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Bifurcation analysis for CO_2 laser

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Abstract:

The bifurcation behaviors for CO_2 laser is studied using four level system model, by solving the differential equations describing that model. The bifurcation diagram gave very clear idea about dynamic behaviors. This system, it has been drawn as a function of modulation amplitude parameter A. From all this, we obtained a scenario of the diagrams by increasing frequency of modulation signal parameter (ω) within (A-ω) plane. Many of different dynamic behaviors such as period doubling and chaos are observed.

Key words: CO_2 laser, Bifurcation Diagrams, Period doubling, Chaos.

Introduction:

Instabilities and chaos were observed in many systems such as mechanics, communication, physics, chemistry, biochemistry, biology, economics, and medicine [1-7].These phenomena have been observed in different laser systems which include lasers with modulated losses. Lasers with an injected signal, feedback, or containing a saturable absorber and bidirectional ring of parameters, each of those systems present deterministic irregular behaviors of different kinds [8-16].This chaos is a purely deterministic phenomenon and the transition from periodic to chaotic regimes follows well-known Scenarios: the Feigenbaum cascade or Shilnkov-type chaos. Shilnkov chaos was previously observed in many systems [17-22] and normally appears when a parameter is varied towards the homoclinic condition associated with a saddle focus [17,23,24].The nonlinear dynamics of a CO_2 laser with modulated parameters has been widely studied over the past two decades. The main reason for such an interest is the desire for a deeper understanding of a coherent dynamical behavior in class-B laser, because CO_2 lasers are among the most convenient systems to study nonlinear physical phenomena[25].

More accurate results can be obtained by using the so-called four-level model (4LM) which takes into account the collisional coupling of each same vibrational band. Recently, some of the dynamic behaviors of Co2 laser using four level model have been studied such as time series, attractors and Fast Fourier Transform (FFT) [26].In this paper, we used 4LM differential equations to describe the behaviors of the output CO_2 laser using bifurcation diagrams which are plotted as a maximum of intensity respect to the modulation amplitude A. These equations were studied numerically over a wide range of different control parameters (A, ω , k and p).

Model:

The nonlinear dynamics of a CO_2 laser can be described by the three-dimensional model where variables have already been scaled to be dimensionless [27]. The model equations are given by

$$\dot{y}_1 = k(y_2 - 1 - \alpha \sin^2\{B[1 + f(t)]\})$$

$$\dot{y}_2 = -\gamma_1 y_2 - 2ke^{y_1}y_2 + y_3 + p \quad ..(1)$$

$$\dot{y}_3 = -\gamma_2 y_3 + zy_2 + zp$$

Where y_1 is the natural logarithm of intensity y_2 which is the main population difference and y_3 is the difference inrotational levels.The function f(t) is given by:

$f(t) = A\sin(\omega t)$, where A and ω are respectively, the modulation amplitude and the frequency of modulation signal. The parameters γ_1 and γ_2 represent, the decay rates, P is the pump parameter, the parameter $k \propto 1/L$ where L is the cavity length and $\alpha \propto (1 - 2T)/2T$, where T is the total transmission coefficient. To solve the model equations we assume that each manifold contains z=10 sublevels. The values of parameters are collected in Table (1) (all parameters are normalized).

Table (1): The parameters used in the study.

ω	A	k	P	γ_1	γ_2	A	B
0.5 up to 1.25	0.01 up to 0.4	25,30,35 and 40	0.082 and 0.09	10.0643	1.0643	4	0.21

Results and Discussion:

We study the effect of the parameters (A, ω, k and p)on the behavior of the 4LM CO_2 laser by solving the equation (1) numerically using the fourth-order Runge-Kutta method.

Numerical bifurcation diagrams of the maximum intensity with respect to the parameter A are shown in Figs.(1 to 4). These diagrams are obtained when ω parameter incremented from 0.5 to 1.25 for the values of the parameter k (k=25,30,35 and 40).

Fig.1, illustrates a scenario of the bifurcation diagrams at k=25,this happens when ω increasing from 0.5 to 1.1. For details, Fig.(1a) illustrates the behavior of CO_2 laser at ω =0.5 that shows different nonlinear dynamics due to incrementing A value, also the appearance of period doubling nature (P1,P2 and P4) leads to chaotic behavior. After that, keeping increment for higher values, the system will again show periodic scenario. These assortments of behaviors happened were all within A range from 0.01up to 0.4. ω increased up to 0.5, the output of the laser system shows chaotic respect to laser range of A value as compared with that appeared at ω=0.5 (see Fig.(1,b-e)). After that, the keep ω increasing parameter , one can notice a narrowing behavior occurrence in chaotic region . At this point, the period doubling becomes the dominant behavior (see Fig.(1,f-j)).

In order to study the effect of k parameter on the bifurcations scenario that has been discussed previously in Fig.(1) , we have repeated our prior work . Thus, Figs. (2 to 4) show these scenarios at k = 30,35 and 40. The scenarios illustrates that the laser system behavior become more chaotic in some of A-ranges compared with that happened in Fig.(1). The increasing in chaotic regimes occurred gradually with k increment. We can notice that the appearance of chaos will terminate according to the values of k and ω (see Table (2)).

Table (2): Illustrate of parameters of the termination of the chaos regimes.

Fig. No.	k-value	ω-value
1	25	1.0
2	30	1.1
3	35	1.2
4	40	1.25

Eventually, we have study the effect of pumping parameter (P) on the bifurcation Scenario described in Fig.(2) (this is all well-illustrated in Fig.(5) when P was increased to 0.09). From this figure we can observe that the dynamic behavior of CO_2 laser have awarded to amore periodicity are compared with the other situations that are shown in Fig.(2).

Conclusion:

We investigate the effect of the main parameters on the bifurcation scenario of the four-level model CO_2 laser. We have found that the bifurcation diagrams show many of dynamics behavior such as period doubling, quasi-periodic, and chaos when the control parameters (A and ω) are changed. For all the studied values of the parameter k and ω, we observed that the laser system will always show only period doubling behavior when the values of A less than 0.18. The most interesting case of our study is the behavior that the lasers system it increasing the parameter p from 0.082 to 0.09,Where output of laser system becomes highly periodicity.

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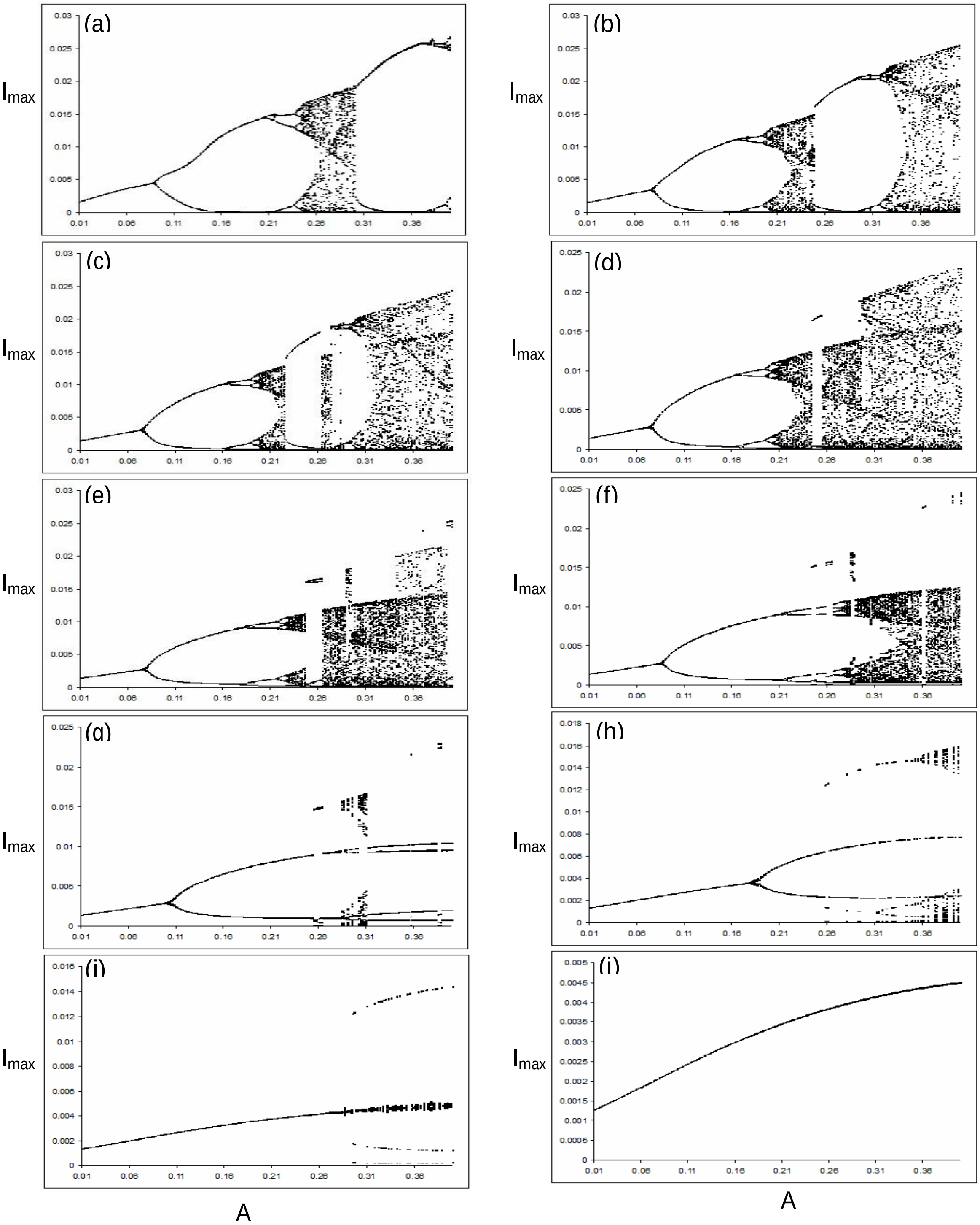

Fig.(1) Bifurcation diagram at k=25 & p=.082

(a) ω =0.5 (b) ω=0.6 (c) ω=0.65 (d) ω=0.7 (e) ω=0.75 (f) ω=0.8 (g) ω=0.85 (h) ω=0.95 (i) ω=1.0 (j) ω=1.1.

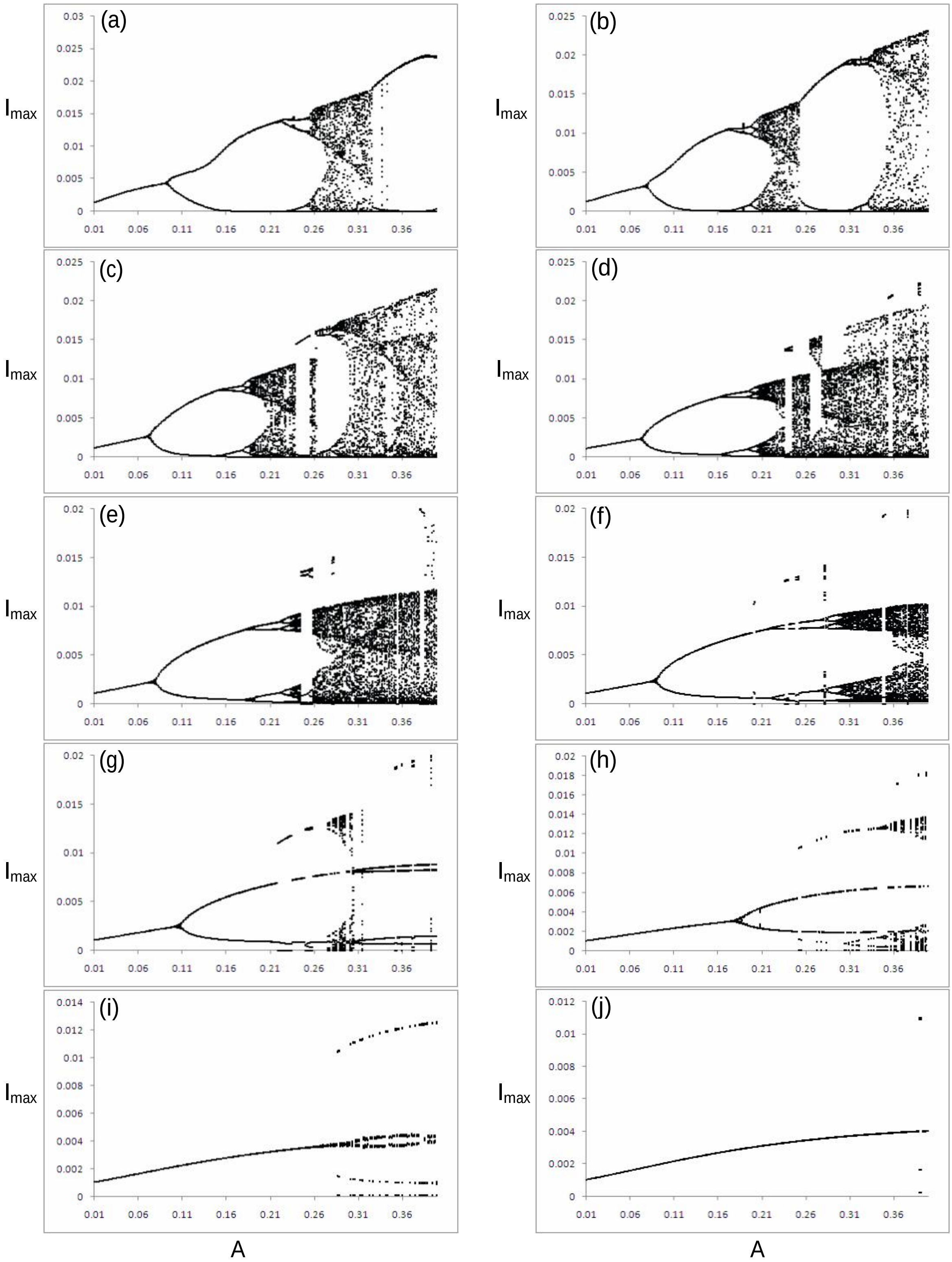

Fig.(2) Bifurcation diagram at k=30 & p=.082

(a) ω =0.5 (b) ω=0.6 (c) ω=0.7 (d) ω=0.8 (e) ω=0.85 (f) ω=0.9 (g) ω=0.95 (h) ω=1.05 (i) ω=1.1 (j) ω=1.15.

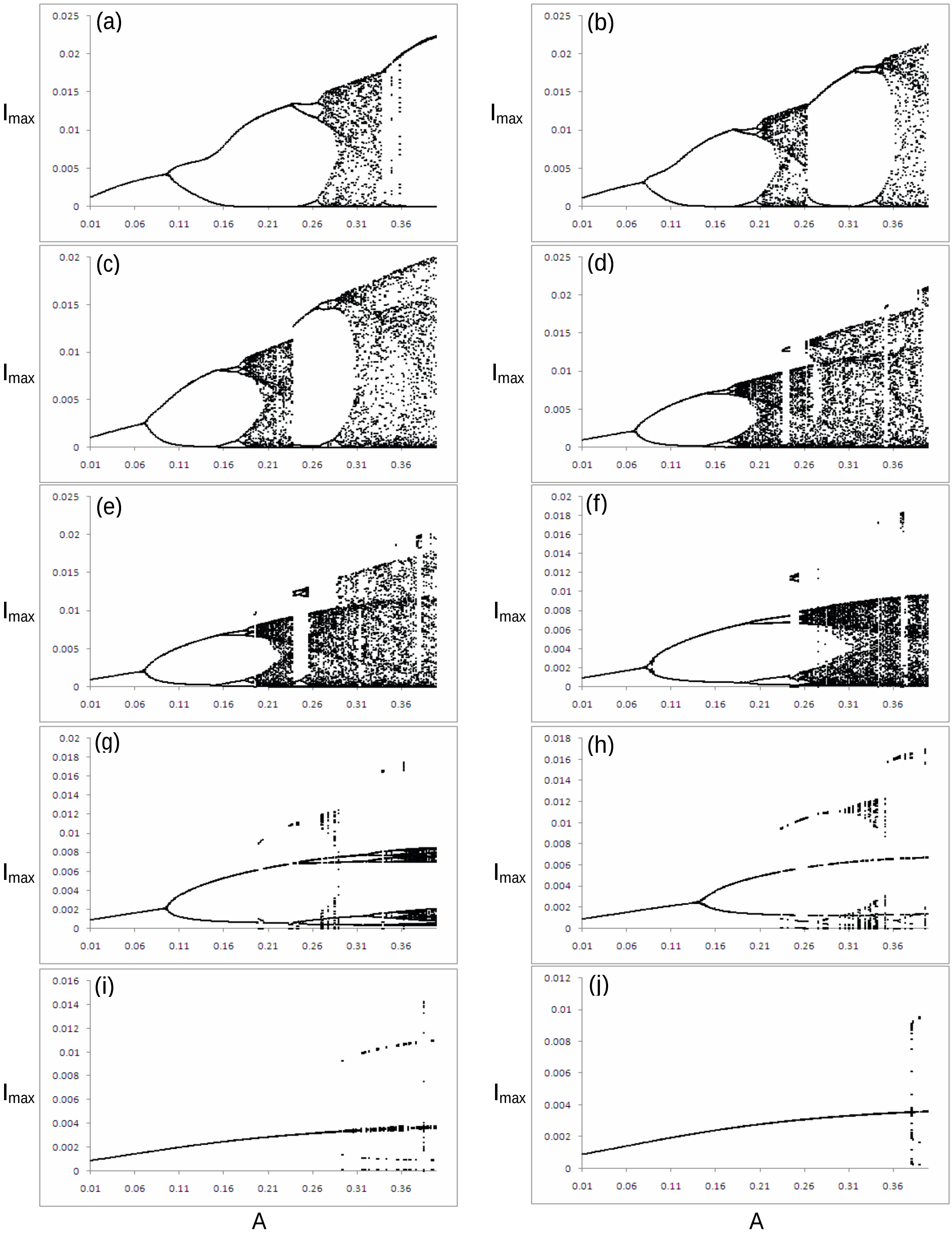

Fig.(3) Bifurcation diagram at k=35 & p=.082

(a) ω =0.5 (b) ω=0.6 (c) ω=0.7 (d) ω=0.8 (e) ω=0.85 (f) ω=0.95 (g) ω=1.0 (h) ω=1.1 (i) ω=1.2 (j) ω=1.25.

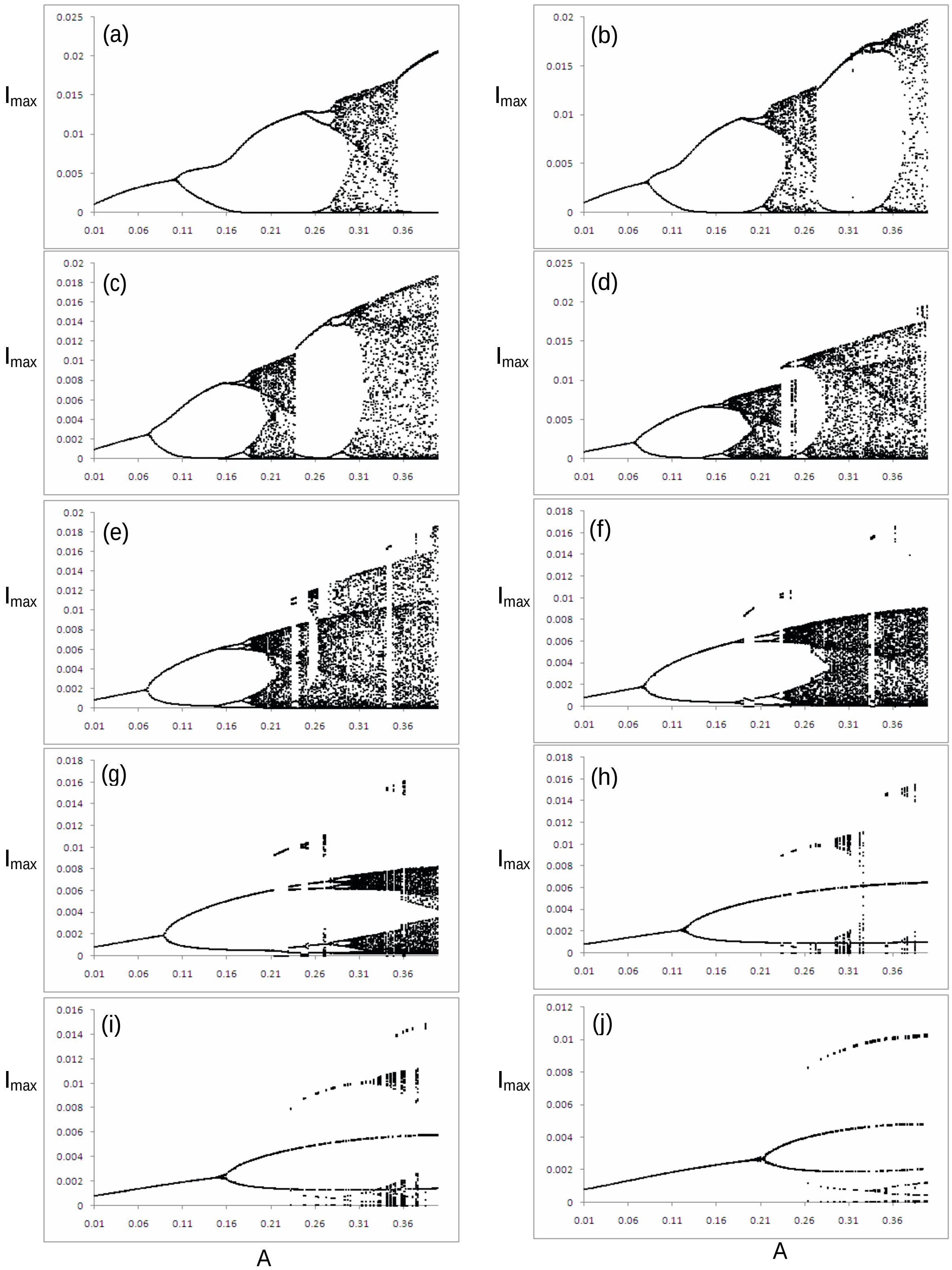

Fig.(4) Bifurcation diagram at k=40 & p=.082

(a) ω =0.5 (b) ω=0.6 (c) ω=0.7 (d) ω=0.8 (e) ω=0.9 (f) ω=1.0 (g) ω=1.05 (h) ω=1.15 (i) ω=1.2 (j) ω=1.25.

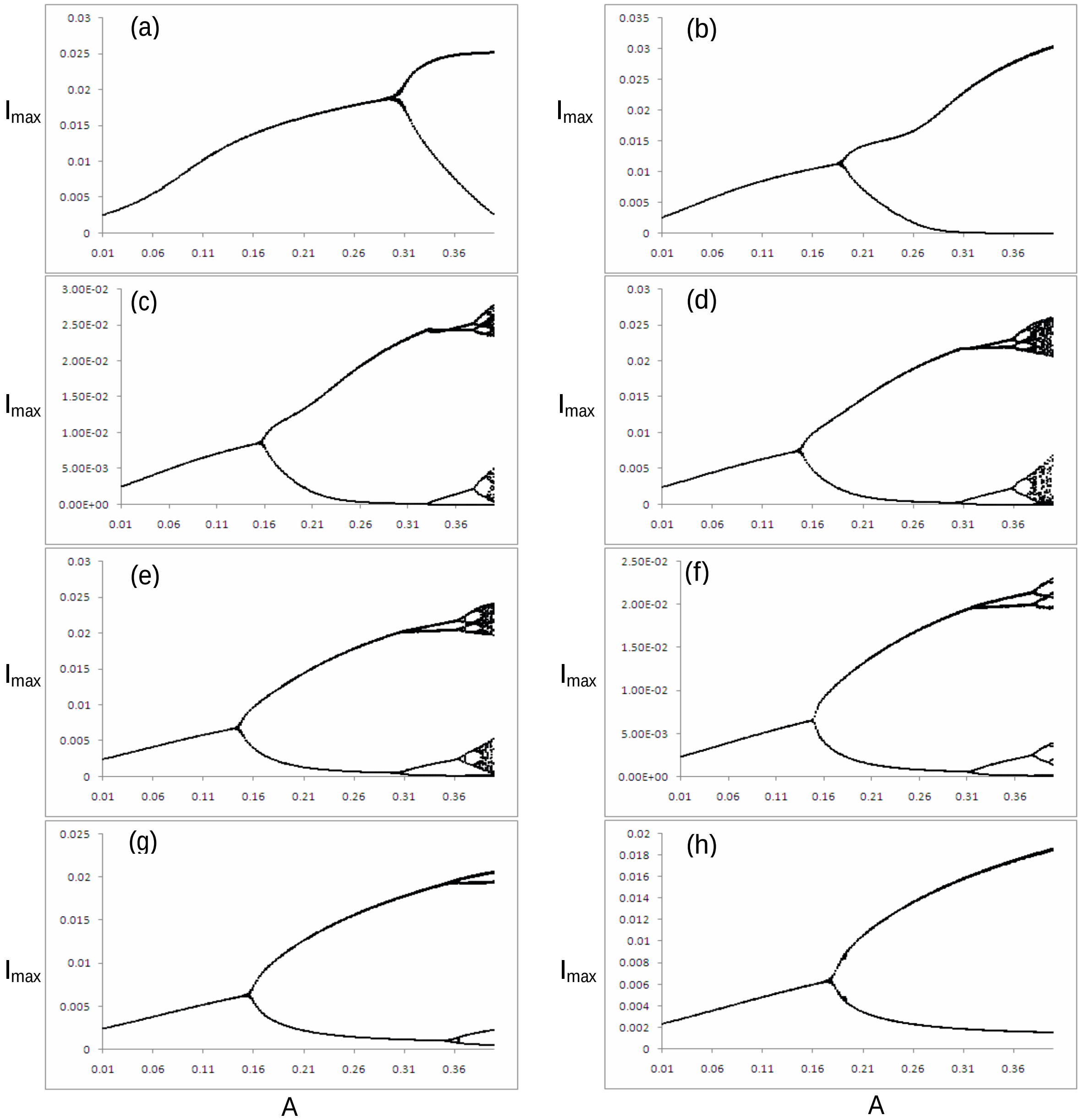

Fig.(5) As in Fig.2 but at p=0.09.
(a) ω =0.5 (b) ω=0.7 (c) ω=0.85 (d) ω=0.95 (e) ω=1.05 (f) ω=1.1
(b) (g) ω=1.2 (h) ω=1.3.

الخلاصة:

تمت دراسة تصرفات التفرعات لموديل ليزر ثنائي اوكسيد الكاربون CO2 ذات الاربع مستويات 4LM وذلك بحل المعادلات الخاصة بذلك النظام عدديا. لقد اعطى مخطط التفرع فكرة واضحة عن التصرفات الديناميكية لهذا النظام وذلك عن طريق رسمها كدالة لعامل سعة التضمين A . وللحصول على سيناريو لعدد من هذه التفرعات تمت زيادة عامل تردد اشارة التضمين ω في المستوي (A-ω) حيث وجدت تصرفات ديناميكية مختلفة مثل تضاعف زمن الدورة والفوضى.