Parameters estimation for modified weibull distribution based on singly type one censored samples

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Abstract

The three parameters distribution called modified Weibull distribution (MWD) introduced by sarhan and zaindin (2009). In this paper, we deal with point estimation to estimate the parameters of modified Weibull distribution based on complete data, using maximum likelihood method Ordinary least squares estimator method and rank set sampling estimator method to obtain the estimate parameters for modified Weibull distribution, then estimate the death function , survival function and hazard function obtained these estimate functions by applied on set of real data which taken for Leukemia disease in the Baghdad general hospital.

Key Words:

Modified Weibulldistribution(MWD), Maximum likelihood estimation method(MLEM),Ordinary least squares estimator method(OLSEM), Rank set sampling estimator method(RSSEM),Newtone-Raphson method , Survival function , Hazard function .

1- Introduction

The modified Weibull distribution was first introduced by Sarhan and Zaindin (2009)[1].Which is a very important distribution that it can be used to describe several reliability models. This distribution contain three parameters one scale parameter α and two shape parameters are λ,γ respectively.Sarhan and Zaindin ;(2009)[1, 2] introduced MWD(α,λ,γ) and prove some basic properties ,Sarhan and Zaindin ;(2009)[1] estimate parameters by MLE based on type two censored data,Said in(2010)[3] considered rank sampling to estimate parameters based on MLE, Mazen ; (2010)[4] presented estimators for parameter based on type one censored data by using MLE , Soufiane and Maher ;(2011)[5] presented that the hazard rat function ofMWD(α,λ,γ) is constant if $\gamma = 1$, increasing if $\gamma > 1$ and decreasing if $\gamma < 1$. our aim of the this paper is interested in MWD(α,λ,γ) and defines the properties of this distribution, and then estimates the three parameters in this distribution by using three Non-Basyian methods(Maximum likelihood estimator method. Ordinary least squares estimator method and Rank set sampling estimator method), after that finding and estimating the death density function, survival probability function and hazard probability function based on complete data .At least applying the mentioned probability functions for areal data which are collected for the Leukemia disease in hospital. The rest of paper is organized as follows: In section two definition and some properties ofMWD(α,λ,γ). In section three deriving point estimation for the parameters of MWD(α,λ,γ)by using MLEM,OLSEM and RSSM . In section four apply the real set data compute the estimation of death density function, survival function and hazard function. Finally we conclude the paper in section five.

2-definition and properties of MWD:

The cdf of MWD(α,λ,γ) take the follows from:

$$F(t;\alpha,\lambda,\gamma) = 1 - \exp(-\alpha t - \lambda t^{\gamma})$$

$t > o$ ... (2.1)

The pdf of the MWD(α,λ,γ) is:

$$f(t;\alpha,\lambda,\gamma) = \begin{cases} (\alpha + \lambda\gamma\, t^{\gamma-1}) \exp(-\alpha t - \lambda t^{\gamma}), & t > 0 \\ 0 & ,o.w \end{cases} \quad (2.2)$$

Where the parameter space is

$\Omega = \{ (\alpha , \lambda ,\gamma) ; \alpha ,\lambda \geq 0 ; \gamma > 0 \}$

The mean of MWD(α,λ,γ) is:

$$\mu_1 = E(t) = \sum_{i=0}^{\infty} \frac{(-\lambda)^i}{i!} \left[\frac{\Gamma(\gamma i + 2)}{\alpha^{1+\gamma i}} + \frac{\lambda\gamma\Gamma(\gamma(1+i)+1)}{\alpha^{1+\gamma(1+i)}} \right] \quad (2.3)$$

The variance of the MWD(α,λ,γ) is:

$$\sigma^2 = var(t) = \sum_{i=0}^{\infty} \frac{(-\lambda)^i}{i!} \left[\frac{\Gamma(i\gamma + 3)}{\alpha^{2+i\gamma}} + \frac{\lambda\gamma\Gamma((1+i)\gamma+2)}{\alpha^{2+(1+i)\gamma}} \right] - \sum_{i=0}^{\infty} \frac{(-\lambda)^{2i}}{(i!)^2} \left[\frac{\Gamma(i\gamma + 2)}{\alpha^{2+i\gamma}} + \frac{\lambda\gamma\Gamma\left((1+i)\gamma+1\right)}{\alpha^{1+(1+i)\gamma}} \right]^2 \quad (2.4)$$

The survival function of the MWD(α,λ,γ) takes following form :

$$s(t;\alpha,\lambda,\gamma) = e^{(-\alpha t - \lambda t^{\gamma})} , \quad t \geq 0 \quad (2.5)$$

The hazard rate function of MWD(α,λ,γ) is:

$$h(t;\alpha,\lambda,\gamma) = (\alpha + \lambda\gamma t^{\gamma-1}) , \quad t \geq 0 \quad (2.6)$$

3-Parameters Estimation:

In this section we shall clarify how to derive and estimate the parameter by using Non-Bayisan methods which are as follows

3.1- The Maximum Likelihood Estimators:

The likelihood function for MWD(α,λ,γ)

$$L = \prod_{i=1}^{n}\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right) e^{\left(-\alpha\sum_{i=1}^{n} t_i - \lambda\sum_{i=1}^{n} t_i^{\gamma}\right)} \quad (3.1)$$

Taking the logarithm for the likelihood function so we get function:

$$\ln L = \ln\prod_{i=1}^{n}\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right) - \alpha\sum_{i=1}^{n} t_i - \lambda\sum_{i=1}^{n} t_i^{\gamma} \quad (3.2)$$

We take the first derivative of equation (3.2) with respect to α,λ,γ and equating each equation to zero, then we get three nonlinear equation for α,λ,γ respectively as follows:

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{n}\frac{1}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \sum_{i=1}^{n} t_i$$

$$\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{n}\frac{\gamma t_i^{\gamma-1}}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \sum_{i=1}^{n} t_i^{\gamma}$$

$$\frac{\partial \ln L}{\partial \gamma} = \sum_{i=1}^{n}\frac{\lambda\left[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \lambda\sum_{i=1}^{n} t_i^{\gamma}\ln t_i$$

$$\sum_{i=1}^{n}\frac{1}{\left(\hat{\alpha} + \hat{\lambda}\hat{\gamma} t^{\hat{\gamma}-1}\right)} - \sum_{i=1}^{n} t_i = 0 \quad (3.3)$$

$$\sum_{i=1}^{n}\frac{\hat{\gamma} t_i^{\hat{\gamma}-1}}{\left(\hat{\alpha} + \hat{\lambda}\hat{\gamma} t_i^{\hat{\gamma}-1}\right)} - \sum_{i=1}^{n} t_i^{\hat{\gamma}} = 0 \quad (3.4)$$

$$\sum_{i=1}^{n}\frac{\hat{\lambda}\left[\hat{\gamma} t_i^{\hat{\gamma}-1}\ln t_i + t_i^{\hat{\gamma}-1}\right]}{\left(\hat{\alpha} + \hat{\lambda}\hat{\gamma} t_i^{-1}\right)} - \hat{\lambda}\sum_{i=1}^{n} t_i^{\hat{\gamma}}\ln t_i = 0 \quad (3.5)$$

To find the maximum likelihood estimations for α,λ,γ we must solve the system of three nonlinear equation (3.2),(3.4),(3.5) by using the iterative method . Such as Newtone-Raphson method to obtain the solution which is as follows:

$$\begin{bmatrix}\alpha_{i+1}\\ \lambda_{i+1}\\ \gamma_{i+1}\end{bmatrix} = \begin{bmatrix}\alpha_i\\ \lambda_i\\ \gamma_i\end{bmatrix} - J_i^{-1}\begin{bmatrix}f(\alpha)\\ g(\lambda)\\ z(\gamma)\end{bmatrix} \quad (3.6)$$

$$f(\alpha) = \sum_{i=1}^{n}\frac{1}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \sum_{i=1}^{n} t_i$$

$$g(\lambda) = \sum_{i=1}^{n}\frac{\gamma t_i^{\gamma-1}}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \sum_{i=1}^{n} t_i^{\gamma}$$

$$z(\gamma) = \sum_{i=1}^{n}\frac{\lambda\left[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \lambda\sum_{i=1}^{n} t_i^{\gamma}\ln t_i$$

Thus, J_i^{-1} is Jacobean matrix which is defined as follows:

$$J_i^{-1} = \begin{bmatrix}\frac{\partial f(\alpha)}{\partial \alpha} & \frac{\partial f(\alpha)}{\partial \lambda} & \frac{\partial f(\alpha)}{\partial \gamma}\\ \frac{\partial g(\lambda)}{\partial \alpha} & \frac{\partial g(\lambda)}{\partial \lambda} & \frac{\partial g(\lambda)}{\partial \gamma}\\ \frac{\partial z(\gamma)}{\partial \alpha} & \frac{\partial z(\gamma)}{\partial \lambda} & \frac{\partial z(\gamma)}{\partial \gamma}\end{bmatrix}$$

Now, we find the formulas of Jacobean matrix as follows:

$$\frac{\partial f(\alpha)}{\partial \alpha} = \sum_{i=1}^{n}\frac{-1}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2}$$

$$\frac{\partial f(\alpha)}{\partial \lambda} = \sum_{i=1}^{n}\frac{-\gamma t_i^{\gamma-1}}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2}$$

$$\frac{\partial f(\alpha)}{\partial \gamma} = \sum_{i=1}^{n}\frac{-\lambda\left[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2}$$

$$\frac{\partial g(\lambda)}{\partial \alpha} = \sum_{i=1}^{n} \frac{-\gamma t_i^{\gamma-1}}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2}$$

$$\frac{\partial g(\lambda)}{\partial \lambda} = \sum_{i=1}^{n} \frac{-\gamma^2 t_i^{2(\gamma-1)}}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2}$$

$$\frac{\partial g(\lambda)}{\partial \gamma} = \sum_{i=1}^{n} \frac{\left[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \sum_{i=1}^{n} \frac{\lambda\gamma\left[\gamma t_i^{2(\gamma-1)}\ln t_i + t_i^{2(\gamma-1)}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} - \sum_{i=1}^{n} t_i^{\gamma}\ln t_i$$

$$\frac{\partial z(\gamma)}{\partial \alpha} = \sum_{i=1}^{n} \frac{-\lambda\left[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2}$$

$$\frac{\partial z(\gamma)}{\partial \lambda} = \sum_{i=1}^{n} \frac{\left[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \sum_{i=1}^{n} \frac{\lambda\gamma\left[\gamma t_i^{2(\gamma-1)}\ln t_i + t_i^{2(\gamma-1)}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} - \sum_{i=1}^{n} t_i^{\gamma}\ln t_i$$

$$\frac{\partial z(\gamma)}{\partial \gamma} = \sum_{i=1}^{n} \frac{\lambda\left[\gamma t_i^{\gamma-1}(\ln t_i)^2 + 2t_i^{\gamma-1}\ln t_i\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)} - \sum_{i=1}^{n} \frac{\lambda^2\left[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}\right]^2}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} - \lambda\sum_{i=1}^{n} t_i^{\gamma}(\ln t_i)^2$$

The absolute values of the difference between the new founded values of parameters and initial values are error terms, where the error are very small value and calculate as follows:

$$\begin{bmatrix}\epsilon(\alpha)_{i+1}\\ \epsilon(\lambda)_{i+1}\\ \epsilon(\gamma)_{i+1}\end{bmatrix} = \left\| \begin{bmatrix}\alpha_{i+1}\\ \lambda_{i+1}\\ \gamma_{i+1}\end{bmatrix} - \begin{bmatrix}\alpha_{i}\\ \lambda_{i}\\ \gamma_{i}\end{bmatrix} \right\| \quad \dots (3.7)$$

3.2 Ordinary Least Squares Estimator Method:

The idea of this method is to minimize the sum squares differences between observed sample values and the expected estimate values by linear approximation which is as follows:

$$\sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} [y_i - E(\hat{y}_i)]^2 \dots (3.8)$$

To apply OLS method we use the CDF of three parameters MWD(α,λ,γ) which define as:

$$F(t_i) = 1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})}$$

$$e^{(-\alpha t_i - \lambda t_i^{\gamma})} = 1 - F(t_i) (3.9)$$

By taking the logarithm for equation (3.9) getting:

$$\ln[1 - F(t_i)] = -\alpha t_i - \lambda t_i^{\gamma}$$

$$\ln[1 - F(t_i)] = t_i\left(-\alpha - \lambda t_i^{\gamma-1}\right)$$

$$\frac{\ln[1 - F(t_i)]}{t_i} = 3.10)$$

Comparing the equation (3.10) with the simple linear mode

$$y = \beta_0 + \beta_1 t + \epsilon$$

$$\epsilon = y - \beta_0 - \beta_1$$

$$\beta_0 = -\alpha, \beta_1 = -\lambda, x = t_i^{\gamma-1}, y_i = \frac{\ln[1 - F(t_i)]}{t_i}$$

$$\epsilon = \frac{\ln[1 - F(t_i)]}{t_i} + \alpha + \lambda t_i^{\gamma-1} \qquad (3.11)$$

By applying the formula (3.11), we get:

$$\sum_{i=1}^{n}\epsilon_i^2=\sum_{i=1}^{n}\left[\frac{\ln[1-F(t_i)]}{t_i}+\alpha+\lambda t_i^{\gamma-1}\right]^2 \quad (3.12)$$

Now, deriving the equation (3.12) with respect to the unknown parameters (α,λ,γ),we get three functions which respectively as follows:

$$f(\alpha)=\frac{\partial\epsilon_i^2}{\partial\alpha}=2\sum_{i=1}^{n}\frac{\ln[1-F(t_i)]}{t_i}+2n\alpha+2\lambda\sum_{i=1}^{n}t_i^{\gamma-1}$$

$$g(\lambda)=\frac{\partial\epsilon_i^2}{\partial\lambda}=2\sum_{i=1}^{n}t_i^{\gamma-2}\ln[1-F(t_i)]+2\alpha\sum_{i=1}^{n}t_i^{\gamma-1}+2\lambda\sum_{i=1}^{n}t_i^{2(\gamma-1)}$$

$$z(\gamma)=\frac{\partial\epsilon_i^2}{\partial\gamma}=2\lambda\sum_{i=1}^{n}t_i^{\gamma-2}\ln t_i\ln[1-F(t_i)]+2\alpha\lambda\sum_{i=1}^{n}t_i^{\gamma-1}\ln t_i+2\lambda^2\sum_{i=1}^{n}t_i^{2(\gamma-1)}\ln t_i$$

These three functions are the system of nonlinear equations and cannot solve it simulateaneasily, so we can solve it by employing Newton-Raphson method such as in maximum likelihood method by applying the Jacobean matrix as in equation (3.6), then:

$$\frac{\partial f(\alpha)}{\partial\alpha}=2n$$

$$\frac{\partial f(\alpha)}{\partial\lambda}=2\sum_{i=1}^{n}t_i^{\gamma-1}$$

$$\frac{\partial f(\alpha)}{\partial\gamma}=2\lambda\sum_{i=1}^{n}t_i^{\gamma-1}\ln t_i$$

$$\frac{\partial g(\lambda)}{\partial\alpha}=2\sum_{i=1}^{n}t_i^{\gamma-1}$$

$$\frac{\partial g(\lambda)}{\partial\lambda}=2\sum_{i=1}^{n}t_i^{2(\gamma-1)}$$

$$\frac{\partial g(\lambda)}{\partial\gamma}=2\sum_{i=1}^{n}t_i^{\gamma-2}\ln t_i\ln[1-F(t_i)]+2\alpha\sum_{i=1}^{n}t_i^{\gamma-1}\ln t_i+4\lambda\sum_{i=1}^{n}t_i^{2(\gamma-1)}\ln t_i$$

$$\frac{\partial z(\gamma)}{\partial\alpha}=2\lambda\sum_{i=1}^{n}t_i^{\gamma-1}\ln t_i$$

$$\frac{\partial z(\gamma)}{\partial\lambda}=2\sum_{i=1}^{n}t_i^{\gamma-2}\ln t_i\ln[1-F(t_i)]+2\alpha\sum_{i=1}^{n}t_i^{\gamma-1}\ln t_i+4\lambda\sum_{i=1}^{n}t_i^{2(\gamma-1)}\ln t_i$$

$$\frac{\partial z(\gamma)}{\partial\gamma}=2\lambda\sum_{i=1}^{n}t_i^{\gamma-2}(\ln t_i)^2\ln[1-F(t_i)]+2\alpha\lambda\sum_{i=1}^{n}t_i^{\gamma-1}(\ln t_i)^2+4\lambda^2\sum_{i=1}^{n}t_i^{2(\gamma-1)}(\ln t_i)^2$$

Where $F(t_i)$ is empirical cumulative distribution function, then we can find it by using the following formula:

$$F(t_i) = \frac{i - 0.5}{n}$$

Also, the Jacobean matrix here are non-singular and symmetric matrix .Now applying the equation (3.6) to get the estimators of the three parameters modified Weibull distribution by ordinary least squares method also the absolute distance values between the next founded values with thelast found values are the error term where∈is very small value then we compute error term by equation (3.7).

3.3 Rank Set Sampling Estimator Method:

Let $t_1, t_2, \dots, t_n$ be random samplesfromMWD(α,λ,γ),and assume that order statistics obtained by arranging the sample in increasing order where$t_{(1)}, t_{(2)}, \dots, t_{(n)}$form the theory of order statistics [3]. The probability density function (pdf) of y_i which is an order statistic formulated as follows:

$$g(y_i; n) = \frac{n!}{(i-1)!\,(n-i)!}[F(y_i)]^{i-1}[1 - F(y_i)]^{n-i} f(y_i) \qquad a < y_i < b \qquad (3.13)$$

$$g(y_i; n) = \frac{n!}{(i-1)!\,(n-i)!}\left[1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})}\right]^{i-1}\left[e^{(-\alpha t_i - \lambda t_i^{\gamma})}\right]^{n-i}(\alpha + \lambda\gamma t_i^{\gamma-1}) e^{(-\alpha t_i - \lambda t_i^{\gamma})} \dots (3.14)$$

The likelihood function of samplet$_{(1)}, t_{(2)}, \dots, t_{(n)}$are:

$$L(t_{(1)}, t_{(2)}, \dots, t_{(n)}; \alpha, \lambda, \gamma) = k^n \prod_{i=1}^{n} (\alpha + \lambda\gamma t_i^{\gamma-1}) e^{[\sum_{i=1}^{n}(n-i+1)(-\alpha t_i - \lambda t_i^{\gamma})]} \prod_{i=1}^{n}\left[1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})}\right]^{i-1} \quad (3.15)$$

where:$k = \frac{n!}{(i-1)!(n-i)!}$

Taking the logarithm for likelihood function we get the following function:

$$\ln L = n \ln k + \sum_{i=1}^{n} \ln(\alpha + \lambda\gamma t_i^{\gamma-1}) - \sum_{i=1}^{n}(n-i+1)(\alpha t_i + \lambda t_i^{\gamma}) + \sum_{i=1}^{n}(i-1)\ln\left[1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})}\right]$$

The partial derivatives for the log-likelihood function with respect to unknown parameters α, λ, γ and setting them to zero, the following equations are found:

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{n} \frac{1}{(\alpha + \lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{n}(n-i+1)t_i + \sum_{i=1}^{n} \frac{(i-1)t_i e^{(-\alpha t_i - \lambda t_i^{\gamma})}}{1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})}}$$

$$\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{n} \frac{\gamma t_i^{\gamma-1}}{(\alpha + \lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{n}(n-i+1)\, t_i^{\gamma} + \sum_{i=1}^{n} \frac{(i-1)t_i^{\gamma} e^{(-\alpha t_i - \lambda t_i^{\gamma})}}{1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})}}$$

$$\frac{\partial \ln L}{\partial \gamma}=\sum_{i=1}^{n}\frac{\lambda\left[\gamma t_i^{\gamma-1}\ln t_i+t_i^{\gamma-1}\right]}{\left(\alpha+\lambda\gamma t_i^{\gamma-1}\right)}-\sum_{i=1}^{n}\lambda(n-i+1)\,t_i^{\gamma}\ln t_i+\sum_{i=1}^{n}\frac{(i-1)\lambda t_i^{\gamma}\ln t_i e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}{1-e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}$$

$$\sum_{i=1}^{n}\frac{1}{\left(\hat{\alpha}+\hat{\lambda}\hat{\gamma}t_i^{\hat{\gamma}-1}\right)}-\sum_{i=1}^{n}(n-i+1)t_i+\sum_{i=1}^{n}\frac{(i-1)t_i e^{\left(-\hat{\alpha}t_i-\hat{\lambda}t_i^{\hat{\gamma}}\right)}}{1-e^{\left(-\hat{\alpha}t_i-\hat{\lambda}t_i^{\hat{\gamma}}\right)}}=0$$

$$\sum_{i=1}^{n}\frac{\hat{\gamma}t_i^{\hat{\gamma}-1}}{\left(\hat{\alpha}+\hat{\lambda}\hat{\gamma}t_i^{\hat{\gamma}-1}\right)}-\sum_{i=1}^{n}(n-i+1)\,t_i^{\hat{\gamma}}+\sum_{i=1}^{n}\frac{(i-1)t_i^{\hat{\gamma}}e^{\left(-\hat{\alpha}t_i-\hat{\lambda}t_i^{\hat{\gamma}}\right)}}{1-e^{\left(-\hat{\alpha}t_i-\hat{\lambda}t_i^{\hat{\gamma}}\right)}}=0$$

The three-function $f(\alpha)$, $g(\lambda)$, $z(\gamma)$ are the first derivative of log-likelihood function with respectively to unknown parameters α, λ, γ respectively.

$$f(\alpha)=\sum_{i=1}^{n}\frac{1}{\left(\alpha+\lambda\gamma t_i^{\gamma-1}\right)}-\sum_{i=1}^{n}(n-i+1)t_i+\sum_{i=1}^{n}\frac{(i-1)t_i e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}{1-e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}$$

$$g(\lambda)=\sum_{i=1}^{n}\frac{\gamma t_i^{\gamma-1}}{\left(\alpha+\lambda\gamma t_i^{\gamma-1}\right)}-\sum_{i=1}^{n}(n-i+1)\,t_i^{\gamma}+\sum_{i=1}^{n}\frac{(i-1)t_i^{\gamma}e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}{1-e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}$$

$$z(\gamma)=\sum_{i=1}^{n}\frac{\lambda\left[\gamma t_i^{\gamma-1}\ln t_i+t_i^{\gamma-1}\right]}{\left(\alpha+\lambda\gamma t_i^{\gamma-1}\right)}-\sum_{i=1}^{n}\lambda(n-i+1)\,t_i^{\gamma}\ln t_i+\sum_{i=1}^{n}\frac{(i-1)\lambda t_i^{\gamma}\ln t_i e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}{1-e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}$$

These functions are system of three non-linear equations and cannot solve it simultaneously, so we can solve it by Newton-Raphson .Thus, we must found the Jacobean matrix J_i as in equation(3.6).

$$\frac{\partial f(\alpha)}{\partial \alpha}=\sum_{i=1}^{n}\frac{-1}{\left(\alpha+\lambda\gamma t_i^{\gamma-1}\right)^2}-\sum_{i=1}^{n}\frac{(i-1)t_i^2 e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}{\left(1-e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}\right)^2}$$

$$\frac{\partial f(\alpha)}{\partial \lambda}=\sum_{i=1}^{n}\frac{-\gamma t_i^{\gamma-1}}{\left(\alpha+\lambda\gamma t_i^{\gamma-1}\right)^2}-\sum_{i=1}^{n}\frac{(i-1)t_i^{\gamma+1}e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}{(1-e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)})^2}$$

$$\frac{\partial f(\alpha)}{\partial \gamma}=\sum_{i=1}^{n}\frac{-\lambda[\gamma t_i^{\gamma-1}\ln t_i+t_i^{\gamma-1}]}{\left(\alpha+\lambda\gamma t_i^{\gamma-1}\right)^2}-\sum_{i=1}^{n}\frac{\lambda(i-1)t_i^{\gamma+1}\ln t_i e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}{\left(1-e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}\right)^2}$$

$$\frac{\partial g(\lambda)}{\partial \alpha}=\sum_{i=1}^{n}\frac{-\gamma t_i^{\gamma-1}}{\left(\alpha+\lambda\gamma t_i^{\gamma-1}\right)^2}-\sum_{i=1}^{n}\frac{(i-1)t_i^{\gamma+1}e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)}}{(1-e^{\left(-\alpha t_i-\lambda t_i^{\gamma}\right)})^2}$$

$$\frac{\partial g(\lambda)}{\partial \lambda} = \sum_{i=1}^{n} \frac{-\gamma^2 t_i^{2(\gamma-1)}}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} - \sum_{i=1}^{n} \frac{(i-1) t_i^{2\gamma} e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}}{\left(1 - e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right)^2}$$

$$\frac{\partial g(\lambda)}{\partial \gamma} = \sum_{i=1}^{n} \frac{\alpha\gamma t_i^{\gamma-1} \ln t_i + \alpha t_i^{\gamma-1}}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} - \sum_{i=1}^{n} (n-i+1) t_i^{\gamma} \ln t_i + \sum_{i=1}^{n} \frac{(i-1) t_i^{\gamma} \ln t_i^{\gamma} \left[-\lambda e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)} + e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)} - e^{2\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right]}{\left(1 - e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right)^2}$$

$$\frac{\partial z(\gamma)}{\partial \alpha} = \sum_{i=1}^{n} \frac{-\lambda\left[\gamma t_i^{\gamma-1} \ln t_i + t_i^{\gamma-1}\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} - \sum_{i=1}^{n} \frac{\lambda (i-1) t_i^{\gamma+1} \ln t_i e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}}{\left(1 - e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right)^2}$$

$$\frac{\partial z(\gamma)}{\partial \lambda} = \sum_{i=1}^{n} \frac{\alpha\gamma t_i^{\gamma-1} \ln t_i + \alpha t_i^{\gamma-1}}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} - \sum_{i=1}^{n} (n-i+1) t_i^{\gamma} \ln t_i + \sum_{i=1}^{n} \frac{(i-1) t_i^{\gamma} \ln t_i^{\gamma} \left[-\lambda t_i^{\gamma} e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)} + e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)} - e^{2\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right]}{\left(1 - e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right)^2}$$

$$\frac{\partial z(\gamma)}{\partial \gamma} = \sum_{i=1}^{n} \left[\frac{\lambda\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)\left[\gamma t_i^{\gamma-1} (\ln t_i)^2 + 2 t_i^{\gamma-1} \ln t_i\right]}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} - \frac{\lambda^2 \left[\gamma t_i^{\gamma-1} \ln t_i + t_i^{\gamma-1}\right]^2}{\left(\alpha + \lambda\gamma t_i^{\gamma-1}\right)^2} \right] - \sum_{i=1}^{n} (n-i+1) t_i^{\gamma} (\ln t_i)^2 + \sum_{i=1}^{n} \left[\frac{\lambda (i-1) \ln t_i \left[-\lambda t_i^{2\gamma} \ln t_i^{\gamma} e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right]}{\left(1 - e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right)^2} + \frac{\lambda (i-1) \ln t_i \left[t_i^{\gamma} \ln t_i e^{\left(-\alpha t_i - \lambda t_i^{\gamma}\right)} - t_i^{\gamma} \ln t_i \, e^{2\left(-\alpha t_i - \lambda t_i^{\gamma}\right)}\right]}{\left(1 - e^{\left(-\alpha t_i - \lambda t_i^{\gamma-1}\right)}\right)^2} \right]$$

Also, the Jacobean matrix is non-singular and symmetric matrix because finding it depending upon the first derivative. Now, by applying the equation (3.6), we get the estimators of three-parameter MWD(α,λ,γ) by using Rank set sampling method.The absolute value of the difference between the next founded value with the last founded value is the error term, then we can find the errorterm by the equation (3.7)

4-<u>Result and discussion</u>:

In this paper, depending on real data for the Leukemia disease, choosing this type of disease because it is widespread and deadly in time in Iraq and this type of diseases has failure time (death time) occurs which is interesting phenomenon in this paper. To collect data for the Leukemia disease, returning the Baghdad general hospital.The time of study point in this paper determined form (1-4-2012) until (31-12-2012) ,that means the duration time of study is constant and fixed for (9) months or (275) days . The number of patients in the experiment for the above duration time is (50) patients. (22) Patients left the hospital and any follow-up could not be done for them, but all (28) patients were dead during the time of study. When applying the test statistic (chi – square) in order to fit MWD(α,λ,γ) data, it is discovered that the calculated value is(9.331645), when comparing thisvalue with tabulated value (14.07)we find out that the calculated value is less than the tabulatedvalue at levelof significant (0.05) with(7) degree of freedom that means the data is distributed according toMWD(α,λ,γ).

4.1-**Estimation the parameters:**

In this section, we shall use MLEM,OLSEM and RSSEM to estimate the three parameters in MWD(α,λ,γ) for complete data. Applying the Newton-Raphson method to estimate the parameters which requires the initial values through assuming them.Trying and considering many initial values of the three parameters inMWD(α,λ,γ), which gives us best results with smallest value of error term and smallest number of iterations. The assumed initial value for three parameters are follows:

Table (3-1)

Initial values of parameters

Initial value of MLE	Initial value of OLS	Initial value of RSS
$\alpha_0 = 0.046$	$\alpha_0 = 0.122$	$\alpha_0 = 0.002$
$\lambda_0 = 0.578$	$\lambda_0 = 0.109$	$\lambda_0 = 0.015$
$\gamma_0 = 0.523$	$\gamma_0 = 1.959$	$\gamma_0 = 1.809$

ByusingMATLABprogram, we've got the following estimated parameters values for MLEM, OLSEM and RSSEM

Table (3-2)

Estimated values for the parameters

Estimate values of MLEM	Estimate valuesof OLSEM	Estimate values of RSSEM
$\hat{\alpha}$= 0.0781	$\hat{\alpha}$ = 0.0233	$\hat{\alpha}$ = 0.0065
$\hat{\lambda}$ = 0.2462	$\hat{\lambda}$= 0.0756	$\hat{\lambda}$ = 0.008
$\hat{\gamma}$ = 0.9442	$\hat{\gamma}$ = 1.3764	$\hat{\gamma}$ = 1.994

Then computing the numerical values for probability death density function f(t) , survival function s(t) and hazard function h(t) for MLEM,OLEM and RSSEM

Table (4-3)

Estimated Values for Functions f(t) , s(t), h(t) for MLE Method

Failure Time /days	$\hat{f}(t)$	$\hat{s}(t)$	$\hat{h}(t)$
11	0.286095824	0.883189041	0.32393498
12	0.282029769	0.873815407	0.322756691
21	0.250345984	0.794156804	0.315234954
37	0.207123177	0.672781482	0.307860996
38	0.204735679	0.665779985	0.307512517
43	0.193578896	0.632731002	0.305941855
53	0.173341859	0.571531344	0.303293704
54	0.17148475	0.565841771	0.303061313
54	0.17148475	0.565841771	0.303061313
60	0.160718704	0.532635078	0.30174262
75	0.137042299	0.458373437	0.298975219
86	0.122148899	0.41086979	0.297293454
107	0.098408517	0.333998382	0.294637706
121	0.085383031	0.29125143	0.293159182
143	0.068435184	0.23504162	0.291161983
164	0.055537946	0.191814902	0.289539268
169	0.053913709	0.186342737	0.289325516
170	0.052338779	0.181030517	0.289115781
177	0.048851678	0.169246671	0.288641883
177	0.048851678	0.169246671	0.288641883
178	0.048377635	0.1676424	0.288576366
191	0.042568118	0.147934972	0.287748848
203	0.037851626	0.131870068	0.287037281
207	0.036405602	0.126932605	0.286810484
228	0.029678834	0.103885405	0.285688192
238	0.026940885	0.094465764	0.285192054
252	0.023530543	0.082699077	0.284532114
263	0.021162618	0.074505736	0.284040119

Table (4-4)

Estimated values for functions f(t) , s(t), h(t) for OLSE method

Failure Time /days	$\hat{f}(t)$	$\hat{s}(t)$	$\hat{h}(t)$
11	0.092077611	0.97280025	0.094652101
12	0.094066049	0.969728736	0.097002436
21	0.107350399	0.939336388	0.114283232
37	0.119374501	0.87845559	0.135891333
38	0.119835045	0.874388947	0.137050045
43	0.121695911	0.854333335	0.142445467
53	0.123799058	0.81328084	0.152221781
54	0.123906013	0.809193662	0.153122818
54	0.123906013	0.809193662	0.153122818
60	0.124224636	0.784371505	0.158374743
75	0.122970585	0.72246433	0.170209905
86	0.120624987	0.677733657	0.177982878
107	0.113885495	0.595511811	0.191239691
121	0.10829943	0.543717071	0.199183428
143	0.098515026	0.46776334	0.210608694
164	0.088699944	0.402227309	0.220521933
169	0.08729066	0.39342779	0.221872127
170	0.085883267	0.384769114	0.223207279
177	0.082617207	0.365139242	0.226262198
177	0.082617207	0.365139242	0.226262198
178	0.082156644	0.362420475	0.226688749
191	0.076166505	0.328069308	0.232165898
203	0.070791295	0.298683148	0.237011346
207	0.069042142	0.289384452	0.238582762
228	0.060205658	0.244185753	0.2465568
238	0.056243037	0.2248016	0.250189667
252	0.050969551	0.199780606	0.255127623
263	0.047065485	0.181797716	0.258889309

Table (3-5)

Estimated values for function f(t) , s(t) , h(t) for RSSE method

Fuiler Time /days	$\hat{f}(t)$	$\hat{s}(t)$	$\hat{h}(t)$
11	0.012346789	0.9965365	0.0123897
12	0.012865869	0.996120488	0.012915977
21	0.017540971	0.991557442	0.017690322
37	0.025622286	0.980040348	0.026144114
38	0.026126459	0.979160615	0.026682506
43	0.028566057	0.974620623	0.029309925
53	0.033355307	0.964275112	0.034591069
54	0.033819205	0.963166728	0.035112514
54	0.033819205	0.963166728	0.035112514
60	0.036592404	0.956124454	0.038271591
75	0.043214109	0.936153456	0.046161352
86	0.047762041	0.919450784	0.051946272
107	0.055613788	0.883203456	0.062968263
121	0.060189481	0.856200222	0.070298372
143	0.066261436	0.809705806	0.081833964
164	0.070704052	0.761690066	0.092825225
169	0.071229586	0.754593156	0.094394689
170	0.071727806	0.747445059	0.095963984
177	0.072782894	0.730606693	0.099619801
177	0.072782894	0.730606693	0.099619801
178	0.072920391	0.728202581	0.100137507
191	0.074455198	0.696203755	0.106944551
203	0.075425931	0.666213514	0.113215853
207	0.075656313	0.65616604	0.115300562
228	0.076136077	0.602968047	0.126268842
238	0.075950479	0.577638503	0.131484446
252	0.075270775	0.542309851	0.138796621
263	0.074414809	0.51483423	0.1445413

5-conclusions

Note we can make the following mention about the results above tables:

1. The values of death density function f(t) of maximum likelihood method are decreasing progressively with the increasing of the failure time for the Leukemia patients in the hospital ,that means there is an opposite relationship between failure times and death density function. While the estimate values of death functions for ordinary least squares method are increasing until (t = 60) then it became decreasing to the end of failure times, but the estimated values of death density function for rank set sampling are increasing until failure time (t = 228) then the values become decreasing until the end of failure time.
2. For all estimation methods, observing that the estimate values of probability survival functions are decreasing with the increasing of failure times, which means that there are all estimation methods an opposite relationship between failure times and probability survival functions.
3. For maximum likelihood estimation methods, noting that the estimate values of probability hazard functions are decreasing with increasing of failure times, that means there is an opposite relationship between failure times and probability hazard functions, it is known that the value of h(t) depends on the shape parameter values such that ($\hat{\gamma}$= 0.9442) thus, the hazard function is decreasing function as(t) increasing when ($\hat{\gamma}$< 1), while ordinary least squares method and the rank set sampling method are increasing with increasing failure times, that means there is a direct relationship between failure times and probability hazard functions it is known that the value of h(t) depends on the shape parameter values such that ($\hat{\gamma}$=1.3764) of OLSEM ($\hat{\gamma}$=1.994) of RSSEM , thus, the hazard function is increasing function as (t) increasing when ($\hat{\gamma}$> 1) .

References

(1) A. M. Sarhan and M. Zaindin , (2009),"Modified Weibull Distribution" .*Applied Science*, V0l 11 ,pp 123-136.

(2) M. Sarhan and M. Zaindin ,(2009), "Parameters Estimation of the Modified Weibull Distribution" . *Applied Mathematical Sciences*,Vol. 11,pp 541-550.

(3) S. A . Al-Hadhrami ,(2010)," Parameters Estimation of the Modified Weibull Distribution

Based on Ranked Set Sampling" .*European Journal of Scientific Research* ISSN 1450-216X Vol.44 No.1,pp.73-78 .

(4) M .Zaindin ,(2010) ,"Parameter estimation of the modified Weibull model based on grouped and censored data".*International Journal of Basic & Applied Sciences IJBAS-IJENS*Vol :10 No:02,pp.122-132

(5) S .Gasmi and M .Berzig ,(2011),"Parameters Estimation OF the Modified Weibull Distribution Based on Type I Censored Samples" .Applied Mathematical Sciences, Vol. 5 No. 59, pp 2899-2917.

معلمات توزيع وبيل المعدل بالاعتماد على عينات مراقبة من النوع الاول

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الخلاصة

لقد اوجد الباحثان (sarhan and zaindin)في عام (2009) توزيع معدل عن توزيع وبيل وسمي توزيع وبيل المعدل . في هذا البحث سوف تقدر معلمات الثلاثة لهذا التوزيع بالتقدير النقطي بالاعتماد على بيانات الكاملة باستخدام طريقة الإمكان الأعظم وطريقة مقدر المربعات الصغرى الاعتيادية وطريقة مقدر معاينة مجموعة الرتبوبعد ذلك تم حساب وإيجاد القيم العددية للمعالم المقدرة لهذا التوزيع واستخدمت تقديرات هذه المعالم لإيجاد الدوال الاحتمالية الثلاثة (الوفاة ،البقاء ،المخاطرة) من خلال استخدام مجموعة من البيانات الحقيقية.