EFFECT OF SOLENOID MAGNETIC FIELD DISTRIBUTION ON BEAM FOCUSING

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ABSTRACT
Beam transport system requires best control on charged particles beam passing through it that means good focusing for charged particles beam.
In present work we study the effect of solenoid magnetic field distribution on charged particles beam focusing using matrix representation method. Focusing strength strongly affected with magnetic field so the beam envelop. Best focusing obtained when magnetic field equal (0.45 kGauss).

INTRODUCTION
Particle optical systems are usually comprised of electric and magnetic bending elements, focusing elements, and high-order multipoles for correction of aberrations [1]. Each of quadrupole and solenoidal beam transport system are feasible, but we will focus on the latter one, trying to find the best formation to obtain the maximum magnetic field strength. To be able to describe the function of different devices along the beam path, general relations for the motion of charged particles in electromagnetic fields are required [2]. In applications, solenoids are working to limit or to focus. They are also used in plasma ion sources. Large-volume solenoid fields are generated by current- carrying coils (normal or superconducting). Permanent magnets have the advantages that do not require a current supply and cooling [3].
For several years short solenoid lens has been used as focusing elements in (electron, ion) microscopes and other beam devices, due to their ability to provide quality focusing. [4]. since the ion beam emerging from the ion source is divergent, electrostatic or magnetic lenses are required in order to keep the ion beam within the evacuated beam tube. Thus the lenses prevent the ion beam from hitting and being stopped by the beam tube walls, and help to bring as many of the ions from the ion source all the way to the experimental setup [6]. In a solenoid, the longitudinal magnetic field on the axis is peaked at the center of the solenoid, decreases toward the ends, and approaches zero far away from the solenoid. In contrast, the radial magnetic field is peaked near the ends of the solenoid [7]. In general optical elements are either magnetic or electrostatic devices depending upon the applications. In the case of high energy beams magnetic devices are used, for low energy beams, this is not an issue and other considerations come into play. Quadrupoles and bending magnets along with field free drift spaces are the main building blocks of a beam transport system. [8]. The method to study the dynamics of intense beam in the elliptical solenoid is to find out the transfer matrix.

CHARGED PARTICLE MOTION IN MAGNETIC FIELD
By the nature of the cross product, the magnetic force is vertically to the velocity of the particle. Lacking strength extension a differential element of path length, magnetic fields performance no work and do not change the kinetic energy of the particle. The force vertical on the
magnet field, that means no force extension of the z axis due to particles transmission in this trend with constant velocity. Force is of constant magnitude and it is vertically of the particle motion. The projection of particle motion in the x-y plane is subsequently a circle [5, 9].

SOLENOID MAGNETS

Solenoid magnets are often employed for focusing in low energy beam transport lattices in the front-end of a machine. Solenoid magnets are appealing due to their simple field structure in idealized form [10, 11]. Solenoids are widely used to transport or focus particle beams. Usually, they are assumed as being ideal solenoids with a high axial-symmetry magnetic field. Also, it used extensively for focusing and transporting the beams in modern accelerators [12, 13]. The solenoid lens is available magnetic lens geometry agree with cylindrical paraxial beams. Magnetic field is static, there is no change of particle energy passing through the lens. Therefore, it could lead to perform relativistic deriving without complex mathematics [9]. There are three optically important regions in a solenoid magnet. At the entrance and exit regions the magnetic field has a radial and axial component, except on the magnet axis [14]. The transfer matrix describe the beam transport in near optical elements facilitates the study of periodic focusing.

\[
B(\theta) = \frac{\mu_0 I N}{2L} \left[ \frac{x_2}{\sqrt{x_2^2 - r^2}} - \frac{x_1}{\sqrt{x_1^2 - r^2}} \right]
\]

\[
B = \mu_0 I N / (L^2 + 4r^2)^{3/2}
\]

The magnetic field increases linearly with current. To calculate magnetic field at any point on the axis, the law of Biot Savart, integrated over a circular current loop is used [2].

\[
B = \mu_0 r^2 / 2(r^2 + x^2)^{3/2}
\]

To a full understanding of the behavior of charged particles passed through solenoid, one could be divide the beam path to three regions each one can describe using matrix in term of focusing strength \( k \). Transfer matrix \((M)\) of the all solenoid is the product of three different matrices \(M_1, M_2\) and \(M_3\) conformity to the entrance fringe field, the constant axial magnetic field and the output fringe field respectively [16, 17].

\[
M = M_1 \cdot M_2 \cdot M_3
\]

\[
M_1 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & \theta / L & 0 \\
0 & 0 & 1 & 0 \\
-\theta & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
M_2 = \begin{bmatrix}
1 & \sin(\theta)(\theta / 2L) & 0 & (1 - \cos(\theta)(\theta / 2L)) \\
0 & \cos(\theta) & 0 & \sin(\theta) \\
0 & (1 - \cos(\theta)(\theta / 2L)) & 1 & \sin(\theta)(\theta / 2L) \\
-\sin(\theta) & 0 & 0 & \cos(\theta) \\
\end{bmatrix}
\]

\[
M_3 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & -\theta / L & 0 \\
0 & 0 & 1 & 0 \\
\theta & 0 & 0 & 1 \\
\end{bmatrix}
\]

Then the total transfer matrix \((M)\) becomes:
\[
M = \begin{bmatrix}
\cos(\theta)^2 & \cos(\theta)\sin(\theta) & \sin(\theta) \\
-(\sin(\theta)^2) & \cos(\theta) & \sin(\theta) \\
-(\sin(\theta)) & -(\sin(\theta)^2) & \cos(\theta) \\
-(\sin(\theta)^2) & -(\sin(\theta)) & \cos(\theta)
\end{bmatrix}
\]

Where \( \theta = L\sqrt{k} \) and \( k \) is the focusing strength which is \( k=B/(2B_{o}R_{o})^2 \).

RESULTS AND DISCUSSION

In present work the behavior of charged particles beam passing through solenoid with radius \( R=20 \) mm, \( L=200\) mm, \( N=500 \) and \( \mu_o=1.26\times10^{-6} \) H/m was studied. The current of solenoid change for range \((10-100) \) A, magnetic field for these current found to be from equation (2) as shown in table (1),which indicates the direct proportional of magnetic field as function of solenoid current. There is increasing of magnetic field by increasing solenoid current.

Table (1): The values of magnetic field for different values of solenoid current.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Magnetic Field (kGauss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.040</td>
</tr>
<tr>
<td>20</td>
<td>0.069</td>
</tr>
<tr>
<td>30</td>
<td>0.095</td>
</tr>
<tr>
<td>40</td>
<td>0.118</td>
</tr>
<tr>
<td>50</td>
<td>0.154</td>
</tr>
<tr>
<td>60</td>
<td>0.182</td>
</tr>
<tr>
<td>70</td>
<td>0.219</td>
</tr>
<tr>
<td>80</td>
<td>0.250</td>
</tr>
<tr>
<td>90</td>
<td>0.289</td>
</tr>
<tr>
<td>100</td>
<td>0.308</td>
</tr>
</tbody>
</table>

Also, we investigate the distribution of magnetic field out the solenoid. Magnetic field calculated for different distance out solenoid. At constant current, the magnetic field of solenoid decreased as we move away from the center of solenoid as shown in table (2).

Table (2): Magnetic field for different distance.

<table>
<thead>
<tr>
<th>Magnetic Field (B) (kGauss)</th>
<th>The Distance (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.330</td>
<td>0.020</td>
</tr>
<tr>
<td>0.989</td>
<td>0.030</td>
</tr>
<tr>
<td>0.670</td>
<td>0.040</td>
</tr>
<tr>
<td>0.413</td>
<td>0.050</td>
</tr>
<tr>
<td>0.259</td>
<td>0.060</td>
</tr>
<tr>
<td>0.112</td>
<td>0.070</td>
</tr>
</tbody>
</table>

Focusing strength \( k \), beam width \( X \) represented by the term \( m_{11} \) of the total transport matrix and beam magnification was calculated as indicates in the table (3). Which the behavior of focusing strength as function of magnetic field, there is increasing in the focusing strength of the beam with increasing magnetic field which allowed to more control on charged particles beam. Also shown in table the magnification of charged particles beam passing through the solenoid as function of magnetic field, one could be conclude that the best focusing occur at magnetic field equal to \( (0.45 \text{ kGauss}) \).
Table (3): Properties charged particles beam passing through solenoid.

<table>
<thead>
<tr>
<th>Magnetic Field B (kGauss)</th>
<th>Focusing Strength $k$ (mm$^{-2}$) $\times 10^{-6}$</th>
<th>Beam Width $X$ (mm)</th>
<th>Magnification $X_{out}/X_{in}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>0.439</td>
<td>19.824</td>
<td>3.9648</td>
</tr>
<tr>
<td>0.25</td>
<td>0.548</td>
<td>14.031</td>
<td>2.8062</td>
</tr>
<tr>
<td>0.30</td>
<td>0.656</td>
<td>9.247</td>
<td>1.8494</td>
</tr>
<tr>
<td>0.35</td>
<td>0.768</td>
<td>7.110</td>
<td>1.422</td>
</tr>
<tr>
<td>0.40</td>
<td>0.877</td>
<td>5.733</td>
<td>1.1466</td>
</tr>
<tr>
<td>0.45</td>
<td>0.987</td>
<td>5.618</td>
<td>1.1236</td>
</tr>
<tr>
<td>0.50</td>
<td>1.101</td>
<td>6.107</td>
<td>1.2214</td>
</tr>
<tr>
<td>0.55</td>
<td>1.210</td>
<td>6.899</td>
<td>1.3798</td>
</tr>
<tr>
<td>0.60</td>
<td>1.324</td>
<td>7.906</td>
<td>1.5812</td>
</tr>
<tr>
<td>0.65</td>
<td>1.432</td>
<td>8.738</td>
<td>1.7436</td>
</tr>
<tr>
<td>0.70</td>
<td>1.548</td>
<td>9.677</td>
<td>1.9354</td>
</tr>
<tr>
<td>0.75</td>
<td>1.643</td>
<td>10.313</td>
<td>2.0626</td>
</tr>
<tr>
<td>0.80</td>
<td>1.750</td>
<td>11.108</td>
<td>2.2216</td>
</tr>
<tr>
<td>0.85</td>
<td>1.862</td>
<td>11.866</td>
<td>2.3732</td>
</tr>
<tr>
<td>0.90</td>
<td>1.971</td>
<td>12.196</td>
<td>2.4392</td>
</tr>
<tr>
<td>0.95</td>
<td>2.083</td>
<td>12.863</td>
<td>2.5726</td>
</tr>
<tr>
<td>1.00</td>
<td>2.209</td>
<td>13.074</td>
<td>2.6148</td>
</tr>
</tbody>
</table>
CONCLUSIONS
Beam focusing strongly affected with solenoid magnetic field, best control on charged particles beam required good focusing of beam that means minimum size for beam. Here best focusing of charged particles beam passing through solenoid obtain at magnetic field equal to (0.45 kGauss).

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