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A comparison among methods for estimation of the parameter of the Maxwell- Boltzmann distribution using simulation

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Abstract

The Maxwell or Maxwell- Boltzmann distribution was invented to solve problems related to physics, chemistry and plays an important role in and other allied sciences. So in this paper Bayesian using special priorinformation for estimating the scale parameter of Maxwell distribution, the maximum likelihood estimation andthree different types of moments are presented for this. The simulation by matlab program is used to compare these estimators with respect to the Mean Square Error (MSE) and Mean Absolute Percentage Error (MAPE), the results of comparison showed that for all the varying sample size, the estimators of Bayes method with special prior distribution is followed by the Maximum likelihood estimatorhas smaller MSE and MAPE compared to others, and in all cases the statistical hypotheses had been satisfied for both methods the MSEand MAPE decrease as sample size increases.

الملخص

يعتبر توزيع Maxwell or Maxwell- Boltzmann من التوزيعات المهمة التي وضعت لحل المشاكل العلمية ضمن علوم الفيزياء والكيمياء وكذلك يلعب دورا مهما ضمن علوم تطبيقية اخرى لذلك فقد تم في هذا البحث استخدام طريقة بيز اعتمادا على معلومات سابقة خاصة و طريقة الامكان الاعظم وكذلك طريقة العزوم بثلاث حالات وباستخدام المحاكاة اعتمادا على برنامج ماتلاب تم تقدير المعلمةله ضمن كل طريقة وتمت المقارنة بين النتائج اعتمادا على Mean Square Error (MSE) وMean Absolute Percentage Error (MAPE) اظهرت النتائج ان افضل تقدير هو بيز وياتي بعده الامكان الاعظم ثم طريقة العزوم ولجميع حجوم العينة حيث حصلنا على اقل قيم للخطأ وتم استيفاء النظرية الاحصائية في هذه التقديرات حيث كان الخطأ يقل كلما ازداد حجم العينة

1- Introduction

The Maxwell distribution isa continuous probability distribution with application in physics and chemistry. The most frequentapplication is in the field of statistical mechanics.to determine the speeds of moleculesThe Maxwell distribution gives the distribution of the speeds of molecules as it is given by statistical mechanics in thermal equilibrium when the temperature is high enough under some conditions as defined in statistical mechanics. For example, this distribution explains many fundamental gas properties in kinetic theory of gasesThe temperature of any (massive) physical system is the result of the motions of the molecules and atoms which make up the system. These particles have a range of different velocities, and the velocity of any single particle constantly changes due to collisions with other particles. However, the fraction of a large number of particles within a particular velocity range is nearly constant. Then Maxwell distribution of velocities specifies this fraction, for any velocity range as a function of the temperature of the systemThe Maxwell distribution was first introduced in the literature by J.C. Maxwell (1860) and again described by Boltzman (1870) with a few Assumptions.Tyagi and Bhattacharya (a) [8], Tyagi and Bhattacharya(b) [9] considered Maxwell distribution as a lifetime modelfor the first time. They obtained Bayes estimates and minimumvariance unbiased estimators of the parameter and reliabilityfunction for the Maxwell distribution. Chaturvedi andRani [10] generalized Maxwell distribution and they obtainedClassical and Bayesian estimators for generalizeddistribution. Bekker and Roux [11] studied Empirical Bayesestimation for Maxwell distribution. These studies givemathematical handling to Maxwell distribution but ignorethe application aspect of the Maxwell distribution. In (2005) Bekker and Roux[1], studied empirical Bayes estimation for Maxwell distribution, and we have assumed that complete sample information is available, SankuDey[6] (2011) studies on Bayes estimators of the parameter of a Maxwell distribution and obtain associated based on conjugate prior under scale invariant symmetric and a symmetric loss functions.

2-Model properties

The Maxwell (or Maxwell – Boltzmann) distribution gives the distribution of speeds of molecules in thermal equilibrium as given by statistical mechanics.
Defining α =KT/M , where K is the Maxwell constant, T is temperature, m is the mass of α molecule. The probability density function of Maxwell distribution over the rang $x \in [0;\infty)$ is given by:

$$f(x;\alpha)=\frac{1}{\alpha^3}\sqrt{\frac{2}{\pi}}x^2.e^{-\frac{x^2}{2\alpha^2}} \qquad \text{…………….(1)}$$

To prove it's a p.d.f we take the integration as the following :

$$\int_0^\infty \frac{1}{\alpha^3}\sqrt{\frac{2}{\pi}}x^2 e^{-\frac{x^2}{2\alpha^2}}dx \qquad \text{…………….(2)}$$

$$\text{let } y=\frac{x^2}{2\alpha^2}$$

$$x^2=2\alpha^2 y$$

$$dx=\sqrt{2}\alpha\frac{1}{2}\frac{1}{\sqrt{y}}dy$$

$$\int_\infty^0 \frac{1}{\alpha^3}\sqrt{\frac{2}{\pi}}2\alpha^2 y e^{-y}\sqrt{2}\alpha\frac{1}{2}\frac{1}{\sqrt{y}}dy$$

$$=\frac{2\Gamma\frac{3}{2}}{\sqrt{\pi}}=\frac{2\frac{1}{2}\sqrt{\pi}}{\sqrt{\pi}}=1$$

3-Methods of estimation:

3.1 Method of moments

The method of moments is a method of estimation of population parameters of interest. So a sample is drawn and the population moments are estimated from the sample. using the sample moments in place of the (unknown) population moments. This results in estimates of those parameters. The method of moments was introduced by Karl Pearson in 1894[7].

Suppose that the problem is to estimate k unknown parameters $\alpha_1, \alpha_2, ..., \alpha_k$ characterizing the distribution f(x, α) of the random variable X. Suppose the first k moments of the true distribution (the "population moments") can be expressed as functions of the α s:

$$\mu_k = E[X^k] = g_k(\alpha_1, \alpha_2, ..., \alpha_k) \text{ ………..(3)}$$

Suppose a sample of size k is drawn, resulting in the values x_j .

For j = 1, …., k, let

$$\hat{\mu}_j = \frac{1}{n}\sum_{i=1}^{n} x_i^j \text{ …………..(4)}$$

be the j-th sample moment, an estimate of μ_i. The method of moment's estimator for $\alpha_1, \alpha_2, ..., \alpha_k$ denoted by $\hat{\alpha}_1, \hat{\alpha}_2, ..., \hat{\alpha}_k$ is defined as the solution (if there is one) to the equations:

$$\hat{\mu}_k = g_k(\hat{\alpha}_1, \hat{\alpha}_2, ..., \hat{\alpha}_k) \text{ ……….(5)}$$

We estimate here three ways of estimation:

a. ***Momentestimator depend on the mean:***

For the p.d.f:

$$f(x;\alpha) = \frac{1}{\alpha^3}\sqrt{\frac{2}{\pi}}x^2.e^{-\frac{x^2}{2\alpha^2}}$$

$$E(x) = \int_0^{\infty} \frac{1}{\alpha^3}\sqrt{\frac{2}{\pi}}x^3 e^{-\frac{x^2}{2\alpha^2}}.dx \text{ ………..…(6)}$$

let $\frac{x^2}{2\alpha^2} = y \Rightarrow x^2 = 2\alpha^2.y$

$x = \sqrt{2}\alpha\sqrt{y}$

$$dx = \sqrt{2}\alpha \frac{1}{2}\frac{1}{\sqrt{y}}.dy$$

$$E(x) = \int_0^\infty \frac{1}{\alpha^3} 2^{\frac{3}{2}} \alpha^3 . y^{\frac{3}{2}} . e^{-y} \sqrt{2}\alpha \frac{1}{2}\frac{1}{\sqrt{y}}.dy$$

$$= \frac{\alpha 2^{\frac{3}{2}}}{\sqrt{\pi}} \int_0^\infty y.e^{-y}.\, dy = \frac{\alpha 2^{\frac{3}{2}} \Gamma 2}{\sqrt{\pi}} = 2\alpha\sqrt{\frac{2}{\pi}}$$

$$2\alpha\sqrt{\frac{2}{\pi}} = \bar{x}$$

Hence the moment mean estimator for α is:

$$\hat{\alpha}_{mon\ mean} = \frac{\bar{x}}{2\sqrt{\frac{2}{\pi}}} \qquad \text{………….…(7)}$$

b- Moment depend on the variance:

$$v(x) = E(x^2) - \left(E(x)\right)^2 \qquad \text{….………..…(8)}$$

$$E(x^2) = \int_0^\infty \frac{1}{\alpha^3}\sqrt{\frac{2}{\pi}} x^4 e^{-\frac{x^2}{2\alpha^2}} dx \qquad \text{………….…(9)}$$

$$\text{let } \frac{x^2}{2\alpha^2} = y \Rightarrow x^2 = 2\alpha y$$

$$x^4 = 4\alpha^4 y^2$$

$$x = \sqrt{2}\alpha\sqrt{y}$$

$$dx = \sqrt{2}\alpha \frac{1}{2}.\frac{1}{\sqrt{y}} dy$$

$$E(x^2) = \int_0^\infty \frac{1}{\alpha^3}\sqrt{\frac{2}{\pi}} 4\alpha^4 y^2 e^{-y}.\sqrt{2}\alpha \frac{1}{2}.\frac{1}{\sqrt{y}} dy$$

$$= \frac{1}{\sqrt{\pi}} 4\alpha^2 \int_0^\infty y^{\frac{3}{2}} e^{-y} dy$$

$$= \frac{4\alpha^2}{\sqrt{\pi}} \Gamma\frac{5}{2}$$

$$= \frac{4\alpha^2}{\sqrt{\pi}} \frac{3}{2} . \frac{1}{2} \sqrt{\pi}$$

$$E(x^2) = 3\alpha^2$$

$$\because E(x) = 2\alpha\sqrt{\frac{2}{\pi}}$$

$$\therefore v(x) = 3\alpha^2 - \left(2\alpha\sqrt{\frac{2}{\pi}}\right)^2$$

$$= 3\alpha^2 - 4\alpha^2 \frac{2}{\pi}$$

$$\therefore v(x) = \alpha^2\left(3 - \frac{8}{\pi}\right)$$

$$\therefore \sigma^2 = \alpha^2\left(3 - \frac{8}{\pi}\right)$$

$$\alpha^2 = \frac{\sigma^2}{\left(3 - \frac{8}{\pi}\right)}$$

Hence the moment variance estimator for α is:

$$\hat{\alpha}_{mon\,var} = \frac{\sigma}{\sqrt{\left(3 - \frac{8}{\pi}\right)}} \qquad \ldots\ldots\ldots\ldots\ldots(10)$$

c- Moment depend on thecoefficient of variation (C.V):

we know that :

$$C.V = \frac{\sigma^2}{\bar{x}} \qquad \ldots\ldots\ldots\ldots\ldots(11)$$

So by substituting we get :

$$\frac{\sigma^2}{\bar{x}} = \frac{\alpha^2\left(3 - \frac{8}{\pi}\right)}{2\alpha\sqrt{\frac{2}{\pi}}} = \alpha\frac{\left(3 - \frac{8}{\pi}\right)}{2\sqrt{\frac{2}{\pi}}}$$

Hence the moment C.V estimator for α is:

$$\hat{\alpha}_{mon\ c.v}=\frac{\frac{\sigma^2}{\bar{x}}}{\frac{\left(3-\frac{8}{\pi}\right)}{2\sqrt{\frac{2}{\pi}}}}=\frac{\sigma^2 2\sqrt{\frac{2}{\pi}}}{\bar{x}\left(3-\frac{8}{\pi}\right)} \quad \ldots\ldots\ldots\ldots..\ldots(12)$$

3.2Method of maximum Likelihood estimation

The maximum-likelihood estimation (MILE) is a method of estimating the parameters of a statistical model, maximum-likelihood estimation provides estimates for the model's parameters.

In general, for a fixed set of data and underlying statistical model, the method of maximum likelihood selects the set of values of the model parameters that maximizes the likelihood function. Intuitively, this maximizes the agreement of the selected model with the observed data, and for discrete random variables it indeed maximizes the probability of the observed data under the resulting distribution. Maximum-likelihood estimation gives a unified approach to estimation,

Suppose there is a sample x_1, x_2, ...,x_n of n independent and identically distributed observations, coming from a distribution with an unknown probability density function $f_o(.)$. It is however surmised that the function f_o belongs to a certain family of distributions $\{f(.|\alpha), \alpha\in\Theta\}$ (where α is a vector of parameters for this family), called the parametric model, so that $f_o = f(.|\alpha_0)$. It is desirable to find an estimator which would be as close to the true value x_1 as possible. Both the observed variables x, and the parameter α can be vectors.

To use the method of maximum likelihood, one first specifies the joint density function for all observations. For an independent and identically distributed sample, this joint density function is by considering the observed values X_1, X_2, ...,X_n, to be fixed "parameters" of this function, whereas G will be the function's variable and allowed to vary freely; this function will be called the likelihood:

$$L(\alpha;\, x_1,....,x_n)= f(x_1,x_2,...,x_n \mid \alpha)=\prod_{i=1}^{n} f(x_i \mid \alpha) \quad \ldots\ldots.....(13)$$

Denotes a separation between the two input arguments: α and the vector- valued input x_1, …..x_n, .

In practice it is often more convenient to work with the logarithm of the likelihood function, called the log-likelihood{10}:

$$\ln L(\alpha, x_1, \ldots, x_n) = \sum_{i=1}^{n} \ln(x_i \mid \alpha) \ldots\ldots\ldots(14)$$

or the average log-likelihood:

$$\hat{\ell} = \frac{1}{n} \ln L \ldots\ldots\ldots(15)$$

The method of maximum likelihood estimates α_0 by finding a value of α that maximizes $\hat{\ell}(\alpha \mid x)$ this method of estimation defines a maximum-likelihood estimator (MILE) of

$$\{\hat{\alpha}_{mle}\} \subseteq \{\arg\max_{\theta\in\Theta} \hat{\ell}(\alpha; x_1, \ldots, x_n)\} \quad \ldots\ldots\ldots(16)$$

So for our distribution Let $x_1, x_2, \ldots . x_n$ be arandom sample of size in have the p.d.f. maxwell distribution:

$$f(x;\alpha) = \frac{1}{\alpha^3}\sqrt{\frac{2}{\pi}}\, x^2 e^{-\frac{x^2}{2\alpha^2}}$$

$$L(x,\alpha) = \frac{1}{\alpha^{3n}}\left(\sqrt{\frac{2}{\pi}}\right)^n \pi x_i^2 e^{-\frac{\sum x_i^2}{2\alpha^2}} \quad \ldots\ldots\ldots\ldots(17)$$

$$\log L = -3n\log\alpha + \frac{n}{2}\log\left(\frac{2}{\pi}\right) + 2\sum \log x_i - \frac{\sum x_i^{\,2}}{2\alpha^2}$$

$$\frac{\partial \log L}{\partial x} = \frac{-3n}{\alpha} + \frac{\sum x_i^{\,2}}{2\alpha^2}$$

$$3n = \frac{\sum x_i^{\,2}}{\alpha^2} \Rightarrow \alpha^2 = \frac{\sum x_i^{\,2}}{3n}$$

Hence the MLE estimator for α is:

$$\hat{\alpha}_{MLE} = \sqrt{\frac{\sum x_i^{\,2}}{3n}} \quad \ldots\ldots\ldots(18)$$

3.3Bayes Estimator

In estimation and decision theory, a Bayes estimator or a Bayes action is an estimator or decision rule that minimizes the posterior expected value of a loss function (i.e., the posterior expected loss). Equivalently, it maximizes the posterior expectation of a utility function. An alternative way of formulating an estimator within Bayesian statistics is Maximum a posteriori estimation.

Suppose an unknown parameter α is known to have a prior distribution π Let $\hat{\alpha} = \hat{\alpha}(x)$ be an estimator of $\hat{\alpha}$ (based on some measurements x, and let $L(\alpha, \hat{\alpha})$ be a loss function, such as squared error: The Bayes risk of α is defined as $E_\pi\{L(\alpha, \hat{\alpha})\}$, where the expectation is taken over the probability distribution of α : this defines the risk function as a function of $\hat{\alpha}$. An estimator $\hat{\alpha}$. is said to be a Bayes estimatorif it minimizes the Bayes risk among all estimators. Equivalently, the estimator which minimizes the posterior expected loss $E_\pi\{L(\alpha, \hat{\alpha}) \mid x\}$for each x also minimizes the Bayes risk and therefore is a Bayes estimator[5]

If the prior is improper then an estimator which minimizes the posterior expected loss for each xis called a generalized Bayes estimator, the prior may be informative or non-informative,

so for our informative priordistribution :

$$g(\alpha) = \alpha \qquad \text{…………(19)}$$

$$L(\alpha) = \left(\sqrt{\frac{2}{\pi}}\right)^n \frac{1}{\alpha^{3n}} \pi x_i^2 \; e^{-\frac{\sum x_i^2}{2\alpha^2}}$$

$$L(\alpha).g(\alpha) = \left(\sqrt{\frac{2}{\pi}}\right)^n \frac{1}{\alpha^{3n}} \pi x_i^2 \; e^{-\frac{\sum x_i^2}{2\alpha^2}} \alpha$$

$$c = \left(\sqrt{\frac{2}{\pi}}\right)^n \pi x_i^2 \int_{\infty}^{0} \alpha^{-3n+1} e^{-\frac{\sum x_i^2}{2\alpha^2}} dx$$

$$\text{let } \frac{\sum x_i^2}{2\alpha^2} = y \Rightarrow 2\alpha^2 y = \sum x_i^2$$

$$\Rightarrow \alpha^2 = \frac{\sum x_i^2}{2y}$$

$$\Rightarrow \alpha = \sqrt{\frac{\sum x_i^2}{2}} y^{\frac{-1}{2}}$$

$$\Rightarrow d\alpha = \sqrt{\frac{\sum x_i^2}{2}}\,\frac{-1}{2}\,y^{\frac{-3}{2}}.dy$$

$$c = \left(\sqrt{\frac{2}{\pi}}\right)^{n} \pi\, x_i^2 \int_0^{\infty} \left(\sqrt{\frac{\sum x_i^2}{2}}\right)^{-3n+1} y^{\frac{3n}{2}-\frac{1}{2}}\, e^{-y} \sqrt{\frac{\sum x_i^2}{2}}\,\frac{1}{2}\,y^{\frac{-3}{2}}.dy$$

$$= \left(\sqrt{\frac{2}{\pi}}\right)^{n} \pi\, x_i^2 \left(\sqrt{\frac{\sum x_i^2}{2}}\right)^{-3n+1} \frac{1}{2}\Gamma\left(\frac{3}{2}n-1\right) \qquad \ldots\ldots\ldots\ldots(20)$$

Then the posterior distribution for α given $x_1, x_2 .... x_n$ is:

$$h(\alpha / x) = \frac{\left(\sqrt{\frac{2}{\pi}}\right)^{n} \pi\, x_i^2\, \alpha^{-3n+1}\, e^{-\frac{\sum x_i^2}{2\alpha^2}}}{\left(\sqrt{\frac{2}{\pi}}\right)^{n} \pi\, x_i^2 \left(\sqrt{\frac{\sum x_i^2}{2}}\right)^{2n+1} \frac{1}{2}\Gamma\left(\frac{3}{2}n-1\right)} \qquad \ldots\ldots\ldots\ldots(21)$$

$$= \frac{2\alpha^{-3n+1} \left(\sqrt{\frac{\sum x_i^2}{2}}\right)^{3n-2} e^{-\frac{\sum x_i^2}{2\alpha^2}}}{\Gamma\left(\frac{3}{2}n-1\right)}$$

By using the squared error loss function the expected posterior is:

$$E(\alpha / x) = \int_0^{\infty} \frac{2\alpha^{-3n+2} \left(\sqrt{\frac{\sum x_i^2}{2}}\right)^{3n-2} e^{-\frac{\sum x_i^2}{2\alpha^2}}}{\Gamma\left(\frac{3}{2}n-1\right)} d\alpha \qquad \ldots\ldots\ldots\ldots(22)$$

$$= \frac{2\left(\sqrt{\frac{\sum x_i^2}{2}}\right)^{3n-2}}{\Gamma\left(\frac{3}{2}n-1\right)} \int_0^{\infty} \alpha^{-3n+2}\, e^{-\frac{\sum x_i^2}{2\alpha^2}}\, d\alpha$$

$$\text{let} - \frac{\sum x_i^2}{2\alpha^2} = y \Rightarrow 2\alpha^2 y = \sum x_i^2 \Rightarrow \alpha^2 = \frac{\sum x_i^2}{2y}$$

$$= \alpha = \sqrt{\frac{\sum xi^2}{2}}\,y^{-\frac{1}{2}}$$

$$\Rightarrow d\alpha = \sqrt{\frac{\sum x_i^2}{2}}\,\frac{-1}{2}\,y^{-\frac{3}{2}}dy$$

$$=\frac{2\left(\frac{\sum x_i^2}{2}\right)^{3n-2}}{\sqrt{\frac{3}{2}n-1}}\int_0^{\infty}\left(\frac{\sum x_i^2}{2}\right)^{3n+2} y^{\frac{3}{2}n-1} e^{-y}\left(\frac{\sum x_i^2}{2}\right)^{3n-2}\frac{1}{2}y^{-\frac{3}{2}}\,dy$$

$$=\frac{\sqrt{\frac{\sum x_i^2}{2}}\,\Gamma\left(\frac{3}{2}n-\frac{3}{2}\right)}{\Gamma\left(\frac{3}{2}n-1\right)}$$

Hence the bayes estimator for α is:

$$\hat{\alpha}_{bay}=\frac{\sqrt{\frac{\sum x_i^2}{2}}\,\Gamma\left(\frac{3}{2}n-\frac{3}{2}\right)}{\Gamma\left(\frac{3}{2}n-1\right)} \qquad \ldots\ldots\ldots\ldots(23)$$

5-The simulation:

After we estimate the parameter α by the preceding methods we use the matlab to simulate the methods to study the difference between them by comparing the results using the mean squared errors (MSE) once and mean absolutepercentage errors (MAPE) once as:

a- Mean squared error (MSE) which is defined by the formula:

$$MSE(\alpha)=\frac{\sum_{i=i}^{n}\left(\alpha_i-\alpha^{\alpha}\right)^2}{n} \qquad \ldots\ldots\ldots\ldots(24)$$

b- Mean absolute percentage error (MAPE)which is defined by the formula:

$$MAPE(\alpha)=\frac{\sum_{i=1}^{n}\left|(\alpha-\hat{\alpha})/\alpha\right|}{n} \qquad \ldots\ldots\ldots\ldots(25)$$

We use the cumulative distribution putting

$$F(x)=\xi \Rightarrow$$

$$so$$

$$P\left(\frac{3}{2},\frac{x^2}{a\alpha^2}\right)=\xi \Rightarrow$$

After simplifying we get :

$$x=\alpha\sqrt{2P^{-1}\left(\frac{3}{2},\xi\right)} \qquad \ldots\ldots\ldots\ldots(26)$$

For generating the values of X, where $P^{-1}(\alpha\ , p)$, as above, denotes the value x where

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$P(\alpha, x) = p$ ..………...(27)

So for the estimated α from the last methods for equations 7,10,12,18,23we simulate the program and calculate the estimated errors for every combination,
The number of replications used was 1000 samples of different sizes , small samples with sizes10,15 and medium samples with sizes 25, 50 and large samples of the size 100 within different values of α which they are 0.5, 1, 1.5, 2, 5, 7 the results for the two methods MSE and MAPE were summarized and tabulated in tables (1) and (2) as the following:

Table(1): Results for the different estimators using (MSE)

	Method / Error	**MLE**	**Mom mem**	**Mam var**	**Mom C.V**	**Bay**	**Best method**
α= 0.5	n= 10	0.0075	0.0087	0.0129	0.0577	0.0079	MLE
	n= 15	0.0042	0.0055	0.0148	0.1124	0.0047	MLE
	n= 25	0.0027	0.0030	0.0091	0.0425	0.0029	MLE
	n= 50	0.0014	0.0014	0.0069	0.0312	0.0014	MLE,MOM MEAN,BAY
	n= 100	0.0005	0.0010	0.0084	0.0526	0.0005	MLE,BAY
α= 1	n= 10	0.0235	0.0177	0.1415	0.6745	0.0335	MOM MEAN
	n= 15	0.0284	0.0331	0.0765	0.4652	0.0284	MLE,BAY
	n= 25	0.0124	0.0166	0.0426	0.3021	0.0104	BAY
	n= 50	0.0022	0.0051	0.0232	0.17101	0.0019	BAY
	n= 100	0.0013	0.0056	0.0357	0.2917	0.0011	BAY
α= 1.5	n= 10	0.0433	0.0509	0.0559	0.2298	0.0587	MLE
	n= 15	0.0294	0.0366	0.1293	0.8612	0.0301	MLE
	n= 25	0.0336	0.0476	0.0959	0.7325	0.0303	BAY
	n= 50	0.0086	0.0110	0.669	0.3781	0.0079	BAY
	n= 100	0.0059	0.0118	0.0506	0.3529	0.0055	BAY
α= 2	n= 10	0.0782	0.0733	0.3708	1.5286	0.0890	MOM MEAN
	n= 15	0.0282	0.0361	0.1879	1.0910	0.0369	MLE
	n= 25	0.0463	0.0650	0.1077	0.6989	0.0392	BAY
	n= 50	0.0227	0.0321	0.2157	1.3379	0.0231	MLE
	n= 100	0.0088	0.0067	0.1612	0.8299	0.0102	MOM MEAN
α= 5	n= 10	0.3639	0.4590	1.5038	0.9515	0.5221	MLE
	n= 15	0.2540	0.3749	1.6313	13.4677	0.2754	MLE
	n= 25	0.1912	0.2430	0.9612	5.9134	0.1813	BAY
	n= 50	0.1517	0.1919	1.3183	7.9647	0.1483	BAY
	n= 100	0.0559	0.1825	0.8226	0.8117	0.0475	BAY
α= 7	n= 10	1.3603	1.7149	1.4782	9.4008	1.5406	MLE
	n= 15	0.4867	0.5933	1.9889	11.8219	0.4408	BAY
	n= 25	0.5936	0.6583	1.7973	8.8107	0.5821	BAY
	n= 50	0.3212	0.5151	1.5973	11.3904	0.3079	BAY
	n= 100	0.1204	0.2173	1.0747	6.9718	0.1089	BAY

Table(2): Results for the different estimators using (MAPE)

Parameter value	Method / Error	MLE	Mom mem	Mam var	Mom C.V	Bay	Best method
α= 0.5	n= 10	0.1356	0.1500	0.1844	0.3939	0.1392	MLE
	n= 15	0.1064	0.1194	0.2063	0.4848	0.1171	MLE
	n= 25	0.0810	0.0818	0.1695	0.3729	0.0878	MLE
	n= 50	0.0530	0.0571	0.1424	0.3153	0.0518	BAY
	n= 100	0.0408	0.0509	0.1679	0.4238	0.0411	MLE
α= 1	n= 10	0.1218	0.1117	0.2890	0.6436	0.1255	MOM MEAN
	n= 15	0.1531	0.1632	0.2017	0.5172	0.1478	BAY
	n= 25	0.0874	0.1029	0.1611	0.4058	0.0799	BAY
	n= 50	0.0419	0.0615	0.1368	0.3730	0.0384	BAY
	n= 100	0.0320	0.0646	0.1727	0.4829	0.0287	BAY
α= 1.5	n= 10	0.1078	0.1211	0.1235	0.2552	0.1277	MLE
	n= 15	0.0957	0.1121	0.2080	0.4626	0.0982	MLE
	n= 25	0.1035	0.1188	0.1634	0.4542	0.1004	BAY
	n= 50	0.0473	0.0562	0.1518	0.3581	0.0503	MLE
	n= 100	0.0402	0.0595	0.1364	0.3654	0.0387	BAY
α= 2	n= 10	0.1149	0.1130	0.2621	0.5118	0.1270	MOM MEAN
	n= 15	0.0691	0.0749	0.1840	0.4360	0.0880	MLE
	n= 25	0.0909	0.1109	0.1301	0.3589	0.0830	BAY
	n= 50	0.0597	0.0734	0.2044	0.5320	0.0588	BAY
	n= 100	0.0407	0.0316	0.1877	0.4242	0.0433	MOM MEAN
α= 5	n= 10	0.1043	0.1193	0.2014	0.4665	0.1238	MLE
	n= 15	0.0842	0.1054	0.1684	0.4855	0.0851	MLE
	n= 25	0.0615	0.0721	0.1656	0.3951	0.0648	MLE
	n= 50	0.0714	0.0797	0.1880	0.4836	0.0675	BAY
	n= 100	0.0386	0.0743	0.1714	0.4910	0.0349	BAY
α= 7	n= 10	0.1322	0.1550	0.1476	0.3311	0.1288	BAY
	n= 15	0.0816	0.0774	0.1726	0.4028	0.0796	MOM MEAN
	n= 25	0.0894	0.0881	0.1616	0.3340	0.0820	BAY
	n= 50	0.0619	0.0785	0.1421	0.3967	0.0622	MLE
	n= 100	0.0609	0.0568	0.1315	0.3378	0.0400	BAY

6-Conclusion:

1. Weconclude that when he sample size increase the MSE and MAPE decrease and that is apply the statistical hypotheses.
2. We found that for small samples for both MSE and MAPE the M.L.E is the best.
3. For medium sample for both MSE and MAPE the bayesestimator is the best.
4. For large sample for both MSE and MAPE the bayesestimator is the best.
5. We recommend to use M.LE for small samples.
6. We recommend to use bayes estimator for medium and large samples.
7. We recommend toTest anothermethods to estimate the parameter.

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Apendex
Matlab program

```
%%%%%%% Maxwell Distribution%%%%%%
clc
clearall
n=10;
elpha=4;
%%%%%%%%%%%%%%%%%%%%%%
for q=1:10
x=elpha.**((2.*gamrnd(1,1.5,1,n)).^(0.5))
elpha_mle(q)=sqrt(sum(x.^2)/(3*n));
elpha_mommean(q)=mean(x)/(2*sqrt(2/pi));
elpha_momvar(q)=sqrt(var(x))/sqrt(3-8/pi);
elpha_cv(q)=(2*var(x)*sqrt(2/pi))/(mean(x)*(3-8/pi));
elpha_bay(q)=(sqrt(sum(x.^2)/2)*gamma(1.5*n-1.5))/gamma(1.5*n-1)

end
elphahat=[mean(elpha_mle) mean(elpha_mommean) mean(elpha_momvar) mean(elpha_cv)
mean(elpha_bay)]
mse=[mean((elpha-elpha_mle).^2) mean((elpha-elpha_mommean).^2) mean((elpha-
elpha_momvar).^2) mean((elpha-elpha_cv).^2) mean((elpha-elpha_bay).^2)]
mape=[mean(abs((elpha-elpha_mle)./elpha)) mean(abs((elpha-elpha_mommean)./elpha))
mean(abs((elpha-elpha_momvar)./elpha)) mean(abs((elpha-elpha_cv)./elpha))
mean(abs((elpha-elpha_bay)./elpha))]
```