Numerical Study Of The Effect Of Some Forming Parameters
In Hemispherical Punch Stretching

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ABSTRACT

In this work a finite element simulation is used for the hemispherical punch stretching test. The effect of strain hardening exponent, the original thickness of the sheet metal and the coefficient of friction between the punch and the blank on the formability of sheet metal are investigated. The results of this simulation are used to derive an empirical formula combining the simultaneous effect of the three parameters on the final thickness of the sheet. This formula is used to derive a numerical criterion which can be used in a separate finite element program for predicting the initiation and the position of localized necking. The effect of the three parameters on the level of the forming limit diagram is also investigated. An emphasis is put on the relation between the coefficient of friction and the strain path.

Keywords: Localized necking, empirical formula, strain path

دراسة عدديه لتأثير بعض عوامل التشكيك في عملية المط بواسطة الخرامة نصف الكروية
Introduction

The development of the finite element method for the simulation of sheet metal forming necessitates the introduction of a numerical criterion into the program as a control to determine when and where localized necking will take place. Many criteria have been developed in the past; for example, Chow et al [1] used a new theory of damage mechanics and Takuda et al[2] used a ductile fracture criterion. In this work a new criterion is developed based on the assumption that localized necking is formed at any point in the sheet when the final thickness $t_f$ at that point reaches a critical value and becomes unable to withstand the stresses applied on it.

The existing metal forming simulation finite element code LS_DYNA [3] was used. A different selected specimen size [4] is used for the hemispherical punch stretching test [5]. In this test a hemispherical punch of 25 mm in radius, a blank of 100*100 mm and a die corner radius of 5 mm are used.

In order to study the effect of the strain hardening exponent $n$, the original thickness $t_o$ and the coefficient of friction $\mu$ on the value of the final thickness, the values of the critical thickness $t_f$, the major $\varepsilon_1$ and minor $\varepsilon_2$ principal strains for the elements where localized necking is starting are all recorded for different values of $n$ by keeping $t_o$ and $\mu$. 

الخلاصة:

تستخدم في هذا البحث المحاكات بطريقة العناصر المحددة لعملية إزالة الصفائح بواسطة الخرامة نصف الكروية. يتم التحقق من تأثير كل من أس الأصلاد الإنفعالى و السمك الأصلى للصفيفة و معامل الاحتكاك بين الخرامة و الصفيفة على قابلية الصفيفة لتشكيل. استخدمت نتائج المحاكات لاشتقاق معادلة تطبيقية تجمع سوية تأثير هذه العوامل الثلاث على السمك النهائي للصفيفة. استخدمت هذه المعادلة لتطوير صيغة رياضية يمكن استخدامها في برامج عناصر محددة أخرى للكشف و تحديد موقع النقص الموضعى حال وقوعه. كذلك تم التحقق من تأثير هذه العوامل الثلاث على مستوى منحنى حد التشکيل. يتم التأكيد هنا على العلاقة بين معامل الاحتكاك و مسار الإنفعال.
constant first, and for different values of $t_o$ keeping $n$ and $\mu$ constant and finally for different values of $\mu$ keeping $n$ and $t_o$ constant.

It is known that increasing the values of $n$ raises the level of the forming limit curve (FLC). This was already proved experimentally by Keeler[6] Nie and Lee [7], and Charpontier [8]. The increase in $t_o$ also raises the level of the forming limit curve as was proved by Heyer [9], Haberfield [10], Keeler [6], Charpontier [8] and Hobbs [11].

Marciniak [12] suggested that increasing $\mu$ permits greater depth of forming. Up to now there is no quantitative relationship between the value of $\mu$ and the strain path of the deformed sheet.

**Finite Element Simulation**

Fig (1) shows schematics of tool set up geometry, punch, blank holder, die and tested specimen.

![Fig (1). Schematics of tool set up geometry](image)

Fig (2) shows the finite element model for the test which has been created using ANSYS (5.3) package [13]. Due to the two fold symmetry of the process, FE discretization is considered for one quarter of the specimen. The element used is the shell 4-node quadrilateral with five integration points through the thickness. A uniform mesh of 400 elements is used for the blank, 192 for the punch, 320 for the blank holder and 640 for the die. The analysis has been carried out with the punch velocity of 10 mm/ms. The blank holding force is 100 KN and concentrated at the center of the blank holder. The contact element is of surface to surface type and the sliding and impact algorithm along interface is by the penalty method.
Uniaxial tension tests were carried on a low carbon steel sheet metal. From these tests, the stress strain curve and the true stress-true strain curve on logarithmic coordinates were drawn in order to obtain the mechanical properties of the sheet metal. Table (1) shows the mechanical properties of the tested material. These properties are used in the FE calculations.

Table (1)

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness t (mm)</th>
<th>Density $\rho$</th>
<th>Modulus of elasticity</th>
<th>Poisson ratio</th>
<th>Strength coefficient $K$ (GP)</th>
<th>Strain hardening exponent</th>
</tr>
</thead>
</table>
RESULTS AND DISCUSSION

The effect of the three parameters \( n \), \( t_o \) and \( \mu \) on the ratio \( (t_f/t_o) \)

To generalize the study for sheets having different initial thicknesses, the dimensionless ratio \( t_f/t_o \) is considered instead of \( t_f \).

The effect of the strain hardening exponent, \( n \)

Fig (3) shows the relation between \( n \) and the ratio of the final thickness (at the instant of the formation of localized necking) to the original thickness \( t_f/t_o \) while keeping \( t_o \) and \( \mu \) constant. It is found that a decrease in \( t_f/t_o \) is about 22% for an increase in \( n \) equals to 100%. This result coincides completely with the results which had been obtained by Keeler [6], Nie and Lee [7], Charpontier [8].

<table>
<thead>
<tr>
<th></th>
<th>Kg/mm³</th>
<th>E (GPa)</th>
<th>( \nu )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild Steel</td>
<td>0.8</td>
<td>7.83-6</td>
<td>0.3</td>
<td>0.924</td>
</tr>
</tbody>
</table>
The effect of the initial thickness, $t_o$

Fig (4) shows the variation of the ratio of the final thickness (at the instant of the formation of localized necking) to the initial thickness $t_f / t_o$ with respect to $t_o$ while keeping $n$ and $\mu$ constant.

Better thinning will be obtained by starting the stretching with a thicker sheet. The decrease in the ratio $t_f / t_o$ is about 11 % for an increase in $t_o$ equal to 100 %. This result agrees strongly with the results which had been obtained by Heyer [9], Haberfield [10], Keeler [6], Charpontier [8] and Hobbs [11].

The effect of the coefficient of friction, $\mu$

Fig (5) shows the variation of $t_f / t_o$ with respect to $\mu$ while keeping $n$ and $t_o$ constant. The decrease in the ratio $t_f / t_o$ is about 15 % for a decrease in $\mu$ equal to 100 %.
By comparison between the effects of the three parameters on the ratio $t_f / t_o$ it can be deduced that the strain hardening exponent has the greatest effect on the final thickness of the sheet.

**Empirical Formula**

The simultaneous effect of the three parameters $n$, $t_o$ and $\mu$ on the ratio $t_f / t_o$ was found by using the statistical package SPSS assuming that

$$t_f / t_o \leq a_0 + a_1 t_o + a_2 n + a_3 n^2 + a_4 \mu$$

where $a_i$ are arbitrary constants

The results obtained are shown in table (2)
Table (2)

Statistical results

<table>
<thead>
<tr>
<th>Model</th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>Std. Error</td>
<td>Beta</td>
</tr>
<tr>
<td>Constant</td>
<td>1.0740</td>
<td>.012</td>
<td>-.335</td>
</tr>
<tr>
<td>t</td>
<td>.0901</td>
<td>.005</td>
<td>-.121</td>
</tr>
<tr>
<td>n</td>
<td>-2.6137</td>
<td>.118</td>
<td>-.482</td>
</tr>
<tr>
<td>n**2</td>
<td>3.059</td>
<td>.309</td>
<td>.491</td>
</tr>
<tr>
<td>μ</td>
<td>0.7182</td>
<td>.021</td>
<td></td>
</tr>
</tbody>
</table>

This shows that all the parameters are significant, and \( n \) has the highest influence as it has the biggest value of \( \text{Beta} \).

The best fitting is given then by the following formula:

\[
t_r / t_o \leq 1.07405 - 0.0901 t_o - 2.6137 n + 3.059 n^2 + 0.7182 \mu
\]

(1)

When the values of \( t_o, n \) and \( \mu \) are known this formula can be applied to determine the critical thickness at which localized necking will initiate.

A Criterion For Predicting Localized Necking

The strain through thickness is given by
\[ \varepsilon_3 = \ln \left( \frac{t_f}{t_o} \right) \]

and from the constancy of volume

\[ \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0 \]

thus localized necking will initiate at any element whenever:

\[ \varepsilon_1 + \varepsilon_2 \geq \ln \left( \frac{t_f}{t_o} \right) \]

(2)

Where \( \frac{t_f}{t_o} \) is calculated from equation (1).

This formula can be considered as a criterion for the indication of the initiation of localized necking based on a critical thickness ratio.

Equation (2) is very simple to imply and may be easily implemented in a separate FE program to predict localized necking.

The Effect Of The C.O.F. On The Strain Path

In stretching process as the punch is pushed into the sheet, tensile forces are generated through the sheet at its center. These are the forces that cause the deformation. The contact stress between the punch and the sheet is very much lower than the yield stress of the sheet so it does not effect the deformation.

In the case of no friction between the punch and the sheet, as in the bulge test, the tensile stresses are maximum at the pole and their values are:
\[ \sigma_1 = \sigma_2 = \frac{P \rho}{2t} \]

where

- **\( \sigma_1 \)**: the stress in the radial direction or the major stress, resulted from the radial tensile stress \( T_1 \)
- **\( \sigma_2 \)**: the stress in the circumferential direction or the minor stress resulted from the circumferential tensile stress \( T_2 \)
- **\( P \)**: is the pressure exerted by the punch
- **\( \rho \)**: is the radius of curvature at that point
- **\( t \)**: is the thickness of the sheet

The failure is expected at the center of the dome by tearing [12].

When friction is considered it will introduce a tangential contact force on the sheet in the radial direction towards its center. The pressure producing this force is a part of the punch pressure. As a result both the stresses \( \sigma_1 \) and \( \sigma_2 \) will be reduced thereby reducing \( \varepsilon_1 \) and \( \varepsilon_2 \). Figs.(6),(7) and (8) show the variation of \( \varepsilon_1 \) and \( \varepsilon_2 \) along a meridian at the instant of the initiation of localized necking for three different values of coefficient of friction.
Fig. (6). The variation of the major and minor strains along a meridian at localized necking when the c.o.f. = 0.075

Fig. (7). The variation of the major and minor strains along a meridian at localized necking when the c.o.f. = 0.175
Fig. (8). The variation of the major and the minor strains along a meridian at localized necking when the c.o.f. = 0.25

The effect of friction is to stick the punch with the sheet to a certain degree depending on the value of the c.o.f. between the two. This reduces relatively the values of $\varepsilon_1$ and $\varepsilon_2$ in the contacted part of the sheet and transfers the effect to the outer ring of contact at that instant. At the ring the effect of the frictional force is added to the tensile force $T_1$. As a result $\varepsilon_1$ is enhanced while $\varepsilon_2$ is still constrained because of friction [In the case of high c.o.f., as an element approaches contact with the punch, the rate of increase of $\varepsilon_2$ will lessen because of circumferential constraint by neighboring elements on the punch. $\varepsilon_1$ will not be so constrained. As an element makes contact with the punch, $d\varepsilon_2 / d\varepsilon_1 \rightarrow 0$ so that necking can occur].[15]

From fig. (6) it can be seen that at low c.o.f. both values of $\varepsilon_1$ and $\varepsilon_2$ are high and close to each other. For higher c.o.f., figures. (7) and (8), the two values are lower near the center. $\varepsilon_1$ increases gradually while $\varepsilon_2$ continues to be small along the meridian from the center to the outer ring of contact. The result is higher difference between $\varepsilon_1$ and $\varepsilon_2$. This difference increases with the increase of the c.o.f. and vice versa. This explains the variation of the strain path by changing the c.o.f.[The strain
path in all regions from uniaxial strain state to balanced biaxial state in
the forming limit diagram (FLD) can be changed by improving the
lubrication, but up to now there is no quantitative understanding on the
corresponding relationship between lubrication and the strain path ]
[16], [small variation in lubrication can affect the path of deformation]
[17]

By more movement of the punch a new ring starts to contact the
punch. $\varepsilon_1$ further increases while $\varepsilon_2$ is always constrained. An instant is
reached when $\varepsilon_3$ reaches the critical value indicated by equation (1). As
$\sigma_1$ is larger than $\sigma_2$ so its effect is overwhelming and a state of localized
uniaxial tension takes place at the ring of contact. As a result $\varepsilon_1$ increases
at the expense of the thickness while $\varepsilon_2$ remains constant ($\frac{d\varepsilon_2}{d\varepsilon_1} \to 0$
). The rate of the reduction in cross-sectional area is higher than the rate of
strain hardening. The remaining area is not able to withstand $\sigma_1$. Failure
takes place by splitting in a circle around the ring perpendicular to $\sigma_1$. It
worth's mentioning here that ($\frac{d\varepsilon_2}{d\varepsilon_1} \to 0$) is a result of the initiation of
failure and not its cause.

The history of the elements to be necked is followed at three different
values of coefficient of friction (0.05, 0.15, 0.30). The values of $\varepsilon_1$ and
$\varepsilon_2$ at each stage of the movement of the punch are recorded and then
plotted on a forming limit diagram to indicate the strain path in each case.
Fig.(9) shows three distinct strain paths for the three different values of
the c.o.f.
This result agrees with the experimental result, fig (10), obtained by Graf and Hosford [18] which had been presented for another purpose but it shows clearly the change of strain path for two metals after using lubricants.
two lubrication conditions. From Graf and Hosford[12]

It is clear that reducing the c.o.f. changes the strain path from the region of plain strain state towards the region of equibiaxial strain state. This is the same effect which can be obtained by increasing the width of the test specimen in the experimental procedure for determining the FLC. Increasing the c.o.f. or reducing the width of the specimen produces a reversed effect.

**Other Effects Of The C.O.F.**

From the previous discussion it can be noticed that in addition to changing the strain path, reducing the c.o.f. moves the position where necking is taking place towards the center of the deformed cup as shown in fig.(11).

![Graph showing the effect of the c.o.f. on necking position](image)

Fig.(11). The effect of the c.o.f. on necking position

In the same time, it will result in more thinning and the final thickness of the sheet will be more homogenous, fig.(12)
Fig. (12). The variation of the reduction in thickness with the distance from the center for different values of the c.o.f.

The Effect Of $N$, $T_0$ And $M$ On The Level Of The FLC

The Effect Of The Strain Hardening Exponent, $N$

Fig (13) shows the relation between $n$ and the major strain $\varepsilon_1$ at plane strain ($\varepsilon_2 = 0$) or the level of the forming limit diagram, FLD. It is clear that $n$ has a considerable effect on the level of the FLD. Increasing the value of $n$ by 100% will raise the FLC level by 100%. This agrees with the fact that the major strain at plane strain equals to the strain hardening exponent [19].
Fig (13). The relation between $n$ and $\varepsilon_1$

**The Effect Of Initial Thickness $T_o$**

Fig (14) shows the relation between the value of $t_o$ and the value of major strain $\varepsilon_1$ at plain strain ($\varepsilon_2 = 0$). Increasing $t_o$ by 100% will raise the FLC level by 20%.

Fig(14). The relation between $t_o$ and $\varepsilon_1$ at plain strain

**The Effect Of $\mu$**

According to what was proved earlier any decrease in the value of $\mu$ will change the strain path towards the equibiaxial strain state region. This means that it is impossible to vary $\mu$ and still being in the plain strain state. Changing the strain path towards the equibiaxial
strain state increases $\varepsilon_1$ and $\varepsilon_2$ as shown in fig(9). This means that the formability of the sheet is increased. It can be proved that the points whose coordinates are the values of $\varepsilon_1$ and $\varepsilon_2$ at localized necking for different values of $\mu$ fall always on the FLC. So the level of the FLC is not affected by the variation of $\mu$. This coincides with the fact that $\mu$ is not an intrinsic property of the material.

Therefore both $n$ and $t_o$ raise the level of the FLC and $n$ has the greatest effect. Reducing the c.o.f. does not have an effect on the level of the FLC but indirectly increases the level of the forming limit diagram.

**Concluding Remarks**

The finite element simulation LS-DYNA was used to study the effect of varying $n$, $t_o$, and $\mu$ (between the punch and the blank) on the formability of sheet metal in the hemispherical punch test. An empirical formula was deduced which can give the combined effect of the three factors $n$, $t_o$, and $\mu$ on the ratio $t_f/t_o$ simultaneously. This formula can be used to calculate the critical thickness at which localized necking occurs for any sheet whenever $n$, $t_o$, and $\mu$ are known.

A numerical criterion for predicting the initiation and place of localized necking based on the critical final thickness is also developed. This criterion can be easily implemented in a separate FE program.

It was found that increasing $n$ or $t_o$ raises the level of the FLC and reduces the final thickness. $n$ has the greatest effect. Changing the value of $\mu$ will not alter the level of the FLC, but results in thinner sections and better thickness homogeneity, and above all, it changes the strain path. Decreasing the value of $\mu$ changes the strain path from the region of plain strain towards the region of equibiaxial strain state resulting in better formability and vice versa.

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