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# Analysis of Boltzmann Equation for SF<sub>6</sub> and Some Gas-Mixture Discharges at Critical Field Condition

*The energy electron distribution function at a particular reduced electric field  $E/p$  (where  $E$  is the electric field and  $p$  is the gas pressure) for SF<sub>6</sub>+He and SF<sub>6</sub>+Ar mixtures has been studied. The distribution function has been calculated as a function of percentage mixture ratio ( $k$ ) ranging from zero (pure SF<sub>6</sub>) to 100% (for He or Ar) at  $(E/p)_{crit}$ . On the other stage, these distribution functions are also studied as a function of ( $k$ ) at a particular  $(E/p)_{crit}$  under which the breakdown occurs for the same gas mixtures. The  $(E/p)_{crit}$  calculation is adopted to explain the discrepancy of the distribution function for gas mixtures. It is also found that the distribution function for gas mixtures at  $(E/p)_{crit}$  and for any value of ( $k$ ) are identical.*

**Keywords:** Gas discharge; Boltzmann equation; Effective ionization coefficient

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## 1. Introduction

In the advancement of high voltage technology and applications in gas insulated substations, gas lasers, and plasma etching, sulfur hexafluoride (SF<sub>6</sub>) and its mixtures have been widely used because of the excellent electrical insulation and other physical and chemical properties [1-5]. Most theoretical studies are concerned with various properties that illustrate good understanding and detailed analysis of the electron swarm parameters for SF<sub>6</sub> and its mixtures before using them in the above applications as well as in electronic devices, stabilizing valves and spark chambers [6-8].

Under various experimental conditions and design considerations, the behavior of the electrical insulation of SF<sub>6</sub> might be changed with certain additives of other gases to form mixtures capable of sustaining higher electrical insulation and larger values of breakdown voltages [9].

The use of mixtures of SF<sub>6</sub> with other inert gases can eliminate some of the technical problems associated with pure SF<sub>6</sub> and may reduce the cost of high voltage cables and circuit breakers [10-11].

Various studies have been carried out on the electrical breakdown of SF<sub>6</sub> mixtures in different electric field configurations. Typical measurements [12] in SF<sub>6</sub>-He and SF<sub>6</sub>-Ar mixtures in uniform field have illustrated that there is a certain limit of  $(E/p)_{crit}$ , the value of which is reduced as the ionization coefficient becomes closer or equals to the attachment coefficient. Such behavior is similar to that demonstrated by pure SF<sub>6</sub>, i.e., for all values of ionization coefficient equal to the attachment coefficient, the performance of  $(E/p)_{crit}$  indicates the same tendency for SF<sub>6</sub>, SF<sub>6</sub>-He and SF<sub>6</sub>-Ar mixtures under the conditions considered there.

In order to predict the distribution function and swarm parameters of particles during the breakdown of gas mixtures, one may solve the Boltzmann equation for a particular range of  $(E/p)$  for which the mixing ratio ( $k$ ) is an important function [12-13].

The ratio of electric field to gas pressure  $E/p$  plays an important role in the mechanism of the breakdown of gases since each individual electron gains its energy from the electric field and this energy, after a certain period of time, may be higher than that lost by a terminating collision [14-16].

For non-attaching gases such as Ar and He this will result in a larger value of ionization coefficient corresponding to small increment of  $E/p$ . In the case of attaching gases such as SF<sub>6</sub>, the ionization coefficient is equal to the attachment coefficient at a certain values of  $E/p$  which is considered in this study as  $(E/p)_{crit}$ .

In the present article, the BOLSIG+ and EEDF codes were used to solve the Boltzmann equation for both SF<sub>6</sub>-Ar and SF<sub>6</sub>-He mixtures with ranges of  $E/p$  and  $(E/p)_{crit}$  that satisfy the breakdown criteria only.

## 2. Theory

Pitchford et al. [17] have solved Boltzmann equation using numerical approach. The calculation is performed over a range of energies between zero and a typically maximum value where the electron energy distribution function is sufficiently small. Therefore, the influence of electron having energies above the maximum level may be neglected.

Schoeffner and Graf [18] studied the breakdown voltage of a quasi-uniform electric field with gas-mixture insulation (SF<sub>6</sub>+N<sub>2</sub>). They adopted the Wieland approximation which involves the linear

addition method for calculating  $(E/p)_{\text{crit}}$  and swarm parameters [19].

The distribution function is calculated by using the EEDF program which solves numerically the Boltzmann equation for  $\text{SF}_6+\text{He}$  and  $\text{SF}_6+\text{Ar}$  mixtures. This requires the input data of cross-sections of these gases. At the final stage, swarm parameters are calculated by inserting the result of distribution function in the equations of swarm parameters [16].

The present calculation is carried out by varying  $E/p$  from 40 to 125 V/cm.torr for  $\text{SF}_6$ , 5 to 50 V/cm.torr for He, and 5 to 90 V/cm.torr for Ar gases to ensure that  $(E/p)_{\text{crit}}$  is in the ranges of these gas mixtures.

### 3. Results and Discussion

Hughes [20] has shown that the electron energy distribution function for He at a certain  $E/p$  shifts towards higher energy in comparison with that for  $\text{SF}_6$  which was reported by Itoh [21] at the same  $E/p$ . Figure (1) displays the distribution function as a function of energy for  $\text{SF}_6$  at  $E/p=40, 60, 80$  and  $125$  V/cm.torr (Fig.1a), He at  $E/p=5, 20, 35$  and  $50$  V/cm.torr (Fig.1b), and Ar at  $E/p=5, 18, 35, 60$  and  $90$  V/cm.torr (Fig.1c). Figures (1b) and (1c) also show the peaks of distribution function in He and Ar at different values of  $E/p$  which are not shifted towards higher energy as reported by Hughes [20], but towards lower energy in comparison with that for  $\text{SF}_6$  (Fig. 1a). Figure (2a) represents the distribution function of 100%  $\text{SF}_6$ , 50%  $\text{SF}_6+50\%$  Ar, and 100% Ar as a function of energy at  $E/p=30\text{V/cm.torr}$  while Figure (2b) represents it for  $\text{SF}_6$  and He gas mixtures at the same value of  $E/p$ . Figures (2a) and (2b) also illustrate that, the distribution function of  $\text{SF}_6$  larger than that for He and Ar at low energies. This indicates very strong attachment and vibrational cross-sections and lower value at high energies which reflects very high excitation and ionization cross-sections at the same value of  $E/p$ .

Figures (2a) and (2b) demonstrate the distribution function for 100%  $\text{SF}_6$ , 50%  $\text{SF}_6+50\%$  He, and 100% He and for 100%  $\text{SF}_6$ , 50%  $\text{SF}_6+50\%$  Ar, and 100% Ar at  $E/p=30\text{V/cm.torr}$ . They also display the distribution function for  $\text{SF}_6$  as a larger value than that for He and Ar at low energies and lower value at higher energies. They are in agreement with the  $\text{SF}_6$  characteristics such as strong attachment cross section at low energies and strong ionization cross section at higher energies. It is also noticed that the linear addition of distribution function (F) is valid for  $\text{SF}_6+\text{Ar}$  mixtures (Fig. 2b) and not valid for  $\text{SF}_6+\text{He}$  mixtures. Therefore,

$$F_{\text{SF}_6+\text{Ar}} = k F_{\text{SF}_6} + (1-k) F_{\text{Ar}} \quad (1)$$

is applied for  $\text{SF}_6+\text{Ar}$  mixtures only. From this conclusion, the linear addition is not a suitable method for predicting the distribution function for gas mixtures.

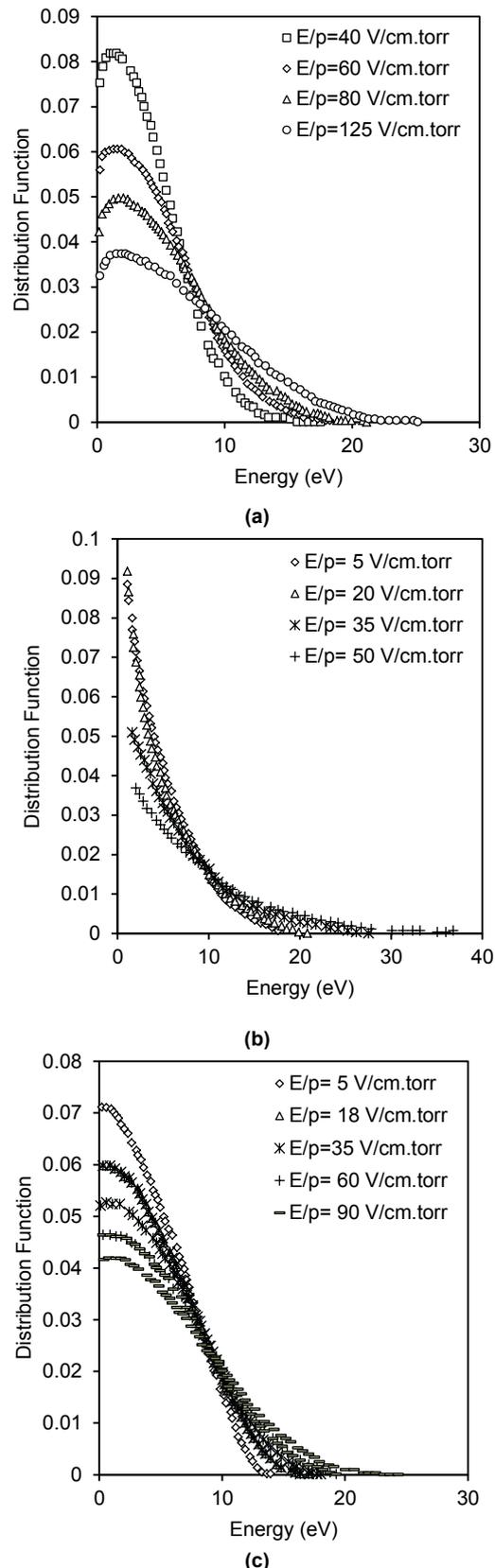
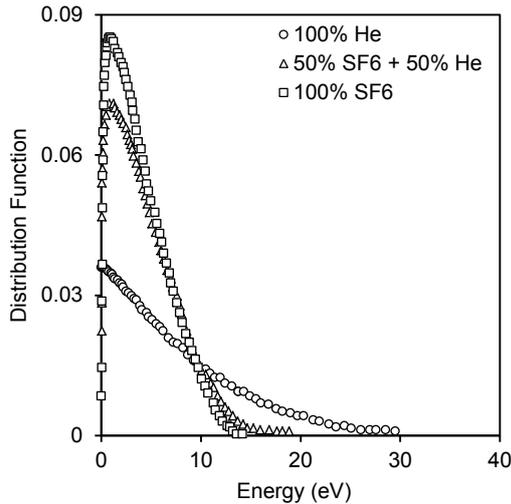
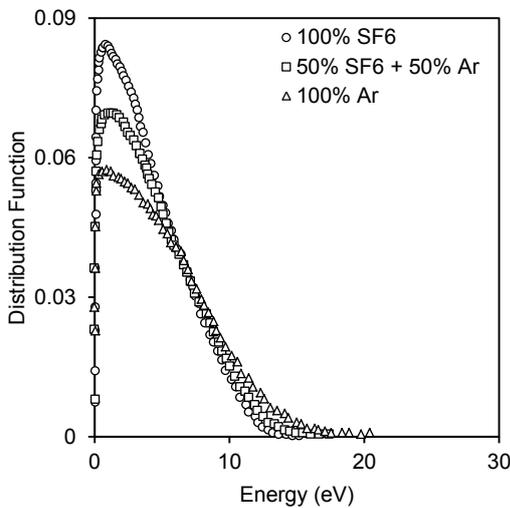


Fig. (1) Distribution function of (a)  $\text{SF}_6$ , (b) He and (c) of Ar as a function of energy



(a)



(b)

Fig. (2) Distribution function of SF<sub>6</sub> and (a) He mixture and (b) Ar mixture as a function of energy at E/p = 30 V/cm.torr

Figure (3) demonstrates the effective ionization coefficient as a function of k in SF<sub>6</sub>+He and SF<sub>6</sub>+Ar mixture at E/p=60 V/cm.torr. When the value of the effective ionization coefficient is equal to zero, we can deduce the (E/p)<sub>crit.</sub> value at any value of (k) for a definite E/p value.

Figure (4) displays the (E/p)<sub>crit.</sub> for SF<sub>6</sub>+He and SF<sub>6</sub>+Ar mixtures as a function of k which is deduced from Fig. (3) when the effective ionization coefficient is equal to zero. The value of (E/p)<sub>crit.</sub> is the same for two mixture gases for k smaller than 0.3. When the value of k exceeds 0.3, (E/p)<sub>crit.</sub> shows values for SF<sub>6</sub>+He lower than that for SF<sub>6</sub>+Ar mixtures. It can be concluded from Fig. (4) that the linear addition of (E/p)<sub>crit.</sub> is valid for SF<sub>6</sub>+He mixtures only as expressed in the equation  $(E/p)_{crit.m} = k ((E/p)_{crit.SF6}) + (1-k) (E/p)_{crit.He}$  (2) while in SF<sub>6</sub>+Ar mixtures, it is applied only at k values smaller than 0.3. It is obvious from Fig. (4) that for k values greater than 0.3, the curve of

SF<sub>6</sub>+Ar mixtures changes its profile from linear to parabola.

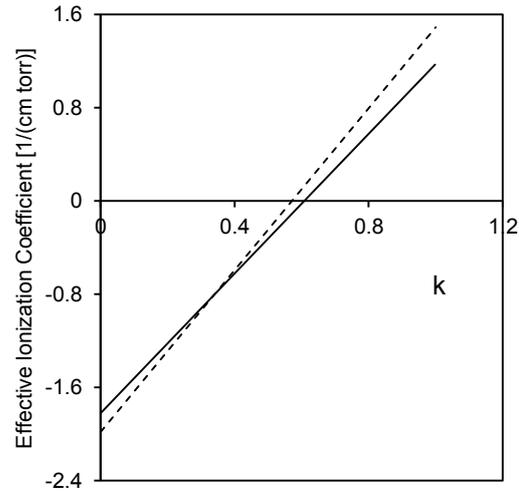


Fig. (3) Effective ionization coefficient as a function of k in SF<sub>6</sub>+He and SF<sub>6</sub>+Ar mixtures at E/p = 60 V/cm.torr

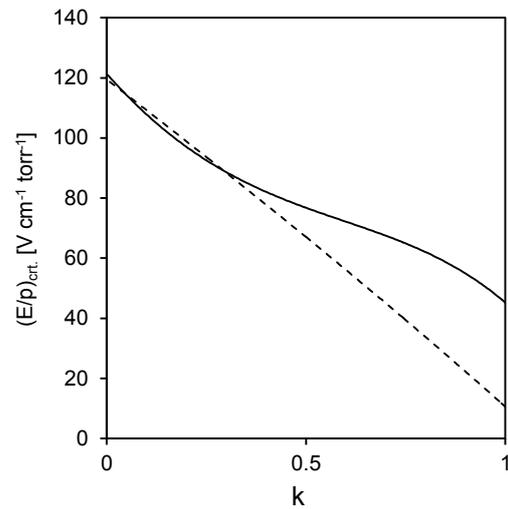


Fig. (4) The variation of (E/p)<sub>crit.</sub> as a function of k for SF<sub>6</sub>+He and SF<sub>6</sub>+Ar mixtures

It can be deduced from Fig. (4) that (E/p)<sub>crit.</sub> is equal to 120 V/cm.torr for SF<sub>6</sub>, 9 V/cm.torr in He, and 45 V/cm.torr in Ar. Figures (5a) and (5b) demonstrate respectively the distribution function of SF<sub>6</sub> with He and SF<sub>6</sub> with Ar, as a function of energy at (E/p)<sub>crit.</sub> only. Such behaviours quite consistent for all pure gases in the present calculations at E/p=(E/p)<sub>crit.</sub> only. They also show the distribution function of any percentage of SF<sub>6</sub> at E/p=(E/p)<sub>crit.</sub> with He or Ar at E/p=(E/p)<sub>crit.</sub> almost same. Figures (5a) and (5b) may confirm that the distribution function of these gas mixtures are the same at E/p=(E/p)<sub>crit.</sub> Such conclusion which is demonstrated in Fig. (5) implies that the distribution functions of (90, 80, 60, 40, 20)% SF<sub>6</sub>+(10, 20, 40, 60, 80)% Ar and He at E/p=(E/p)<sub>crit.</sub> as a function of energy are nearly the same. From Figs. (5a) and (5b), one can rewrite Eq. (1) as;

$F_{SF_6+M} = k F_{SF_6} + (1-k) F_M$  (3)  
 at  $E/p=(E/p)_{crit}$ , only, where M represents the He or Ar gas. Equation (3) is valid at  $E/p=(E/p)_{crit}$ , only. Therefore, the linear addition method is valid at  $E/p=(E/p)_{crit}$ , only.

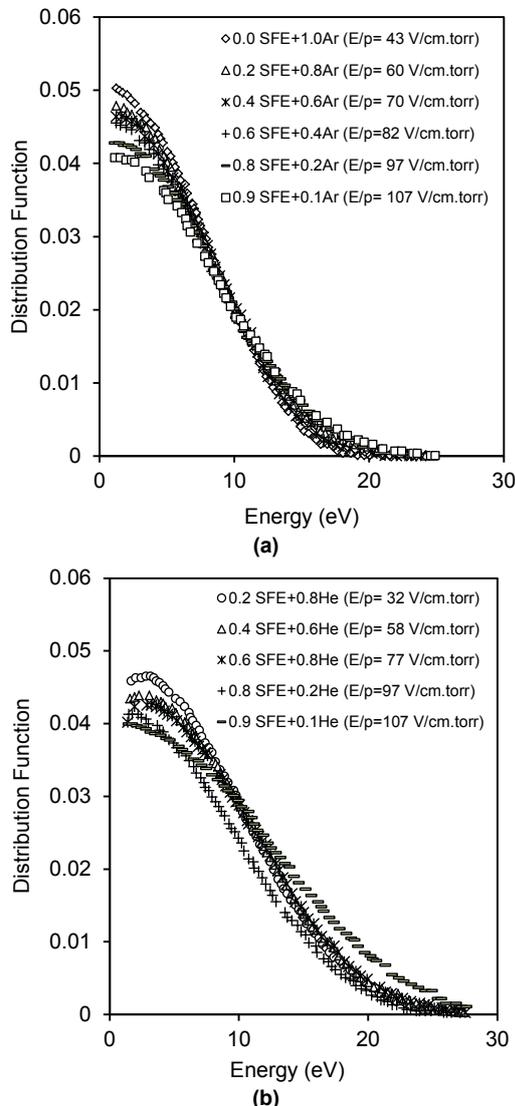


Fig. (5) Distribution function of SF<sub>6</sub> and (a) Ar mixture and (b) He mixture as function of energy at  $(E/p)_{crit}$ .

It is useful to use this conclusion for solving Boltzmann equation for gas mixtures at  $E/p=(E/p)_{crit}$ , only and then all swarm parameters can be calculated. The difficulty in solving Boltzmann equation may be arisen by the collision term for these mixtures.

#### 4. Conclusion

The BOLSIG+ and EEDF codes have been used to study the electron distribution function at  $E/p=(E/p)_{crit}$  in SF<sub>6</sub>+He and SF<sub>6</sub>+Ar mixtures. The peak position of the distribution function in He and Ar at different values of  $E/p$  are shifted towards lower energy in comparison with that for SF<sub>6</sub>. The

$(E/p)_{crit}$  for SF<sub>6</sub>+He appears linear while in SF<sub>6</sub>+Ar is not at  $k$  values greater than 0.3. It is also found that the distribution function is highly consistent for pure and gas mixtures (SF<sub>6</sub>, He, Ar) at a condition  $E/p=(E/p)_{crit}$ , only. Therefore, the linear addition method is valid for predicting the distribution function of gas mixtures (SF<sub>6</sub>+He and SF<sub>6</sub>+Ar) only at  $E/p=(E/p)_{crit}$ .

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