STUDY OF TOW DIMENSIONAL LAMINAR NATURAL CONVECTION INSIDE CAVITY +

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Abstract:

An incompressible fluid flow and heat transfer by natural convection inside a rectangular cavity had been studied numerically. The study focused on the variation of velocity vectors and temperature values due to the change of Grashof number and an enclosure aspect ratio. The Grashof numbers was ranging from 10^3 to 10^5. Nusselt number dependence on temperature field especially near hot and cold walls is verified. The continuity, momentum and energy equations have been solved using finite volume method (FVM). The results show that the central core region is distorted with increase of Grashof number. Also the results indicated that the rate of heat transfer is increased with the increase of Grashof number. The validation of the computed results is verified through the comparison with the published results.

المستخلص:

دراسة انتقال الحرارة بالحمل الحر الطباقي ثنائي البعد داخل فجوة جريان المائع وانتقال الحرارة نتيجة للحمل الحر داخل حيز مستطيل تمت دراسته عدديا. ركزت الدراسة على التغيرات التي تحصل في متجهات السرع ودرجات الحرارة نتيجة لتغير رقم كراشوف والنسبة الباعية للحيز.تراوحت أرقام كراشوف التي تم دراستها بين($١٠^٣$) و($١٠^٥$). بينت هذه الدراسة اعتماد عدد نسلت على مجال درجات الحرارة وخاصة بالقرب من الجدران الساخنة والباردة.معادلات الجريان(الاستمرارية والزخم والطاقة) تم حلها باستخدام تقنية الحجم المحدد. بينت النتائج أن منطقة القلب الساكن تشوهت وأصبحت غير منتظمة مع زيادة عدد كراشوف. أيضا بينت النتائج أن معدل انتقال الحرارة يزداد مع زيادة عدد كراشوف.تم التحقق من فعالية الطريقة العددية المستخدمة من خلال المقارنة مع النتائج المنشورة .

Nomenclature:

Ar	aspect ratio, —
Gr	Grashof number, —
Nu	Local Nusselt number, —
Nu_{av}	average Nusselt number, —
L	length, m
Pr	Prandtle number, —
T_c	cold wall temperature, Ċ
T_h	hot wall temperature,Ċ
T_o	average temperature,Ċ
ν	kinematic viscosity, m^2/s^2
Φ	dependent variable , —
Γ	diffusion term , —

Introduction:

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Natural convection flow and heat transfer inside cavity is of some interest in many engineering applications such as air-conditioning, cooling of electronic equipment and outdoor applications. The literature includes many investigation relating the topic of natural convection inside an enclosure. Batchelor [1] gave the limit of transition from the conduction regime to the boundary layer regime and the transition from laminar to turbulent. Gill [2] solved this problem theoretically assuming an isothermal rotating core. His results was validated with experimental results. Yin et al.[3] gave experimental data on heat transfer results with different aspect ratios. Elsherbiny et al.[4] performed an experimental study for natural convection in a rectangular enclosure for different aspect ratios. Roas[5] studied the natural convection in an enclosure in detail, different temperature and boundary conditions were used. Tang [6] used a multi scale thermal analysis approach to demonstrate the mixed convection from discrete heat sources in enclosure. Nusselt numbers and temperature distribution were obtained by Lees [7] for different geometries and orientations. Yang[8] presented results on 3-D velocity and temperature field in addition to local and average Nusselt numbers. Jalal [10] solve this problem numerically by using primitive variable method. Teetstra [11] used different models for dealing with laminar natural convection from heated bodies in an enclosure. The aim of this study is to get an insight into the natural convection flows inside an enclosure. The solution of the governing equations is done by numerical method using finite volume method (FVM) with pressure correction .This method provides a set of discretisation equations which means that the system of partial differential equations is transformed to the system of algebraic equations. In this paper orthogonal grid with different points depending on Grashof number is used.

Problem Description:

The problem to be considered is shown in fig.1. The geometry is specified by the height H and length L. The vertical walls are maintained at uniform temperature, T_h and T_c .The top and bottom walls are adiabatic.

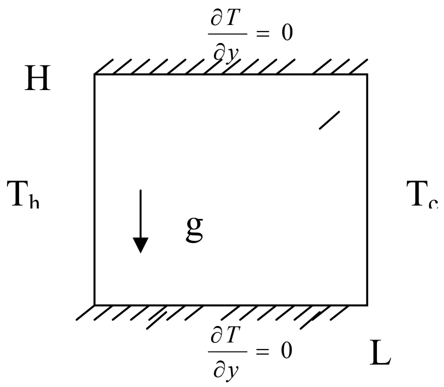


Fig. (1): problem definition

Mathematical Model:

Equations of conservation of mass, momentum and energy are the basic equations of the mathematical model. Fluid is assumed to be a Newtonian and gravity is only the external force. Constant thermophysical properties are assumed except for the density that appears in the buoyancy force. The variables under consideration can be cast in the following form:

$$X = \frac{x}{H} \;,\; Y = \frac{y}{H} \;,\; U = \frac{u}{\upsilon/H} \;,\; V = \frac{v}{\upsilon/H}$$

$$P = \frac{p}{\rho(\upsilon/H)^2} \;,\; \theta = \frac{T - T_o}{T_h - T_c}$$

where T_o is the average temperature $= (T_h + T_c)/2$

The governing equations upon introducing dimensionless variables have the forms[10].

$$\frac{\partial}{\partial X}(U) + \frac{\partial}{\partial Y}(V) = 0 \qquad (1)$$

$$\frac{\partial}{\partial X}(UU) + \frac{\partial}{\partial Y}(UV) = -\frac{\partial P}{\partial X} + \nabla^2 U \qquad (2)$$

$$\frac{\partial}{\partial X}(UV) + \frac{\partial}{\partial Y}(VV) = -\frac{\partial P}{\partial Y} + \nabla^2 V + Gr\theta \qquad (3)$$

$$\frac{\partial}{\partial X}(U\theta) + \frac{\partial}{\partial Y}(V\theta) = \frac{1}{\Pr}\nabla^2 \theta \qquad (4)$$

where $\Pr = \frac{\mu Cp}{k}$

The predicted temperature field is used to calculate the values of local and average Nusselt numbers especially on the hot and cold walls. The local and average Nusselt numbers on the isotherm walls are expressed as follows:

$$Nu = -\frac{H}{\Delta T}\left(\frac{\partial \theta}{\partial X}\right) \qquad (5) \quad , \qquad Nu_{av} = \frac{1}{H}\int_{y=0}^{y=H} Nu\,dY \qquad (6)$$

Boundary Conditions:

In order to get a complete mathematical model, the boundary conditions are defined as follows:

$0 \le x \le L(y = 0, y = H)$ u = v = 0 $\quad \frac{\partial \theta}{\partial y} = 0$

$x = 0 \quad 0 \le y \le H \quad$ u = v = 0

$x = L \quad 0 \le y \le H \quad$ u = v = 0

The thermal boundary conditions on the hot and cold walls are:

Hot wall $\quad \theta = 0.5$

cold wall $\quad \theta = -0.5$

Solution Procedure:

Finite volume method (FVM) is used to solve the above mentioned mathematical model. This gives a system of discretisation equations which means that the system of elliptic partial differential equations is transformed to the system of algebraic equations. Common Gauss-siedl method helps in solving algebraic equations system. The essence of the discretisation is to select appropriate discretisation scheme (appropriate option of balance between convective and diffusion terms through the boundary of each control volume.) The general governing differential equations takes the form[9]:

$$div(\rho U\Phi) = div(\Gamma grad\Phi) + S_{\Phi} \qquad (7)$$

In this paper orthogonal grid with hybrid scheme is used. The one dimensional convective and diffusive fluxes can be expressed as follows:

$$J_x = \rho U\Phi - \Gamma\frac{\partial \Phi}{\partial x} \qquad (8)$$

by using finite volume method the discretisation of east side flux by hybrid scheme gives the form [11]:

$$\left(\rho U\Phi - \Gamma\frac{\partial \Phi}{\partial x}\right)_e = (\rho U)_e\frac{\Phi_p + \Phi_e}{2} - \Gamma_e\frac{\Phi_p + \Phi_e}{\Delta x_e} \qquad (9)$$

A computer program is built to obtain the results of numerical procedure using SIMPLE algorithm. The first calculation is started from zero field and the every one takes the previous as initial condition.

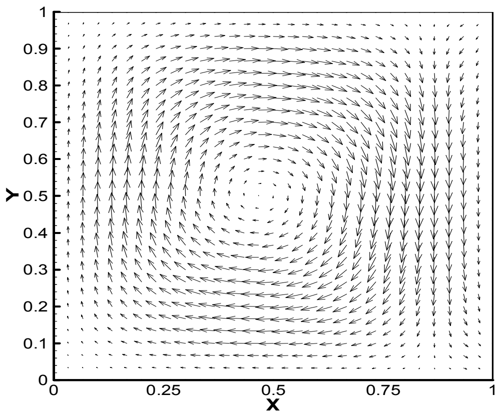
Results:

Two dimensional steady calculation has been performed for laminar regime of fluid through figs 2 to 8. As we might see, Figs(2,3) exhibits the calculated velocity vectors for different Grashof numbers. It is clear that the flow is characterized by non–interacting separated boundary layers on hot and cold walls and central core region. The stagnant core region is greatly affected with increasing of Grashof number where at higher Grashof number, the central core becomes non uniform. The region of the secondary flow is increased with increasing of Grashof number. The values of axial velocity is higher at upper part of the enclosure.

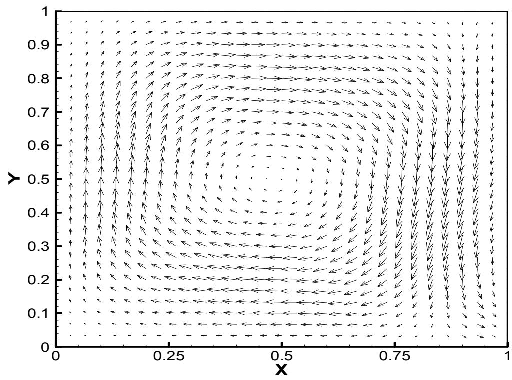
Fig.4.shows the distribution of thermal field for different Grashof numbers. At lower Grashof number the central core region is not stratified with uniform temperature and temperature values is concentrated on the hot and cold walls. With increasing of Grashof number the core region is stratified with nearly uniform temperature because the convection is dominant. The higher values of temperature is found in the upper region and this behavior increases with increase of Grashof number. The effect of enclosure aspect ratio on the flow structure is seen in fig.5 as flow field and fig.6 as isotherm contours. A typical flow structure demonstrate two re circulation regions, the upper region where the density decreases with temperature and lower on with common density variation. The secondary flow regions and the size of the core region is increased with increase of aspect ratio. The stagnant core region is stratified with nearly uniform temperature and this behavior is better at Ar=1.The local Nusselt number versus Grashof number can be seen in fig.7. As the figure shows, the values of local Nusselt number depend on Grashof number. The local Nu is nearly starts from one which means that the main heat transfer mechanism is conduction. As Grashof number increases the rate of heat transfer is increased. In fig.8, the computed results is compared with published results [10]. The comparison indicated acceptable agreement.

Conclusions:

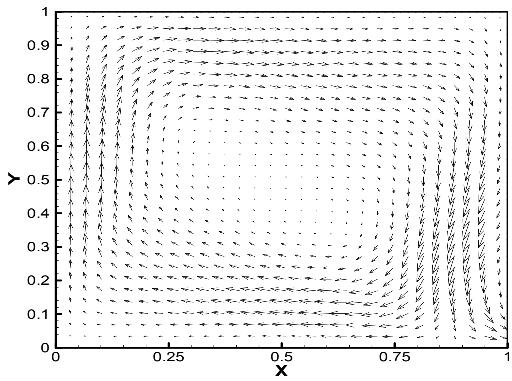
- The results indicated that the core region has stable flow and stable isothermal stratification.
- There is no significant change in flow structure and thermal field as aspect ratio changes except for the central core region.



a. Gr =10^2

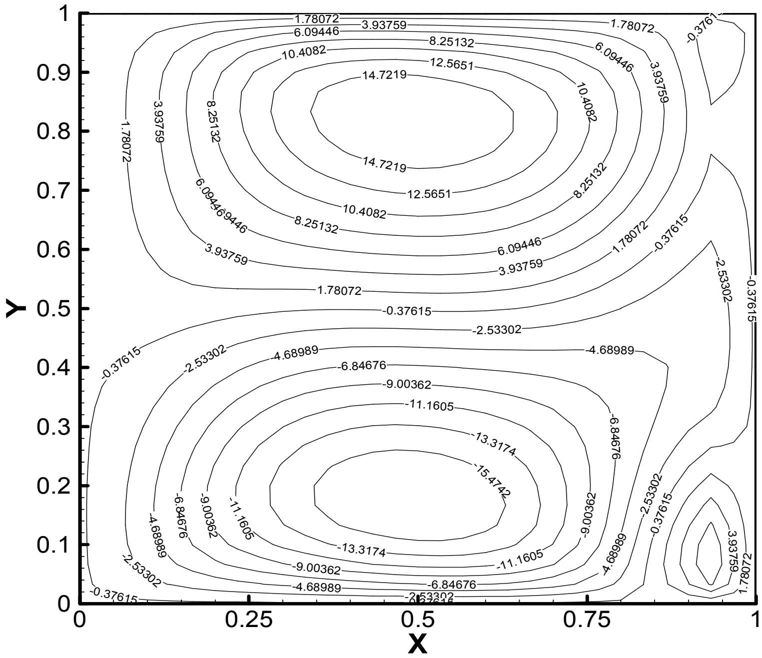


b. Gr =10^4

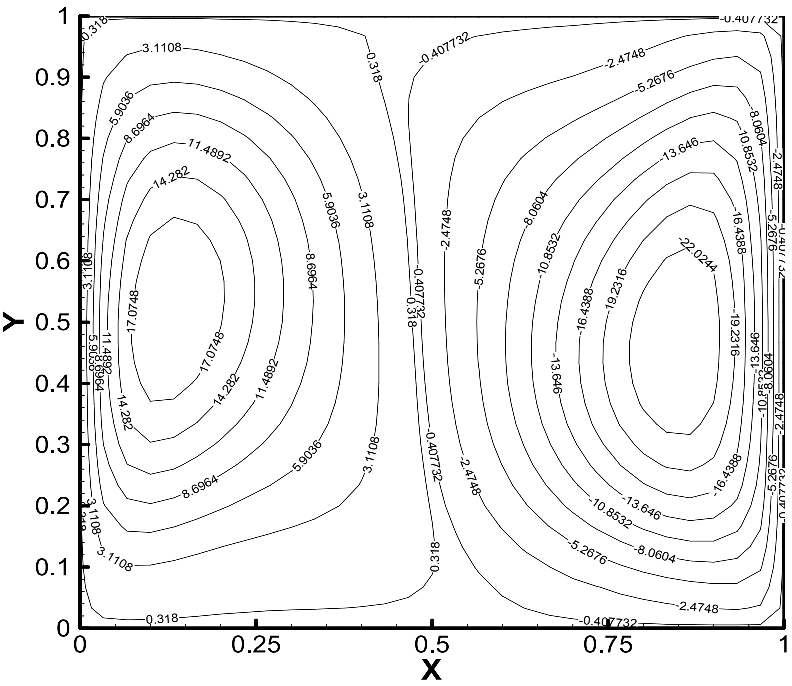


c. Gr =10^5

Fig. (2): Flow field calculated for adiabatic boundary conditions and A_p=

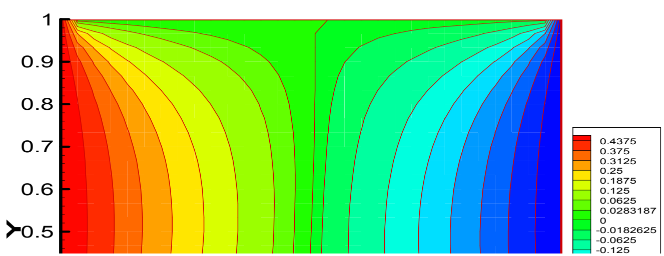


a. Gr = 10^4



b. Gr = 10^4

Fig. (3): Contours of calculated flow field: a. U-velocity, b. V-velocity



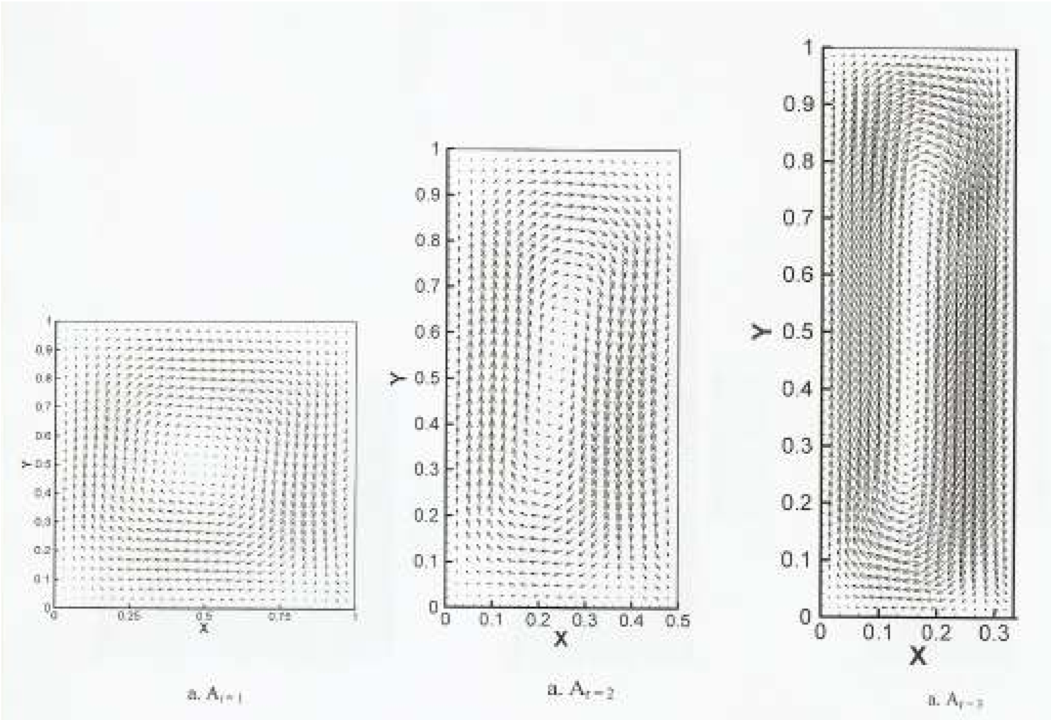
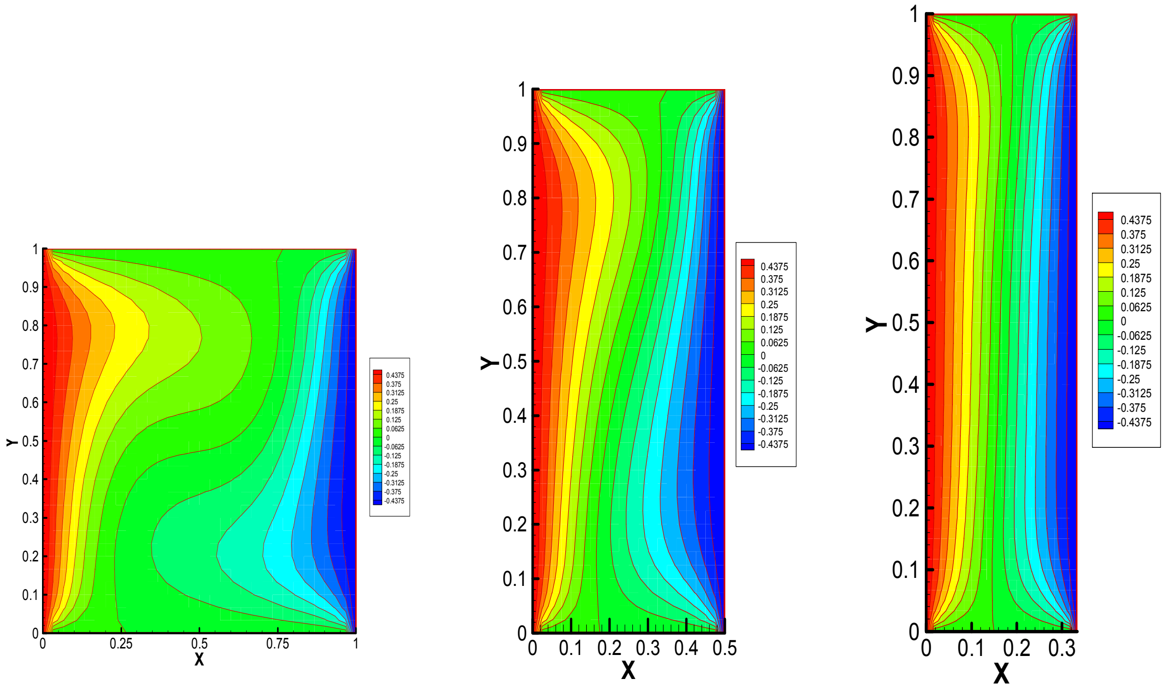
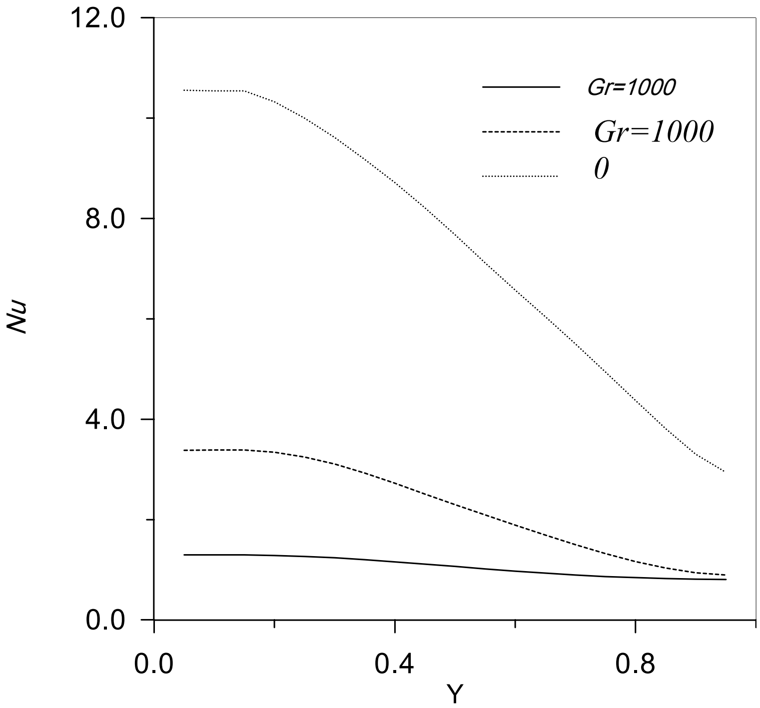


Fig. (5): Calculated flow field Gr = 10^4 and for different aspect ratio



a. A_r =1 **b. A_r =2** **c. A_r**

Fig. (6): Isotherm contours for different aspect ratios and Gr = 10^4



Fi g.(7):.calculated local Nusselt numbers on the hot wall

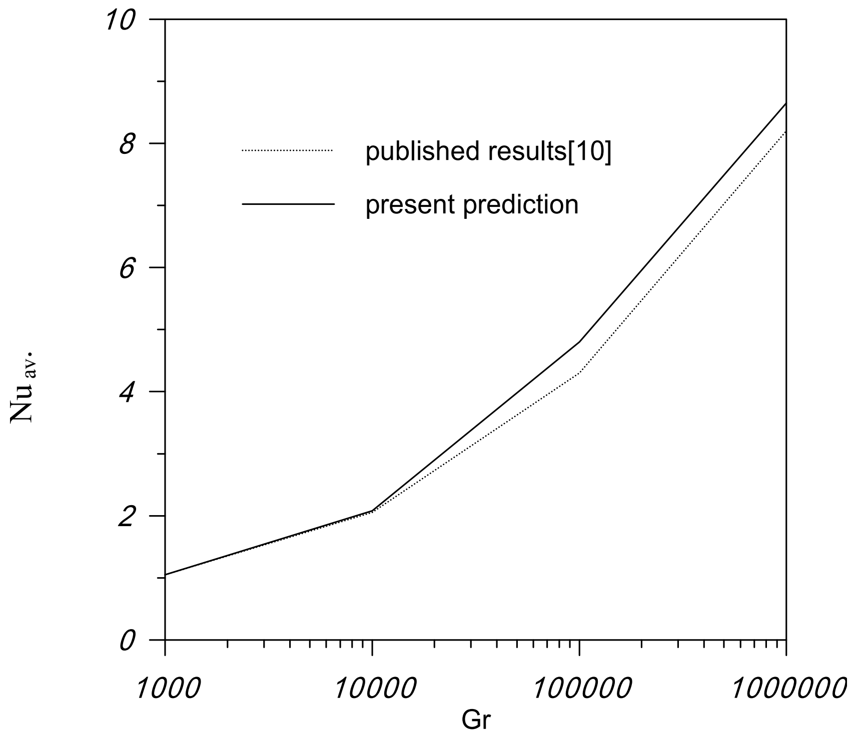


Fig.(8): comparison between published results and present study

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