

Some Results of Reverse θ - Centralizers on prime Rings

Ikram A. Saed
Department of Applied Sciences
Branch of Applied Mathematics
University of Technology

Abstract:

Let R be an associative ring with center $Z(R)$. In this paper we introduce the

definition of reverse θ - centralizers of R which is a generalization of reverse-

centralizers of R then we proved some results concerning reverse θ - centralizers

in prime and semiprime rings. Then we introduce the definition of double

reverse θ -centralizers which is a generalization of a double reverse- centralizers

after that we shall generalized some results of double reverse-centralizers to a

double reverse θ -centralizers.

Keywords:

Prime ring, semiprime ring, θ -centralizers, reverse-centralizers, reverse θ -

centralizers, double reverse θ -centralizers.

Introduction:

Throughout this paper, R will represent an associative ring with center $Z(R)$. This paper consists of three sections. In section one, we recall some basic definitions and other concepts which will be used in this paper. Also we shall give some necessary remarks and some examples that illustrate these concepts.

In section two, we give the definition of reverse θ -centralizer, and we proved

some results when R is prime or semiprime ring. In section three, we shall

introduce the definition of double reverse θ -centralizer, and we prove some

results when R is prime and semiprime ring.

§1 Basic concepts:

Definition 1.1: [1]

A ring R is called a prime ring if for any $a, b \in R$, $aRb = \{0\}$, implies that either $a=0$ or $b=0$.

Definition 1.2: [1]

A ring R is called a semiprime ring if for any $a \in R$, $aRa = \{0\}$, implies that $a=0$.

Remark 1.3: [1]

Every prime ring is a semiprime ring, but the converse in general is not true.

The following example justifies this remark.

Example 1.4: [6]

Z_6 is a semiprime ring but is not prime. Let $a \in R$ such that $aRa = \{0\}$, implies that $a^2 = 0$, hence $a=0$, therefore R is a semiprime ring. But R is not prime, since $2 \neq 0$ and $3 \neq 0$ and implies that $2R3 = \{0\}$.

Definition 1.5: [4]

Let R be an arbitrary ring. If there exists a positive integer n such that $na=0$, for all $a \in R$, then the smallest positive integer with this property is called characteristic of the ring, by symbols we write $\text{char } R=n$. If no such positive integer exists (that is, $n=0$ is only integer for which $na=0$, for all a in R), then R is said to be of characteristic zero.

Definition 1.6: [4]

A ring R is said to be n -torsion free where $n \neq 0$ is an integer if whenever $na=0$ with $a \in R$, then $a=0$.

Remark 1.7: [4]

A characteristic not equal n is equivalent to n -torsion free in a prime ring.

Definition 1.8: [2]

Let R be a ring. Define a Lie product $[.,.]$ on R as follows:

$$[x,y] = xy - yx, \text{ for all } x,y \in R.$$

Properties 1.9: [2]

Let R be a ring, then for all $x,y,z \in R$, we have:

1- $[x,yz] = y[x,z] + [x,y]z$

2- $[xy,z] = x[y,z] + [x,z]y$

3- $[x+y,z] = [x,z] + [y,z]$

4- $[x,y+z] = [x,y] + [x,z]$

Definition 1.10: [3]

Let R be a ring. Define a Jordan product on R as follows: $aob = ab + ba$, for all $a,b \in R$.

Definition 1.11: [2]

Let R be a ring, the center of R denoted by $Z(R)$ and is defined by:

$$Z(R) = \{x \in R / xr = rx, \text{ for all } r \in R\}$$

Definition 1.12: [2]

Let R be ring with center $Z(R)$. A mapping $f:R \rightarrow R$ is said to be centralizing if $[f(x), x] \in Z(R)$, For all $x \in R$, and f is said to be commuting if $[f(x), x] = 0$ for all $x \in R$.

Definition 1.13: [6]

Let R be ring, an additive mapping $f:R \rightarrow R$ is called a homomorphism if $f(xy) = f(x) f(y)$, for all $x,y \in R$. And is called anti-homomorphism if $f(xy) = f(y) f(x)$, for all $x,y \in R$.

Definition 1.14: [6]

Let R be ring, an additive mapping $f:R \rightarrow R$ is said to be Jordan homomorphism if $f(x^2) = f(x) f(x)$ holds for all $x \in R$.

Remark 1.15: [6]

Every homomorphism is a Jordan homomorphism. But the converse in general is not true.

The Following example illustrate this remark

Example 1.16: [6]

Let F be field, and let $M_2(F)$ be a ring of all matrices of order 2 over F .

Define $f:R \rightarrow R$ as $F \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$, for all $a,b,c,d \in F$.

Then f is a Jordan homomorphism but is not homomorphism.

Definition 1.17: [3]

A left (right) centralizer of a ring R is an additive mapping $T:R \rightarrow R$ which satisfies $T(xy) = T(x) y$ ($T(xy) = xT(y)$), for all $x,y \in R$. A centralizer of a ring R is both left and right centralizer.

Remark 1.18: [3]

Let R be a ring with an identity element, $T:R \rightarrow R$ is a left (right) centralizer if and only if T is of the form $T(x) = ax$ ($T(x) = xa$), for some fixed element $a \in R$.

Example 1.19: [1]

Let F be field, and let $D_2(F)$ be a ring of all diagonal matrices of order 2 over F .

Let $T:D_2(F) \rightarrow D_2(F)$ be an additive mapping defined as

$$T\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\right) = \begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix}, \text{ for all } a,b \in F.$$

Then T is a centralizer.

Definition 1.20: [5]

Let R be a ring and let $T, S: R \rightarrow R$ be additive mappings, then a pair (T, S) is called a double centralizer if T is a left centralizer, S is a right centralizer, and they satisfy a balanced condition $x T(y) = S(x)y$ for all $x, y \in R$.

Example 1.21: [5]

Define $T, S: M_2(F) \rightarrow M_2(F)$ by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} c & c \\ c & d \end{bmatrix}, \text{ for all } a, b, c, d \in F.$$

$$S\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} c & b \\ c & d \end{bmatrix}, \text{ for all } a, b, c, d \in F.$$

Then (T, S) is a double centralizer.

Remark 1.22: [5]

Let R be a ring and let $T: R \rightarrow R$ be centralizer, then it is clear that (T, T) is a double centralizer.

Definition 1.23: [4]

An additive mapping $T:R \rightarrow R$ is called a left (right) θ - centralizer if for all $x,y \in$

R , $T(xy) = T(x) \theta(y)$ ($T(xy) = \theta(x) T(y)$). A θ - centralizer of R is both left and

right θ - centralizer, where θ is a homomorphism on R .

Definition 1.24: [5]

Let R be a ring. An additive mapping $T:R \rightarrow R$ is said to be left (right) reverse-centralizer of R , if $T(xy) = T(y)x$ (resp. $T(xy) = yT(x)$), for all $x,y \in R$. A reverse- centralizer of R is both left and right reverse- centralizer.

Example 1.25: [5]

Let F be a Field, and R be ring of triangular matrices of the form.

$$x = \begin{bmatrix} a & c & b \\ 0 & 0 & c \\ 0 & 0 & a \\ 0 & 0 & 0 \end{bmatrix}, \text{ for all } a,b,c,d \in F, x \in R .$$

Define $T:R \rightarrow R$ by $T(x) = \begin{bmatrix} \theta\theta\theta c \\ \theta\theta\theta\theta \\ \theta\theta\theta\theta \\ \theta\theta\theta\theta \end{bmatrix}$, for all $c \in F$ and $x \in R$.

Then T is a reverse-centralizer.

Definition 1.26: [5]

Let R be a ring, and let $T, S :R \rightarrow R$ be additive mappings then a pair (T, S) is called a double reverse-centralizer, if T is a left reverse- centralizer, S is a right reverse-centralizer and they satisfy the condition $xT(y) = S(x)y$, for all $x, y \in R$.

Definition 1.27: [6]

A nonempty subset U of R is said to be a (two-sided) ideal of R if U is a subgroup of R under addition and for every $u \in U$ and $r \in R$, both ur and ru are in U .

§2 Reverse θ -centralizers:

Definition 2.1:

Let R be a ring, and θ be an anti-homomorphism on R . An additive mapping

$T:R \rightarrow R$ is called a left (right) reverse θ -centralizer if for all $x,y \in R$, $T(xy) =$

$T(y) \theta(x)$ ($T(xy) = \theta(y) T(x)$). A reverse θ -centralizer of R is both left and right

reverse θ -centralizer.

Example 2.2:

Let F be a field, and let $M_2(F)$ be a ring of all matrices of order 2 over F .

Define $T: M_2(F) \rightarrow M_2(F)$ as $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} c & a \\ d & b \end{bmatrix}$, for all $a,b,c,d \in F$.

and Define $\theta: M_2(F) \rightarrow M_2(F)$ as $\theta \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$, for all $a, b, c, d \in F$.

Then T is a reverse θ -centralizer.

Theorem 2.3:

Let R be a prime ring, I be an ideal of R , and T is a left reverse θ -centralizer, if

$T = \theta$ on I , then $T = \theta$ on R , where θ be a non-zero surjective anti-

homomorphism.

Proof:

By the hypothesis, we have

$$T(x) = \theta(x) \quad \text{for all } x \in I \quad (1)$$

Replacing x by xr in (1), when $r \in R$, $x \in I$, we get

$$T(xr) = \theta(xr)$$

$$T(r) \theta(x) = \theta(r) \theta(x)$$

$$(T(r)-\theta(r)) \theta(x) = 0 \text{ for all } r \in R, x \in I \quad (2)$$

Again, replace x by xt in (2), when $t \in R, x \in I$, to get

$$(T(r)-\theta(r)) \theta(xt) = 0 \text{ for all } t, r \in R, x \in I.$$

$$(T(r)-\theta(r)) \theta(t)\theta(x) = 0$$

By the surjectivity of θ and the primeness of R it follows that $T = \theta$.

Theorem 2.4:

Let R be a semiprime ring and let $T: R \rightarrow R$ be a mapping satisfying $T(y)\theta(x) =$

$\theta(y)T(x)$, for all $x,y \in R$, then T is reverse θ -centralizer where θ is a surjective

anti-homomorphism on R .

proof:

we need to show that T is an additive and $T(xy) = T(y)\theta(x) = \theta(y)T(x)$, for all

$x,y, z \in R$. So, let $x,y,z \in R$. Then

$$(T(x+y) - T(x) - T(y))\theta(z) = T(x+y)\theta(z) - T(x)\theta(z) - T(y)\theta(z)$$

$$= \theta(x+y)T(z) - \theta(x)T(z) - \theta(y)T(z)$$

$$= \theta(x) T(z) + \theta(y) T(z) - \theta(x) T(z) - \theta(y) T(z) = 0$$

By the surjectivity of θ and the semiprimeness of R , we get

$$T(x+y) = T(x) + T(y), \text{ for all } x, y, \in R.$$

To prove the second property, consider

$$(T(xy) - T(y)\theta(x))\theta(z) = T(xy)\theta(z) - T(y)\theta(x)\theta(z)$$

$$= \theta(xy)T(z) - \theta(y)T(x)\theta(z)$$

$$= \theta(y)\theta(x)T(z) - \theta(y)\theta(x)T(z) = 0$$

Again, by the surjectivity of θ and the semiprimeness of R , we get

$$T(xy) = T(y) \theta(x) = \theta(y) T(x), \text{ for all } x, y, \in R.$$

Theorem 2.5:

Let R be a 2-torsion free semiprime ring and T be a left reverse θ -centralizer

which satisfies $T(xoy) = 0$, for all $x, y, \in R$, where θ is a surjective anti-

homomorphism on R , then $T=0$

Proof:

we have

$$T(xy+yx) = 0, \text{ for all } x, y, \in R. \tag{1}$$

On the other hand, we obtain

$T(xy) + T(yx) = 0$, for all $x, y \in R$.

$$T(y) \theta(x) + T(x) \theta(y) = 0, \text{ for all } x, y \in R. \quad (2)$$

Replacing x by $xz+zx$ in (2), we find

$$T(y) \theta(xz+zx) + T(xz+zx) \theta(y) = 0 \quad (3)$$

From (1) and (3), we obtain

$$T(y) \theta(xz+zx) = 0, \text{ for all } x, y, z \in R. \quad (4)$$

Again replacing z by $xz+zx$, to get

$$T(y) \theta(x^2z+zx^2) + 2T(y) \theta(xzx) = 0, \text{ for all } x, y, z \in R. \quad (5)$$

From (4) and R is a 2-torsion free, gives

$$T(y) \theta (xzx) = 0, \text{ for all } x, y, z \in R. \quad (6)$$

Since θ is onto, replace $\theta(z)$ with $\theta(z) T(y)$, implies that

$$T(y) \theta (x) \theta(z) T(y) \theta(x) = 0, \text{ for all } x, y, z \in R.$$

By the surjectivity of θ and the semiprimeness of R , we get

$$T(y) \theta(x) = 0, \text{ for all } x, y \in R.$$

Again by the semiprimeness of R , we have $T=0$.

Theorem 2.6:

Let R be a prime ring, and let I be an ideal of R . If $T(xr) = T(r) \theta(x)$, for all $x \in$

R and $x \in I$, then T is a left reverse θ -centralizer on R , where θ is a non-zero

surjective anti-homomorphism on R .

Proof:

By the assumption of theorem, we have

$$T(xr) = T(r) \theta(x), \text{ for all } r \in R \text{ and } x \in I.$$

This reduces to

$$T(xsr) = T(sr) \theta(x) = T(r) \theta(s) \theta(x), \text{ for all } r, s \in R \text{ and } x \in I.$$

It follows that: $(T(sr) - T(r)\theta(s))\theta(x) = 0$, for all $r, s, x \in R$.

i.e. $(T(sr) - T(r)\theta(s))\theta(t)\theta(x) = 0$, for all $r, s, t, x \in R$.

By the surjectivity of θ and the primeness of R , we obtain $T(sr) = T(r)\theta(s)$, for

all $r, s \in R$.

Lemma 2.7:

Let R be a noncommutative prim ring, and $T: R \rightarrow R$ be a left reverse θ -

centralizer. If $T(x) \in Z(R)$ holds for all $x \in R$, then $T=0$, where θ is a surjective

anti- homomorphism on R .

Proof:

We have

$$[T(x), y] = 0, \text{ for all } x, y \in R \quad (1)$$

Putting zx for x in (1), we get $[T(zx), y] = 0$

$$[T(x) \theta(z), y] = 0, \text{ for all } x, y, z \in R \quad (2)$$

$$\text{This gives: } T(x) [\theta(z), y] + [T(x), y] \theta(z) = 0, \text{ for all } x, y, z \in R. \quad (3)$$

$$\text{From (1), we have: } T(x) [\theta(z), y] = 0, \text{ for all } x, y, z \in R \quad (4)$$

Putting wx for x in (4), we get

$$T(x) \theta(w) [\theta(z), y] = 0, \text{ for all } x, y, z, w \in R.$$

Since R is a noncommutative prime ring and by the surjectivity of θ , it follows

$T=0$.

Lemma 2.8:

Let R be a ring with an identity element, and $T: R \rightarrow R$ is a left (right) reverse θ –

centralizer if and only if T is of the form $T(x) = a\theta(x)$ ($T(x) = \theta(x) a$) for some

fixed element $a \in R$ where θ is surjective anti- homomorphism on R .

Proof:

Let T be a leftreverse θ –centralizer, then $T(yx) = T(x) \theta(y)$, for all

$x,y \in R$. Replace x by 1 we get $T(y) = a\theta(y)$, for all $y \in R$.

Where $a=T(1)$. If $T(yx) = \theta(x) T(y)$, we obtain the assertion of the lemma with

similar approach as above. To show the converse, assume

$T(x) = a\theta(x)$, for all $x \in R$, then

$T(yx) = a \theta(yx) = a \theta(x) \theta(y) = T(x) \theta(y)$, for all $x,y \in R$,

Hence T is a left reverse θ –centralizer.

Similarly, we can show that T is a right reverse θ –centralizer if $T(x) = \theta(x) a$,

for all $x \in R$.

Theorem 2.9:

Let R be a prime ring, and T is a left reverse θ –centralizer of R which satisfies

$T(x) = a \theta(x) + \theta(x) a$, for all $x \in R$ and fixed $a \in R$. Then $a \in Z(R)$ where θ is a

non-zero surjective anti- homomorphism on R .

Proof:

We have

$$T(yx) = T(x) \theta(y), \text{ for all } x, y \in R \quad (1)$$

i.e. $T(yx) = (a \theta(x) + \theta(x) a) \theta(y)$

On the other hand, we obtain

$$T(yx) = a \theta(yx) + \theta(yx) a, \text{ for all } x, y \in R \quad (2)$$

On combining last two equations, we obtain

$$\theta(x) [a, \theta(y)] = 0, \text{ for all } x, y \in R \quad (3)$$

Further taking $x=rx$ in (3), we get

$$\theta(x) \theta(r)[a, \theta(y)] = 0, \text{ for all } x, y \in R \quad (4)$$

By the surjectivity of θ and the primeness of R , yields $[a, \theta(y)] = 0$, for all $y \in R$.

§3 Double Reverse θ -centralizers:

Definition 3.1:

Let R be a ring, and let $T, S: R \rightarrow R$ be additive mappings then a pair (T, S) is

called a double reverse θ –centralizer, if T is left reverse θ –centralizer, S is a

right reverse θ –centralizer and they satisfy the condition $\theta(x) T(y) = S(x) \theta(y)$,

for all $x,y \in R$. where θ is an anti-homomorphism on R .

Example 3.2:

Let F be a field, and let $M_2(F)$ be a ring of all matrices of order 2 over F .

And θ on $M_2(F)$ defined by

$$\theta \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, \text{ for all } a,b,c,d \in F.$$

Let $T,S: M_2(F) \rightarrow M_2(F)$ be additive mappings defined as.

$$T \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} a & a \\ b & d \end{bmatrix}, \text{ for all } a,b,c,d \in F.$$

$$S \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} \theta & \theta \\ \theta & \theta \end{bmatrix}, \text{ for all } a,b,c,d \in F.$$

Then (T,S) is a double reverse θ –centralizer.

Theorem 3.3:

Let R be a semiprime ring and $T,S: R \rightarrow R$ be mappings satisfying

$$\theta(x) T(y) = S(x) \theta(y), \text{ for all } x,y \in R \dots(1). \text{ Where } \theta \text{ is an anti-homomorphism}$$

on R . Then (T,S) is a double reverse θ –centralizer.

Proof:

We need to show that T,S are additive mappings, and

$$T(xy) = T(y) \theta(x), \text{ for all } x,y \in R.$$

$$S(xy) = \theta(y) S(x), \text{ for all } x,y \in R.$$

Now replacing y by $y+z$ in (1), we get

$$\theta(x) T(y+z) = S(x) \theta(y+z), \text{ for all } x,y,z \in R.$$

$$\theta(x) T(y+z) = S(x) \theta(y) + S(x) \theta(z)$$

$$\theta(x) T(y+z) = \theta(x) T(y) + \theta(x) T(z), \text{ for all } x,y,z \in R. \quad (2)$$

$$\text{Hence } \theta(x) (T(y+z) - T(y) - T(z)) = 0, \text{ for all } x,y,z \in R. \quad (3)$$

By the semiprimeness of R , we get, $T(y+z) = T(y) + T(z)$, for all $y,z \in R$.

Similarly, we can show that

$$S(x+y) = S(x) + S(y), \text{ for all } x,y \in R.$$

Now, replacing y with yz in (1), we obtain

$$\theta(x) T(yz) = S(x) \theta(yz), \text{ for all } x,y,z \in R.$$

$$\theta(x) T(yz) = S(x) \theta(z) \theta(y), \text{ for all } x,y,z \in R.$$

$$\theta(x) T(yz) = \theta(x) T(z) \theta(y), \text{ for all } x,y,z \in R.$$

$$\text{Implies that: } \theta(x) (T(yz) - T(z) \theta(y)) = 0, \text{ for all } x,y,z \in R. \quad (4)$$

By the semiprimeness of R we get, $T(yz) = T(z) \theta(y)$, for all $y,z \in R$.

Similarly we can show that: $S(xy) = \theta(y) S(x)$, for all $x,y \in R$.

Then (T,S) is a double reverse θ –centralizer.

Theorem 3.4:

Let R be a prime ring, I be non-zero ideal of R. Let $T,S: R \rightarrow R$ be additive mappings such that T is a left reverse θ –centralizer, S is a right reverse θ –

centralizer. Where θ is an anti- homomorphism on R , and they satisfying $\theta(x)$

$T(y) = S(x) \theta(y)$, for all $x,y \in I$. Then (T,S) is a double reverse θ –centralizer.

Proof:

We have: $\theta(x) T(y) = S(x) \theta(y)$, for all $x,y \in I$. (1)

Replace x with rx in (1), when $x \in I$ and $r \in R$, we get

$\theta(x) (\theta(r) T(y) - S(r) \theta(y)) = 0$, for all $r \in R$ and $x,y \in I$. (2)

i.e. $\theta(x) R(\theta(r)T(y) - S(r) \theta(y)) = 0$, for all $r \in R$ and $x,y \in I$.

By the primeness of R and since I be a non-zero ideal of R , we get

$$\theta(r) T(y) = S(r) \theta(y), \text{ for all } r \in R \text{ and } y \in I. \quad (3)$$

Replace y with yt in (3), where $t \in R$ and $y \in I$, we get

$$(\theta(r) T(t) - S(r) \theta(t)) \theta(y) = 0, \text{ for all } t, r \in R \text{ and } y \in I.$$

Implies that: $(\theta(r) T(t) - S(r) \theta(t)) RI = 0$, for all $t, r \in R$.

By the primeness of R , we get.

$$\theta(r) T(t) = S(r) \theta(t), \text{ for all } t, r \in R.$$

Then (T,S) is a double reverse θ –centralizer.

Theorem 3.5:

Let R be a prime ring, I be a non-zero ideal of R and (T,S) be a double reverse

θ –centralizer. Where θ is an anti- homomorphism on R .

If $T=S$ on I , then $T=S$ on R

Proof:

We have: $T(x) = S(x)$, for all $x \in I$. (1)

Replacing x with xr in (1), when $r \in R$ and $x \in I$, we get, $T(xr) = S(xr)$

$T(r) \theta(x) = \theta(r) S(x) = \theta(r) T(x)$, for all $r \in R$ $x, \in I$. (2)

Since (T,S) is a double reverse θ –centralizer, then

$$T(r)\theta(x) = S(r)\theta(x), \text{ for all } r \in R, x \in I. \quad (3)$$

i.e. $(T(r) - S(r))RI = 0$, for all $r \in R$.

Since R is a prime ring and I be a non-zero ideal of R , we get $T=S$.

Theorem 3.6:

Let R be a semiprime ring, and (T,S) be a double reverse θ –centralizer. Where

θ is an anti- homomorphism on R . Then T (or S) is a reverse θ –centralizer if

and only if $T=S$.

Proof:

We have: $T(yx) = T(x) \theta(y) = \theta(x) T(y)$, for all $x,y \in R$.

On the other hand, $\theta(x) T(y) = S(x) \theta(y)$, for all $x,y \in R$.

This gives: $(T(x) - S(x)) \theta(y) (T(x) - S(x)) = 0$, for all $x,y \in R$.

By the semiprimeness of R , we get $T=S$.

Conversely, suppose $T(x) = S(x)$, for all $x \in R$.

i.e. $T(x) \theta(y) = \theta(x) S(y) = \theta(x) T(y)$, for all $x,y \in R$.

Implies that $T(xy) = T(x) \theta(y) = \theta(x) T(y)$, for all $x,y \in R$.

Theorem 3.7:

Let R be a prime ring, I be a non-zero ideal of R , and (T,S) be a double reverse

θ –centralizer. Where θ is an anti- homomorphism on R .

If $T(xr) = S(r) \theta(x)$, for all $r \in R, x \in I$, then $T=S$.

Proof:

We have: $T(xr) = T(r) \theta(x) = S(r) \theta(x)$, for all $r \in R, x \in I$. (1)

This reduces to: $(T(r)- S(r))\theta(x) = 0$, for all $r \in R, x \in I$. (2)

Replacing x with xt in (2), when $t \in R, x \in I$. Leads to

$$(T(r)- S(r)) \theta(xt) = 0, \text{ for all } r, t \in R, x \in I.$$

$$(T(r)- S(r)) \theta(t) \theta(x) = 0, \text{ for all } r, t \in R, x \in I.$$

i.e. $(T(r)- S(r)) RI = 0, \text{ for all } r \in R.$

since R is a prime ring and I be a non-zero ideal, we have $T=S$

References:

[1] E. Albas: On τ -centralizers of semiprime Rings, Siberian Math. J., 48 (2) (2007), 191-196.

[2] S. Ali and C. Haetinger: Jordan α -centralizers in Rings and some Applications, Bol. Soc. paran. Math., 26 (35) (2008), 71-80.

[3] M. N. Daif, M. S. Tammam El-Saiyad and N. M. Muthana: An Equation

Related to θ -centralizers in semiprime Rings, International Mathematical

Forum, 3 (24) (2008), 1164-1177.

[4] M. N. Daif, M. S. Tammam El-Saiyad and N. M. Muthana: An Identity on

θ -centralizers in semiprime Rings, International Mathematical Forum, 3 (19)

(2008), 937-944.

[5] F. A. Fadhl: Double centralizers on prime and semiprime Rings, Baghdad University (Iraq). Msc. Thesis. (2010).

[6] I. N. Herstein: Topics in Ring Theory, Univ. of Chicago press 1969.

بعض النتائج حول تمرکزات θ - العكسية

على الحلقات الاولية

اكرام احمد سعيد
قسم العلوم التطبيقية
فرع الرياضيات التطبيقية
الجامعة التكنولوجية

المستخلص:

لتكن R حلقة تجميعية مركزها $Z(R)$. في هذا البحث قدمنا تعريف تمرکزات θ - العكسية على الحلقة R والذي هو تعميم التمرکزات العكسية على الحلقة R . وبعد ذلك برهنا بعض النتائج المتعلقة بتمرکزات θ - العكسية في الحلقات الاولية وشبه الاولية. بعد ذلك قدمنا تعريف تمرکزات θ - العكسية المزدوجة والذي هو تعميم التمرکزات العكسية المزدوجة. وبعد ذلك سنقوم بتعميم بعض النتائج المتعلقة بالتمرکزات العكسية المزدوجة الى التمرکزات θ - العكسية المزدوجة.