New types of contra-precontinuous functions

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Abstract:
in this work, we introduce new types of contra-precontinuous functions by using preopen sets, also we study some properties of these types and shows relationships between these types.

Keywords:- ((preopen set, contra- continuous function, contra-pre continuous function))

1-Introduction:
The contra-continuous function has been studied first one in 1996 by J. Dontchev [1]. After that other searchers present some modifying of this function like, S. Jafari and T. Noiri in 2002, introduced and studied some class of contra-continuous function which is contra-precontinuous functions [5], and in 2005 another class of contra-continuous function is said to be contra- semi-continuous have been studied by Veera Kumar, [6].

In this work, we given some new types of contra-precontinuous functions. This types derivative from contra-precontinuous function exist in [5],and we study the composition of these types as well as introduce the preservation theorems by using these types.

2- Some Basic Concepts:
Here, we give some basic concepts of this work and we will needed in other sections.

Definition (2.1),[3]:-
Let \((X,T)\) be a topological space, a subset \(A\) of \(X\) is said to be preopen set if satisfy \(A \subseteq \text{int cl}A\).

From above definition it is easy to check that, every open set is preopen but the converse is not true in general. To illustrate that consider the irrational numbers in
usual topological space. The following corollary given the condition to make the converse is true

**Definition (2.2), [6]:**
A topological space \((X, \tau)\) is said to be a sub maximal if every preopen subset of \(X\) then \(A\) is open set.

Now, the following remarks give some properties of preopen sets.

**Remarks (2.3), [3]:**
1. The complement of preopen set is preclosed set.
2. The union of any family of preopen sets is preopen.
3. The intersection of two preopen sets is not necessary to be preopen set.

Next, we give the following definition of contra-continuous function, this definition appear first one in by Dontchev J., [1].

**Definition (2.4), [1]:**
A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be contra-continuous if \(f^{-1}(U)\) is closed set in \(X\), whenever \(U\) is open in \(Y\).

**Definitions (2.5), [6], [4]:**
1. A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be precontinuous if each open (closed) set \(U\) in \(Y\), \(f(U)\) is preopen (preclosed) set in \(X\).
2. A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be \(P^*\)-continuous if each preopen (preclosed) set \(U\) in \(Y\), \(f(U)\) is open (closed) set in \(X\).
3. A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be \(P^{**}\)-continuous if each preopen (preclosed) set \(U\) in \(Y\), \(f(U)\) is preopen (preclosed) set in \(X\).

**Definitions (2.6), [5], [4]:**
1. A topological space \((X, \tau)\) is said to be precompact space, if for every preopen (preclosed) cover of \(X\) has finite sub cover.
2. A topological space \((X, \tau)\) is said to be preconnected space, if \(X\) is not the union of two disjoint preopen (preclosed) sets.

**3- Certain types of Contra-precontinuous Functions:**
Now, we introduce some new class of contra-precontinuous functions with some properties of these types. Moreover shows the relations between these types and we start by the following definition.
**Definition (3.1), [5]:**

A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be contra-precontinuous if each open (closed) set \( U \) in \( Y \), \( f(U) \) is preclosed (preopen) set in \( X \).

It is clear that, every contra-continuous is contra-precontinuous functions, but the converse may to be not true. To illustrate that consider the following example.

**Example (3.2):**

Let \( X = \{a, b, c\} \)  
\( T = \{X, \phi\} \) is a topology define on \( X \) and \( \sigma = \{X, \phi, \{a\}\} \) be a topology define on \( X \).  
A function \( f : (X, T) \rightarrow (X, \sigma) \) defined by \( f(x) = x \) is contra-precontinuous but not contra-continuous function.

Next, the following proposition given the condition to make the converse is true

**Proposition (3.3):**

Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) is contra-precontinuous. if \( X \) is sub maximal space then \( f \) is contra-continuous function.

Now, the following definition introduces other type of contra-precontinuous function which is strong contra-precontinuous function

**Definition (3.4):**

A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be strong contra-precontinuous if for each preopen (preclosed) set \( U \) in \( Y \), \( f(U) \) is closed (open) set in \( X \).

The following proposition shows the relation between strongly contra-precontinuous function and contra-precontinuous function.

**Theorem (3.5):**

If \( f : (X, \tau) \rightarrow (Y, \sigma) \) is strongly contra-precontinuous function, then \( f \) is contra-precontinuous function.

**Proof:**

Let \( f : X \rightarrow Y \) be a strongly contra--precontinuous function and \( U \) be open set in \( Y \) thus \( U \) is preopen and by using definition (3.4) we get \( f^{-1}(U) \) closed in \( X \), therefore \( f \) is contra-continuous function.

The converse of above theorem is not true in general. See the following example
Example (3.6):-

Let $X = R$, $T_u$ be a usual topology define on $X$ and $\sigma = \{R, \phi\}$ be a topology define on $X$.
A function $f : (X, T) \rightarrow (X, \sigma)$ defined by $f(x) = x$ is contra-continuous but not strongly contra-precontinuous function.

Next, the following proposition given the condition to make the converse of theorem ( ) is true the proof of it is easy, thus it is omitted.

Proposition (3.7):-

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ contra-precontinuous function. If $Y$ is sub maximal space then $f$ is strongly contra-precontinuous function.

Now, we introduce new type of contra-precontinuous is said to be irresolute contra-precontinuous by the following definition.

Definition (3.8):-

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be irresolute contra-precontinuous if for each preopen (preclosed) set $U$ in $Y$, $f(U)$ is preclosed (preopen) set in $X$.

Next, the following remarks give the relation between irresolute contra-precontinuous, strong contra-precontinuous and contra-precontinuous functions.

Remarks (3.9):-

(1) Every strong contra-precontinuous function is irresolute contra-precontinuous function.
(2) Every irresolute contra-precontinuous function is contra-precontinuous function.

4- The Composition of Certain Types of Contra-precontinuous Functions:

In this section, we give the composition of certain types of contra-precontinuous functions have been studied. And we start by the following theorem.

Theorem (4.1):-

Let $X, Y$ and $Z$ be a topological spaces and $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be a functions.
(1) If \( f \) is contra-continuous and \( g \) is contra-continuous then \( g \circ f \) is continuous function.

(2) If \( f \) is contra-continuous and \( g \) is contra-precontinuous. If \( Y \) submaximalspace then \( g \circ f \) is continuous function.

(3) If \( f \) is contra-continuous and \( g \) is strongly contra-precontinuous. then \( g \circ f \) is \( P^* \)-continuous function

(4) If \( f \) is contra-continuous and \( g \) is irresolute contra-precontinuous. If \( Y \) then \( g \circ f \) is continuous function

**Proof:-**

(1) Let \( U \) be a open set in \( Z \) and since \( g \) is contra-continuous, thus so, \( g^{-1}(U) \) is closed in \( Y \) also, since \( f \) is contra-continuous then by using definition (2.4), we get \( f^{-1}(g^{-1}(U)) \) is open set in \( X \), thus \( g \circ f \) is a continuous function.

(2) Let \( U \) be open set in \( Z \) and since \( g \) is contra-continuous, thus so, \( g^{-1}(U) \) is preclosed in \( Y \) also, since \( Y \) is sub maximal space so, \( g^{-1}(U) \) is closed set , but \( f \) is contra-continuous by using definition (2.4), we get \( f^{-1}(g^{-1}(U)) \) is open set in \( X \), then \( g \circ f \) is \( P^* \)-continuous function.

(3) Let \( U \) be preopen set in \( Z \) and since \( g \) is strongly contra-continuous, thus so, \( g^{-1}(U) \) is closed in \( Y \) also, since \( f \) is contra-continuous by using definition (2.4), we get \( f^{-1}(g^{-1}(U)) \) is open set in \( X \), then \( g \circ f \) is \( P^* \)-continuous function.

(4) Let \( U \) be preopen set in \( Z \) and since \( g \) is irresolute contra-continuous, so \( g^{-1}(U) \) is preclosed in \( Y \) also, since \( Y \) is sub maximal space so, \( g^{-1}(U) \) is closed set , but \( f \) is contra-continuous by using definition (2.4), we get \( f^{-1}(g^{-1}(U)) \) is open set in \( X \), then \( g \circ f \) is \( P^* \)-continuous function.

Now, the following theorem give the composition when \( f \) is contra-precontinuous and \( g \) be any other types.

**Theorem (4.2):**

Let \( X, Y \) and \( Z \) be a topological spaces and \( f : X \longrightarrow Y \) is contra-precontinuous then

(1) If \( g : Y \longrightarrow Z \) is contra-continuous then \( g \circ f \) is precontinuous function.

(2) If \( g : Y \longrightarrow Z \) is contra-precontinuous and \( Y \) is sub maximal space then \( g \circ f \) is precontinuous function.

(3) If \( g : Y \longrightarrow Z \) is strongly contra-precontinuous then \( g \circ f \) is \( P^* \)-continuous function

(4) If \( g : Y \longrightarrow Z \) is irresolute contra-pre continuous and \( Y \) is sub maximal space then \( g \circ f \) is \( P^* \)-continuous function
Proof:-

(1) Let $U$ be a open set in $Z$ and since $g$ is contra-continuous, thus so, $g^{-1}(U)$ is closed in $Y$ also, since $f$ is contra-precontinuous then, we get $f^{-1}(g^{-1}(U))$ is preopen set in $X$, thus $gof$ is a precontinuous function.

(2) Let $U$ be open set in $Z$ and since $g$ is contra- continuous, thus so, $g^{-1}(U)$ is preclosed in $Y$ also, since $Y$ is sub maximal space so, $g^{-1}(U)$ is closed set, but $f$ is contra-precontinuous by using definition (3.1), we get $f^{-1}(g^{-1}(U))$ is preopen set in $X$, then $gof$ is a precontinuous function.

(3) Let $U$ be preopen set in $Z$ and since $g$ is strongly contra- continuous, thus so, $g^{-1}(U)$ is closed in $Y$ also, since $f$ is contra-precontinuous by using definition (3.1), we get $f^{-1}(g^{-1}(U))$ is pre open set in $X$, then $gof$ is a $P^*$ - continuous function.

(4) Let $U$ be preopen set in $Z$ and since $g$ is irresolute contra- continuous, so $g^{-1}(U)$ is preclosed in $Y$ also, since $Y$ is sub maximal space so, $g^{-1}(U)$ is closed set, but $f$ is contra-precontinuous by using definition (3.1), we get $f^{-1}(g^{-1}(U))$ is preopen set in $X$, then $gof$ is a $P^*$ - continuous function.

Next, the following theorem give the composition when $f$ is strongly contra-precontinuous and $g$ be any other types.

Theorem (4.3):-

Let $X,Y$ and $Z$ be a topological spaces and $f : X \rightarrow Y$ is strongly contra-precontinuous function then

(1) If $g : Y \rightarrow Z$ is contra-continuous then $gof$ is continuous function.

(2) If $g : Y \rightarrow Z$ is contra-precontinuous then $gof$ is continuous function.

(3) If $g : Y \rightarrow Z$ is strongly contra-precontinuous then $gof$ is $P^*$ - continuous function.

(4) If $g : Y \rightarrow Z$ is irresolute contra-pre continuous then $gof$ is $P^{**}$ - continuous function.

Proof:-

(1) Let $U$ be open set in $Z$ and since $g$ is contra-continuous, thus so, $g^{-1}(U)$ is closed, thus $g^{-1}(U)$ is preclosed in $Y$ also, since $f$ is strongly contra-precontinuous then, we get $f^{-1}(g^{-1}(U))$ is open set in $X$, thus $gof$ is a continuous function.

(2) Let $U$ be open set in $Z$ and since $g$ is contra-pre continuous, thus so, $g^{-1}(U)$ is preclosed in $Y$ also, since $f$ is strongly contra-precontinuous by using definition (3.4), we get $f^{-1}(g^{-1}(U))$ is open set in $X$, then $gof$ is a continuous function.

(3) Let $U$ be preopen set in $Z$ and since $g$ is strongly contra- continuous, thus so, $g^{-1}(U)$ is closed in $Y$, thus $g^{-1}(U)$ is preclosed also, since $f$ is strongly contra-
continuous by using definition (), we get $f^{-1}(g^{-1}(U))$ is open set in $X$, then $gof$ is a $P^*$–continuous function.

(4) Let $U$ be preopen set in $Z$ and since $g$ is irresolute contra-continuous, so $g^{-1}(U)$ is preclosed in $Y$ also, since $f$ is strongly contra-continuous by using definition (3.4), we get $f^{-1}(g^{-1}(U))$ is open set in $X$, then $gof$ is a $P^*$–continuous function.

Finally, then following theorem give the composition when $f$ is irresolute contra-precontinuous and $g$ be any other types.

**Theorem (4.4):**

Let $X, Y$ and $Z$ be a topological spaces and $f : X \longrightarrow Y$ is irresolute contra-precontinuous function then

1. If $g : Y \longrightarrow Z$ is contra-continuous and $Y$ is sub maximal space then $gof$ is $P^*$–continuous function.
2. If $g : Y \longrightarrow Z$ is contra-precontinuous and $Y$ sub maximal space then $gof$ is $P^{**}$–continuous function.
3. If $g : Y \longrightarrow Z$ is strongly contra-precontinuous then $gof$ is $P^*$–continuous function.
4. If $g : Y \longrightarrow Z$ is irresolute contra-pre continuous then $gof$ is $P^{**}$–continuous function.

**Proof:**

1. Let $U$ be open set in $Z$ and since $g$ is contra-continuous, thus so, $g^{-1}(U)$ is closed, thus $g^{-1}(U)$ is preclosed in $Y$ also, since $f$ is irresolute contra-precontinuous then, we get $f^{-1}(g^{-1}(U))$ is preopen set in $X$, thus $gof$ is a $P^*$–continuous function.

2. Let $U$ be open set in $Z$ and since $g$ is contra-pre continuous, thus so, $g^{-1}(U)$ is preclosed in $Y$ also, since $f$ is irresolute contra-precontinuous by using definition (3.9), we get $f^{-1}(g^{-1}(U))$ is preopen set in $X$, then $gof$ is a $P^{**}$–continuous function.

3. Let $U$ be preopen set in $Z$ and since $g$ is strongly contra-continuous, thus so, $g^{-1}(U)$ is closed in $Y$, thus $g^{-1}(U)$ is preclosed also, since $f$ is irresolute contra-continuous by using definition (), we get $f^{-1}(g^{-1}(U))$ is preopen set in $X$, then $gof$ is a $P^*$–continuous function.

4. Let $U$ be preopen set in $Z$ and since $g$ is irresolute contra-continuous, so $g^{-1}(U)$ is preclosed in $Y$ also, since $f$ is irresolute contra-continuous by using definition (3.9), we get $f^{-1}(g^{-1}(U))$ is open set in $X$, then $gof$ is a $P^{**}$–continuous function.
5- Preservation Theorems:

In this section, we use the functions have been studied in this search in order to study some of topological property between two topological spaces.

**Theorem (5.1):**

Let \( f : X \rightarrow Y \) be irresolute contra-precontinuous bijective function. If \( X \) is precompact space then \( Y \) is precompact.

**Proof:**

Let \( f : X \rightarrow Y \) be irresolute contra-precontinuous bijective function, and let \( \{A_n\}_{n=1}^{\infty} \) be any preopen cover of \( Y \), so \( f^{-1}(\{A_n\}_{n=1}^{\infty}) \) is preclosed cover of \( X \) and since \( X \) is precompact space then \( X \subseteq f^{-1}(A_1) \cup f^{-1}(A_2) \cup \ldots \cup f^{-1}(A_n) \). Thus \( Y \subseteq G_1 \cup G_2 \cup \ldots \cup G_n \) therefore; \( Y \) precompact space.

Now, the following theorem given anther property can be transformation by using irresolute contra-precontinuous function.

**Theorem (5.2):**

Let \( f : X \rightarrow Y \) be irresolute contra-precontinuous bijective function. If \( X \) is preconnected space then \( Y \) is preconnected space.

**Proof:**

Let \( f : X \rightarrow Y \) be irresolute contra-precontinuous bijective and let \( X \) be preconnected space and suppose \( Y \) is not preconnected space thus, there is nonempty disjoint preopen sets \( A_1 \) and \( A_2 \) such that \( Y = A_1 \cup A_2 \) also, since \( f \) is irresolute contra-precontinuous one-to-one and onto thus, \( f^{-1}(A_1) \) and \( f^{-1}(A_2) \) are disjoint preclosed sets such that, \( X = f^{-1}(A_1) \cup f^{-1}(A_2) \) then \( X \) is not preconnected space, which is contradiction. Therefore; \( Y \) is preconnected space.

**References:**