

A Geometric Construction of a (56,2)-Blocking Set in PG(2,19) and on Three Dimensional Linear [325, 3, 307]₁₉Griesmer Code

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ABSTRACT

In this paper we give a geometrical construction of a (56, 2)-blocking set in PG(2, 19) and We obtain a new (325,18)- arc and a new linear code [325,3,307]₁₉ and apply the Griesmer rule so that we prove it an optimal or non-optimal code, giving some examples of field 19 arcs Theorem (2.1)

Keywords: Arc , Bounded Griesmer , double Blocking set , projection $[n, k, d]_q$ code. Projective Plane , Optimal Linear code.

البناء الهندسي للمجاميع القالبية – (56, 2) في PG(2, 19) وفي الشفرات Griesmer [325,3,307]₁₉ الخطية ثلاثية الابعاد.

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المخلص

في هذا البحث سوف نعطي بناء هندسي للمجموعة القالبية – (56,2) في المستوى الاسقاطي PG(2,19) ونحصل على قوس جديد – (325,18) وشفرة خطية [325,3,307]_q جديدة ونطبق قاعدة Griesmer عليها لكي نبرهن بأنها شفرة مثلى او غير مثلى مع اعطاء بعض الامثلة على اقواس الحقل 19 ميرهنه (2.1). الهدف من البحث الحصول على قوس جديد.

الكلمات المفتاحية: القوس، قيود (حدود) Griesmer ، المجموعة القالبية المزدوجة، الشفرة $[n, k, d]_q$ الاسقاطية، المستوى الاسقاطي، الشفرة الخطية المثلى.

1 . Introduction

Give GF(q) a chance to indicate the Galois field of q components and V (3, q) be the vector space of column vectors of length three with sections in GF(q). Let PG(2, q) be the comparing projective plane. The purposes of PG(2, q) are the non-zero vectors of V (3, q) with the standard that X =(x1, x2, x3) and Y=(λx1,λx2,λx3) speak to a similar point, where λ ∈ GF(q)/{0}. The quantity of purposes of PG(2, q) is q²+q+1.If the point P(X) is the proportionality class of

the vector X , at that point we will state that X is a vector speaking to $P(X)$. A subspace of measurement one is an arrangement of focuses the majority of whose speaking to vectors shape a subspace of measurement two of $V(3, q)$. Such subspaces are called lines. The quantity of lines in $PG(2, q)$ is $q^2 + q + 1$. There are $q + 1$ [5][6].

Definition 1.1 . A (k, r) - circular segment is an arrangement of k purposes of a projective plane to such an extent that some r , however no $r+1$ of them, are collinear [3]

Definition 1.2 A (l, n) - blocking set S in $PG(2, q)$ is an arrangement of l focuses to such an extent that each line of $PG(2, q)$ crosses S in at any rate n focuses, and there is a line meeting S inaccurately n focuses Note that a (k, r) - circular segment is the supplement of a $(q^2 + q + 1 - k, q + 1 - r)$ - blocking set in a projective plane Let $V(n, q)$ signify the vector space of all arranged n -tuples over \mathbb{F}_q and alternately $GF(q)$. A direct code C over $GF(q)$ of length n and measurement k is a k -dimensional subspace of $V(n, q)$. The vectors of C are called code words. The Hamming separation between two code words is characterized to be the quantity of facilitate puts in which they differ. The least separation of a code is the littlest of the separations between particular code words. Such a code is called a $[n, k, d]_q$ -code if its minimum Hamming separation is d . A focal issue in coding hypothesis is that of streamlining one of the parameters n, k and d for given estimations of the other two and q -settled. One of the variants is [2][9]

Problem. Find $nq(k, d)$, the littlest estimation of n for which there exists an $[n, k, d]_q$ -code which accomplishes this esteem is called ideal. The outstanding lower destined for the capacity $nq(k, d)$ is the accompanying Griesmer bound

$$nq(k, d) \geq gq(k, d) = \sum_{j=0}^{k-1} \left\lfloor \frac{d}{q^j} \right\rfloor \text{ and so are optimal.}$$

Now give some examples using Griesmer base to see the optimal code and the non-optimal code.

Example(1.4):- Let the code be linear is $[325, 3, 307]_{19}$

Solution:- by best Griesmer $nq(k, d) \geq gq(k, d) = \sum_{j=0}^{k-1} \left\lfloor \frac{d}{q^j} \right\rfloor$.

$$\begin{aligned} n &= nq(k, d) = \sum_{j=0}^{3-1} \left\lfloor \frac{307}{19^j} \right\rfloor \\ &= \frac{307}{19^0} + \frac{307}{19^1} + \frac{307}{19^2} \end{aligned}$$

$= 307 + 16.1578947368 + 0.8504155125 \cong 325$ So that code is optimal.

Example(1.5):- Let the code be linear is $[145, 3, 133]_{13}$.

Solution:- by best Griesmer $nq(k, d) \geq gq(k, d) = \sum_{j=0}^{k-1} \left\lfloor \frac{d}{q^j} \right\rfloor$.

$$\begin{aligned} n &= nq(k, d) = \sum_{j=0}^{3-1} \left\lfloor \frac{133}{13^j} \right\rfloor \\ &= \frac{133}{13^0} + \frac{133}{13^1} + \frac{133}{13^2} \end{aligned}$$

$= 133 + 10.23076923 + 0.786982248 \cong 145$

So that code is optimal.

Example(1.6):- Let the code be linear is $[336,3,317]_{19}$.

Solution:- by best Griesmer $nq(k, d) \geq gq(k, d) = \sum_{j=0}^{k-1} \left\lceil \frac{d}{q^j} \right\rceil$.

$$n = nq(k, d) = \sum_{j=0}^{3-1} \left\lceil \frac{317}{19^j} \right\rceil$$

$$= \frac{317}{19^0} + \frac{317}{19^1} + \frac{317}{19^2}$$

$$= 317 + 16.68421053 + 0.87116343 \leq 336$$

So that code is non-optimal

Codes with parameters are called Griesmer codes. There exists a relationship between (n,r)-circular segments in PG(2,q) and [n,3,d]codes, given by the following hypothesis.[gq(k,d)]q

Theorem 1.7 There exists a projective $[n, 3, d]_q$ code if and only if there exists an (n, n-d)-arc in PG(2, q). In this paper we consider the case q =19 and the elements of GF(19) are denoted by 0,1,2,3, 4,5,6,7, 8, 9,10 ,11,12,13,14,15,16,17,18.

2.The construction

It is obvious that in PG(2, q) (q is prime) three lines as a rule position frame a (3q, 2)- blocking set. The issue of finding a 2-blocking set with under 3q components had for since quite a while ago stayed unsolved as of not long ago Braun et al. [2] found the principal case of such a set. They developed the (57, 2)-blocking set in PG(2, 19), comprising of the accompanying focuses on

The complement of the (57,2)-blocking set

{ (0, 1,5), (0, 1, 8), (0, 1, 11), (1, 0, 7), (1, 0,12), (1, 1,1), (1, 1, 4), (1, 2; 9), (1, 2,10)
 (1, 2, 15), (1, 3,7),(1; 3, 17), (1, 4, 6), (1, 4, 18), (1, 5, 5), (1, 5,10), (1, 5, 14),
 (1,6, 2)
 (1, 6, 3), (1, 6, 16), (1, 7, 8), (1, 7, 11),(1,7,13), (1, 8, 0), (1, 8, 5), (1, 9, 8), (1, 9, 11)
 (1, 9, 16), (1,10, 8), (1, 10, 16), (1, 11, 0), (1, 1,15),(1, 11, 14), (1, 12, 8), (1, 12, 11)
 (1, 12, 13), (1, 13, 2), (1, 13, 3), (1, 13, 17), (1, 14,9), (1, 14,10), (1, 14, 14),(1, 15, 1)
 (1,15,6), (1,15, 18), (1, 16, 3), (1, 16, 7), (1, 16, 12), (1, 16, 17), (1, 17, 4), (1, 17, 9)
 (1, 17, 10),(1, 17,13), (1, 17, 15), (1, 18, 1), (1, 18, 4)}

All the more unequivocally, this (57, 2)- blocking set emerges as the supplement of a (324, 18)- circular segment in PG(2, 19) which they built by the strategy displayed in their article [2] (see additionally [1]. In the following hypothesis we in demonstrate Lower bound,(324,18)- circular segment and Give us the accompanying:

- 1) Get new code $[325,3,307]_{19}$
- 2) Get a new (325,18) – arc

- a) There is no line in $PG(2, 19)$ which is unique in relation to l_i and which contains four of the expelled focuses
- b) The lines that contain three of the evacuated focuses meet at most four new focuses A_1, A_2, A_3 ,
- c) The lines that contain only two of the evacuated focuses don't go through the crossing point focuses M_1, M_2, N_1, N_2, M_6 and P_{11}

The conditions (a)– (c) will ensure that including the focuses A_1, A_2, A_3, A_4 to the arrangement of outstanding purposes of the lines, we will acquire a 2-blocking set without any than 56. Clearly we ought not expel any focuses from the quadruples $M_i, N_i, P_i, Q_i, I = 1, 2$; generally, the lines $p_2: x = 0$ and $p_1: y = 0$ will move toward becoming 1-or 0-secants. Correspondingly, it isn't alluring to expel any focuses from the quadruples $M_i, N_i, P_i, Q_i, I = 6, 11$, on the grounds that expelling a crossing point purpose of the lines L_i will either not diminish enough the quantity of focuses, or the lines $p_5: x + 3y = 0$ or $p_{12}: x + 10y = 0$ will move toward becoming 1-or 0-secants. We require to observe the accompanying guideline: on the off chance that we have officially expelled two points from a fourfold M_i, N_i, P_i, Q_i , we ought to leave the staying two in the set. Else, one of the lines p_i will turn into a 1-or 0-secant. Let us take out the accompanying 22: from the line L_{17} – $M_1, M_4, M_5, M_8, M_{11}, M_{12}, M_{14}, M_{20}$ from the line L_{18} – $N_2, N_6, N_{11}, N_{12}, N_{14}, N_{17}, N_{16}$, from the line L_{23} – P_1, P_8, P_{14}, P_{17} and from the line L_{24} – Q_2, Q_3, Q_{22}

Now we select four lines intersecting six points and lines are $L_{17}, L_{18}, L_{23}, L_{24}$ such that

$$L_{17} \cap L_{18} = 18 = (1, 12, 14)$$

$$L_{17} \cap L_{23} = 356 = (1, 6, 12)$$

$$L_{17} \cap L_{24} = 284 = (1, 13, 8)$$

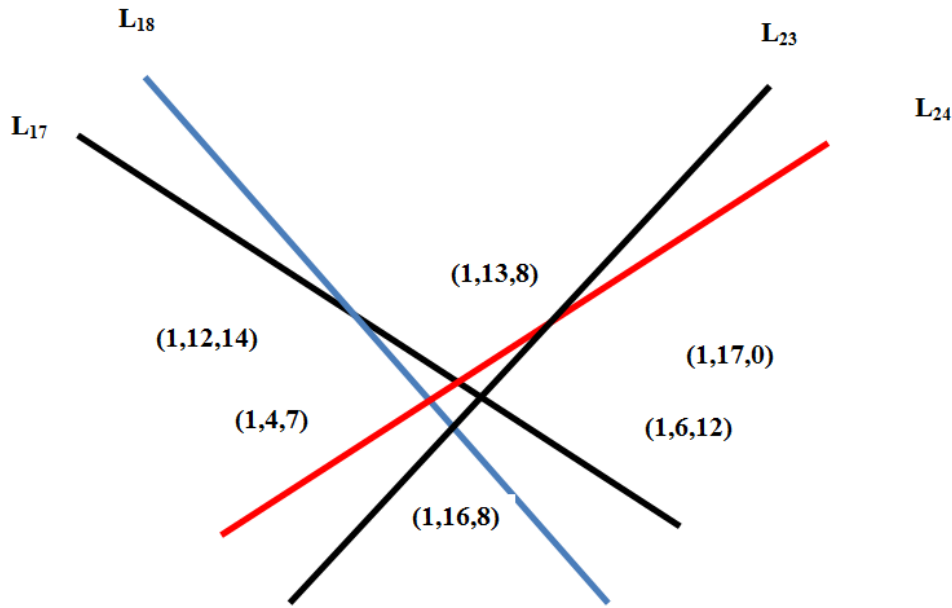
$$L_{18} \cap L_{23} = 290 = (1, 10, 14)$$

$$L_{18} \cap L_{24} = 357 = (1, 6, 8)$$

$$L_{23} \cap L_{24} = 24 = (1, 17, 0)$$

So the six common points are the sequence points $[18, 356, 284, 290, 357, 24]$

Now we draw the intersection points and show the intersection



The set of removed points

$$A = \left\{ \begin{array}{l} (1,1,4), (0,1,13), (1,9,13), (1,7,6), (1,16,9), (1,10,7), (1,5,18), (1,4,5) \\ (1,17,16), (1,14,11), (1,18,5) \\ (1,15,10), (1,9,9), (1,1,2), (1,2,10), (1,1,14) \\ (1,11,10), (1,18,11), (1,11,15), (0,1,17), (1,15,4), (1,12,10) \end{array} \right\}$$

is a(22,8)-arc in PG(2,19) and has the following secant distribution:

$$T_0 = 111, T_2 = 83, T_1 = 162, T_3 = 22, T_4 = 1, T_5 = 1, T_8 = 1, T_7 = 1$$

condition (a) is satisfied. The forty-eight 2-secants of A are

$X + 5Y = 0$	$X + 14Y + Z = 0$	$X + 2Y + 10Z = 0$
$X + 3Y + 15Z = 0$	$X + 11Y + 5Z = 0$	$X + 16Z = 0$
$X + 7Y + 5Z = 0$	$X + 12Y + 15Z = 0$	$X + 9Y + 18Z = 0$
$X + Y + 11Z = 0$	$Y + 3Z = 0$	$X + 14Y + 15Z = 0$
$X + 18Y + 18Z = 0$	$X + 8Y + 4Z = 0$	$X + 13Y + 15Z = 0$
$X + 2Y + 12Z = 0$	$X + 11Y + 11Z = 0$	$X + 6Y + 13Z = 0$
$X + 2Z = 0$	$X + 5Y + 2Z = 0$	$X + 13Y + 12Z = 0$
$X + 17Y + 11Z = 0$	$X + 8Y + 5Z = 0$	$X + 9Y + Z = 0$
$X + 7Y = 0$	$X + 8Y + 13Z = 0$	$X + 18Y + 6Z = 0$
$X + Y + 12Z = 0$	$X + 1Y + 13Z = 0$	$X + 18Y + 10Z = 0$
$X + 9Y + 3Z = 0$	$X + 17Y + 9Z = 0$	$X + 3Y + 5Z = 0$
$X + 8Y + 7Z = 0$	$X + 3Y + Z = 0$	$X + 18Y + 14Z = 0$
$X + 9Y + 2Z = 0$	$Y + 16Z = 0$	$X + 10Y + 8Z = 0$
$X + 16Y + 7Z = 0$	$Y + 8Z = 0$	$X + 12Y + 7Z = 0$
$X + 9Y + 3Z = 0$	$X + 10Y + 9Z = 0$	$X + 2Z = 0$
$X + 15Y + 12Z = 0$	$X + 13Y + 7Z = 0$	$X + 15Y + 8Z = 0$
$X + 12Y + 18Z = 0$	$X + 18Y + 6Z = 0$	$X + 6Y + Z = 0$
$X + 17Y + 15Z = 0$	$X + 12Y + Z = 0$	$X + 16Y + 18Z = 0$
$X + 12Y + 8Z = 0$	$X + 6Y + Z = 0$	$X + 16Y + 18Z = 0$
$X + 5Y + 7Z = 0$	$X + 4Y + 17Z = 0$	$X + 13Y + 4Z = 0$
$X + 6Y = 0$	$X + 11Y + 13Z = 0$	$X + 5Y + 9Z = 0$

$$\begin{array}{lll}
 X + 13Y + 11Z = 0 & X + 8Y + 10Z = 0 & X + 7Y + 5Z = 0 \\
 X + 10Y + 3Z = 0 & X + 5Y + 8Z = 0 & X + 15Y + 7Z = 0 \\
 X + 4Y + 16Z = 0 & X + 9Y + 12Z = 0 & X + 7Y + 7Z = 0 \\
 X + 3Y + 2Z = 0 & X + 17Y + 2Z = 0 & X + 9Y + 18Z = 0 \\
 X + 18Y + 4Z = 0 & X + 11Y + Z = 0 & X + 7Y + 10Z = 0 \\
 X + 11Y + 2Z = 0 & X + 7Y + 14Z = 0 & X + 5Y + 9Z = 0 \\
 X + 6Y + 14Z = 0 & X + 18Y + 10Z = 0 &
 \end{array}$$

It is anything but difficult to check now that none of them contains a crossing point purpose of lines li ; therefore condition (c) is fulfilled. Let us take a gander at condition (b). The 3-secants of An , i.e. the lines not the same as li , with the end goal that each contains three of the evacuated focuses, are

$$\begin{array}{ll}
 g1 = x + y + 4z = 0 & , \quad N_7, M_{10}, P_3 \in g1 \\
 g2 = 0x + y + 13z = 0 & , \quad M_{11}, Q_{18}, P_4 \in g2 \\
 g3 = 1x + 9y + 13z = 0 & , \quad M_9, N_{11}, P_{13} \in g3 \\
 g4 = x + 7y + 6z = 0 & , \quad M_{13}, Q_6, N_5 \in g4 \\
 g5 = x + 16y + 9z = 0 & , \quad M_4, N_{15}, P_{16} \in g5 \\
 g6 = x + 10y + 7z = 0 & , \quad Q_1, P_2, N_9 \in g6 \\
 g7 = x + 5y + 18z = 0 & , \quad Q_7, P_{18}, M_{19} \in g7 \\
 g8 = x + 4y + 5z = 0 & , \quad Q_{13}, P_8, N_8 \in g8 \\
 g9 = x + 17y + 16z = 0 & , \quad M_9, N_4, P_3 \in g9 \\
 g10 = x + 14y + 11z = 0 & , \quad N_1, M_2, P_{19} \in g10 \\
 g11 = x + 18y + 5z = 0 & , \quad P_{14}, Q_{13}, N_{18} \in g11 \\
 g12 = x + 15y + 0z = 0 & , \quad Q_4, P_{11}, N_3 \in g12 \\
 g13 = x + 9y + 9z = 0 & , \quad M_{15}, N_7, P_8 \in g13 \\
 g14 = x + y + 2z = 0 & , \quad Q_{18}, P_{12}, N_{19} \in g14 \\
 g15 = x + 2y + 10z = 0 & , \quad M_{19}, N_9, P_{18} \in g15 \\
 g16 = x + y + 14z = 0 & , \quad M_2, N_1, P_8 \in g16 \\
 g17 = x + 11y + 10z = 0 & , \quad Q_7, P_1, M_{20} \in g17 \\
 g18 = x + 18y + 11z = 0 & , \quad M_{15}, N_{11}, P_{20} \in g18 \\
 g19 = x + 11y + 15z = 0 & , \quad M_{19}, N_7, Q_2 \in g19 \\
 g20 = 0x + y + 17z = 0 & , \quad Q_{15}, P_{17}, N_{16} \in g20 \\
 g21 = x + 15y + 4z = 0 & , \quad Q_{14}, P_{12}, N_5 \in g21 \\
 g22 = x + 12y + 10z = 0 & , \quad M_2, N_1, P_9 \in g22
 \end{array}$$

Each line gi intersects some line li at a point not in the set A. Indeed:

$$\begin{array}{ll}
 g1 \cap l2 = (1,14,11) & , \quad g2 \cap l1 = (1,10,7) \\
 g3 \cap l3 = (1,11,10) & , \quad g4 \cap l4 = (1,1,13) \\
 g5 \cap l2 = (1,17,16) & , \quad g6 \cap l1 = (1,9,13) \\
 g7 \cap l4 = (0,1,17) & , \quad g8 \cap l3 = (1,2,6) \\
 g9 \cap l1 = (0,1,13) & , \quad g10 \cap l4 = (1,12,10) \\
 g11 \cap l1 = (1,4,5) & , \quad g12 \cap l1 = (1,7,9) \\
 g13 \cap l4 = (1,15,4) & , \quad g14 \cap l2 = (1,0,13) \\
 g15 \cap l2 = (1,6,11) & , \quad g16 \cap l4 = (1,13,6) \\
 g17 \cap l2 = (1,2,5) & , \quad g18 \cap l4 = (1,9,16) \\
 g19 \cap l3 = (1,9,9) & , \quad g20 \cap l2 = (1,3,18) \\
 g21 \cap l2 = (1,18,6) & , \quad g22 \cap l1 = (1,10,7)
 \end{array}$$

Furthermore, the lines g_i intersect one another in quadruples at the points $(1,12,10), (1,11,15), (1,3,2), (1,1,1)$.

More precisely

$$\begin{aligned} g_1 \cap g_3 \cap g_5 \cap g_7 \cap g_{16} \cap g_{18} \cap g_{21} &= (1,12,10) \\ g_6 \cap g_9 \cap g_{12} \cap g_{13} \cap g_{14} \cap g_{15} \cap g_{21} \cap g_{22} &= (1,11,15) \\ g_2 \cap g_7 \cap g_8 &= (1,3,2) \\ g_4 \cap g_{14} \cap g_{17} \cap g_{20} &= (1,1,1) \end{aligned}$$

Therefore, $(1,12,10), (1,11,15), (1,3,2), (1,1,1)$ are the points A_1, A_2, A_3, A_4 .

Adding these four points to the rest 52 points, we obtain the set

$$B = \left\{ \begin{array}{l} (1,12,14), \\ (1,14,2), (1,2,17), (1,18,16), (1,3,11), (1,15,15), (1,8,0), (1,17,3), (1,0,10) \\ (1,11,1) \\ (1,5,15) \\ (1,0,13), (1,11,6), (1,3,18), (1,8,1), (1,13,3), (1,7,12), (0,1,8), (1,6,4), (1,10,17) \\ (1,9,7), (1,12,2), (1,13,13), (1,15,16), (1,14,5), (1,8,15), (1,4,9), (1,5,1), (1,3,17), \\ (0,1,11) \\ (1,10,18) \\ (1,11,12) \\ (1,6,2)(1,7,4)(1,5,5)(1,3,9)(1,2,11)(1,1,13)(1,7,1) \\ (1,18,17)(1,10,14)(1,14,6)(1,0,15)(1,6,3)(1,9,16)(1,7,3)(1,8,18)(1,13,8)(1,6,12) \\ (1,16,8)(1,4,7)(1,17,0) \\ , (1,12,10), (1,11,15), (1,3,2), (1,1,1) \end{array} \right\}$$

The supplement of the set B is a $(325, 18)$ - bend

It pursues now by Theorem 1.7 that there exists a $[325,3,307]_{19}$ Griesmer code

$$\begin{aligned} M_{18}(2,19) = & \{(1,0,0), (0,1,0), (0,0,1), (1,0,2), (1,5,2), (1,5,8), (1,6,13) \\ & (1,4,16), (1,3,14), (1,17,15), (1,7,7), (1,15,12), (1,4,5), (1,2,10), (1,1,4) \\ & (1,3,15), (1,12,10), (1,1,14), (0,1,17), (1,0,16), (1,3,2), (1,5,18), (1,14,15) \\ & (1,7,5), (1,2,16), (1,3,8), (1,6,1), (1,16,15), (1,7,0), (0,1,7), (1,0,17), (1,7,18) \\ & (1,9,8), (1,6,18), (1,9,18), (1,15,4), (1,12,11), (1,13,6), (1,8,11), (1,13,11) \\ & (1,13,0), (0,1,13), (1,11,2), (1,5,0), (0,1,5), (1,0,4), (1,2,12), (1,4,10), (1,1,6) \\ & (1,8,10), (1,1,10), (1,1,3), (1,16,18), (1,9,13), (1,17,5), (1,14,11), (1,4,8) \\ & (1,18,18), (1,9,12), (1,4,0), (0,1,4), (1,0,14), (1,17,2), (1,15,11), (1,13,10) \\ & (1,1,15), (1,7,9), (1,18,14), (1,17,4), (1,12,16), (1,3,0), (0,1,3), (1,0,18) \\ & (1,9,2), (1,5,9), (1,9,10), (1,1,11), (1,13,15), (1,13,11), (1,18,8), (1,6,15) \\ & (1,7,6), (1,10,6), (1,8,6), (1,8,9), (1,18,14), (1,11,10), (1,11,13), (1,11,9) \\ & (1,18,10), (1,1,1), (1,10,12), (1,4,4), (1,12,12), (1,4,12), (1,4,18), (1,9,0) \\ & (0,1,9), (1,0,1), (1,10,2), (1,5,14), (1,17,11), (1,13,11), (1,17,14), (1,17,6) \\ & (1,8,5), (1,2,18), (1,9,1), (1,10,16), (1,3,16), (1,16,1), (1,10,10), (1,1,12) \\ & (1,4,6), (1,10,15), (1,7,15), (1,7,13), (1,11,3), (1,16,7), (1,15,14), (1,17,10) \\ & (1,1,0), (0,1,1), (1,4,2), (1,5,3), (1,16,6), (1,8,16), (1,3,7), (1,15,9), (1,18,6) \\ & (1,8,13), (1,11,14), (1,17,18), (1,9,3), (1,16,13), (1,11,7), (1,10,5), (1,6,9) \\ & (1,2,3), (1,13,16), (1,3,3), (1,16,12), (1,14,7), (1,15,3), (1,16,4), (1,17,8) \\ & (1,18,15), (1,7,14), (1,17,7), (1,1,17), (1,14,16), (1,3,6), (1,8,7), (1,15,8) \\ & (1,6,16), (1,3,1), (1,10,13), (1,11,17), (1,14,4), (1,12,18), (1,9,15), (1,7,8) \end{aligned}$$

(1,6,6),(1,8,12),(1,4,15),(1,7,11),(1,13,17),(1,14,13),(1,11,4),(1,12,1)
 (1,8,14),(1,10,8),(1,6,5),(1,2,14),(1,17,17),(1,14,12),(1,4,1),(1,10,4)
 (1,12,8),(1,6,17),(1,14,10),(1,1,16),(1,3,5),(1,2,8),(1,6,14),(1,17,9),
 (1,18,4),(1,12,9),(1,18,9),(1,18,3),(1,16,5),(1,2,15),(1,7,16),(1,3,4)
 (1,12,0),(0,1,12),(0,1,6),(1,8,2),(1,5,4),(1,12,5),(1,2,7),(1,15,13)
 (1,11,15),(1,16,0),(0,1,16),(1,0,6),(1,2,2),(1,5,12),(1,4,3),(1,16,9)
 (1,18,5),(1,2,0),(0,1,2),(1,0,7),(1,15,2),(1,17,1),(1,10,1),(1,10,7)
 (1,15,0),(0,1,15),(1,0,9),(1,18,2),(1,5,16),(1,0,8),(1,6,2),(1,5,13)
 (1,11,0),(1,7,2),(1,5,18),(1,9,9),(1,18,12),(1,4,17),(1,3,13),(1,11,16)
 (1,14,1),(1,10,9),(1,18,11),(1,12,17),(1,14,18),(1,9,14),(1,16,3),(1,16,11)
 (1,13,1),(1,3,10),(1,1,5),(1,2,4),(1,12,7),(1,15,11),(1,15,7),(1,6,7)
 (1,15,18),(1,9,4),(1,12,15),(1,7,10),(1,1,9),(1,18,1),(1,10,11),(1,13,18)
 (1,9,5),(1,2,1),(1,10,3),(1,16,10),(1,4,11),(1,1,18),(1,9,11),(1,13,5)
 (1,2,9),(1,18,0),(0,1,18),(1,0,11),(1,13,2),(1,5,10),(1,1,7),(1,15,17)
 (1,14,3),(1,16,17),(1,14,17),(1,14,8),(1,6,10),(1,1,8),(1,6,0),(0,1,6)
 (1,1,2),(1,5,7),(1,15,1),(1,10,0),(0,1,10),(1,0,3),(1,16,2),(1,5,6)
 (1,8,4),(1,12,3),(1,16,4),(1,12,4),(1,12,13),(1,14,9),(1,18,7),(1,11,11)
 (1,15,6),(1,8,8),(1,15,5),(1,2,13),(1,11,5),(1,2,5),(1,9,17),(1,14,14)
 (1,17,12),(1,4,13),(1,11,8),(1,6,11),(1,13,4),(1,12,6),(1,8,3),(1,16,16)
 (1,3,12),(1,4,14),(1,17,13),(1,11,18),(1,9,6),(1,8,17),(1,14,0),(0,1,14)
 (1,13,12),(1,17,16)}.

Table (1)
Projection Level Points in PG(2,19)

i	pi	i	pi	i	pi	i	pi	i	pi
1	(1,0,0)	29	(1,14,15)	57	(0,1,5)	85	(1,17,2)	113	(1,8,9)
2	(0,1,0)	30	(1,7,5)	58	(1,0,4)	86	(1,15,11)	114	(1,18,14)
3	(0,0,1)	31	(1,2,16)	59	(1,12,2)	87	(1,13,10)	115	(1,11,10)
4	(1,0,2)	32	(1,3,8)	60	(1,5,5)	88	(1,1,15)	116	(1,1,13)
5	(1,5,2)	33	(1,6,1)	61	(1,2,12)	89	(1,7,9)	117	(1,11,13)
6	(1,5,8)	34	(1,10,5)	62	(1,4,10)	90	(1,18,14)	118	(1,11,9)
7	(1,6,13)	35	(1,2,3)	63	(1,1,6)	91	(1,17,4)	119	(1,18,10)
8	(1,11,11)	36	(1,16,15)	64	(1,8,10)	92	(1,12,16)	120	(1,1,1)
9	(1,13,12)	37	(1,7,0)	65	(1,1,10)	93	(1,3,0)	121	(1,10,12)
10	(1,4,16)	38	(0,1,7)	66	(1,1,3)	94	(0,1,3)	122	(1,4,4)
11	(1,3,14)	39	(1,0,17)	67	(1,16,18)	95	(1,0,18)	123	(1,12,12)
12	(1,17,15)	40	(1,14,2)	68	(1,9,13)	96	(1,9,2)	124	(1,4,12)
13	(1,7,7)	41	(1,5,15)	69	(1,11,6)	97	(1,5,9)	125	(1,4,18)
14	(1,15,12)	42	(1,7,18)	70	(1,8,14)	98	(1,18,16)	126	(1,9,0)
15	(1,4,5)	43	(1,9,8)	71	(1,17,5)	99	(1,3,18)	127	(0,1,9)
16	(1,2,10)	44	(1,6,18)	72	(1,2,17)	100	(1,9,10)	128	(1,0,1)
17	(1,1,4)	45	(1,9,18)	73	(1,14,11)	101	(1,1,11)	129	(1,10,2)
18	(1,12,14)	46	(1,9,7)	74	(1,13,13)	102	(1,13,15)	130	(1,5,14)

19	(1,17,16)	47	(1,15,4)	75	(1,11,12)	103	(1,7,17)	131	(1,17,11)
20	(1,3,15)	48	(1,12,11)	76	(1,4,8)	104	(1,14,5)	132	(1,13,14)
21	(1,7,4)	49	(1,13,6)	77	(1,6,7)	105	(1,2,11)	133	(1,17,14)
22	(1,12,10)	50	(1,8,11)	78	(1,15,16)	106	(1,13,11)	134	(1,17,6)
23	(1,1,14)	51	(1,13,11)	79	(1,3,9)	107	(1,18,8)	135	(1,8,5)
24	(1,17,0)	52	(1,13,0)	80	(1,18,18)	108	(1,6,15)	136	(1,2,18)
25	(0,1,17)	53	(0,1,13)	81	(1,9,12)	109	(1,7,6)	137	(1,9,1)
26	(1,0,16)	54	(1,0,13)	82	(1,4,0)	110	(1,8,1)	138	(1,10,16)
27	(1,3,2)	55	(1,11,2)	83	(0,1,4)	111	(1,10,6)	139	(1,3,13)
28	(1,5,18)	56	(1,5,0)	84	(1,0,14)	112	(1,8,6)	140	(1,11,16)

	pi	i	pi	i	pi	i	pi	i	pi
141	(1,3,16)	176	(1,13,16)	211	(1,10,8)	246	(1,0,6)	281	(1,14,1)
142	(1,3,11)	177	(1,3,3)	212	(1,6,5)	247	(1,2,2)	282	(1,10,9)
143	(1,13,3)	178	(1,16,12)	213	(1,2,14)	248	(1,5,12)	283	(1,18,11)
144	(1,16,1)	179	(1,4,9)	214	(1,17,17)	249	(1,4,3)	284	(1,13,8)
145	(1,10,10)	180	(1,18,17)	215	((1,14,12)	250	(1,16,9)	285	(1,6,4)
146	(1,1,12)	181	(1,14,7)	216	(1,4,1)	251	(1,18,5)	286	(1,12,17)
147	(1,4,6)	182	(1,15,3)	217	(1,10,4)	252	((1,2,0)	287	(1,14,18)
148	(1,8,15)	183	(1,16,14)	218	(1,12,8)	253	(0,1,2)	288	(1,9,14)
149	(1,7,1)	184	(1,17,8)	219	((1,6,17)	254	(1,0,7)	289	(1,17,3)
150	(1,10,15)	185	(1,6,9)	220	(1,14,10)	255	(1,15,2)	290	(1,16,8)
151	(1,7,15)	186	(1,18,15)	212	(1,1,16)	256	(1,5,1)	291	(1,6,3)
152	(1,7,13)	187	(1,7,14)	222	(1,3,5)	257	(1,10,14)	292	(1,16,3)
153	(1,11,3)	188	(1,17,7)	223	(1,2,8)	258	((1,17,1)	293	(1,16,11)
154	(1,16,7)	189	(1,15,10)	224	(1,6,14)	259	(1,10,1)	294	(1,13,1)
155	(1,15,14)	190	(1,1,17)	225	(1,17,9)	260	(1,10,7)	295	(1,10,18)
156	(1,17,10)	191	(1,14,16)	226	(1,18,4)	261	(1,15,0)	296	(1,9,16)
157	(1,1,0)	192	(1,3,6)	227	((1,12,9)	262	(0,1,15)	297	(1,3,10)
158	(0,1,1)	193	(1,8,7)	228	(1,18,9)	263	(1,0,9)	298	(1,1,5)
159	(1,0,12)	194	(1,15,8)	229	(1,18,3)	264	(1,18,2)	299	(1,2,4)
160	(1,4,2)	195	(1,6,16)	230	(1,16,5)	265	(1,5,16)	300	(1,12,7)
161	(1,5,3)	196	(1,3,1)	231	(1,2,15)	266	(1,3,17)	301	(1,15,11)
162	(1,16,6)	197	(1,10,13)	232	(1,7,16)	267	(1,14,6)	302	(1,13,7)
163	(1,8,16)	198	(1,11,17)	233	(1,3,4)	268	(1,8,0)	303	(1,15,7)
164	(1,3,7)	199	(1,14,4)	234	(1,12,0)	269	(0,1,8)	304	(1,15,18)
165	(1,15,9)	200	(1,12,18)	235	(0,1,12)	270	(1,0,8)	305	(1,9,4)
166	(1,18,6)	201	(1,9,15)	236	(1,0,6)	271	(1,6,2)	306	(1,12,15)
167	(1,8,13)	202	(1,7,8)	237	(1,8,2)	272	(1,5,13)	307	(1,7,10)
168	(1,11,14)	203	(1,6,6)	238	(1,5,4)	273	(1,11,0)	308	(1,1,9)
169	(1,17,18)	204	(1,8,12)	239	(1,12,5)	274	(0,1,11)	309	(1,18,1)
170	(1,9,3)	205	(1,4,15)	240	(1,2,7)	275	(1,0,15)	310	(1,10,11)
171	(1,16,13)	206	(1,7,11)	241	(1,15,13)	276	(1,7,2)	311	(1,13,18)
172	(1,11,7)	207	(1,13,17)	242	(1,11,15)	277	(1,5,18)	312	(1,9,5)
173	(1,15,15)	208	(1,14,13)	243	(1,7,3)	278	(1,9,9)	313	(1,2,1)
174	(1,7,12)	209	(1,11,4)	244	(1,16,0)	279	(1,18,12)	314	(1,10,3)
175	(1,4,11)	210	(1,12,1)	245	(0,1,16)	280	(1,4,17)	315	(1,16,10)

i	pi	i	pi	i	pi	i	pi	i	pi
316	(1,1,18)	330	(1,14,8)	344	(1,5,6)	358	(1,15,5)	372	(1,8,3)
317	(1,9,11)	331	(1,6,10)	345	(1,8,4)	359	(1,2,13)	373	(1,16,16)
318	(1,13,5)	332	(1,1,8)	346	(1,12,3)	360	(1,11,5)	374	(1,3,12)
319	(1,2,9)	333	(1,6,8)	347	(1,16,4)	361	(1,2,5)	375	(1,4,14)
320	(1,18,0)	334	(1,6,0)	348	(1,12,4)	362	(1,2,6)	376	(1,17,3)
321	(0,1,18)	335	(0,1,6)	349	(1,12,13)	363	(1,8,18)	377	(1,11,18)
322	(1,0,11)	336	(1,0,10)	350	(1,11,1)	364	(1,9,17)	378	(1,9,6)
323	(1,13,2)	337	(1,1,2)	351	(1,10,17)	365	(1,14,14)	379	(1,8,17)
324	(1,5,10)	338	(1,5,7)	352	(1,14,9)	366	(1,17,12)	380	(1,14,0)
325	(1,1,7)	339	(1,15,1)	353	(1,18,7)	367	(1,4,13)	381	(0,1,14)
326	(1,15,17)	340	(1,10,0)	354	(1,15,6)	368	(1,11,8)		
327	(1,14,3)	341	(0,1,10)	355	(1,8,8)	369	(1,6,11)		
328	(1,16,17)	342	(1,0,3)	356	(1,6,12)	370	(1,13,4)		
329	(1,14,17)	343	(1,16,2)	357	(1,4,7)	371	(1,12,6)		

Table (2)
Projection Level Lines in PG(2,19)

L_1	1,2,24,37,52,56,82,93,126,157,234,244,252,261,268,273,320,334,340,380
L_2	2,3,25,38,53,57,83,94,127,158,235,245,253,262,289,274,321,335,341,381
L_3	3,4,26,39,54,58,84,95,128,159,236,246,254,263,290,275,322,336,342,1
L_4	4,5,27,40,55,59,84,96,129,160,237,247,255,264,291,276,323,337,343,2
\vdots	\vdots
L_{381}	381,1,23,36,51,55,81,92,125,156,233,243,251,260,267,272,319,333,379

Table(3) The bounds of linear codes :

q \ r	11	13	16	17	19
2	12	14	18	18	20
3	21	23	28	28-33	31-39
4	32	38-40	52	48-52	52-58
5	43-45	49-53	65	61-69	68-77
6	56	64-66	78-82	79-86	86-96
7	67	79	93-97	95-103	105-115
8	78	92	120	114-120	126-134
9	89-90	105	129-131	137	147-153
10	100-102	118-119	142-148	154	172
11		132-133	159-164	166-171	191

12		143-147	180-181	183-189	204-210
13			195-199	205-207	225-230
14			210-214	221-225	243-250
15			231	239-243	265-270
16				256-261	286-290
17					305-310
18					324-330

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- [11] ندى ياسين قاسم يحيى, مصطفى ناظم سالم, طرق هندسية جديدة لبرهان وجود الشفرات الخطية ثلاثية الابعاد $[97, 3, 87]_{11}$, $[143, 3, 131]_{13}$ سيظهر في مجلة التربية والعلم, جامعة الموصل, العراق