Iraqi Journal of Science
ISSN: 0067-2904

Nucleon momentum distributions and elastic electron scattering form factors for ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes

Ahmed N. Abdullah*
Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq

Abstract:
In the framework of correlation method so-called coherent density fluctuation model (CDFM) the nucleon momentum distributions (NMD) of the ground state for some even mass nuclei of *fp*-shell like ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes are examined. Nucleon momentum distributions are expressed in terms of the fluctuation function ($|f(x)|^2$) which is evaluated by means of the nucleon density distributions (NDD) of the nuclei and determined from theory and experiment. The main characteristic feature of the NMD obtained by CDFM is the existence of high-momentum components, for momenta $k \geq 2\ fm^{-1}$. For completeness, also elastic electron scattering form factors, F(q) are evaluated within the same framework.

Keywords: Nucleon density distributions, Nucleon momentum distributions, Elastic electron scattering, Coherent density fluctuation model.

توزيعات زخم النيكليون وعوامل التشكل للاستطارة الالكترونية المرنة لنظائر الكروم ^{50}Cr، ^{52}Cr و ^{54}Cr

أحمد نجم عبدالله*
قسم الفيزياء ، كلية العلوم، جامعة بغداد، بغداد، العراق

الخلاصة
تم استخدام أنموذج تموج الكثافة المترابط في حساب توزيعات زخم النيكليون (NMD) للحالة الارضية لبعض النوى الزوجية الواقعة ضمن القشرة النووية *fp* مثل نظائر الكروم ^{50}Cr، ^{52}Cr و ^{54}Cr. لقد تم التعبير عن NMD بدلالة دالة التموج ($|f(x)|^2$) والتي تحسب من خلال النتائج النظرية والعملية لتوزيعات كثافة النيكليون. تميزت نتائج توزيعات زخم النيكليون بخاصية الذيل الطويل عند قيم الزخوم العالية. في هذه الدراسة تم ايضاً حساب عوامل التشكل للاستطارة الالكترونية المرنة لهذه النوى.

1. Introduction

The systematic investigations of the nucleon momentum distributions in nuclei extend the scope of the nuclear ground-state theory. Until the mid-seventies more attention in the theory had been paid to the study of quantities such as the binding energy and the nuclear density distribution $\rho(r)$. This is related to the ability of the widely used Hartree-Fock theory to describe successfully these quantities, which, however, are not very sensitive to the dynamical short-range correlations. The experimental situation in recent years concerning the interaction of particles with nuclei at high energies, in particular the nuclear photo effect, meson absorption by nuclei, inclusive proton production in proton-nucleus collisions, and even some phenomena at low energies such as giant multipole resonances, makes it possible to study additional quantities. One of them is the nucleon momentum distribution $n(k)$ [1,2] which is specifically related to the processes mentioned above. However, it has been shown [3] that, in principle, it is impossible to describe correctly both momentum and density distributions simultaneously in the Hartree-Fock theory. The reason is that the nucleon momentum distribution is sensitive to short-range and tensor nucleon-nucleon correlations. It reflects the peculiarities of the

*Email:Ahmednajim1979@yahoo.com

nucleon-nucleon forces at short distances which are not included in the Hartree-Fock theory. This requires a correct simultaneous description of both related distributions ρ (*r*) and *n*(*k*) in the framework of nuclear correlation methods.

The main characteristic feature of the nucleon momentum distribution obtained by various correlation methods [1,2,4-7] is the existence of high-momentum components, for momenta $k \geq 2$ fm^{-1}, due to the presence of short-range and tensor nucleon correlations. This feature of n(k) has been confirmed by the experimental data on inclusive and exclusive electron scattering on nuclei. In general, the knowledge of the momentum distribution for any nucleus is important for calculations of cross-sections of various kinds of nuclear reactions. The coherent density fluctuation model (CDFM) has been suggested in [1,2] as a model for studying characteristics of nuclear structure and nuclear reactions based on the local density distribution as a variable of the theory and using the essential results of the infinite nuclear matter theory.

Hamoudi *et al*. [8-10] have studied the NMD and elastic electron scattering form factors for *p*-shell [8], *sd*-shell [9] and *fp*- shell [10] nuclei using the framework of CDFM. They [8, 9,10] derived an analytical form for the NDD based on the use of the single particle harmonic oscillator wave functions and the occupation number of the states. The derived NDD's, which are applicable throughout the whole *p*-shell [8], *sd*-shell [9] and *fp*- shell [10] nuclei, have been used in the CDFM. The calculated NMD and elastic form factors of all considered nuclei have been in very good agreement with experimental data.

In the present study, we follow the work of Hamoudi *et al*. [8-10] and utilize the CDFM with weight functions originated in terms of theoretical NDD of some fp-shell nuclei such as ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes. It is found that the theoretical weight function ($|f(x)|^2$) based on the derived NDD is capable to give information about the NMD and elastic charge form factors as do those of the experimental data [11].

2. Theory

The nucleon density distribution NDD of one body operator can be written as [8]:

$$\rho(r) = \frac{1}{4\pi}\sum_{n\ell}\xi_{n\ell}\,4(2\ell+1)\,|R_{n\ell}|^2 \qquad \textbf{(1)}$$

Where $\xi_{n\ell}$ is the nucleon occupation probability of the state $n\ell$ ($\xi_{n\ell}=0$ or 1 for closed shell nuclei and $0<\xi_{n\ell}<1$ for open shell nuclei) and $R_{n\ell}$ is the radial part of the single particle harmonic oscillator wave function.

The NDD form of *Cr*-isotopes is derived on the assumption that there are filled 1*s*, 1*p* and 1*d* orbitals and the nucleon occupation numbers in 2*s*, 1*f* and 2*p* orbitals are equal to, respectively, ($4-\alpha_1$), α_2 and ($A-40-\alpha_2+\alpha_1$) and not to 4, (A-40) and 0 as in the simple shell model. Using this assumption in Eq. (1), an analytical form for the ground state NDD of *Cr*-isotopes is obtained as:

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{10-\frac{3}{2}\alpha_1+\left[\frac{11}{3}\alpha_1+\frac{5}{3}(A-40-\alpha_2)\right]\left(\frac{r}{b}\right)^2+\left[8-2\alpha_1-\frac{4}{3}(A-40-\alpha_2)\right]\left(\frac{r}{b}\right)^4 + \left[\frac{8}{105}\alpha_2+\frac{4}{15}(A-40-\alpha_2)+\frac{4}{15}\alpha_1\right]\left(\frac{r}{b}\right)^6\right\} \qquad (2)$$

where *A* is the nuclear mass number, *b* is the harmonic oscillator size parameter, the parameter α_1 characterizes the deviation of the nucleon occupation numbers from the prediction of the simple shell model (α_1=0). The parameter α_2 in Eq. (2) is assumed as a free parameter to be adjusted to obtain agreement with the experimental NDD.

The normalization condition of the $\rho(r)$ is given by [12]

$$A = 4\pi\int_0^\infty \rho(r) r^2 dr \qquad \textbf{(3)}$$

and the mean square radius (MSR) of the considered nuclei is given by [12]

$$\langle r^2\rangle = \frac{4\pi}{A}\int_0^\infty \rho(r) r^4 dr \qquad \textbf{(4)}$$

The central NDD, $\rho(r=0)$ is obtained from Eq. (2) as

$$\rho(0)=\frac{1}{\pi^{3/2}b^3}\left(10-\frac{3}{2}\alpha_1\right) \quad \textbf{(5)}$$

then α_1 is obtained from Eq. (5) as

$$\alpha_1=\frac{2}{3}\left(10-\rho(0)\pi^{3/2}b^3\right) \quad \textbf{(6)}$$

Substituting Eq. (2) into Eq. (4) and after simplification gives:

$$\langle r^2\rangle=\frac{b^2}{A}\left(\frac{9A-120}{2}+\alpha_1\right) \quad \textbf{(7)}$$

In Eq's (5) and (7), the values of the central density $\rho(0)$ and $\langle r^2\rangle$ are taken from the experiments while the parameter *b* is chosen in such a way as to reproduce the experimental root mean square radii of nuclei.

The NMD, *n*(*k*), of the considered nuclei is studied using two distinct methods. In the first, it is determined by the shell model using the single particle harmonic oscillator wave functions in momentum representation and is given by [10]:

$$n(k)=\frac{b^3}{\pi^{3/2}}e^{-b^2k^2}\left[10+8(bk)^4+\frac{8(A-40)}{105}(bk)^6\right] \quad \textbf{(8)}$$

k is the momentum of the particle.
In the second method, the NMD is determined by the Coherent Density Fluctuation Model (CDFM), where the mixed density is given by [1,2]

$$\rho(r,r')=\int_0^\infty |f(x)|^2\rho_x(r,r')dx \quad \textbf{(9)}$$

where:

$$\rho_x(r,r')=3\rho_0(x)\frac{j_1(k_F(x)|\bar{r}-\bar{r}'|)}{k_F(x)|\bar{r}-\bar{r}'|}\theta(x-\frac{1}{2}|\bar{r}+\bar{r}'|) \quad \textbf{(10)}$$

is the density matrix for *A* nucleons uniformly distributed in a sphere with radius *x* and density $\rho_0(x)=3A/4\pi x^3$. The Fermi momentum is defined as [1,2]:

$$k_F(x)=\left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3}\equiv\frac{V}{x}; \qquad V=\left(\frac{9\pi A}{8}\right)^{1/3} \quad \textbf{(11)}$$

and the step function θ, is defined by

$$\theta(y)=\begin{cases}1, & y\geq 0\\ 0, & y<0\end{cases} \quad \textbf{(12)}$$

The diagonal element of Eq. (9) gives the one-particle density as

$$\rho(r)=\rho(r,r')\big|_{r=r'}=\int_0^\infty |f(x)|^2\rho_x(r)dx \quad \textbf{(13)}$$

In eq. (13), $\rho_x(r)$ and $|f(x)|^2$ have the following forms [1,2]:

$$\rho_x(r)=\rho_0(x)\theta(x-r) \quad \textbf{(14)}$$

$$|f(x)|^2=\frac{-1}{\rho_0(x)}\frac{d\rho(r)}{dr}\bigg|_{r=x} \quad \textbf{(15)}$$

The weight function of Eq. (15), determined in terms of the NDD satisfies the following normalization condition [1,2]

$$\int_0^\infty |f(x)|^2dx=1, \quad \textbf{(16)}$$

and holds for monotonically decreasing density NDD distribution, i.e. $\frac{d\rho(r)}{dr} < 0.$

On the basis of eq. (13), the NMD, $n(k)$, is expressed as [1,2]:

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k)dx, \quad \textbf{(17)}$$

where

$$n_x(k) = \frac{4}{3}\pi x^3 \theta(k_F(x) - |\vec{k}|), \quad \textbf{(18)}$$

is the Fermi-momentum distribution of the system with density $\rho_0(x)$. By means of Eqs. (15), (17) and (18), an explicit form for the NMD is expressed in terms of $\rho(r)$ as

$$n_{CDFM}(k) = \left(\frac{4\pi}{3}\right)^2 \frac{4}{A} \int_0^{V/k} \left[6\rho(x)x^5 dx - \left(\frac{V}{k}\right)^6 \rho\left(\frac{V}{k}\right)\right], \quad \textbf{(19)}$$

with normalization condition

$$A = \int n_{CDFM}(k) \frac{d^3k}{(2\pi)^3} \quad \textbf{(20)}$$

The elastic monopole form factor $F(q)$ of the target nucleus is also expressed in the CDFM as [1,2]:

$$F(q) = \frac{1}{A}\int_0^\infty |f(x)|^2 F(q,x)dx \quad \textbf{(21)}$$

where $F(q,x)$ is the form factor of uniform charge density distribution given by:

$$F(q,x) = \frac{3A}{(qx)^2} \cdot \left[\frac{\sin(qx)}{(qx)} - \cos(qx)\right] \quad \textbf{(22)}$$

Inclusion the corrections of the nucleon finite size $F_{fs}(q)$ and the center of mass corrections $F_{cm}(q)$ in the calculations requires multiplying the form factor of equation (22) by these corrections. Here, $F_{fs}(q)$ is considered as free nucleon form factor which is assumed to be the same for protons and neutrons. This correction takes the form [12]:

$$F_{fs}(q) = e^{\left(\frac{-0.43q^2}{4}\right)} \quad \textbf{(23)}$$

The correction $F_{cm}(q)$ removes the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [12]:

$$F_{cm}(q) = e^{\left(\frac{b^2q^2}{4A}\right)} \quad \textbf{(24)}$$

It is important to point out that all physical quantities studied above in the framework of the CDFM such as NMD and $F(q)$, are expressed in terms of the weight function ($|f(x)|^2$). Therefore, it is worthwhile trying to obtain the weight function firstly from the NDD of two- parameter Fermi (2PF) model extracted from the analysis of elastic electron-nuclei scattering experiments and secondly from theoretical considerations. The NDD of 2PF is given by [13]

$$\rho_c(r) = \frac{\rho_0}{1 + e^{(r-c)/z}} \quad \textbf{(25)}$$

Introducing Eq. (25) into Eq. (15), we obtain the experimental weight function $|f(x)|^2_{2PF}$ as

$$|f(x)|^2_{2PF} = \frac{4\pi x^3 \rho_0}{3Az}\left(1 + e^{\frac{x-c}{z}}\right)^{-2} e^{\frac{x-c}{z}} \quad \textbf{(26)}$$

Moreover, introducing the derived NDD of Eq. (2) into Eq. (15), we obtain the theoretical weight function $|f(x)|^2$ as

$$|f(x)|^2_{th} = \frac{8\pi x^4}{3Ab^2}\rho(x) - \frac{16x^4}{3A\pi^{1/2}b^5}\left\{\left[\frac{11}{6}\alpha_1 + \frac{5}{6}(A-40-\alpha_2)\right] + \left[8-2\alpha_1 - \frac{4}{3}(A-40-\alpha_2)\right]\left(\frac{x}{b}\right)^2 + \left[\frac{4}{35}\alpha_2 + \frac{2}{5}(A-40-\alpha_2) + \frac{2}{5}\alpha_1\right]\left(\frac{x}{b}\right)^4\right\}e^{-x^2/b^2} \quad (27)$$

3. Results and Discussion

The nucleon momentum distributions, $n(k)$, and elastic form factors, F(q), in nuclei like ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes are explicitly calculated within CDFM. In order to calculate the $n(k)$, obtained from Eq. (19), we need to investigate the NDD for both experiment, such as, 2PF [13] and theoretical consideration using Eq.(2), which includes some parameters needed for calculations. These parameters have been calculated and presented in Table -1 together with other parameters employed for the selected isotopes used in the present work. The parameter α_1 is determined by introducing the harmonic oscillator size parameter b, chosen such that to reproduce the measured root mean square radius (rms), and the experimental central density $\rho_{\exp}(0)$ into Eq. (6), while the parameter α_2 is assumed as a free parameter to be adjusted to obtain agreement with the experimental NDD. For comparison the calculated rms radius $\langle r^2\rangle^{1/2}_{cal}$ and the experimental one $\langle r^2\rangle^{1/2}_{\exp}$ are also displayed in Table-1. A remarkable agreement has been shown for all considered nuclei. The (4-α_1), α_2 and (A-40-α_2+α_1) nucleon occupations numbers for 2*s*, 1*f* and 2*p* orbitals, respectively, have been also calculated and tabulated in Table-2.

Table 1- Parameters for the NDD of considered isotopes together with $\langle r^2\rangle^{1/2}_{cal}$ and $\langle r^2\rangle^{1/2}_{\exp}$

Nuclei	2PF [13]		$\rho_{\exp}(0)$ (fm^{-3}) [13]	$\langle r^2\rangle^{1/2}_{cal}$	$\langle r^2\rangle^{1/2}_{\exp}$ [13]	*b* (*fm*)	α_1	α_2
	c (*fm*)	*z* (*fm*)						
^{50}Cr	3.941	0.566	0.16185	3.707	3.707	2.031	1.62998	9.1
^{52}Cr	4.01	0.497	0.16707	3.685	3.684	2.005	1.66472	11.2
^{54}Cr	4.01	0.578	0.16573	3.777	3.776	2.044	1.40939	12.3

Table 2- Calculated occupation numbers of 2s, 1f and 2p orbitals of the considered isotopes

Nuclei	Occupation No. of 2*s* (4-α_1)	Occupation No. of 1*f* (α_2)	Occupation No. of 2*p* (A-40-α_2+α_1)
^{50}Cr	2.37001	9.1	2.52998
^{52}Cr	2.33527	11.2	2.46472
^{54}Cr	2.59060	12.3	3.10939

The NDD calculations for ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes obtained using Eq. (2) with $\alpha_1=\alpha_2=0$ (blue curve) and $\alpha_1\neq\alpha_2\neq 0$ (red curve) are presented in Figure-1 along with experimental data [8] denoted by the filled circle symbols. This figure shows that considering the parameters α_1 and α_2 in the calculations leads to a satisfactory results for NDD between the red curves and the experimental data of 2PF (the filled circle symbols). It also shows a poor agreement between the blue curves and the experimental data especially in the region of small *r*, i.e., $0 \leq r \leq 2.5$ fm.

Figure-2 shows the $n(k)$ (in fm^3) versus k (fm^{-1}) for the selected isotopes calculated with shell model using single particle harmonic oscillator wave function in momentum space. The experimental and theoretical $n(k)$ obtained by CDFM, using experimental and theoretical NDD in Eq. (19), have been also presented in this figure. It is clearly seen that the calculated $n(k)$ distributions using shell model has a steep slope behavior, which are in disagreement with the studies [1, 2, 14-16]. This disagreement refer to the fact that the ground state shell model Slater determinate wave function does not take into account the important effect of the short range dynamical correlation function which is responsible for the behavior of n(k) in the high momentum [15, 16]. The calculated $n(k)$ obtained by CDFM for the interested nuclei are much closer to the experimental data than the shell model calculation. The CDFM corrected the steep slope behavior of the $n(k)$ curves to a long tail manner for momenta k ≥ 2 fm^{-1}. The property of long-tail manner obtained by CDFM is connected to the presence of high densities ρ_x(r) in the decomposition of Eq. (14), though their fluctuation function $|f(x)|^2$ are small.

The elastic electron scattering form factors for the considered isotopes are calculated in the framework of the CDFM by introducing the theoretical weight functions of the Eq. (27) into Eq. (21). The calculated form factors (solid curves) are plotted versus q as shown in Figure-3 for ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes where the filled circle symbols are the experimental data [11]. This figure shows that the experimental form factors of these nuclei are in a good agreement with those of calculated result up to momentum transfer $q \approx 1$ fm^{-1}, whereas for $q > 1$ fm^{-1} the calculated form factors underestimate these experimental data.

4. Conclusions

The nucleon momentum distribution can be easily calculated by means of the weight function which is extracted from experiment and theory. It was shown that the nucleon momentum distribution is close to the other studies [14-16] and has a strongly expressed high-momentum tail. High-momentum components of the distribution $n(k)$ in CDFM are related to the existence of high densities $\rho_x(r)$ in the decomposition of Eq. (14), though their fluctuation function $|f(x)|^2$ are small.

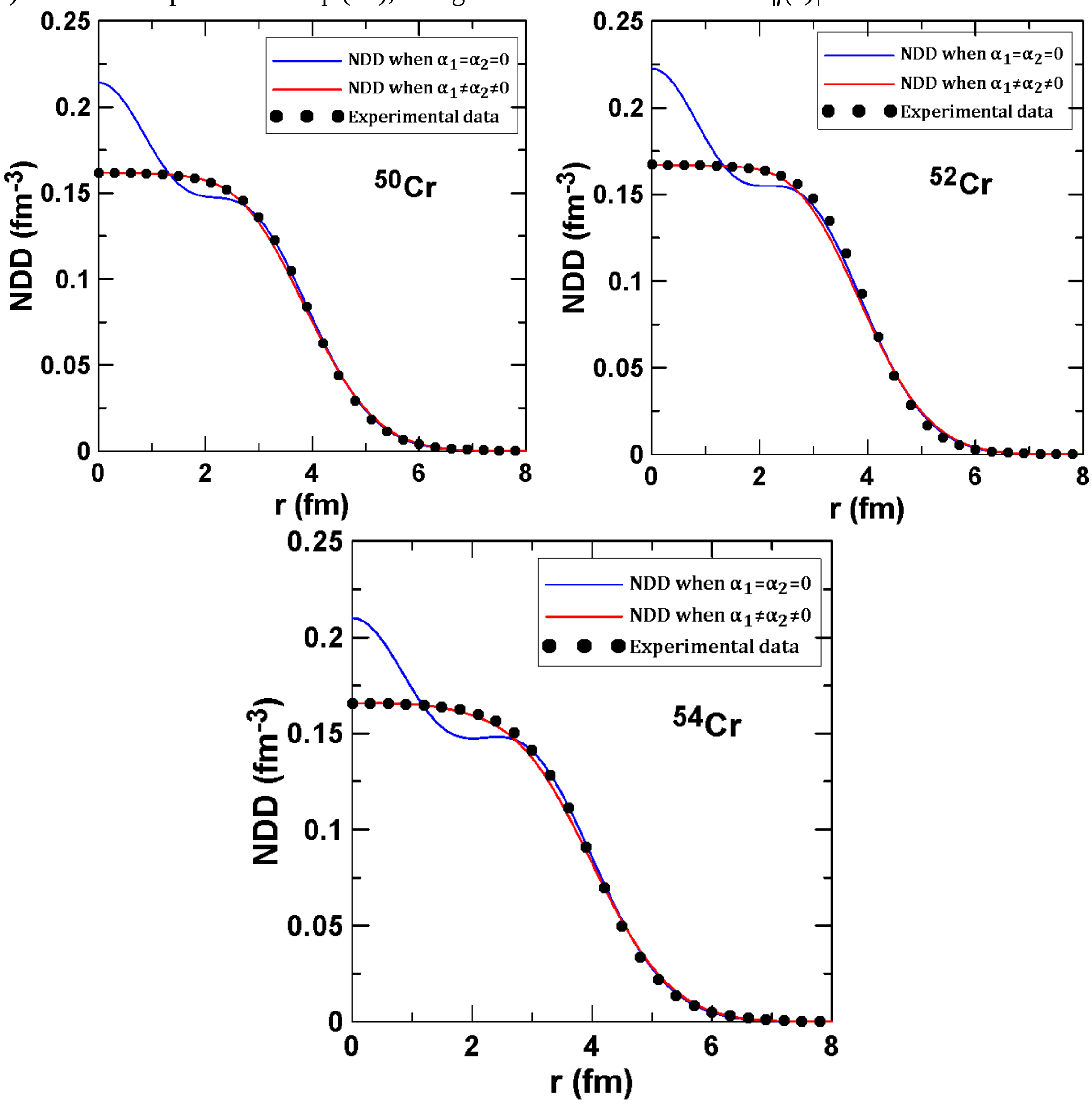

Figure 1- The NDD versus r for ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes. The blue and red curves are the calculated NDD of Eq. (2) when α_1=α_2=0 and $\alpha_1 \neq \alpha_2 \neq 0$, respectively. The filled circle symbols are the experimental data taken from ref. [13].

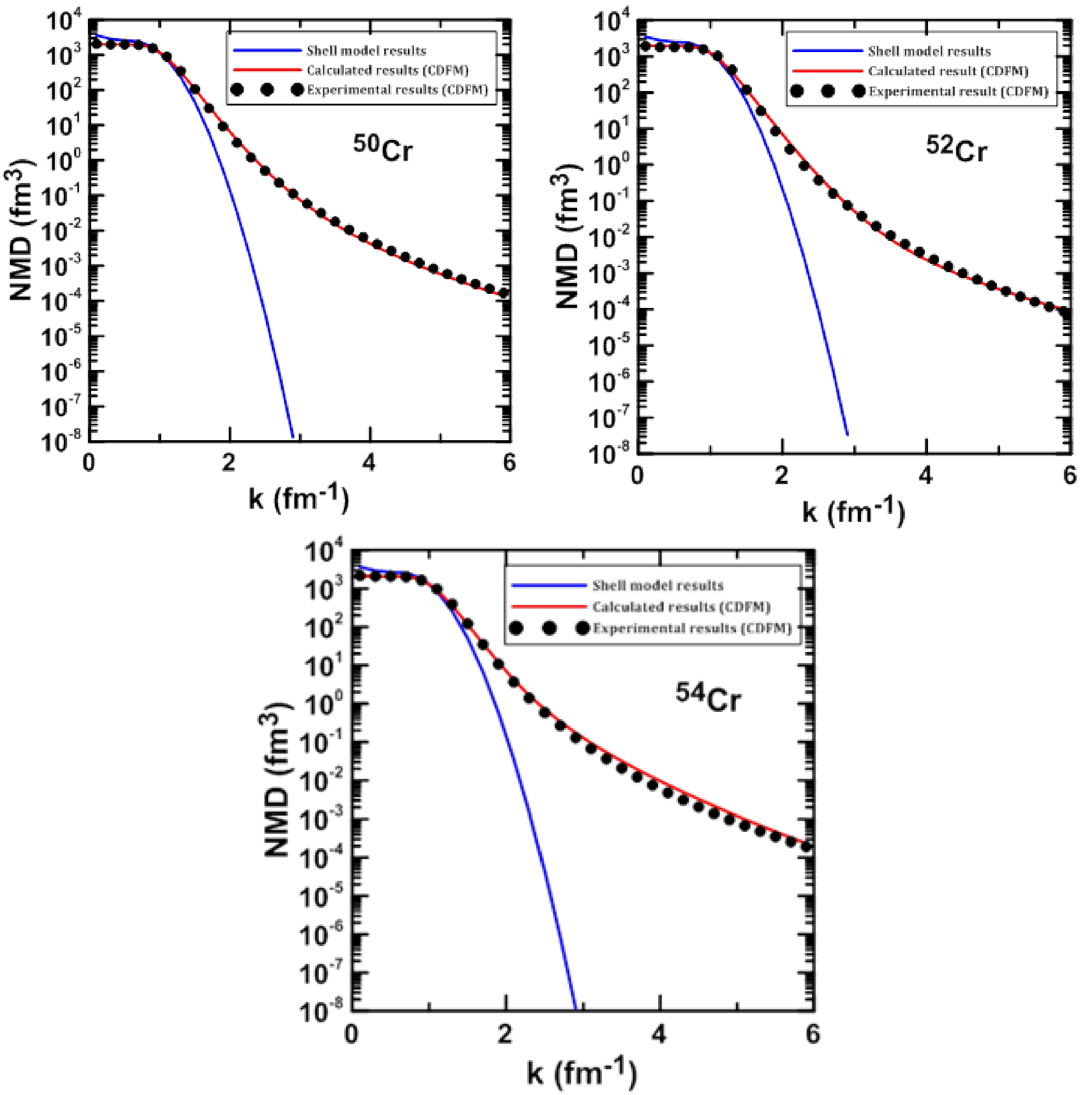

Figure 2- The NMD versus k for ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes. The red curves and filled circle symbols are the calculated NMD expressed by the CDFM of Eq. (19) using the theoretical NDD of Eq. (2) and the experimental data of ref. [13]**,** respectively. The blue curves are the calculated NMD of Eq. (8**)** obtained by the shell model calculation using the single-particle harmonic oscillator wave functions in momentum representation.

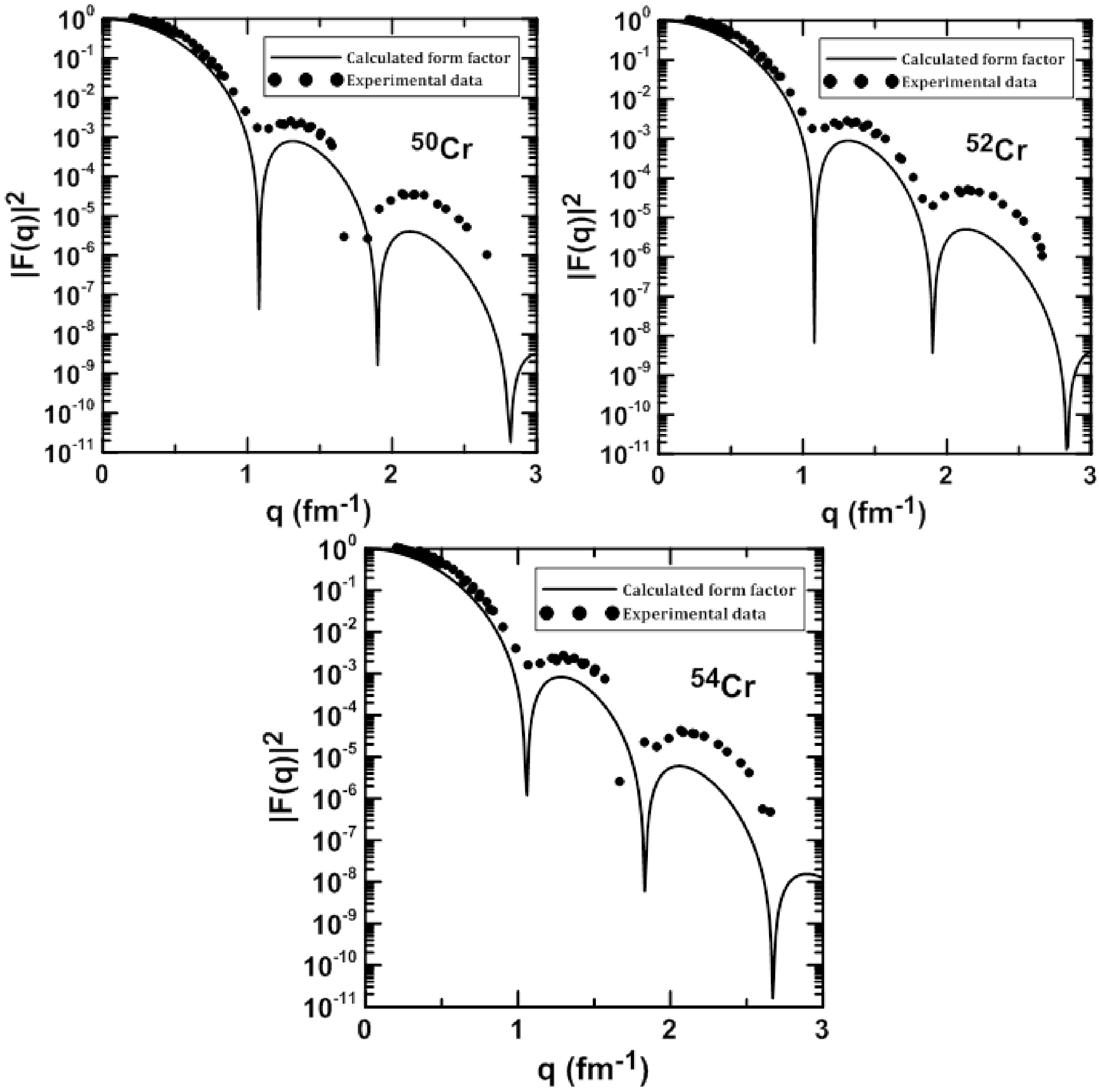

Figure 3-The elastic form factors for ^{50}Cr, ^{52}Cr and ^{54}Cr isotopes. The solid curves are the form factors calculated using Eq. (21). The filled circle symbols are the experimental data taken from ref. [11].

References

1. Antonov A. N., Hodgson P. E. and Petkov I. Z.**1988**. *Nucleon momentum and density distribution in nuclei*. Clarendon, Oxford, pp:1-165.
2. AntonovA. N., Hodgson P. E. and Petkov I. Z. **1993**. *Nucleon correlation in nuclei* Springer-Verlag, Berlin–Heidelberg–New York.
3. Jaminon M., Mahaux C. and Ngo H. **1985**. Inability of any Hartree-Fock approximation to reproduce simultaneously the density and momentum distributions of nuclei. *Phys. Lett.,* 158 B, p:103.
4. Amado R.D. **1976**. Momentum distributions in the nucleus. *Physical Review,* C 14, p:1264.
5. J.G. Zabolitzky, W. Ey. **1978**. Momentum distributions of nucleons in nuclei**.** *Phys. Lett.,* 76 B, p:527.
6. Antonov A. N., Nikolaev V.A., and Petkov I. Z. **1980**. Nucleon momentum and density distributions of nuclei. *Z. Physik A*, 297, pp: 257-260.
7. Antonov A.N., Bonev I.S., Christov C.V. and Petkov I. Z. **1988**. Generator Coordinate Calculations of Nucleon Momentum and Density Distributions in ^{4}He, ^{16}O and ^{40} Ca. *Nuovo Cimento* A100, pp: 779-788.

8. Hamoudi A. K., Hasan M. A. and Ridha A. R. **2012**. Nucleon momentum distributions and elastic electron scattering form factors for some 1p-shell nuclei, *Pramana Journal of Physics* (Indian Academy of Sciences), 78(5), pp: 737-748.

9. Hamoudi A. K., Flaiyh G. N. and Mohsin S. H. **2012**. Nucleon Momentum Distributions and Elastic Electron Scattering Form Factors for some sd-shell Nuclei, *Iraqi Journal of Science*, 53(4), pp:819-826.

10. Hamoudi A. K and Ojaimi H. F. **2014**. Nucleon momentum distributions and elastic electron scattering form factors for ^{58}Ni, ^{60}Ni, ^{62}Ni, and ^{64}Ni isotopes using the framework of coherent fluctuation model, *Iraqi Journal of Physics*, 12(24), pp:33-42.

11. Lightbody J. W., Bellicard J. B., Cavedon J. M., Frois B., Goutte D., Huet M., Leconte P., Nakada A., Ho P. X., Platchkov S. K., Turck-Chieze S., de Jager C. W., Lapikas J. J. and de Witt Huberts P.K. A. **1983**. Elastic and inelastic electron scattering from ^{50,52,54}Cr . *Physical Review*, C27, pp: 113-132.

12. Hamoudi A. K. **2010**.Nucleon Momentum Distributions and Elastic Electron Scattering Form Factors for some sd-shell Nuclei. *Journal of Al-Nahrain University,* 13(2), pp: 105-113.

13. Vries H. D., Jager C.W., and Vries C. **1987**. Nuclear Charge density distribution parameters from elastic electron scattering. *Atomic Data and Nuclear Data Tables*, 36(3), pp: 495-536.

14. Moustakidis C. C. and Massen S. E. **2000**. One-body density matrix and momentum distribution in s-p and s-d shell nuclei. *Physical Review,* C62, pp:34318_1-34318_7.

15. Traini M. and Orlandini G. **1985** . Nucleon momentum distributions in doubly closed shell nuclei. *Z. Physik A*, 321, pp: 479-484.

16. Ri M. D., Stringari S. and Bohigas O. **1982**. Effects of short range correlations on one- and two-body properties of nuclei. *Nuclear Physics*, A376, pp:81-93.