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Robust PID Tuning Rules for General Plant Model

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ABSTRACT

In this paper, a proposed PID tuning rules for general known or unknown plant models are presented. The design procedure used to obtain the tuning rules has been previously used in literature to obtain tuning rules, but for dedicated plant models. Our contribution in this article is that the proposed tuning rules are applicable for general plant models. These rules are designed to be robust for plant gain variations. The design procedure is based on some specification or constraints of frequency response, namely phase margin, gain crossover and robustness condition. The designed rules are given in terms of frequency response parameters of the plant model 'which can be found experimentally' instead of the plant model transfer function (T.F.) parameters. So, these rules do not need the model of controlled process to be known. Simulation study showed clearly the generality, ease of use, good performance and robustness of the obtained tuning rules. Simulation study has included comparison study with other known tuning rules.

Keywords:- PID Controller, Fractional PID controller, Crossover frequency, Phase Margin.

تنغيم صلب لمتحكمات كسب-تكامل-تفاضل لمنظومة عامة النموذج

الخلاصة

ان الهدف في هذا البحث هو تقديم معادلات تنغيم جديدة للمتحكم من نوع الكسب-تكامل-تفاضل (PID) لمنظومات عامة معلومة النموذج او مجهولته. الاجراء المستخدم في الحصول على معادلات التنغيم هو في الحقيقة مستخدم سابقا لهذا الغرض ولكن لمنظومات محددة النموذج وليس لمنظومات عامة. اذن مساهمتنا من خلال هذا البحث هو ان معادلات التنغيم قابلة للتطبيقلمنظومات عامة النموذج. معادلات التنغيم هذه صممت لتكون صلبة تجاه التغيرات في الكسب للمنظومة المتحكم بها. الطريقة المتبعة في التصميم اعتمدت على بعض الخصائص او القيود الموجودة في الاستجابة الترددية، وتحديدا هامش الزاوية والكسب عند تردد الانتقال وشرط الصلابة. معادلات التنغيم التي تم الحصول عليها كانت بدلالة بعض متغيرات الاستجابة

الترددية التي يمكن الحصول عليها تجريبياً بدلا من متغيرات الدالة الانتقالية للمنظومة. وبهذا، فان معادلات التنغيم المستحصلة لاتحتاج الى ان يكون نموذج المنظومة الهدف معلوماً. المحاكاة بينت بشكل واضح العمومية وسهولة الاستخدام والاداء والصلابة الجيدان لمعادلات التنغيم المستحصلة.المحاكاة شملت ايضا مقارنة اداء المتحكمات المصممة من خلال الطريقة المقترحة مع اداء متحكمات صممت باستخدام بعض الطرق المعروفة.

INTRODUCTION

For decades, PID controllers have been widely used in process industries and other control area. According to many articles, it can be said that more than 90% of controllers involve PID controller as a major controller or it is a part of the overall control system. Also, it can be confirmed 'due to surveys on current status in process control [1,2]', that PID controller still plays the same important role, and it is quite reasonable predicting this situation continuity. Structure simplicity and easy implementation are the main two reasons the success of PID controllers return to. Control design or tuning of PID controller means determining its parameters (k_c, T_i and T_d) to meet a given set of closed loop system performance requirements. Often, PID controllers are designed by considering: - 1- performance criteria 2- robustness issues.

In literature, there are many PID tuning methods like the well-known Ziegler-Nichols and Cohen-Coon tuning rules. These rules and also most other rules are dedicated for specified plant models, namely First Order Plus Dead Time (FOPDT) model.

Nearly, all tuning rules reported in literature are based on the approximate plant models derived from the step response of the plant [3]. FOPDT and Second Order Plus Dead Time (SOPDT) are the most commonly models used for this purpose. The popularity of the PID tuning rules comes from their ease to use and their applicability over a wide range of processes.

Recently, tuning methods based on optimization techniques with the aim of ensuring good stability and robustness have received attention in the literature [4-8]. Despite of their effectiveness, these methods rely on somewhat complex numerical optimization procedures and do not provide tuning rules. Instead, the tuning of PID controller is given as the solution of the optimization problem.

Finding simple PID controller tuning with significant performance improvement is still an important research issue and challenging task in process control engineering [9].

Our aim in this paper is to find high performance and robust tuning rules for general known or unknown plant model.

Beside the known classical PID controllers, another class of PIDs controllers have appeared recently. These are the Fractional Order PID (FOPID) controllers. In these controllers, the derivative and the integral parts have a real or fractional orders [10,11]. The design procedures developed for FOPID controllers have been mainly based on obtaining a set of equations in the controller parameters which can be then solved using some optimization techniques[11,12]. In [12], tuning strategies for FOPID controllers based on the time domain and frequency response have been studied and compared. In this paper, higher order plant model have been assumed but not a general and unknown plant model as it is the case for our work. This the key difference for our work with the mentioned paper and other works in this field.

Problem Formulation

Fig.1 shows classical feedback control system.

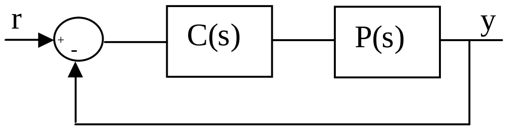

Figure (1) A classical feedback system.

The controlled process P is a single input single output (SISO) general process with known or unknown model. In general its transfer function (T.F.) can be put as:-

$$\mathbf{P(s)} = \frac{\mathbf{k_p N(s) e^{-ls}}}{\mathbf{D(s)}} \qquad \cdots \quad (1)$$

Where, N(s) and D(s) are polynomials in s of the order n and m respectively with n ≥ m, K_p is the process gain, and l the dead time. The controller C is a PID controller with the following T.F.:-

$$C(s) = K_c(1 + \frac{1}{T_i s} + T_d s \qquad \cdots. \quad (2)$$

Our aim is to find tuning rules in term of some frequency response parameters of the controlled process. These parameters are phase margin, crossover frequency and the derivative of phase of process at crossover frequency. All of these parameters can be obtained experimentally by plotting experimental frequency response for the plant. Then, we do not need to know the model of the process to be controlled.

Design Procedure

From Eq.2, the following can easily be obtained:-

$$|C(j\omega)| = \frac{k_c\sqrt{(T_i\omega)^2 + (1 - T_d T_i \omega^2)^2}}{T_i\omega} \qquad \cdots. \quad (3)$$

$$\angle C(j\omega) = tan^{-1}\left(\frac{T_i\omega}{(1 - T_d T_i \omega^2)}\right) - \frac{\pi}{2} \qquad \cdots \quad (4)$$

As mentioned earlier, the purpose of this article is to construct high performance and robust tuning rules. For this purpose, three design constraints have been chosen. These conditions ensure the performance of step response for closed loop system, and robustness against gain variation of plant.

The first condition considered is the phase margin constraint of the open loop system, i.e. at the gain crossover frequency, the open loop T.F. (C(jω).P(jω)) satisfies the following:-

$$\angle c(i\omega_c)P(j\omega_c) = \angle C(j\omega) + \angle P(j\omega) = -\pi + \varphi_m \qquad \cdots (5)$$

Where
φ_m is the desired phase margin of the closed loop and ω_c is the gain crossover frequency. Substituting Eq.4 into 5 and rearranging the resulting equation, the following can be obtained:-

$$\frac{Ti\omega_c}{1-T_dT_i\omega_c{}^2}+C_1=0 \qquad \cdots \quad (6)$$

Where:-

$$C_1=-\tan^{-1}[\varphi_m-\frac{\pi}{2}-\lfloor P(j\omega_c)]$$

The second design specification is the gain condition of the open loop at the gain crossover frequency 'ω_c', that is:-

$$|C(j\omega)P(j\omega)|=1 \qquad \cdots \quad (7)$$

From which, one can obtain the following equation:-

$$\frac{T_i{}^2\omega_c{}^2}{(T_i\omega_c)^2+(1-T_dT_i\omega_c{}^2)^2}-k_c{}^2C_2=0 \qquad \cdots. \quad (8)$$

Where,

$$C_2=(|P(j\omega)|)^2$$

The third design specification is the robustness condition against gain variations of the plant, which can be ensured by satisfying the desirable system property which is known as Iso-damping property, this feature is obtained if the open-loop phase Bode plot is flat at the gain crossover frequency ω_c, i.e., the phase derivative with respect to the frequency is zero at ω_c. If this feature is satisfied, then at ω_c the Nyquist curve of the open-loop system tangentially touches the sensitivity circle and the phase Bode is locally flat which implies that the system will be more robust to gain variations [13]. For systems that exhibit iso-damping property, the overshoots of the closed-loop step responses will remain almost constant for different values of the controller gain. This will ensure that the closed-loop system is robust to gain variations [13]. The following equation represent this condition mathematically [11-14]:-

$$\left.\frac{d\,(\arg[C(j\omega)P(j\omega)])}{d\omega}\right|=0 \qquad \cdots \quad (9)$$

The system which have this constraint satisfied, that the phase derivative with respect to the frequency is zero at ω_c, will ensure the robustness of the closed loop system to plant gain variations.
From Eq.9, the following can be obtained:-

$$\frac{T_i(T_iT_d\omega_c^2+1)}{(1-T_iT_d\omega_c^2)^2+T_i^2\omega_c^2}+C_3 = 0 \qquad \cdots \quad (10)$$

Where,

$$C_3 = \left.\frac{d\angle P(j\omega)}{d\omega}\right| \qquad —$$

The equations 6, 8 and 10 can be solved simultaneously for PID parameters k_c, T_i and T_d. The result of solving these three equations are the following:-

$$K_c = C_1\sqrt{\frac{1}{C_3(C_1^2+1)}} \qquad \cdots \quad (11)$$

$$T_i = -\frac{2C_1^2}{\omega_c(C_2\omega_cC_1^2-C_1+C_2\omega_c)} \qquad \cdots \quad (12)$$

$$T_d = -\frac{C_2\omega_cC_1^2+C_1+C_2\omega_c}{2C_1^2\omega_c} \qquad \cdots\cdot \quad (13)$$

Equations 11 to 13 represent tuning rules for general process which can be described by the general T.F. shown by Eq.1. These tuning rules are in terms of frequency response parameters.

Selecting φ_m and ω_c

To solve tuning equations, we need to select suitable values for phase margin and gain crossover frequency. Suitable initial guess for these two parameters can be obtained by getting Bode plot for controlled plant, then selecting desired phase margin then obtaining the phase value (phase=φ_m-180^o), then obtaining ω_c for this value of phase. Fig.2 shows example for certain controlled plant. Fine adjustment for the obtained values may be needed to obtain optimum tuning parameters.

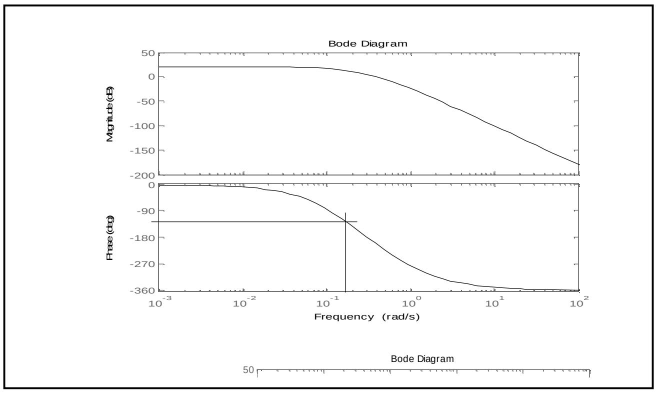

Figure (2) Bode plot for a certain plant model .

Simulation Study

The validity and performance of the obtained tuning rules are investigated by an extensive simulation study. Four different process models have been chosen to be controlled using PID controller tuned by the obtained tuning rules.
Comparison study with other tuning rules have been also made . The comparison study Included FOPDT model only because most designed tuning rules known in literature dedicated for this this model.

1- FOPDT:-

The first models used in our simulation study is the FOPDT model which is widely used to approximate high order process by first order model with delay time. The general T.F. for FOPDT is:-

$$p1(s) = \frac{K_p e^{-ls}}{Ts+1} \qquad \cdots \quad (14)$$

Where
K_p is the process gain, T is the time constant and l the dead time.
This model has been used with K_p=10, T=5 and l=0.5.
In order to investigate the performance of the obtained tuning rules, we have compared the results obtained using our tuning rules with results obtained using some other known tuning rules. These are the well-known Ziegler-Nichols method [15], Cohen-Coon method [16] and the optimal tuning rules proposed by Saeed-Mahdi Tavakoli [17]. We will refer to these methods by Z-N, C-C and S-M respectively and Gn for our method.
To select suitable φ_m and ω_c, bode plot have been obtained as shown in figure below. φ_m has been selected to be 70°, then from Fig.3 the phase at this point and the corresponding frequency are -110 and 0.9 rad/s.

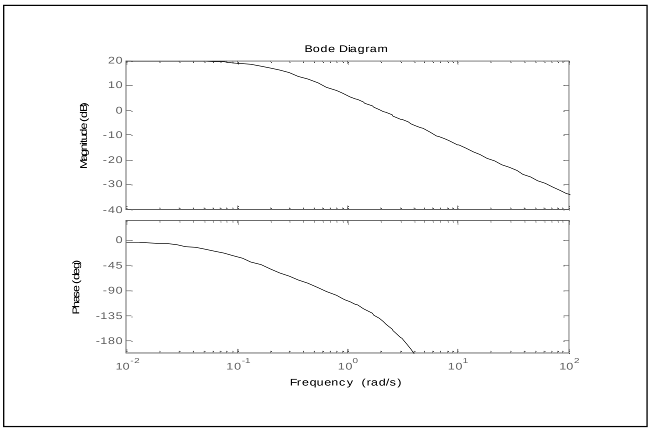

Figure (3) Bode plot for the model p1.

Using ω_c=0.7 and $\varphi_m=\pi/2.5$ C_1, C_2 and C_3 can be easily obtained. Now by applying Eqs.6, 8 and 10, PID parameters can be obtained:- K_c=0.4, T_i=2.6 and T_d=0.21.
Fig.4 shows the step responses for the FOPDT model using the four tuning rules.

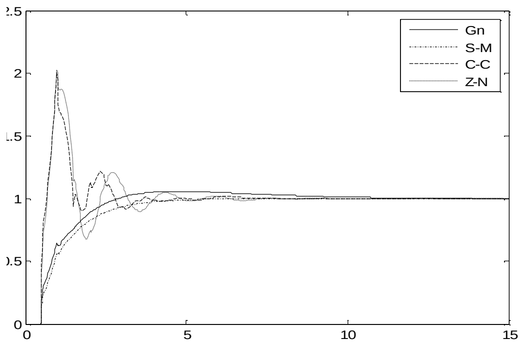

Figure (4) step responses for P1 for the four controllers of comparison study.

To show the robustness of the obtained tuning rules against process gain variations, we have simulated the system for three gain values, they are:- k_p=10 (the gain value the PID controller have been tuned for), 6 and 14.
Fig.5 shows the step response for the three cases.

2- **SOPDT:-** This model is also used to approximate high order processes. The general T.F. for this model is shown below:-

$$p2(s) = \frac{Kpe^{-ls}}{t_2 s^2 + t_1 s + 1} \quad \cdots (15)$$

The used model have been chosen with the following parameters:- K_p=10, t_1=0.5, t_2=3 and l=0.5.
Using similar procedure used previously to select ω_c and φ_m with fine adjustment, we chose ω_c=0.6 and $\varphi_m=\pi/2.4$.Now, applying Eqs.6, 8 and 10, PID parameters have been have been obtained for the model :- K_c=0.183, T_i=2.389, T_i=.4788.
As for FOPDT, the model have been simulated for three different gain (K_p) values:- 10 (nominal value), 6 and 14. Fig.6 shows the step response for the three cases.

3- In this part of simulation study, two general T.Fs. have been selected. Eq.15 and 16 describe these T.Fs.:-

$$p3(s) = \frac{10}{(0.2s+1)(0.3s+1)(0.5s+1)(0.7s+1)} \quad \cdots (15)$$

$$p4(s) = \frac{10(0.8s+1)(0.4s+1)}{(0.2s+1)(0.3s+1)(0.5s+1)(0.7s+1)} \qquad \cdots \quad (16)$$

For P3, we have selected ω_c=0.7 and $\varphi_m=\pi/2.3$ and for P4, ω_c=1.4 and $\varphi_m=\pi/2.3$. Using the presented tuning rules after determining the parameters C_1, C_2 and C_3, PID controllers for the two T.Fs. have been obtained:-

P3: k_c=0.0978, T_i=1.211, T_d=0.627.

P4: k_c =0.1711, T_i =0.9896, T_d =0 .2187

Figs.7 and 8 show the step response for the models with k (the gain of the processes)=10 (nominal value), 6 and 14.

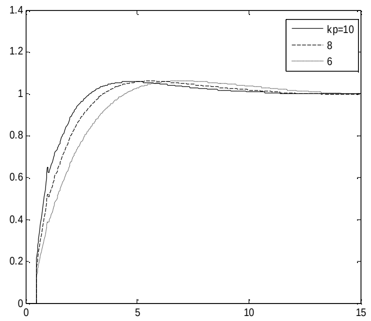

Figure.(5) step responses for P1 for three values of k_p.

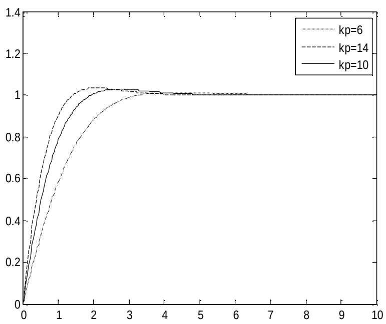

Figure.(6) step responses of P2 for three values of k_p.

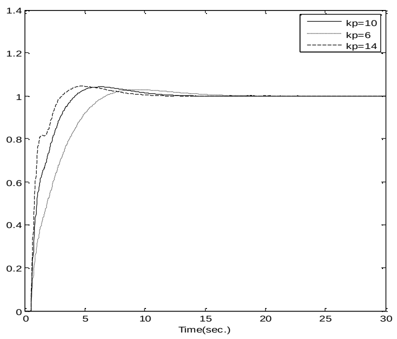

Figure.(7) step responses of P3 for three values of k_p.

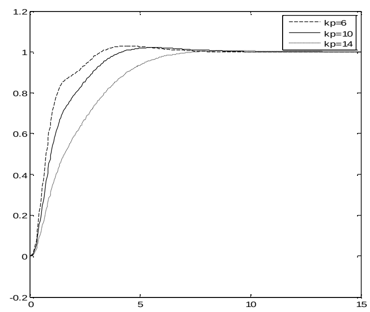

Figure.(8) step responses of P4 for three values of k_p.

CONCLUSIONS

In this work, a design procedure based on some frequency response specifications for open loop T.F. has been used to obtain tuning rules for general process model and not for limited or approximated models such FOPDT and SOPDT as it is the case for nearly all tuning rules presented in literature. The presented tuning rules are given in terms of some frequency response parameters instead of the parameters of the controlled process model. One of the advantages is that the process model for these tuning rules does not need to be known because of all of the parameters constructing the presented tuning rules can be obtained experimentally. An extensive simulation study has showed that the presented tuning rules give good results for the design specifications, where these rules have gave fast response without overshot for step changes and also they provide robustness for process gain variations.

Simulation study included comparing the presented tuning rules with some other rules which dedicated for FOPDT. This comparison showed clearly that our rules are superior for these rules, where these rules give fast response without overshot for step changes and also they provide robustness for process gain variations.

REFERENCES

[1] M. Kano, M. Ogawa, "The state of the art in chemical process control in Japan: good practice and questionnaire survey", Journal of Process Control, vol 20, (2010), PP. 969–982.

[2] L. Desborough, R. Miller, "Increasing customer value of industrial control performance monitoring—Honeywell's experience", Sixth International Conference on Chemical Process Control, AIChE Symposium Series Number 326, vol. 98, 2002.

[3]M. Ramasamya, S. Sundaramoorthy, "PID controller tuning for desired closed loop responses for SISO systems using impulse response",Computers and Chemical Engineering, Vol. 32, (2008), PP. 1773–1788.

[4] A. I. Ribica and M. R. Matausek, "A dead-time compensating PID controller structure and robust tuning", Journal of Process Control, vol. 22, 2012, PP. 1340– 1349.

[5] Sh. Zhang, Cyrus W. Taft, Joseph Bentsman, Aaron Hussey, Bryan Petrus, "Simultaneous gains tuning in boiler/turbine PID-based controller clusters using iterative feedback tuning methodology", ISA Transactions, vol. 51, 2012, PP. 609–621.

[6] John Q. Zhoua , David E. Claridge, "PI tuning and robustness analysis for air handler discharge air temperature control", Energy and Buildings, Vol. 44, 2012, PP. 1–6.

[7] M. Ge, M. Chiu, and Q. Wang, "Robust PID Controller design via LMI approach," Journal of Process Control, vol. 12, 2002, pp. 3 13.

[8] R. Toscano, "A simple PI/PID controller design method via numerical optimization approach," Journal of Process Control, vol. 15, 2005, pp. 81–88,

[9]M. Lee, M. Shamsuzzoha, Truong Nguyen Luan Vu, "IMC–PID Approach: An Effective Way to Get an Analytical Design of Robust PID Controller", International Conference on Control, Automation and Systems, COEX, Seoul, Korea, 2008 Oct., PP. 14-17.

[10] W. K. Ho, C. C. Hang and L. S. Cao, "Tuning of PI controllers based on gain and phase margin specifications", ELSEVIER, Automatica, vol. 31, issue 3, 1995, PP. 496-502.

[11] D. Wang and X. Gao "Analytical Tuning of PI^{λ} Controllers with Phase Margin and Robustness", IEEE the second international conference on intelligent control and information processing, 2011.

[12] S. Das, S. Saha, Sh. Das, A. Gupta,"On the Selection of Tuning Methodology of FOPID Controllers for the Control of Higher Order Processes", ISA Transactions, Vol 50, Iss 3, 2011, PP. 376-388.
[13] Chen, Y. Quan Moore, K. L., "Relay feedback tuning of robust PID controllers with iso-damping property", IEEE Transactions on Systems, Man, and Cybernetics, vol. **35, Iss.** 1,2005. PP. 23–31,
[14]De-Jin Wang and Xue-Li Gao, "Analytical Tuning of PI^{λ} Controllers with Phase Margin and Robustness", the 2nd international conference on intelligent control and information processing, vol. 2, 2011, PP. 595-598.
[15] J.G. Ziegler, N.B. Nichols, "Optimum settings for automatic controllers", ASME Transactions, vol. 64, 1942, PP. 759–768.
[16] G. H. Cohn and G. A. Coon, "Theoretical consideration of related control", Trans. *ASME*, vol. 75, 1953, PP. 827-834.
[17] S. Tavakoli and M. Tavakoli, "Optimal tuning of PID controller for first order plus time delay using dimensional analysis", The fourth international conference on control and automation (ICCA'03), 10-12 June 2003, Montreal, Canada.