Calculation the Cross Sections of $^{9}B(n,\alpha)^{6}Li$ reaction by using the reciprocity theory for the first exited state

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Abstract

In this study light elements ^{6}Li , ^{9}B for $^{9}B(n,\alpha)^{6}Li$ reaction as well as α-particle energy from 6.84 MeV to 9.94 MeV are used according to the available data of reaction cross sections with threshold energy (3.9537MeV). The more recent cross sections data of $^{6}Li(\alpha,n)^{9}B$ reaction is reproduced in fine steps (50keV) in the specified energy range , as well as cross section (α,n) values were derived from the published data of (n,α) as a function of energy in the same fine energy steps by using the principle inverse reaction . This calculation involves only the first exited state of ^{6}Li , ^{9}B in the reactions $^{6}Li(\alpha,n)^{9}B$ and $^{9}B(n,\alpha)^{6}Li$.

الخلاصة

في هذه الدراسة اعيد حساب المقاطع العرضية للنوى الخفيفة (^{6}Li , ^{9}B) للتفاعل $^{9}B(n,\alpha)^{6}Li$ للبيانات المتوفرة في الادبيات العالمية وللمدى الطاقي من MeV (6.84) الى MeV (9.94) وبطاقة عتبه مقدارها (3.9537MeV) كدالة للمقاطع العرضية . بأستخدام نظرية التعاكس اذ اشتقت معادلة لحساب المقاطع العرضية لتفاعل $^{9}B(n,\alpha)^{6}Li$ وللمستو المتهيج الاول وذلك بالاعتماد على المقاطع العرضية لتفاعل $^{9}B(n,\alpha)^{6}Li$ ومن ثم الحصول على معادلة للرسم البياني من خلال استخدام برامج الحاسوب (Matlab-6.5) . تم رسم وجدولة النتائج بالاضافة الى مناقشة النتائج.

Introduction

The interaction of particles with matter is described in terms of quantities known as cross sections which is defined in the following way [1]. Consider a thin target of area (a) and thickness (X) containing (N) atoms per unit volume, placed in a uniform mono-directional beam of incident particles (neutrons for example of intensity I_o, which strikes the entire target normal to its surface as shown in fig.(1). It is found that the rate at which interactions occur within the target is proportional to the beam intensity and to the atom density, area and thickness of the target. Summarizing this experimental result by an equation, we define the interaction rate

(in the entire target)=σINaX --- (1-1)

Where the proportionality constant σ is known as the cross section , Thus

σ = interaction rate / INaX ---- (1-2)

As NaX is equal to the total number of atoms in the target, it follow that σ is the interaction rate per atom in the target per unit intensity of the incident beam [2] .

Reciprocity Theory

If the cross-sections of the reaction **A(α,n)B** is measured as a functions of T**α** (T**α** = Kinetic energy of α-particle) the cross–sections of the inverse reaction **B(n,α)A** can be calculated as a function of Tn (Tn = Kinetic energy of neutron) using the reciprocity theorem [3] which states that :

$$\frac{\sigma_{(\alpha,n)}}{\sigma_{(\alpha,n)}\lambda_{\alpha}^{2}}=\frac{\sigma_{(n,\alpha)}}{g_{(n,\alpha)}\lambda_{n}^{2}} \quad \text{---(1-3)}$$

Where $\sigma_{(\alpha,n)}$ and $\sigma_{(n,\alpha)}$ represent cross-sections of (α,n) and (n,α) reactions respectively , g is a statistical factor and ƛ is the de–Broglie wave length divided by 2π and is given by

$$ƛ=\frac{\hbar}{MV} \quad \text{-----(1-4)}$$

Where ħ is Dirac constant (h /2π) , h is plank constant , M and v are mass and velocity of α or n particle .

From eq.(1-4),we have

$$ƛ^{2}=\frac{\hbar^{2}}{2MT} \quad \text{------(1-5)}$$

The statistical g-factors are givens by[3]

$$g_{(\alpha,n)}=\frac{2J_c+1}{(2I_A+1)(2I_\alpha+1)} \quad \text{---(1-6)}$$

And

$$g_{(n,\alpha)}=\frac{2J_c+1}{(2I_B+1)(2I_n+1)} \quad \text{----(1-7)}$$

The conservation law of the momentum implies that:

$I_A + I_\alpha = J_c = I_B + I_n$ ----(1-8)

And

$\pi_A\pi_\alpha (-1)^{\ell\alpha} = \pi_c = \pi_B\, \pi_n(-1)^{\ell n}$ ----(1-9)

J_c and π_c are total angular momentum and parity of the compound nucleus .

I_A and π_A are total angular momentum and parity of nucleus A.

I_B and π_B are total angular momentum and parity of nucleus B.

I_α and π_α are total angular momentum and parity of α-particle.

I_n and π_n are total angular momentum and parity of neutron .

$\pi_\alpha = \pi_n = +1$ -----(1-10)

$I_\alpha = s_\alpha + \ell_\alpha$ ------(1-11)

Where I_α is the total angular momentum of alpha particle.

s_α is spin of α-particle = 0

l_α is the orbital angular momentum of α-particle.

And

$I_n = s_n + \ell_n$ ------(1-12)

Where I_n is the total angular momentum of the neutron

s_n is spin of neutron = 1/2

l_n is the orbital angular momentum of neutron .

From eq.(1-8),we have:

$|J_c - I_A| \leq l_\alpha \leq J_c + I_A$ -----(1-13)

And

$|J_c - I_B| \leq l_n \leq J_c + I_B$ ------(1-14)

The reactions A(α,n)B and B(n,α)A can be represented with the compound nucleus C as in the following schematic diagram. It is clear that there are some important and useful relations between the kinetic energies of the neutron and alpha particle. One can calculate the separation energies of α-particle (S_α) and neutron (S_n) using the following relations:

S_α and S_n are separation energies of α and n from C. Then

$$E = S_\alpha + \frac{M_A}{M_A + M_\alpha} T_\alpha \quad \text{---(1-15a)}$$

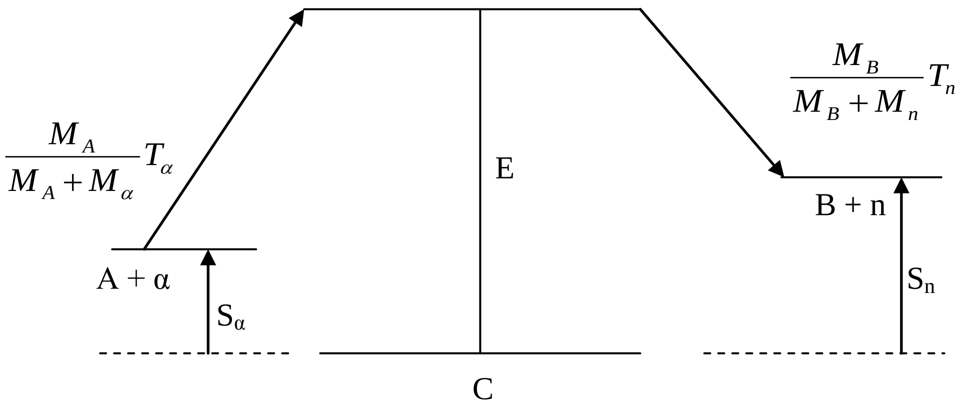
$$E = S_n + \frac{M_B}{M_B + M_n} T_n \quad \text{---(1-15b)}$$

With

$S_\alpha = 931.5\,[M_A + M_\alpha - M_c]$ ----(1-16)

$S_n = 931.5\,[M_B + M_n - M_c]$ ----(1-17)

Combining (1-15a) , (1-15b) , (1-16) and (1-17)



Schematic diagram of the reactions

and as the Q-value of the reaction A(α,n)B is given by :

$Q = 931.5[M_A + M_\alpha - M_B - M_n]$ ---(1-18)

Then

$$Q = \frac{M_B}{M_B + M_n} T_n - \frac{M_A}{M_A + M_\alpha} T_\alpha \quad \text{----(1-19)}$$

Or :

$$T_n = \frac{M_B + M_n}{M_B}\left[\frac{M_A}{M_A + M_\alpha} T_\alpha + Q\right] \quad \text{----(1-20)}$$

The threshold energy E_{th} is given by :

$$E_{th} = -Q\frac{M_A + M_\alpha}{M_A} \quad \text{---(1-21a)} \quad \text{Or}$$

$$Q = -\frac{M_A}{M_A + M_\alpha} E_{th} \quad \text{---(1-21b)}$$

Then

$$T_n = \frac{M_B + M_n}{M_B} \times \frac{M_A}{M_A + M_\alpha}(T_\alpha - E_{th}) \quad \text{---(1-22)}$$

Thus eq.(1-3) can be written as follows :

$$\sigma_{(n,\alpha)} = \frac{g_{(n,\alpha)} M_\alpha T_\alpha}{g_{(\alpha,n)} M_n T_n} \sigma_{(\alpha,n)} \quad \text{----(1-23)}$$

It is clear from this equation that the cross sections of reverse reaction are related by a variable parameters which can be calculated if the nuclear characteristics of the reactions are known.

Results and Discussion

The cross section of (α,n) reactions for the elements **^{6}Li** and **^{9}B** of ^{6}Li(α,n)^{9}B reaction available in the literature[4] , have been taken and re-plotted for a defined energy level as shown in Fig.(2).These plots were analyzed using the Matlab computer program to obtain the cross sections for the selected energies and we get equation for these data as follow:

y = 0.0022*x^10-0.18*x^9 + 6.4*x^8 - 1.4e+2*x^7+ 1.9e+3*x^6 -1.9e+4*x^5 +1.2e+5*x^4-5.6e+5*x^3+1.7e+6*x^2 - 2.9e+6*x + 2.3e+6

The atomic mass of elements and isotopes mentioned in this study have been taken from the latest nuclear wallet cards released by the National Nuclear Data Center(NNDC)[5] and the energy level, parity and spin scheme of isotopes from [6].

By using the reciprocity theory we derive the mathematical formula for ^{9}B(n,α)^{6}Li reaction for first exited state:

$$\sigma_{n,\alpha} = 3.720 \frac{T_\alpha}{T_n} \sigma_{\alpha,n}$$

By using semi empirical formula the evaluated cross sections as a function of neutron energy from (1.9269)MeV to (4.0298)MeV of present work are listed in table (1). From these data which were plotted, we got the mathematical equation representing the cross sections distribution in the indicated range of neutron energy Fig.(3) and percentage error (± 0.1819) barn as follow :

$Y=0.011X^{10} + 0.038X^9 - 0.044X^8 - 0.19X^7 + 0.038X^6 + 0.24X^5 - 0.088X^4 - 0.099X^3 + 0.086X^2 + 0.19X + 0.44$

From cross sections of ^{6}Li(α,n)^{9}B we calculated the neutron yield [7][8] by using stopping power [9] from the Zeigler formula (SRIM-2008)[10].We are plotted in figure(4).

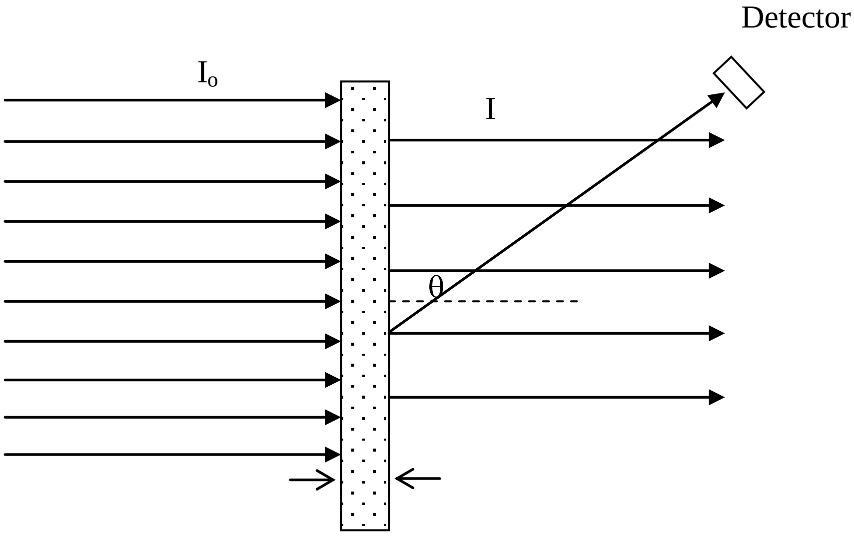


Figure (1): A schematic diagram illustrating the definition of total cross section in terms of the reduction of intensity[1].

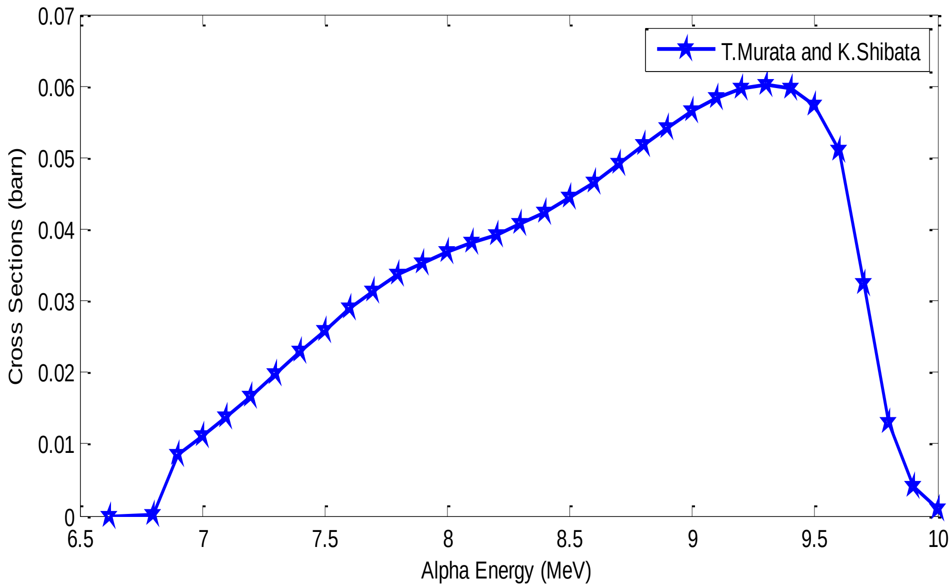


Fig.(2): Cross sections of $^{6}Li(\alpha,n)^{9}B$ Reaction[4]

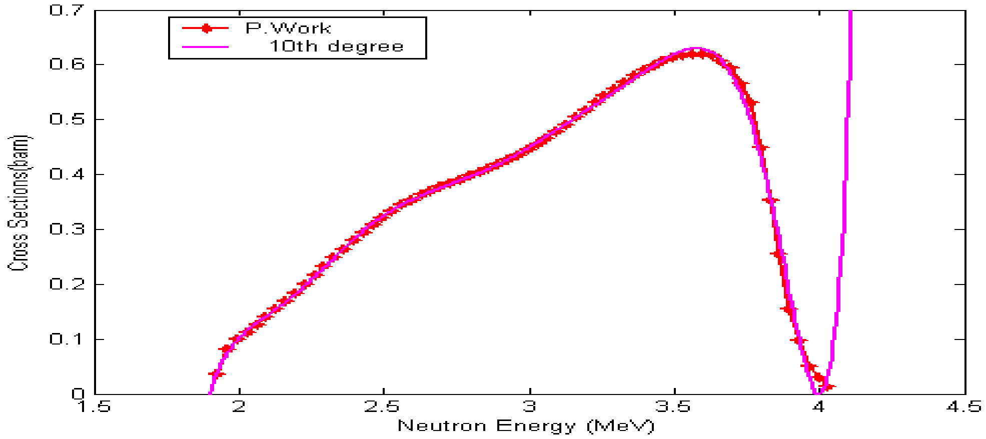


Fig.(3): Cross sections of $^{9}B(n,\alpha)^{6}Li$ Reaction of P.Work

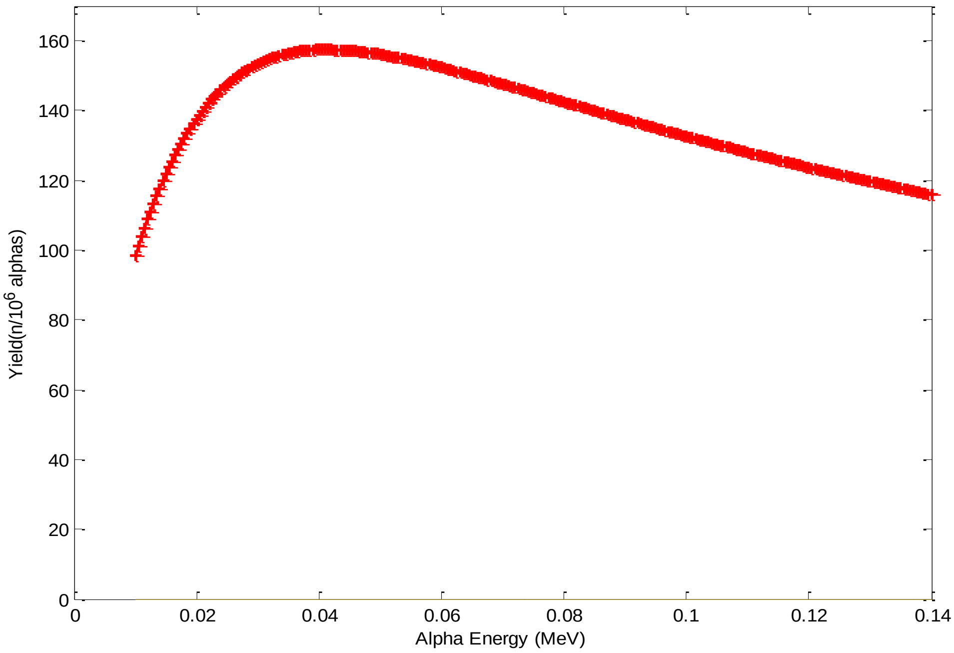


Fig.(4) :Neutron Yield for $\mathbf{^{6}Li\ (\alpha,n)^{9}B}$ reaction

Table (1):The cross sections of $\mathbf{^{9}B(n,\alpha)^{6}Li}$ Reaction as a function of neutron energy present work.

neutron - energy (MeV)	X- sections (barn) P.Work	neutron - energy (MeV)	X- sections (barn) P.Work	neutron - energy (MeV)	X- sections (barn) P.Work
1.9269	0.0359	2.7280	0.3843	3.5291	0.6163
1.9603	0.0802	2.7614	0.3908	3.5625	0.6191
1.9937	0.0993	2.7948	0.3973	3.5959	0.6182
2.0270	0.1121	2.8282	0.4039	3.6293	0.6162
2.0604	0.1255	2.8615	0.4110	3.6627	0.6062
2.0938	0.1392	2.8949	0.4183	3.6960	0.5941
2.1272	0.1537	2.9283	0.4266	3.7294	0.5652
2.1606	0.1685	2.9617	0.4352	3.7628	0.5321
2.1939	0.1840	2.9951	0.4451	3.7962	0.4491

2.2273	0.1996	3.0284	0.4553	3.8296	0.3536
2.2607	0.2157	3.0618	0.4667	3.8629	0.2544
2.2941	0.2319	3.0952	0.4784	3.8963	0.1544
2.3275	0.2480	3.1286	0.4910	3.9297	0.0972
2.3608	0.2640	3.162	0.5039	3.9631	0.0508
2.3942	0.2792	3.1953	0.5172	3.9965	0.0292
2.4276	0.2943	3.2287	0.5306	4.0298	0.0138
2.4610	0.3080	3.2621	0.5437	----	----
2.4944	0.3213	3.2955	0.5567	----	----
2.5277	0.3329	3.3289	0.5686	----	----
2.5611	0.3441	3.3622	0.5802	----	----
2.5945	0.3535	3.3956	0.5901	----	----
2.6279	0.3625	3.429	0.5996	----	----
2.6613	0.3702	3.4624	0.6065	----	----
2.6946	0.3776	3.4958	0.6128	----	----

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