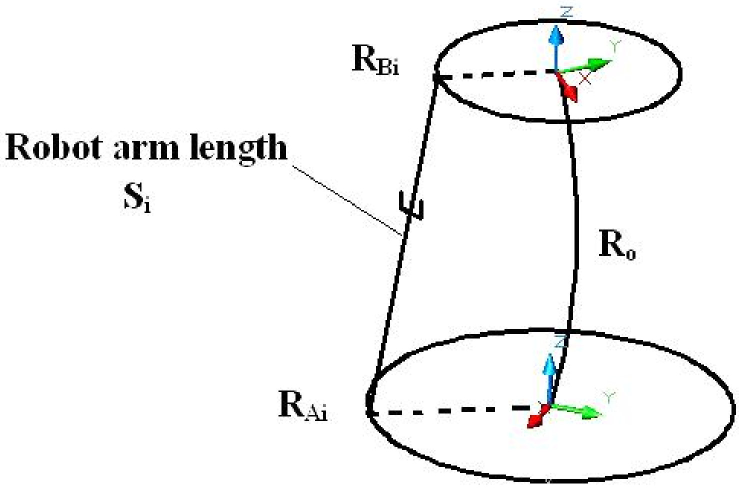


**Figure3 Configuration System of the Robot Arm**