Inverse Kinematics Analysis Using Close Form Solution Method for 5 DOF Robot Manipulate

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ABSTRACT:

This work proposes a close form solution algorithm to solve the inverse kinematics for a five degree of freedom (DOF) robot manipulator, close form solution is preferable to numerical solutions because analytical ones yield complete solutions and are computationally fast and reliable. The motion path of a robot arm is calculated using the geometric analysis. The proposed algorithm is verified using developed simulation modules. Computer simulation is conducted to demonstrate accuracy of the proposed algorithm to generate an appropriate joint angle for reaching desired Cartesian coordinate. The algorithm has been tested yield fair, which have also compared with the robot arm's actual reading.

Keywords: Robot manipulator, Lab_volt R5150 Robot arm, Forward Kinematics, Inverse kinematics.

تحليل الكينيماتي العكسي باستخدام طريقة (الحل المغلق) لذراع روبوت ذو خمس درجات للحرية

الخلاصة:

يقترح هذا البحث أستخدام خوارزمية الحل المغلق لايجاد الكينيماتيكية العكسية لذراع روبوت ذو 5 درجات حرية. يفضل أستخدام هذا النوع من الحلول على مثيلاتها العددية بسبب كونها تعطي حلولا كاملة وسريعة وذات معولية. تم استخدام التحليل الهندسي لايجاد قيم الزوايا المفصلية لمسار حركة ذراع الروبوت , تم أثبات الخوارزمية المقترحة باستخدام نماذج محاكاة مطورة. كما تم أجراء محاكاة بأستخدام الحاسوب للبرهنة على دقة الخوارزمية المقترحة على توليد الزوايا المناسبة للمفاصل للوصول الى الابعاد المطلوبة. اظهر أستخدام طريقة (الحل المغلق) لايجاد الحلول الكينيماتيكية تطابق مع القراءات الفعلية لذراع الروبوت.

INTRODUCTION:

The inverse kinematics solution, it is essential for robots that follow paths. A computer control system called the controller, commands the robot to move through its workspace based on a plan or desired trajectory of the robot end-effector. The plan is developed using an inverse kinematics algorithm which computes the next desired joint configuration given the current configuration and the next desired end effector position. The plan may be pre-computed off-line and stored in the controller's memory or computed dynamically during the assigned task. The robot is raised for multiple applications in the industry by highlighting the line production, which may be used in industrial automation applications; it may add multiple end effectors for different purposes such as suction, welding, painting, manipulating parts between others [1].

Previous Research

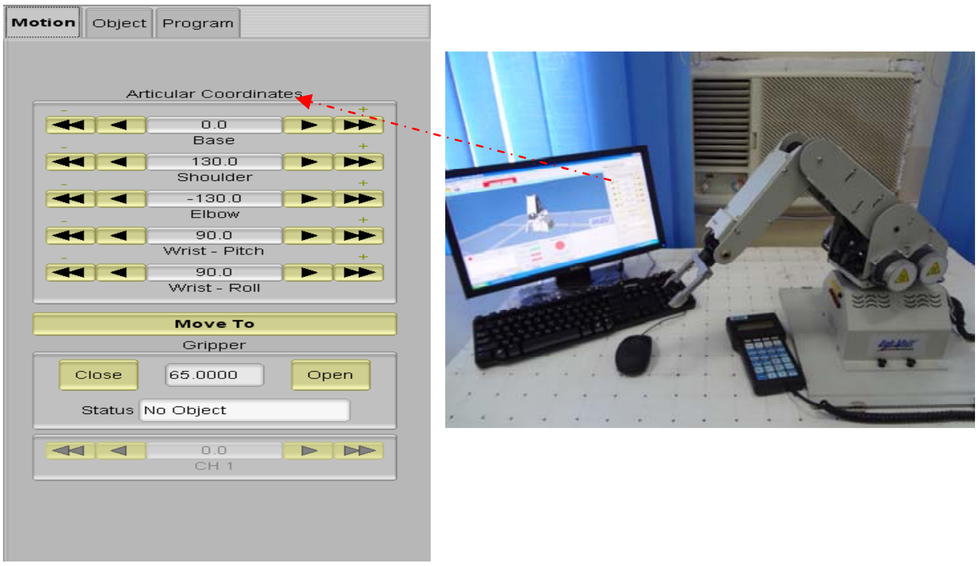
The problem of kinematic analysis had been investigated as found in many literatures [2-3]. Various approaches had been used.

Yu Jie Cui[4] deduced a formulation of modular robot based on Denevit and Hartenberg(D-H) and presented the kinematics simulation based on Matlab, The workspace was not simulated in this work. Jun Xie, and et al [5] established the kinematics model of practical series mechanical arm to act the manipulations with parallel executive mechanism, and solved the problem using Denavit-Hartenberg (D-H) transformation. Tahseen et al[6] used (Denavit - Hartenberg (DH)) as base to an analytical solution for the forward kinematics analysis of Lab_Volt R5150 robot arm and analyze the movement of the robot arm from one point in space to another point, and analyzes its work space.

In this work, close form solution algorithm to solve the inverse kinematics robot manipulator. The developed model of inverse kinematics algorithm is used to solve the inverse kinematics for a 5DOF manipulator robot with wrist offset, to analysis the movement of arm from one point to another point. In this solution the only decision variables are the coordinates of origin and the destination point in space of end-effector besides the geometric parameter of robot manipulator. As shown in Figure (1). This model would achieve provision of a real-time solution, suitable for tracking trajectories.

The Robot Description

The industrial robot used in this work is an elbow Lab-Volt robot Model R5150 with five rotational axes with stepper actuators: base, shoulder, elbow, wrist, and gripper. The base supports the arm mechanism and it houses the motor that provides shoulder rotation. The shoulder houses the motors that move the other sections of the arm. The mechanism that move the forearm, wrist and gripper are carried in the lower end of the upper arm. The upper arm and forearm have up and down motions. The wrist, that can rotate 360-degree clockwise and counterclockwise directions, moves up and down to control the gripper location and direction. The two-fingered parallel jaw gripper is attached to the arm of the robot to enable it to grasp and move materials, parts and tools [7]. The basic challenge associated with the R5150 arm is the limited information available on its governing control model for location placement. Two ways by which control can be effected on R5150 arm, this robot can be programmed by using either a hand-held terminal (teach pendant) or a RoboCIM simulation software. Where, the coordinate system that is used in Lab_volt R5150 robot manipulator to permit the control and visualization of the system motion in an interactive way is only the articulate coordinates system as shown figure (1). This type of robot has a complex inverse kinematics, which needs a long time for such calculation.



Articular Coordinates

Base 0.0

Shoulder 130.0

Elbow -130.0

Wrist - Pitch 90.0

Wrist - Roll 90.0

Gripper 65.0000

Status No Object

CH 1 0.0

Figure (1): Lab_volt R5150 Robot Manipulator

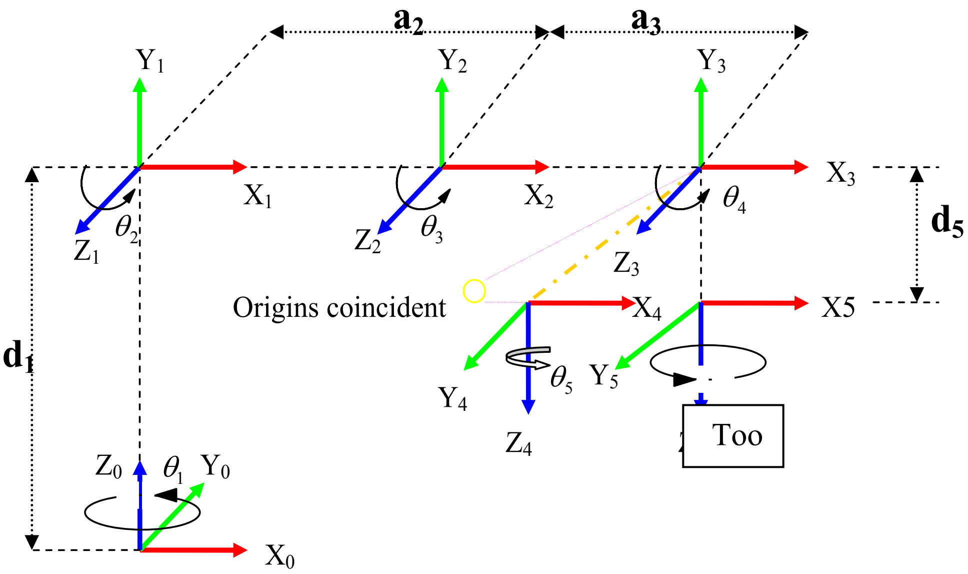


Figure (2): Link Coordinates of a Five_Axis Robot Manipulator [8].

Assigning the Coordinate Frames:

Lab_Volt R5150 has robot manipulator with five rotational joints and a moving grip as shown in Figure (2). Joint 1 represents the waist (base) and its axis of motion is z_0. This joint provides a rotational θ_1 angular motion around z_0 axis in x_0y_0 plane. Joint 2 is identified as the shoulder and its axis is perpendicular to Joint 1 axis. It provides a rotational θ_2 angular motion around z_1 axis in x_1y_1 plane. z_2 axes of Joint 3 (Forearm) and z_3 of Joint 4 (Wrist) are parallel to z_1 axis of Joint 2; they provide θ_3 and θ_4 angular motions in x_2y_2 and x_3y_3 planes respectively. Joint five is identified as the grip rotation. Its z_4 axis is vertical to z_3 axis and it provides θ_5 angular motions in x_4y_4 plane. Figure (2) show the robot at rest position (all joint angles equal to zero).

Transformation Matrix:

After establishing (D-H) coordinate system for each link figure(3), a homogeneous transformation matrix can easily be developed considering frame {i-1} and frame {i} transformation consisting of four basic transformations as shown in Table (1) and the joint link parameter as given in Table (2). The overall complex homogeneous matrix of transformation can be formed by consecutive applications of simple transformations according to Denavit- Hartenberg (D-H) notation [9].

Table (1) Transferring from frame i-1 to frame i

Operation	Description
T_1	A rotation about z_{i-1} axis by an angle θ_i.
T_2	Translation along z_{i-1} axis by distance d_i.
T_3	Translation by distance a_i along x_i axis
T_4	Rotation by angle α_i about x_i axis

Table (2) D-H Parameter for R5150 Robot Arm [9].

Joint i	**joint name**	θ_i	d_i **(mm)**	a_i **(mm)**	α_i	**range**	**motion**
1	**base**	$\theta 1$	255.5	0	90	-185 to +153	rotates the body
2	**shoulder**	$\theta 2$	0	190	0	-32 to +149	raises and lowers (upper arm)
3	**elbow**	$\theta 3$	0	190	0	-147 to +51	raises and lowers (forearm)
4	**wrist pitch**	$\theta 4$	0	0	90	-5 to 180	raises and lowers (gripper)
5	**wrist roll**	$\theta 5$	115	0	0	± 360	rotates the gripper

The overall (4 x 4) coordinate transformation matrix, T (tool, reference), results from multiplying the individual frame-to-frame (T) matrices together. The entries in the T (tool, reference) matrix will, in general be functions of all 5 joint variables.

$$\left.\begin{aligned}
{}^{0}T_{1} &= \begin{bmatrix} c_{\theta 1} & 0 & s_{\theta 1} & 0 \\ s_{\theta 1} & 0 & -c_{\theta 1} & 0 \\ 0 & 0 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad
{}^{1}T_{2} = \begin{bmatrix} c_{\theta 2} & -s_{\theta 2} & 0 & a_{2}c_{\theta 2} \\ s_{\theta 2} & c_{\theta 2} & 0 & a_{2}s_{\theta 2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^{2}T_{3} &= \begin{bmatrix} c_{\theta 3} & -s_{\theta 3} & 0 & a_{3}c_{\theta 3} \\ s_{\theta 3} & c_{\theta 3} & 0 & a_{3}s_{3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad
{}^{3}T_{4} = \begin{bmatrix} c_{\theta 4} & 0 & s_{\theta 4} & 0 \\ s_{\theta 4} & 0 & -c_{\theta 4} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^{4}T_{5} &= \begin{bmatrix} c_{\theta 5} & -s_{\theta 5} & 0 & 0 \\ s_{\theta 5} & c_{\theta 5} & 0 & 0 \\ 0 & 0 & 1 & d_{5} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}\right] \quad \ldots.(1)$$

$${}^{0}T_{5} = {}^{0}T_{1}\ {}^{1}T_{2}\ {}^{2}T_{3}\ {}^{3}T_{4}\ {}^{4}T_{5} = \begin{bmatrix} n_{x} & o_{x} & a_{x} & p_{x} \\ n_{x} & o_{y} & a_{y} & p_{y} \\ n_{z} & o_{z} & a_{z} & p_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots (2)$$

The overall transformation matrix has been modeled and simplified using matlab program as shown in table (3)
From the kinematics modeling equations, we can extract the position and orientation of the end-effector with respect to base.

Kinematics Equations from the Overall Transformation Matrix.
The kinematics model in the form of overall transformation matrix is expressed in 12 kinematics equations as shown in Table (3).

Table (3): Kinematics Equations from the Overall Transformation Matrix.

Equation No	RHS	LHS	Vector	Component
1	nx	$c_1c_{234}c_5 + s_1s_5$		X
2	ny	$s_1s_{234}c_5 - c_1s_5$	normal vector	Y
3	nz	$s_{234}c_5$		Z
4	ox	$s_1c_5 - c_1c_{234}s_5$		X
5	oy	$-s_1c_{234}s_5 - c_1c_5$	orientation vector	Y
6	oz	$-s_{234}s_5$		Z
7	ax	c_1s_{234}		X
8	ay	s_1s_{234}	approach vector	Y
9	az	$-c_{234}$		Z
10	px	$c_1(a_3c_{23} + a_2c_2 + d_5s_{234})$		X
11	py	$s_1(a_3c_{23} + a_2c_2 + d_5s_{234})$	position vector	Y
12	pz	$d_1 + a_3s_{23} + a_2s_2 - d_5c_{234}$		Z

Where are:
n : Normal vector of the hand. Assuming a parallel-jaw hand, it is orthogonal to the fingers of the robot arm.
o : Orientation vector(Sliding vector)of the hand. It is pointing in the direction of the finger motion as the gripper opens and closes.
a : Approach vector of the hand. It is pointing in the direction normal to the palm of the hand (i.e., normal to the tool mounting plate of the arm).
p : Position vector of the hand. It points from the origin of the base coordinate system to the origin of the hand coordinate system, which is usually located at the centre point of the fully closed fingers.
The orientation of the hand is described according to the Euler (RPY) rotation as:

The general position vector(the tool-tip position) of Lab_volt R5150 is given by ,

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} c_1(a_3c_{23} + a_2c_2 + d_5s_{234}) \\ s_1(a_3c_{23} + a_2c_2 + d_5s_{234}) \\ d_1 + a_3s_{23} + a_2s_2 - d_5c_{234} \end{bmatrix} \ldots\ldots\ldots \quad (3)$$

where

$$C_1 = Cos\theta_1 \ , \ S_1 = Sin\theta_1 \ , \ C_{23} = Cos(\theta_2 + \theta_3) \ and \ S_{23} = Sin(\theta_2 + \theta_3)$$

Inverse Kinematic Model:

Inverse Kinematics (IK) analysis determines the joint angles for desired position and orientation of the robot end-effect in Cartesian space, each joint position must be known to obtain the necessary robot motion that achieves the desired end-effect location. The general steps of position control of robot arm are illustrated in Figure (3) as block diagram.

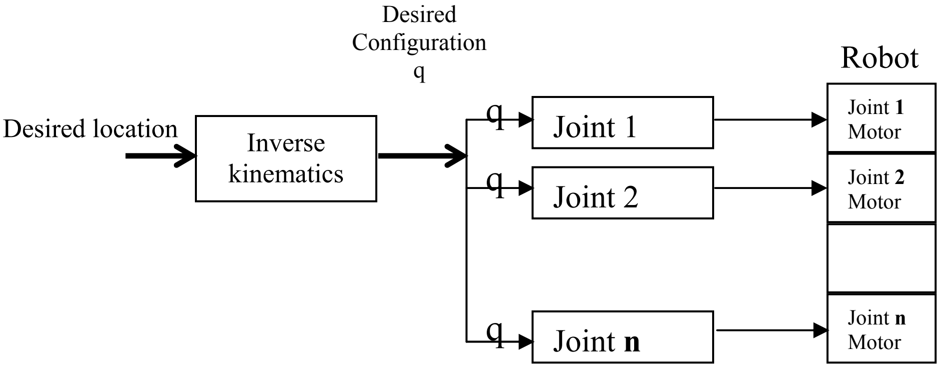


Figure (3): location Control of a Robot

Proposal Approach for Inverse Kinematics:

Inverse Kinematics analysis determines the joint angles for desired position and orientation in Cartesian space. Total transformation matrix equation will be used to calculate inverse kinematics equations. Its solution, however, is much more complex than direct kinematics since there is no unique analytical solution. Each manipulator needs a particular method considering the system structure and restrictions [10]. In the inverse kinematics the user specifies the desired goal position of the end-effector in cartesian space as (x, y, z) where z is the height of the end effector From figure (4) θ_1 can be calculated from the following equation

θ_1 = atan2 (py,px).

The lengths d_1, a_2, a_3 and d_5 correspond to the shoulder height, upper arm length, forearm length and gripper length, respectively are constant. The angles θ_1, θ_2, θ_3, θ_4 and θ_5 correspond to waist rotation, upper arm, forearm, wrist, and end-effector, respectively. These angles are updated as the specified location in space changes. We solve for the joint angles of the arm, θ1:5 given desired position (x, y, and z) and orientation (Pitch, Roll).The geometric approach used to solve for these angles, looking at figure (5) concluding that the relationship between θ_2 , θ_3 ,θ_4 and γ as shown below.

$$\gamma = \theta_1 + \theta_2 + \theta_3 \qquad \ldots(4)$$

Also the calculation of γ can be calculated from the equation 2 as shown below

$\cos(\gamma) = T(3,3)$ ……(5)

$$\sin(\gamma) = a\tan 2(\sqrt{1-\cos(\gamma)^2}\ , -\cos(\gamma))$$

$rw = rg - d5\cos(\gamma)$

$zw = zg - d5\sin(\gamma)$

Or

$rw = a2\cos(\theta_2) + a3\cos(\theta_2 + \theta_3)$

$zw = a2\sin(\theta_2) + a3\sin(\theta_2 + \theta_3)$

$$\cos(\theta_3) = \frac{(zw-d1)^2 + rw^2 - a2^2 - a3^2}{2a2a3}$$

$$\sin(\theta_2) = \sqrt{1-\cos(\theta_3)^2}$$

$$\theta_3 = a\tan 2(\sin(\theta_3), \cos(\theta_2)) \qquad \text{…..(6)}$$

$$\theta_2 = a\tan 2((zw-d1), rw) - a\tan 2(a3\sin(\theta_3), a2 + a3\cos(\theta_3)) \qquad \text{…..(7)}$$

$$\theta_4 = \gamma - \theta_2 - \theta_3 \qquad \text{…(8)}$$

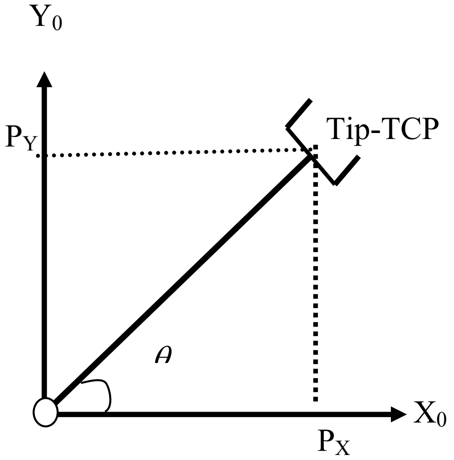


Figure (4): Geometric Analysis

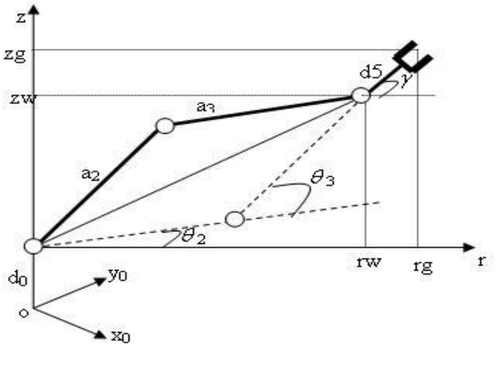


Figure (5): Arm Articulator planar view of robot

VALIDATION:

In order to verify and demonstrate the proposal algorithm to control the end-effector, addition works are performed. The algorithm is implementing by Matlab program to computation time and solution accuracy. Close-loop process is embedded in algorithm , as shown in figure (6).Simulations were conducted using Matlab Robotics Toolbox on an Intel (R) CPU T2080 @ 0.99GHz, 1.00GB Memory (RAM), 32bit Operating System.

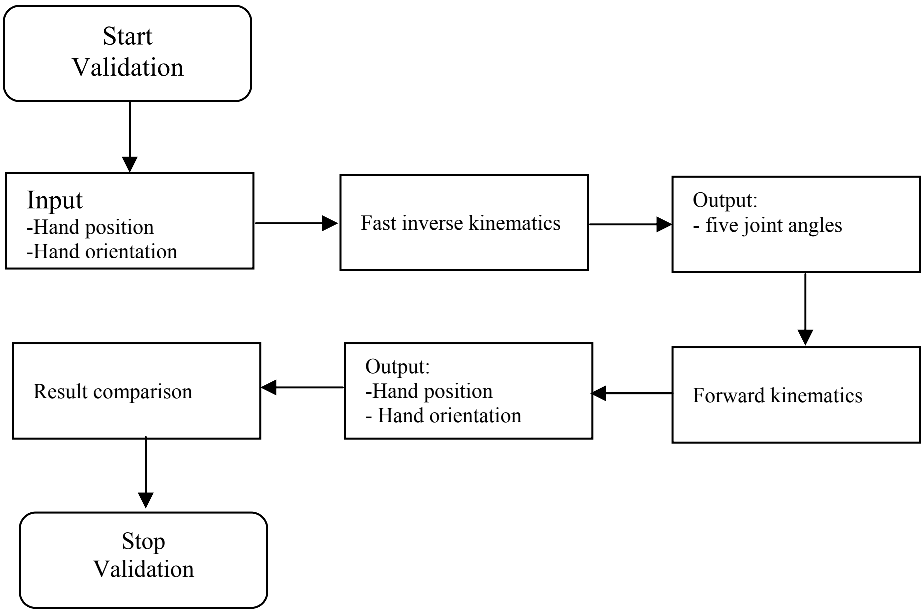


Figure (6): Validation process for the inverse kinematics algorithm.

The adopted program which is based on developed model and I.K algorithm has been used to determine the joint variables for two locations in Cartesian space, through taking the required Cartesian information of target position (Tip-TCP) which is (X, Y, Z) with respect to the base coordinate frame and the orientation of the gripper rotates (pitch angle, roll angle) which are the rotate angles of gripper about pitch and roll rotating axis respectively. There are two set of the joint variables (solutions) of the inverse kinematics of robot.

Results:

For given coordinates of the origin and the destination point's of the end-effector and known values of geometric parameters of the robot find out the joint angles.

Case study No. 1

	Cartesian coordinates			**orientation**	
target	**px(mm)**	**py(mm)**	**pz(mm)**	**pitch**	**roll**
1	**38**	**363**	**83**	**180°**	**-180°**

Knowing the following values:

d1 =255.55 mm; a2 = 190 mm; a3 = 190 mm; d5 = 115 mm; $\theta_{234} = 0^{o}$; $\theta_5 = 0^{o}$;

Analytical solution

	joint coordinates				
target	θ_1	θ_2	θ_3	θ_4	θ_5
1	**84.0239**	**4.5444**	**-27.0099**	**22.4655**	**0**

The final matrix $^{0}T_5$ **is**

$$\begin{bmatrix} 0.1041 & 0.9946 & 0 & 38 \\ 0.9946 & -0.1041 & 0 & 363 \\ 0 & 0 & -1 & 83 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots (9)$$

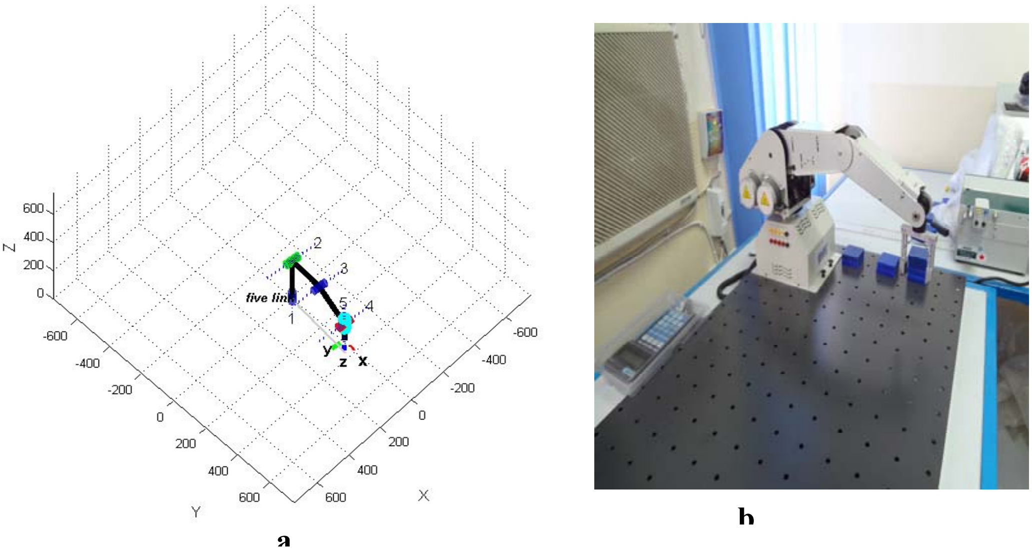


Figure (7): a-Simulation Plot and b- Physical Model for Case Study No. 1

Case study No. 2

	Cartesian coordinates			orientation	
target	**px(mm)**	**py(mm)**	**pz(mm)**	**pitch**	**roll**
2	**320**	**320**	**216**	**135°**	**-90°**

Knowing the following values:

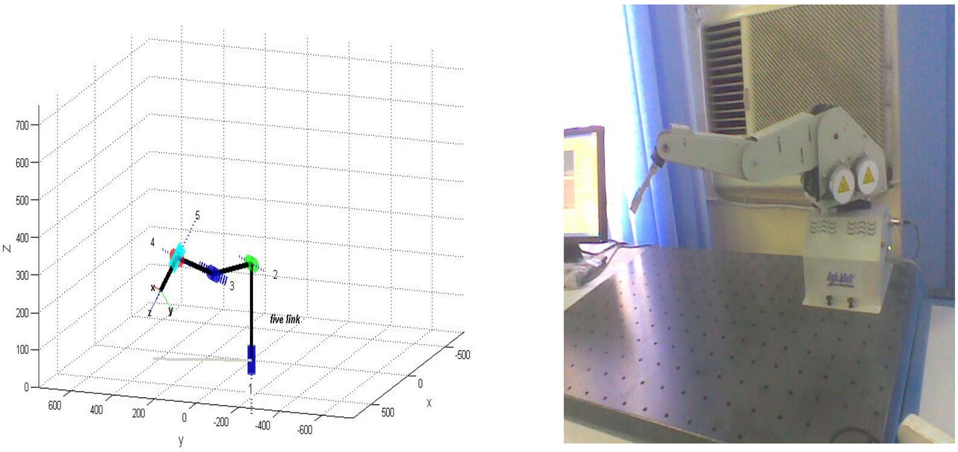
d1 =255.55 mm; a2 = 190 mm; a3 = 190 mm; d5 = 115 mm; $\theta_{234} = 45^{o}$; $\theta_5 = 90^{o}$;

Analytical solution

	joint coordinates				
target	θ_1	θ_2	θ_3	θ_4	θ_5
2	**45**	**-4.1331**	**21.1049**	**28.0282**	**90**

The final matrix $^0T_5$ is

$$\begin{bmatrix} 0.7071 & -0.5 & 0.5 & 320 \\ -0.7071 & -0.5 & 0.5 & 320 \\ 0 & -0.7071 & -0.7071 & 216 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{............ (10)}$$



a

b

Figure (8): a-Simulation Plot and b-Physical Model for Case Study No. 2

Comparisons between the desired and measured position values for robot arm are shown in the following tables.

The value of percentage error is calculated using the formula:-

$$percentage\ error = \left| \frac{measured\ value - desired\ value}{measured\ value} \right| x100\ \% \qquad \text{..... (11)}$$

Table (4): Case study No .1

position values	desired values (mm)	measured values (mm)	Percentage Error
X	38	37.5	≈ 1%
Y	363	361.46	≈ 0.4%
Z	83	79.25	≈ 4.7%

Table (5): Case study No .2

position values	desired values (mm)	measured values (mm)	Percentage Error
X	320	318.29	≈ 0.5%
Y	320	319.36	≈ 0.2%
Z	216	212.14	≈ 1.78%

CONCLUSION:

The inverse kinematics analysis of 5DOF Lab_Volt R5150 robot manipulator is investigated. From difference between desired and actual values of Lab_Volt R5150 robot arm manipulator, the desired values are checked against the physical position of the robot manipulator .When desired values xyz coordinates of target position are compared with the measured values of the position of the end-effector of the robot along the X, Y and Z coordinates due to angular position of each of the 5 joints as output to of the adopted inverse kinematics algorithm shown in tables(4) and (5), it is observed that the values were very close. However there are some deviations in (x, y, z) directions. This error is normal and we can eliminate this error by applying any special controller, but in z direction the error is large and we can eliminate this error by analyzing the dynamic modeling for robot arm. All angles are commutated and the time for solving kinematics of the manipulator is computed using adopted algorithm with check output by forward analysis which is run by Matlab 10.0. From the experimental runs it could be concluded that very small time is needed for the calculation of the joints variable (0.001741s), therefore this algorithm is very good to use for real robot manipulator control system. The desired values and orientate of two cases are checked offline with visual plot of the robot configuration according to a set of solutions to angular position of each of the 5 joints in Matlab environment and are compared with physical configuration of real robot manipulator.

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