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The Dynamics of Discrete System with Constant Rate Harvesting

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Abstract
In this papera prey -predator discrete dynamical system with Holling type I functional response is presented . The existence of all possibleequilibria have been obtained algebraically. The model has four equilibriaand some conditions for the local stability of its equilibria have been established. We see that the proposed model has rich dynamics behavior. Aconstant rate harvesting for a single population is also considered as well as the exitence of the bionomic equilibrium is dicussed and computed. The numerical simulation is given to conforim the theoretical analysis stability of the model.Finally a general disscution is provided.

Keywords: Discrete time, Prey-predator model, Harvesting.

ديناميكية النظام المتقطع مع نسبة الحصاد الثابتة

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الخلاصة
في هذا البحث تمت دراسة السلوك الديناميكي لنموذج متقطع (لفريسة– المفترس) مع دالة استجابة من النوع الاول . كذلك تم ايجاد نقاط الاتزان لهذا النموذج مع وضع الشروط الازمة لتحقيق الاستقرار المحلي لكل نقاط الاتزان . من خلال دراسة هذا النظام لاحظنا انه ذات سلوك ديناميكي ثري ومعقد. كما درسنا في هذا البحث نسبة الحصاد في حالة كونها كمية ثابتة في المجتمع مع مناقشة واثبات وجود التوازن الايكولوجي. اجريت واعطيت امثلة العددية لتعزيز نتائج التحليل الرياضي لاستقرارية النموذج اضافة الى تقديم مناقشة عامة.

Introduction

During the last decades, the dynamics of prey-predator relationship have been received much attention in ecological science as well as mathematical modeling due to its importance and existence in life. Mathematical models have mostly use differential equations or partial differential equations or difference equations. These different types of models are depending on the time scale and space of structure of the problem [1].

Discrete time or difference models can produce and exhibit moreplentifull dynamical behavior than those which have seen in continuous time models of the same type [2].Some of authors have been

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considered stage or structure to formulate the problem in their models for more details see [3-10]. In [11], the author has been studied the same model with optimal control in continuous time form. The existence of periodic solutions and stability of solutions for prey-predator models as well as the Flip bifurcation andHopf bifurcation are extensively studied in the literaturesee [12-19]. Since the type of functional response can greatly effect modelpredations, thus some linear and non-linear functional response are considered to describe this phenomena. For examplesHolling functional response of type I, type II ,and, type III as well as Beddington- De Angelis and Crowley –Martin functional response , [20-22]. The square root functional response is also studied by Liujuan Chen and Fengde Chen in [23]. In this work we will investigate the dynamical behavior of non-linear system of two difference equations. We assume that individuals within a single population are identical and there is also no structuring variable within each population.

In the litrecture a simple nonlinear equation $X_{t+1}=f(x_t)$ is widely used to formulate the growth of populations. These kinds of models are well known to possesa complicated dynamics behavior [24-25]. In [26] F. Marotto has been studied the dynamics of discrete model for a single population and he investigated the existence and behavior of its equilibria the dynamic of his model is given by $x_{t+1} = r_1 x_t^2(1 - x_t)$ where x_t is the density of population at period time t. His model depicts the threshold phenomena.Which means that at very low density many populations are tending to extinction rather than to growth. However, beyond some threshold the population will be growing according to that well known equation, logistic equation.

This paper is organized as follows. In section 2 we formulate our two prey-predator model with Holling type I functional response. The existence of all its equilibria are discussed. We also derive a set of conditions for which the system gives alocal stablility of allequilibria. In section 3 .Aconstant rate harvesting for a single population is also investigated as well as the exitence of the bionomic equilibrium is dicussed and computed. For the harvesting model some conditions are set for the local stability of its equilibria. One can also note that the harvesting improve local stability the reason of that is harvesting will reduce the parameter r to an acceptable level for local stability. In section 4 we present the numerical simulations which clarify and conforim our theoretical analysis results. Finally a general disscution is provided in section 5.

2. The model and the stability of its equilibria.

Let x_t denotes the number of prey density and y_t denotes the number of predator density in thet-th generation. Our model is described by the following two of non-linear difference equations:

$$M\begin{cases} x_{t+1}=r_1 x_t^2(1-x)-b_1 y_t x_t \\ y_{t+1}=-r_2\, y_t+b_2 y_t x_t \end{cases} \qquad (2.1)$$

The parameters r_1 and r_2 represent for intrinsic growth rate of prey species and the predators death rates respectively, while the positive parameters b_1 and b_2 represent the maximum per capitakilling rate and conversion rate of predator respectively,the predator consumes the prey by functional response Holling type I. Starting with initial condition (x_0, y_0) a trajectory of the state of population output is uniquely determined by the iteration the system (2.1) in the following form:

$(x_t, y_t)=M^t\,(x_0, y_0)$, t= 0, 1,2,……..

In order to determine all possible the equilibria of the system (2.1). The following algebraic equations should be solved

$$\begin{aligned} r_1 x^2(1-x) - b_1 y\, x &= x \\ -r_2\, y + b_2 y\, x &= y \end{aligned} \qquad (2.2)$$

After solving the equations (2.2) we get this lemma.

Lemma 1:

The system (2.1) has four equilibriafor all parameters values, namely E_i,i=0,1,2,3. These are:

1- $E_0 = (0,0)$, the trivial equilibrium is always exists.

2- $E_i= (k_i, 0)$, i=1,2 the boundary equilibria are exists for all $r_1 \geq 4$ where $k_1 = \frac{1}{2} + a, k_2 = \frac{1}{2} - a$ and $a = \sqrt{\frac{1}{4} - \frac{1}{r_1}}$.

3- $E_3= (x^*,\ y^*) = \left(\frac{r_2+1}{b_2},\ \frac{r_1 x^*(1-x^*)-1}{b_1}\right)$ the unique positive equilibrium is exist if $r_1 > 4$, and $x^* \epsilon A \sqcup B$where $A= (\frac{1}{2} - a,\ \frac{1}{2})$ and $B=(\frac{1}{2},\ \frac{1}{2} + a)$, For studying the local behavior of the system (2.1) at each equilibrium, one needs to compute the Jacobian's matrix of the system (2.1). This can be supplied by:

$$J(x, y)=\begin{bmatrix} 2r_1x-3r_1x^2-b_1y & -b_1x \\ b_2y & -r_2+b_2x \end{bmatrix}$$

Thus the characteristic polynomial of J (x,y) is

$$F(\lambda)=\lambda^2+P\lambda+Q \qquad (2,3)$$

Where P = -trac(J), and Q = det(J).

Definition[7]: A steady (equilibrium) point (x, y) for a system of two dimension is called a sink if $|\lambda_1|<1$ and $|\lambda_2|<1$, (x, y) is called a source if $|\lambda_1|>1$ and $|\lambda_2|>1$, (x, y) is called a saddle point if either $|\lambda_1|>1$ and $|\lambda_2|<1$ or $|\lambda_1|<1$ and $|\lambda_1|>1|\lambda_1|>1$.Finally (x, y) is called a non-hyperbolic if either $|\lambda_1|$=1 or $|\lambda_2|$=1.

The next lemma gives the local stability of E_0.

Lemma 2:

i- The steady point E_0 is sink point if $r_2<1$

ii-The steady point E_0 is saddle point if $r_2>1$

iii-The steady point E_0 is non-hyperbolic point if $r_2=1$

Proof:

It is easy to check that the roots of the equation (2.3) at E_0 are $\lambda_1=0$ and $\lambda_2=-r_2$.Thereforeall results are obtained.

The Jacobain's matrix at E_1 can be written as

$$J(E_1)=\begin{bmatrix} -\frac{1}{2}r_1-r_1a+3 & -\frac{b_1}{2}-b_1a \\ 0 & -r_2+\frac{b_2}{2}+b_2a \end{bmatrix}$$

So that the roots of (2.3) are $\lambda_1=-\frac{1}{2}r_1-r_1a+3$ and $\lambda_2=-r_2+\frac{b_2}{2}+b_2a$.

It is clear that if $r_1=4$ then the $E_1$is always non-hyperbolic point. The local stability of $E_1$for $r_1>4$ is given in the next lemma.

Lemma 3:

1) The steady point E_1 is sink if one of the following conditions holds:

i) $\frac{1}{2}+\frac{1}{b_2}<\frac{r_2}{b_2}<1-\frac{1}{b_2}$ and $r_1\in M_1\cap M_2$

ii) $\frac{1}{2}-\frac{1}{b_2}\le\frac{r_2}{b_2}<min\left\{\frac{1}{2}+\frac{1}{b_2},1-\frac{1}{b_2}\right\}$ and $r_1\in M_1\cap M_3$

where $M_1=\left(4,\frac{15}{3}\right)$,$M_2=\left(\frac{1}{N_2-N_2^2},\frac{1}{N_1-N_1^2}\right)$ and $M_3=\left(0,\frac{1}{N_1-N_1^2}\right)$where $N_1=\frac{1}{b_2}+\frac{r_2}{b_2}$ and $N_2=\frac{r_2}{b_2}-\frac{1}{b_2}$

2) The steady point E_1 is source if one of the following conditions holds:

i) $r_1>\frac{16}{3}$, and $\frac{r_2}{b_2}<\frac{1}{2}-\frac{1}{b_2}$

ii) $\frac{1}{2}+\frac{1}{b_2}<\frac{r_2}{b_2}<\frac{-1}{b_2}+1andr_1\in I_1\cap(I_2\cup I_3)$ here $I_1=(\frac{16}{3},\infty)$,
$I_2=(0,\frac{1}{N_2-N_2^2})$ and $I_3=\left(\frac{1}{N_1-N_1^2},\infty\right)$

3) The steady point E_1 is saddle point if one of the following conditions holds:

i) $\frac{1}{2}+\frac{1}{b_2}<\frac{r_2}{b_2}<\frac{-1}{b_2}+1$ and $r_1\in I_1\cap M_2$

ii) $\frac{1}{2}+\frac{1}{b_2}<\frac{r_2}{b_2}<\frac{-1}{b_2}+1$ and $r_1\in M_1\cap(I_2\cup I_3)$

iii) $4<r_1<\frac{16}{3}$, and $\frac{r_2}{b_2}<\frac{1}{2}-\frac{1}{b_2}$

iv) $\frac{1}{2}-\frac{1}{b_2}\le\frac{r_2}{b_2}<min\left\{\frac{1}{2}+\frac{1}{b_2},1-\frac{1}{b_2}\right\}$ and $r_1\in I_1\cap M_3$

4) The steady point E_1 is non-hyperbolic point if one of the following conditions holds:

i) $r_1=\frac{16}{3}$

ii) $\frac{1}{2}+\frac{1}{b_2}<\frac{r_2}{b_2}<\frac{-1}{b_2}+1$ and either $r_1=\frac{1}{N_2-N_2^2}$ or $r_1=\frac{1}{N_1-N_1^2}$

iii) $\frac{1}{2}-\frac{1}{b_2} \leq \frac{r_2}{b_2} < min\left\{\frac{1}{2}+\frac{1}{b_2}, 1-\frac{1}{b_2}\right\}$ and $r_1 = \frac{1}{N_1-N_1^2}$

proof

For (1)(i), it is clear that $|\lambda_1| < 1$ if and only if $4 < r_1 < \frac{16}{3}$. Now suppose that $\frac{1}{2}+\frac{1}{b_2} < \frac{r_2}{b_2} < \frac{-1}{b_2}+1$ then both of N_1 and N_2 are greater than $\frac{1}{2}$ as well as both of $N_1 - N_1^2$ and $N_2 - N_2^2$ are greater than zero with $N_1 - N_1^2 < N_2 - N_2^2$.

Therefore $|\lambda_2| < 1 \Leftrightarrow N_2-\frac{1}{2} < a < N_1 - \frac{1}{2} \Leftrightarrow \frac{1}{N_2-N_2^2} < r_1 < \frac{1}{N_1-N_1^2}$. Hence E_1 is local stable point if $r_1 \in M_1 \cap M_2$

For (1)(ii),if $\frac{1}{2}-\frac{1}{b_2} < \frac{r_2}{b_2}$ then $N_2 < \frac{1}{2}$, $N_1 > \frac{1}{2}$ and $N_1 - N_1^2 > 0$. Therefore $|\lambda_2| < 1 \Leftrightarrow 0 < a < N_1 - \frac{1}{2} \Leftrightarrow r_1 < \frac{1}{N_1-N_1^2}$. Hence E_1 is local stable point.

For (2) (i), if $r_1 > \frac{16}{3}$then $|\lambda_1| > 1$.

Now if $\frac{r_2}{b_2} < \frac{1}{2}-\frac{1}{b_2}$ then both of N_1 and N_2 are less than $\frac{1}{2}$. So that $|\lambda_2| > 1$. for all $r_1 > 4$. Hence E_1 is source.

For (2)(ii), from proof (1) we have $N_1 - N_1^2 < N_2 - N_2^2$. So that if $r_1 \in I_1 \cap (I_2 \cup I_3)$ then $|\lambda_1| > 1$and $|\lambda_2| > 1$. Therefore E_1 is source.

The proof of (3) and (4) are clear from proof (1) and (2).

Now we will study the local stability of E_2, so that the Jacobain's matrix at E_2 is given by

$$J(E_2)=\begin{bmatrix} -\frac{1}{2}r_1 + r_1 a + 3 & -\frac{b_1}{2} + b_1 a \\ 0 & -r_2 + \frac{b_2}{2} - b_2 a \end{bmatrix}$$

As before $a=\sqrt{\frac{1}{4}-\frac{1}{r_1}}$. Hence the roots of equation (2.3) are $\lambda_1 = -\frac{1}{2}r_1 + r_1 a + 3$ and $\lambda_2 = -r_2 + \frac{b_2}{2} - b_2 a$.

Remark: It is clear that if $r_1=4$ then E_2 is always non-hyperbolic point.

It remains to investigate the case when the value $r_1 > 4$. in this case the next lemma gives the dynamics behavior of the point E_2.

Lemma 4:

1- The steady point E_2 is never to be sink.

2- The steady point E_2 is saddle point if one of the following conditions holds:

i- $\frac{1}{2}-\frac{1}{b_2} \leq \frac{r_2}{b_2} < \frac{1}{2}+\frac{1}{b_2}$ and $r_1 \in S_1 \cap I_2$ where $S_1=(4, \infty)$, $I_2=(0, \frac{1}{N_2-N_2^2})$.

ii- $\frac{1}{b_2} < \frac{r_2}{b_2} < \frac{1}{2}-\frac{1}{b_2}$ and $r_1 \in I_1 \cap S_2$ where $S_2 = \left(\frac{1}{N_1-N_1^2}, \frac{1}{N_2-N_2^2}\right)$.

iii-If $r_2 \leq 1 (\frac{r_2}{b_2} \leq \frac{1}{b_2})$, $b_2 \geq 4$ and $r_1 \in I_1 \cap S_3$ where $S_3 = \left(\frac{b_2^2}{2b_2-4}, \infty\right)$.

3- The steady point E_2 is source if one of the following conditions holds:

i- $\frac{r_2}{b_2} \geq \frac{1}{2}+\frac{1}{b_2}$

ii- $\frac{1}{2}-\frac{1}{b_2} \leq \frac{r_2}{b_2} < \frac{1}{2}+\frac{1}{b_2}$ and $r_1 \in I_1 \cap I_4$ where $I_4=\left(\frac{1}{N_2-N_2^2}, \infty\right)$.

iii- $\frac{1}{b_2} < \frac{r_2}{b_2} < \frac{1}{2}-\frac{1}{b_2}$ and $r_1 \in I_1 \cap (I_4 \cup I_5)$ where $I_5=(0, \frac{1}{N_1-N_1^2})$.

4- E_2 is non-hyperbolic point if one of the following conditions holds:

i- $\frac{1}{2}-\frac{1}{b_2} \leq \frac{r_2}{b_2} < \frac{1}{2}+\frac{1}{b_2}$ and $r_1 = \frac{1}{N_2-N_2^2}$.

ii- $\frac{1}{b_2} < \frac{r_2}{b_2} < \frac{1}{2}-\frac{1}{b_2}$ and either $r_1 = \frac{1}{N_1-N_1^2}$ or $r_1 = \frac{1}{N_2-N_2^2}$.

iii- $r_2 \leq 1$, $b_2 > 4$ and $r_1 = \frac{b_2^2}{2b_2-4}$.

Proof (1) since for all $r_1 > 4$ then the $|\lambda_1|$ is never to be less than 1. Hence the result is obtained.

Proof (2) (i) if $\frac{1}{2}-\frac{1}{b_2}\le\frac{r_2}{b_2}<\frac{1}{2}+\frac{1}{b_2}$ is hold then we have $\frac{1}{2}-N_1\le 0$ and $\frac{1}{2}-N_2>0$ as well as $N_2-N_2^2>0$. Therefore $|\lambda_2|<1\Leftrightarrow 0<a<\frac{1}{2}-N_2\Leftrightarrow r_1<\frac{1}{N_2-N_2^2}$. Hence E_2 issaddle point . For (2) (ii) if $\frac{1}{b_2}<\frac{r_2}{b_2}<\frac{1}{2}-\frac{1}{b_2}$ then we have $\frac{1}{2}-N_1>0$, $\frac{1}{2}-N_2>0$,$N_1-N_1^2>0N_2-N_2^2>0$ and $N_1-N_1^2>N_2-N_2^2$.

So that $|\lambda_2|<1\Leftrightarrow\frac{1}{2}-N_1<a<\frac{1}{2}-N_2\Leftrightarrow\frac{1}{N_1-N_1^2}<r_1<\frac{1}{N_2-N_2^2}$. Hence E_2 is saddle point.

For (2) (iii) , let r_2=1 ,b_2> 4 then $N_1=\frac{2}{b_2}$ and $N_2=0$.

$|\lambda_2|<1\Leftrightarrow\frac{1}{2}-N_1<a<\frac{1}{2}-N_2\Leftrightarrow 0<\frac{1}{r_1}<\frac{2}{b_2}-\frac{4}{b_2^2}\Leftrightarrow r_1>\frac{b_2^2}{2b_2-4}$.

The same result one can have whenr_2< 1 , so that E_2 is saddle point .

Proof (3) (i) becauseof a is always positive real number for all $r_1>4$. Then $|\lambda_2|<1$ is never to be hold for all $r_1>4$ so that E_2 is source.

For (3) (ii) and (iii) are clear from proof (2) (i) and (ii), respectively.

Proof (4) (i) from proof (2) (i) we have $\frac{1}{2}-N_1<0$ so that $\lambda_2=1\Leftrightarrow$ a= $\frac{1}{2}-N_2\Leftrightarrow r_1=\frac{1}{N_2-N_2^2}$

For (4) (ii) from proof (2) (ii) one can easily get $|\lambda_2|=1\Leftrightarrow r_1=\frac{1}{N_1-N_1^2}$ or $r_1=\frac{1}{N_2-N_2^2}$. Finally (4) (iii) is directly obtained from (2)(iii) .

In order to discuss the local stability of the unique positive steady point E_3, as before we need to compute the Jacobain's matrix at E_3 as well as we also need to the statbility criterion which found in [26] . These criterion are given in the next lemma.

Lemma 5:

Let $F(\lambda)=\lambda^2+P\lambda+Q$ suppose that F(1)>0, λ_1,λ_2 are roots of F(0)=0 then :

1- $|\lambda_1|<1$ and $|\lambda_2|<1$ if and only if F(-1)>0 and Q< 1.

2- $|\lambda_1|$> 1 and $|\lambda_2|<1$ (or $|\lambda_1|<1$ and $|\lambda_2|$> 1) if and only if F(-1) < 0

3- $|\lambda_1|$> 1 and $|\lambda_2|$> 1 if and only if F(-1)> 0 and Q> 1.

4- λ_1=-1 and $|\lambda_2|\neq 1$ if and only if F(-1)=0 and P≠ 0,2.

The Jacobain's matrix at E_3 is given by:

$$J(E_3)=\begin{bmatrix} r_1x^*-2r_1x^{*2}+1 & -b_1x^* \\ \frac{b_2}{b_1}(r_1x^*-r_1x^{*2}-1) & 1 \end{bmatrix}$$

So that the P and Q in equation (2.3) are P= $-r_1x^*+2r_1x^{*2}-2$ and Q=$(2r_1+r_1r_2)x^*-(3r_1+r_1r_2)x^{*2}-r_2$.

The next Lemma shows the dynamics of the E_3.

Lemma 6:

1- The positive steady point E_3 is sink if this condition is hold:

i- $x^*\epsilon I_6\cap I_7\cap I_9$ where $I_6=(\frac{1}{2}-a\ ,\ \frac{1}{2}+a)$ /{ $\frac{1}{2}$ } , $I_7=(\frac{m}{2}-\sqrt{\frac{m^2}{4}-\frac{(r_2-3)}{5r_1+r_2r_1}},\frac{m}{2}+\sqrt{\frac{m^2}{4}-\frac{(r_2-3)}{5r_1+r_2r_1}})$, where$I_9=(\frac{m_1}{2}+\sqrt{\frac{m_1{}^2}{4}-\frac{(r_2+1)}{3r_1+r_2r_1}},\infty)$m=$\frac{(r_2+3)}{5+r_2}$ and $m_1=\frac{(r_2+2)}{3+r_2}$.

2- The positive steady point E_3 is source if $x^*\epsilon I_6\cap I_7\cap I_{10}$ where $I_{10}=(\frac{m_1}{2}-\sqrt{\frac{m_1{}^2}{4}-\frac{(r_2+1)}{3r_1+r_2r_1}},\frac{m_1}{2}+\sqrt{\frac{m_1{}^2}{4}-\frac{(r_2+1)}{3r_1+r_2r_1}})$.

3-The positive steady pointE_3 is saddle point if one of these conditions holds :

i- $x^*\epsilon I_6\cap I_{11}$where I_{11}=$(0,\frac{m}{2}+\sqrt{\frac{m^2}{4}-\frac{(r_2-3)}{5r_1+r_2r_1}})$.

ii- $x^*\epsilon I_6\cap I_{12}$where I_{12}=$(\frac{m}{2}+\sqrt{\frac{m^2}{4}-\frac{(r_2-3)}{5r_1+r_2r_1}},\infty)$.

4-The positive steady point E_3 is non-hyperbolic point if one of these conditions holds :

i- $x^* = \frac{m}{2} - \sqrt{\frac{m^2}{4} - \frac{(r_2-3)}{5r_1+r_2r_1}}$ and either $x^* \neq \frac{1}{4} \mp \sqrt{\frac{(r_1+16)}{16r_1}}$ or $x^* \neq \frac{1}{4} \mp \sqrt{\frac{(r_1+32)}{16r_1}}$

ii- $x^* = \frac{m}{2} + \sqrt{\frac{m^2}{4} - \frac{(r_2-3)}{5r_1+r_2r_1}}$ and either $x^* \neq \frac{1}{4} \mp \sqrt{\frac{(r_1+16)}{16r_1}}$ or $x^* \neq \frac{1}{4} \mp \sqrt{\frac{(r_1+32)}{16r_1}}$.

Proof (1)(i) when $x^* \epsilon I_6$ one can get F(1) > 0, and if $x^* \epsilon I_7 \cap I_9$ then F(-1)> 0 as well as $Q < 1$. So by lemma (5) (1) we have $|\lambda_1| < 1$ and $|\lambda_2| < 1$ hence the positive fixed point is sink.
Proof (2) it is easily to check that if $x^* \epsilon I_6 \cap I_7 \cap I_{10}$ then F(1)> 0 , F(-1)> 0 and Q> 1 then according to lemma (5) ,(3) the positive steady (equilibrium) point is source.
Proof (3) if $x^* \epsilon I_6 \cap I_{11}$ or $x^* \epsilon I_6 \cap I_{12}$ then F(1)> 0 and F(-1) $<$ 0 theref-ore by lemma (5) , (2) the positive steady(equilibrium) point is saddle point.
Proof (4) if the condition (i) or (ii) is satisfied then F(-1)=0 and P ≠0,2 so that the result can be easily obtained.

3-Harvesting

In this section we will consider the resource biomass for a single population is subject to a constant rate harvesting. That means a percentage qEx_k of the population is removed at every period time k. Here we will also assume that the predator population is absent therefore the system (2.1) including harvesting becomes:

$x_{k+1} = rx_k^2(1 - x_k) - qEx_k$ (3.1)

Where E denotes the harvesting effort, and q is constant called the catch ability coefficient, the parameterr is as same as mentioned before. Recall that this system (3.1) without harvesting was investigated by Marotto[18] .

To discuss the equilibrira analysis of the present model. One can easy to see that the model (3.1) has three equilibria, namely

$e_0=0$, $e_1=\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{qE+1}{r}}$, $e_2=\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{qE+1}{r}}$

Note that the e_0 is always exists while e_1, e_2 are exist for all values of

$r \geq 4(qE+1)$ (3.2)

The next lemma gives the local stability of e_0, e_1, e_2 .

Lemma(7):

1- Local stability of e_0

i. e_0 is sink if qE<1 .

ii. e_0 is source if qE>1 .

iii. e_0 is non-hyperbolic point if qE=1 .

2- Local stability of E_1

i. e_1 is sink if $4(1+qE) < \mathbf{r} < \frac{(1+3(qE+1))^2}{1+2(1+qE)}$.

ii. e_1 is source if $4(1+qE) < \mathbf{r} < \frac{(1+3(qE+1))^2}{1+2(1+qE)}$

iii. e_1 is non- hyperbolic if $r = \frac{(1+3(qE+1))^2}{1+2(1+qE)}$

3- Local stability of e_2

e_2 is never to be sink for all values of $r \geq 4(qE+1)$ therefore e_2 is always source point.The proof of this lemma (1) is easy so that it is omitted.

Bionomical equilibrium:

Bionomic equilibrium describes situation which the equilibrium level effort is determined by both of biological and economic parameters.In the literature, the bionomic equilibrium is said to be attained when the total revenue gained by selling the harvested biomass in an economicsteady orequilibrium case equals the total cost to the harvested it.

The net gain or the net economic revenue at time k is the difference between the total revenue which is $pqEx_k$ and the total cost C=cE.

$$N(x, E,k) = (pqx_k-c)E \quad (3.3)$$

wherep is price per unit biomass and c is cost parameter of the harvesting effort.

In order to compute the bionomic equilibrium biomass level, $B(x_\infty, E_\infty)$ where x_∞ and E_∞ are positive values. One has to solve the conditions for equilibrium – effort level , these conditions are :

$$rx^2(1-x)-qEx=x \qquad (3.4)$$
$$(pqx-c)E = 0$$

After solving (3.4) we get

$$E_\infty = \frac{1-\frac{rc}{qp}+\frac{rc^2}{p^2q^2}}{-q} = \frac{rc}{pq^2} - \frac{1}{q} - \frac{rc^2}{p^2q^3}$$

For corresponding stock level $x=x_\infty$ with $x_\infty = \frac{c}{pq}$.

It is clear that the bionomical equilibrium $B(x_\infty, E_\infty)$ is exist only when these condition $\frac{rc}{pq^2} > \frac{1}{q} + \frac{rc^2}{p^2q^3}$ is satisfied and inequality of equation (3.2) holds.

Note that if $c>pqx$ then the cost of harvesting is geater than the revenue so that the whole harvesting work will be closed alternativily if $c<pqx$ then the revenue will be positive and the whole harvesting work will be in operation.

1. Numerical analysis:

Our goal in this section is to present numerical simulations that confirm the above theoretical analysis. At different set of parameters the localstability of equilibria is investigated numerically.

For the steady point E_1 we choose the parameters as follows:

$r_1=4.7, r_2=4.1, b_1=0.01, b_2=6$ and $E_1=(0.693,0)$. So that one can easily see that the condition (i) in 1 of lemma 3 issatisfied and the point is local stable.Since $\frac{1}{2}+\frac{1}{b_2}=0.6667$, $\frac{r_2}{b_2}=0.6833$, $\frac{-1}{b_2}+1=0.8333$ $M_1=\left(4,\frac{16}{3}\right)$ and $M_2=(4.004, 7.8431)$.Hence the E_1 is locally stable according to the lemma 3.

Figure- 2 illustrates the local stability of E_1. The trajectories of the prey population and the predator population are also illustrated in Figure- 1 as a function of time.

Now the condition in Lemma 3 (ii) is satisfied by choosing these parsmeters $r_1=4.3$, $r_2=3.6, b_1=0.01, b_2=6$. Therefore $E_1=(0.6321,0)$ so that $\frac{1}{2}-\frac{1}{b_2}=0.3333$, $\frac{1}{2}+\frac{1}{b_2}=0.6667$, $\frac{r_2}{b_2}=0.6833$, $\frac{-1}{b_2}+1=0.8333$ $M_1=\left(4,\frac{16}{3}\right)$ $M_3=(0,\ 5.5901)$. Figure- 3 illustrates the local stability of E_1 Numerically.

For the point E_3we choose $r_1=8, r_2=5, b_1=0.5, b_2=7.947$. Therefore $E_3=(0.7550,0.9596)$. $I_6=(0.1464, 0.8536)$, $I_7=(0.0326, 0.7674)$, and $I_9=(0.7500,\infty)$

Figure- 4 and Figure- 5 illustrate the trajectories of the prey-predator population as a function of time and the local stability of the point E_3 according to the Lemma 6 respectively.

For the harvesting model we will choose a set $r=7$, $q=0.9$, $E=0.61$. Then $e_1= 0.6732$, $c=0.1$, $p=0.3$ $x_\infty=0.3704$and $E_\infty=8.093$.

Figure- 6 illustrates local stability, while,if $E=0$ accordint to the work of Marottothe chaos or nowhere dense is appeared. This also shows that the harvesting will improve the stability.

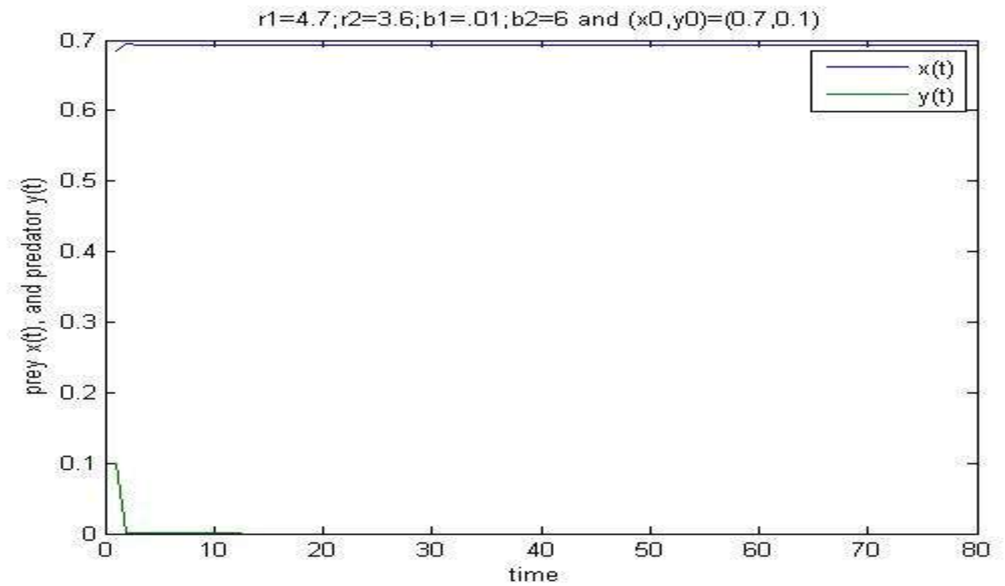

Figure 1- The trajectories of the prey population and the predator population as a function of time for different initial points . This Figure shows That E_1 with all parameters above is local stability..

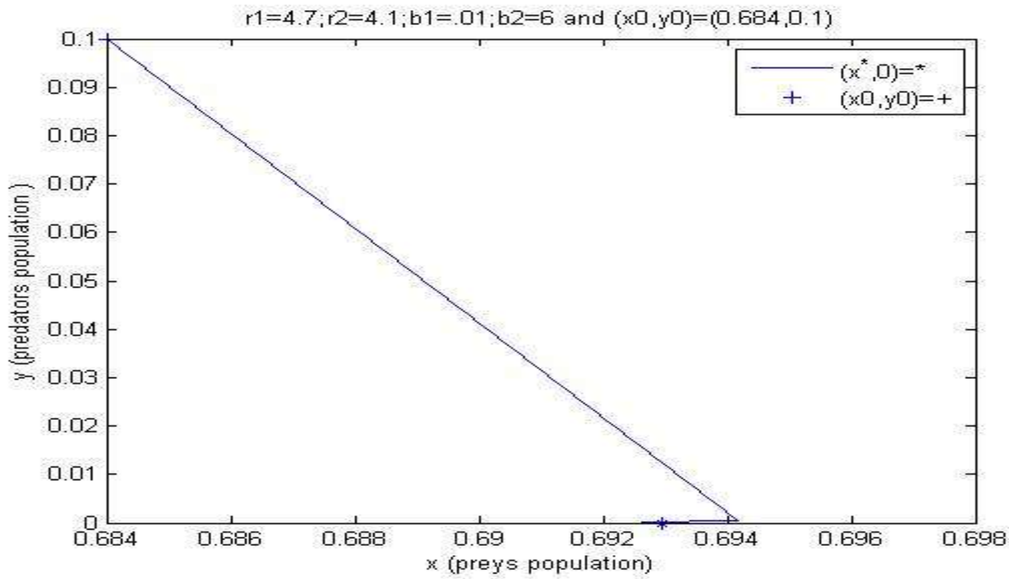

Figure 2-This figure illustrates that $E_1$with all parameters above is local stability according to the condition(i) in lemma 3

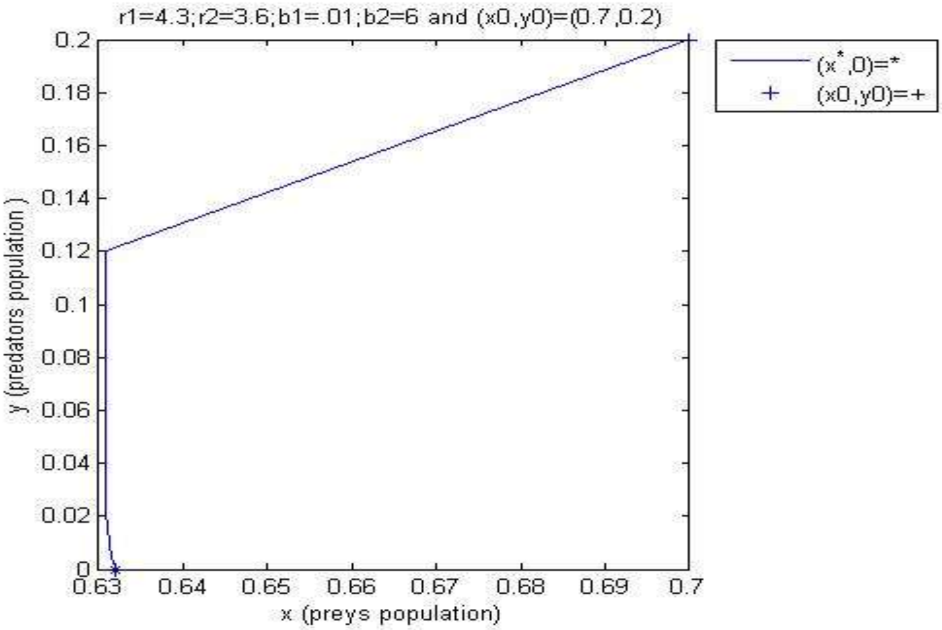

Figure 3- Local stability of E_1 according to the condition(ii) in lemma 3.

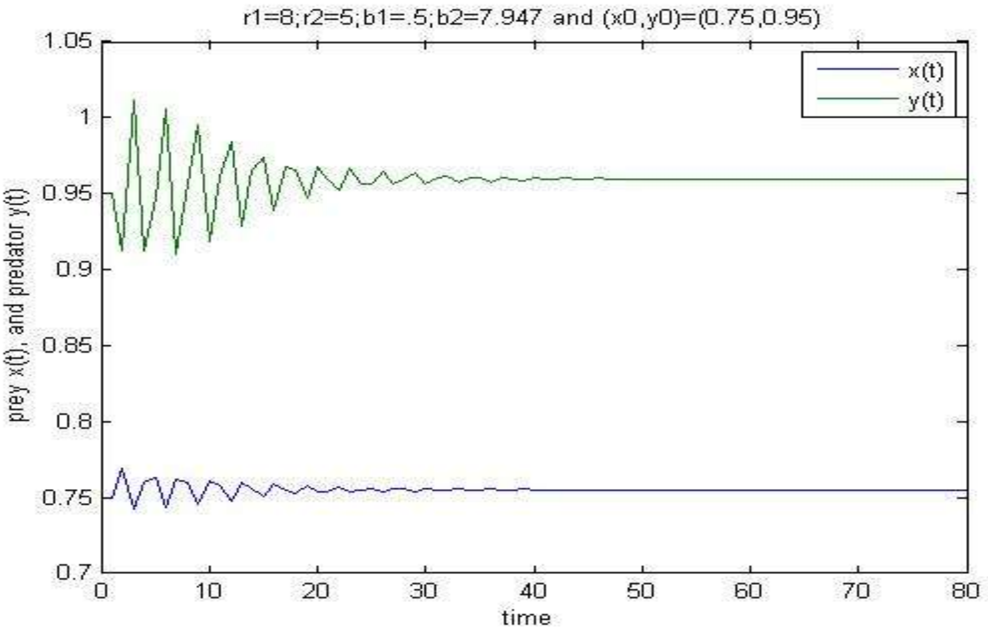

Figure 4-Trajectories of the prey population and the predator population as a function of time which illustrates that E is local stability.

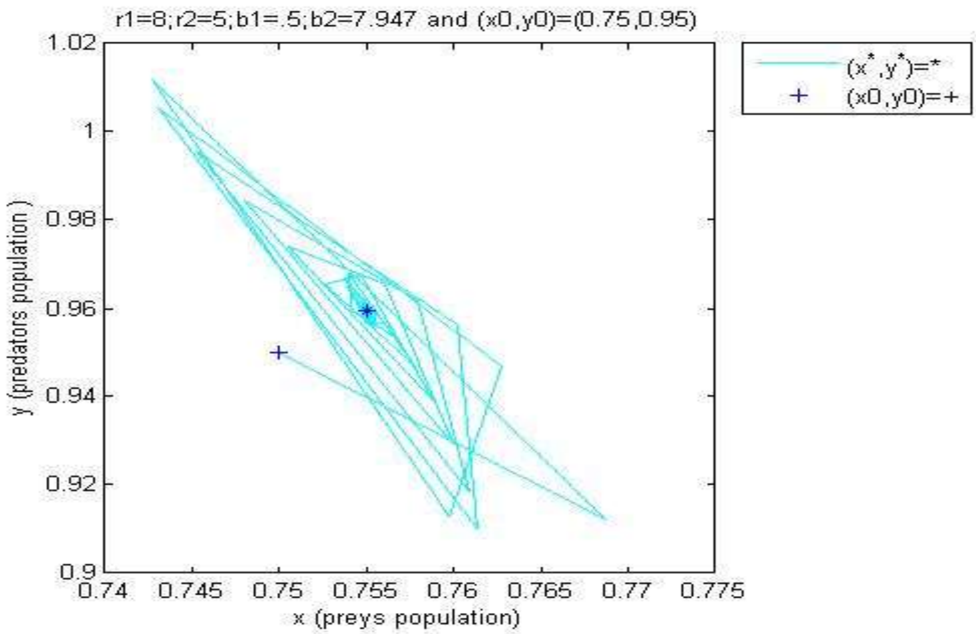

Figure 5- This figure shows that E is local stable according to condition in Lemma 6.

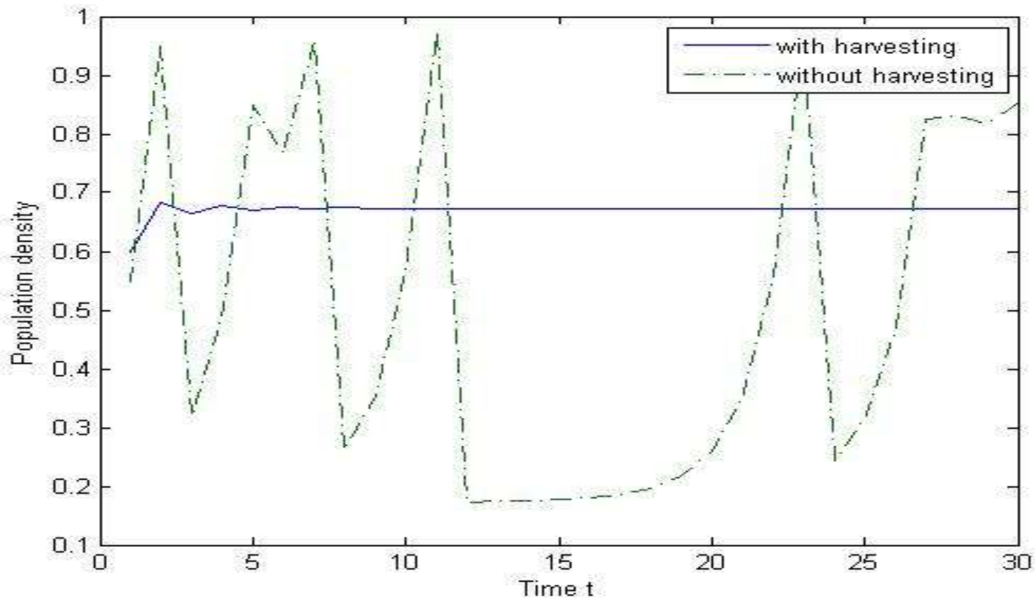

Figure 6- This figure shows that e1 is local stable according to condition in Lemma 7. Here r=7,q=0.9,E=0.61, initail value x_0 =0.55;

Discussion and Conclusions

This work deals with the investigation of the dynamical behavior of non-linear system which defines by two difference equations. The formulation of two pry-predator with Holling type I function response are derived, moreover we give and derive the conditions for the local stability as well as the existence of all the equilibria. We have been also considered a constant rate harvesting for a single population. Some conditions are derived for existence of the bionomical equilibrium point and its value is computed for some values of parameters. The numerical analysis has been given and it conformed the analytic results.

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