Estimation of Reliability in Multi-Component Stress-Strength Model Following Burr-III Distribution

Asst. Prof. Nada Sabah Karam* **Hind Husham Jani***

*** Mathematical Department / College Of Education / AL-Mustanseriya University / Iraq**

Abstract:

In this paper, we estimate the multicomponent S out of K stress-strength system reliability for Burr-III distribution. The research methodology adopted here is to estimate the parameters by using maximum likelihood ML, least square LS, weighted least square WLS, regression Rg and moment MOM estimation. The reliability is estimated using the same methods of estimation and results are compared by Monte-Carlo simulation study using MSE and MAPE criteria, the results show that the ML was the best between them.

المستخلص:

في هذه الورقة، سوف يتم تقدير معوليه نظام متعدد المكونات ***S*** من ***K*** الإجهاد–المتانه لتوزيع بور–***III***. منهجية البحث المعتمدة هنا هي لتقدير المعلمات باستخدام طريقة الامكان الاعظم ***MLE***، طريقة المربعات الصغرى ***LS***، طريقة المربعات الصغرى الموزونة ***WLS***، وطريقة الانحدار ***Rg***، وطريقة العزوم ***MOM***. والمعوليه تقدر باستخدام نفس الأساليب التقدير ومقارنة النتائج وفقا لدراسة محاكاة مونت كارلو باستخدام معايير ***MSE*** و***MAPE*** ،بينت النتائج أن ***ML*** كان الأفضل بينهما.

Key Words: Burr-III distribution, reliability estimation, stress-strength, ML, LS, WLS, Rg, MOM estimation, simulation study.

1- Introduction:

A Burr system of distributions was constructed in 1941 by Irving W. Burr. Since the corresponding density functions have a wide variety of shapes, this system is useful for approximating histograms, particularly when a simple mathematical structure for the fitted cumulative distribution function (CDF) is required. Other applications include simulation quantal response, approximation of distributions, and development of non-normal control charts. A number of standard theoretical distributions are limiting forms of Burr distributions. [15]

The CDF of Br III(α,θ) is:-

$$F(x) = (1 + x^{-\theta})^{-\alpha}; \qquad x > 0;\ \alpha,\theta > 0 \qquad ....(1)$$

Where the parameters $\theta > 0$ and $\alpha > 0$ are the shape parameters of the distribution. Its PDF is:-

$$f(x) = \alpha\theta x^{-(\theta+1)}(1 + x^{-\theta})^{-(\alpha+1)}; \qquad x > 0;\ \alpha,\theta > 0 \qquad ....(2)$$

The stress-strength model is used in many applications in physics and engineering such as, strength failure and the system collapse. This model is of special importance in reliability literature. In the statistical approach to the stress-strength model, most of the considerations depend on the assumption that the component strengths are independently and identically distributed (iid) and are subjected to a common stress. [3]

The system reliability of multicomponent based on X and Y two independent and identical random variable.

The system of multicomponent stress-strength studied by Bhattacharyya and Johnson in 1974 where imposed that a stress-strength model is formulated for s out of k system consisting of identical components have an exponential distribution.

The system of multicomponent have been studied by several authors (Bhattacharyya and Johnson (1974), Kim and Kang (1981), Rao and Kantam (2010), Rao (2012), Hassan and Basheikh (2012), Pandit and Kantu. (2012), Rao and Naidu (2013), Rao (2013), Rao and Kantam el. at. (2013), Rao and Kantam (2013), Nayeban and Roknabadi el. at., (2014) Rao (2014)). (see [2]-[6], [8]-[14]).

The main aim of this article is to discuss the derivate of the mathematical formula of reliability system $R_{(s,k)}$ for Burr type III distribution, and estimate the reliability function $R_{(s,k)}$ by using ML, LS, WLS, Rg and MOM methods, then comparison among the results of the estimation methods of the reliability function of multicomponent stress-strength model by using mean square error (MSE) and mean absolute percentage error (MAPE), that will get from a simulation study.

2- Experimental Aspect of Reliability in multicomponent stress-strength:-

In this article, the reliability of multicomponent stress-strength for Burr-III distribution imposed X and Y fallow the same population with unknown shape parameters α, λ and common and known scale parameter θ. Let the random variables $Y, X_1, X_2, \dots, X_k$ are independent, $F(x)$ be the continuous distribution function (CDF) of $X_i, i = 1,2,\dots,k,$ and $G(y)$ be the common continuous (CDF) of Y then the reliability in a multicomponent stress-strength model is: [2]

$$\left.\begin{aligned} R_{(s,k)} &= \text{Prob (at least s of } X_1, X_2, \dots, X_k \text{ exceed } Y) \\ &= \sum_{a=1}^{k} \mathrm{C}_a^k \int_{-\infty}^{\infty} [1 - F(y)]^a \, F(y)^{k-a} dG(y) \end{aligned}\right\} \quad \dots.(3)$$

Where Y is a strength random variable of multicomponent subjected to a common stress X.

Let $X \sim Br3(\alpha, \theta)$ and $Y \sim Br3(\lambda, \theta)$ with unknown shape parameters α, λ and common and known scale parameter θ, where X and Y are independently distributed, the reliability in multicomponent stress-strength $R_{(s,k)}$ of Burr-III distribution can be obtained by substitution (1) in (3) as:

$$R_{(s,k)} = \sum_{i=s}^{k} \mathrm{C}_i^k \int_0^{\infty} \left[1 - \left(1 + y^{-\theta}\right)^{-\alpha}\right]^i \left[\left(1 + y^{-\theta}\right)^{-\alpha}\right]^{k-i} \lambda\theta y^{-(\theta+1)} \left(1 + \mathrm{y}^{-\theta}\right)^{-(\lambda+1)} \mathrm{dy}$$

$$= \frac{\lambda}{\alpha} \sum_{i=s}^{k} \mathrm{C}_i^k \int_0^1 u^{(k-i+\frac{\lambda}{\alpha}-1]} [1-u]^i \, du \qquad \text{where } \mathrm{u} = \left(1 + y^{-\theta}\right)^{-\alpha}$$

$$= \frac{\lambda}{\alpha} \sum_{i=s}^{k} \mathrm{C}_i^k B\left(\left(k - i + \frac{\lambda}{\alpha}\right), (i+1)\right)$$

Then the $R_{(s,k)}$ of Br III distribution is given by:-

$$R_{(s,k)} = \frac{\lambda}{\alpha} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\lambda}{\alpha} - j\right)\right]^{-1} \qquad \dots.(4)$$

Where s, k, i and j and s are integers.

3- Different method of estimation:

The unknown shape parameters of Br III distribution for the multicomponent reliability function have been estimated by different methods of estimation; Maximum likelihood, Least square, Weighted least square, Regression and Moment method.

3-1 Maximum likelihood function (MLE):-

Let $(x_1, x_2, \dots, x_n)$ strength random sample have $Br3(\alpha, \theta)$ distribution with sample size n, where α is unknown parameter and let Y stress random variable have $Br3(\lambda, \theta)$ with sample size m where λ is unknown parameter, then the likelihood function L, using equation (2) as:-[16]

$$L(x_1, x_2, \dots, x_n; \alpha, \theta) = \prod_{i=1}^{n}\left[\alpha\theta x_i^{-(\theta+1)}\left(1 + x_i^{-\theta}\right)^{-(\alpha+1)}\right]$$

$$= \alpha^n \theta^n \prod_{i=1}^{n} x_i^{-(\theta+1)} \prod_{i=1}^{n}\left(1 + x_i^{-\theta}\right)^{-(\alpha+1)}$$

The first derivatives of the log-likelihood function with respect to α and λ are given, respectively, by

$$\left.\begin{aligned}\frac{\partial \ln L(x_1,x_2,\dots,x_n;\alpha,\theta)}{\partial \alpha} &= \frac{n}{\alpha} - \sum_{i=1}^{n} \ln\left(1 + x_i^{-\theta}\right)\\ \frac{\partial \ln L(y_1,y_2,\dots,y_n;\lambda,\theta)}{\partial \lambda} &= \frac{m}{\lambda} - \sum_{j=1}^{m} \ln\left(1 + y_j^{-\theta}\right)\end{aligned}\right\} \quad \dots.(5)$$

Then by solution of equations (5), the ML's estimator for α and λ, $(\hat{\alpha}_{MLE}, \hat{\lambda}_{MLE})$, respectively, can be obtained as:

$$\hat{\alpha}_{MLE} = \frac{n}{\sum_{i=1}^{n} \ln\left(1+x_i^{-\theta}\right)}, \quad \hat{\lambda}_{MLE} = \frac{m}{\sum_{j=1}^{m} \ln\left(1+y_j^{-\theta}\right)} \quad \dots.(6)$$

Substitution the equations (6) in the equation (3), the ML estimator for $R_{(s,k)}$, $\hat{R}_{BML}$, by the invariant property of ML estimation method, can be obtained as:

$$\hat{R}_{BML} = \frac{\hat{\lambda}_{MLE}}{\hat{\alpha}_{MLE}} \sum_{i=s}^{k} \frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k + \frac{\hat{\lambda}_{MLE}}{\hat{\alpha}_{MLE}} - j\right)\right]^{-1} \quad \dots.(7)$$

3-2:- Least Square Method (LS):-

The least squares method estimators can be produce by minimizing the sum of square error between the value and it's expected value, This estimation method is very popular for model fitting, especially in linear and non-linear regression**.** [3]

$$s_1 = \sum_{i=1}^{n}\left[F(x_{(i)}) - E\left(F\left(x_{(i)}\right)\right)\right]^2 \; S_2 = \sum_{j=1}^{m}\left[F(y_{(j)}) - E\left(F(y_{(j)})\right)\right]^2 \; .(8)$$

Where $E\left(F\left(x_{(i)}\right)\right)$ and $E\left(F(y_{(j)})\right)$ equal to P_i, P_j the plotting position, where $P_i = \frac{i}{n+1}$, for $i = 1,2,\dots,n$ and $P_j = \frac{j}{m+1}$, for $j = 1,2,\dots,m$. ….(9)

Suppose that $x_1, x_2, \dots, x_n$ is a random sample, where X_i is strength random variable $Br3(\alpha,\theta)$ distribution with sample size n, and Y is the stress random variable of $Br3(\lambda,\theta)$ distribution with sample size m.

From a distribution function (1):

$F\left(x_{(i)}\right) = \left(1 + {x_{(i)}}^{-\theta}\right)^{-\alpha}$ and $G\left(y_{(j)}\right) = \left(1 + {y_{(j)}}^{-\theta}\right)^{-\lambda}$

$\rightarrow \left(F\left(x_{(i)}\right)\right)^{-1} = \left(1 + {x_{(i)}}^{-\theta}\right)^{\alpha}$ and $\left(G\left(y_{(j)}\right)\right)^{-1} = \left(1 + {y_{(j)}}^{-\theta}\right)^{\lambda}$

Simplification and changing $F\left(x_{(i)}\right)$ and $G\left(y_{(j)}\right)$ by plotting position P_i, P_j (9), and equal to zero, we obtain:-

$$\left.\begin{aligned} &\ln(P_i)^{-1} - \alpha \ln\left(1 + {x_{(i)}}^{-\theta}\right) = 0 \\ &\ln\left(P_j\right)^{-1} - \lambda \ln\left(1 + {y_{(j)}}^{-\theta}\right) = 0 \end{aligned}\right\} \qquad \text{….(10)}$$

Substitution (10) in (8) and taking the first derivative with respect to the unknown shape parameters α and λ, and equating the result to zero, we get:

$$\hat{\alpha}_{LS} = \frac{\sum_{i=1}^{n}\left(\ln(P_i)^{-1}\ln\left(1+{x_{(i)}}^{-\theta}\right)\right)}{\sum_{i=1}^{n}\left(\ln\left(1+{x_{(i)}}^{-\theta}\right)\right)^2}, \ \hat{\lambda}_{LS} = \frac{\sum_{j=1}^{m}\left(\ln\left(P_j\right)^{-1}\ln\left(1+{y_{(j)}}^{-\theta}\right)\right)}{\sum_{j=1}^{m}\left(\ln\left(1+{y_{(j)}}^{-\theta}\right)\right)^2} \qquad \text{….(11)}$$

Substitution (11) in (3), the LS estimator approximately for $R_{(s,k)}$, $\hat{R}_{BLS}$, can be obtained as:

$$\hat{R}_{BLS} = \frac{\hat{\lambda}_{LS}}{\hat{\alpha}_{LS}} \sum_{i=s}^{k} \frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k + \frac{\hat{\lambda}_{LS}}{\hat{\alpha}_{LS}} - j\right)\right]^{-1} \qquad \text{….(12)}$$

3-3 Weighted Least Square Method (WLS):-

The weighted least squares estimators can be obtained by minimizing the following equation. [3]

$$\left.\begin{aligned} s_1 &= \sum_{i=1}^{n} \omega_i \left[F(x_{(i)}) - E\left(F\left(x_{(i)}\right)\right)\right]^2 \text{ and} \\ S_2 &= \sum_{j=1}^{m} \omega_j \left[F(y_{(j)}) - E\left(F(y_{(j)})\right)\right]^2 \end{aligned}\right\} \quad ....(13)$$

With respect to the unknown parameters α and λ, Where $E\left(F\left(x_{(i)}\right)\right)$ and $E\left(F(y_{(j)})\right)$ equal to P_i, P_j the plotting position, where P_i, P_j as in (9)

$$\left.\begin{aligned} \text{And } \omega_i &= \frac{1}{Var[F(x_{(i)})]} = \frac{(n+1)^2(n+2)}{i(n-i+1)}, i = 1,2,\dots,n \\ and\ \omega_j &= \frac{1}{Var[G(y_{(j)})]} = \frac{(m+1)^2(m+2)}{j(m-j+1)}, j = 1,2,\dots,m \end{aligned}\right\} \quad ....(14)$$

By substitution (10) in (13), and taking the partial derivative with respect to the unknown shape parameters α and λ, and simplify the result we obtain we get:

$$\left.\begin{aligned} \hat{\alpha}_{WLS} &= \frac{\sum_{i=1}^{n} \omega_i \left(\ln(P_i)^{-1} \ln\left(1+x_{(i)}{}^{-\theta}\right)\right)}{\sum_{i=1}^{n} \omega_i \left(\ln\left(1+x_{(i)}{}^{-\theta}\right)\right)^2} \\ \hat{\lambda}_{WLS} &= \frac{\sum_{j=1}^{m} \omega_j \left(\ln(P_j)^{-1} \ln\left(1+y_{(j)}{}^{-\theta}\right)\right)}{\sum_{j=1}^{m} \omega_j \left(\ln\left(1+y_{(j)}{}^{-\theta}\right)\right)^2} \end{aligned}\right\} \quad ....(15)$$

Where ω_i, ω_j as in (14).

Substitution (15) in (3), the WLS estimator approximately for $R_{(s,k)}$, $\hat{R}_{BWL}$, can be obtained as:

$$\hat{R}_{BWL} = \frac{\hat{\lambda}_{WLS}}{\hat{\alpha}_{WLS}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\hat{\lambda}_{WLS}}{\hat{\alpha}_{WLS}} - j\right)\right]^{-1} \quad ....(16)$$

3-4 Regression Method (Rg):-

Regression is one of the important procedures that use auxiliary information to construct estimators with good efficiency. [7]

The standard regression equation:-

$$z_i = a + bu_i + e_i \quad ....(17)$$

Where z_i is dependent variable (response variable), u_i is independent variable (Explanatory Variable) and e_i is the error r.v. independent identically Normal distributed with $(0, \sigma^2)$.

Taking natural logarithm to (1), then changing $F(x_{(i)})$ and $G(y_{(j)})$ by plotting position P_i and P_j (9), we obtain:

$$\left.\begin{array}{l} \ln P_i = -\alpha \ln\left(1 + x_{(i)}{}^{-\theta}\right); i = 1,2, \dots, n \\ \ln P_j = -\lambda \ln\left(1 + y_{(j)}{}^{-\theta}\right); j = 1,2, \dots, m \end{array}\right\} \quad \dots.(18)$$

Comparing the equation (18) with the equation (17), we get:

$$\left.\begin{array}{l} z_i = \ln P_i, a = 0, b = \alpha, u_i = -\ln\left(1 + x_{(i)}{}^{-\theta}\right) \\ z_i = \ln P_j, a = 0, b = \lambda, u_i = -\ln\left(1 + y_{(j)}{}^{-\theta}\right) \end{array}\right\} \quad \dots.(19)$$

Where b can be estimated by minimizing the summation of squared error with respect to b, then we get:

$$\hat{b} = \frac{n \sum_{i=1}^{n} z_i u_i - \sum_{i=1}^{n} z_i \sum_{i=1}^{n} u_i}{n \sum_{i=1}^{n} [u_i]^2 - \left[\sum_{i=1}^{n} u_i\right]^2} \quad \dots.(20)$$

By substation (19) in (20), the Rg estimator for the unknown strength parameter α, ($\hat{\alpha}_{Rg}$) and the unknown stress parameter λ, ($\hat{\lambda}_{Rg}$) can be formulated as:

$$\left.\begin{array}{l} \hat{\alpha}_{Rg} = \dfrac{-n \sum_{i=1}^{n} \ln P_i \ln\left(1+x_{(i)}{}^{-\theta}\right) + \sum_{i=1}^{n} \ln P_i \sum_{i=1}^{n} \ln\left(1+x_{(i)}{}^{-\theta}\right)}{n \sum_{i=1}^{n}\left[\ln\left(1+x_{(i)}{}^{-\theta}\right)\right]^2 - \left[\sum_{i=1}^{n} \ln\left(1+x_{(i)}{}^{-\theta}\right)\right]^2} \\ \hat{\lambda}_{Rg} = \dfrac{m \sum_{j=1}^{m} \ln P_j \ln\left(1+y_{(j)}{}^{-\theta}\right) - \sum_{j=1}^{m} \ln P_j \sum_{j=1}^{m} \ln\left(1+y_{(j)}{}^{-\theta}\right)}{m \sum_{j=1}^{m}\left[\ln\left(1+y_{(j)}{}^{-\theta}\right)\right]^2 - \left[\sum_{j=1}^{m} \ln\left(1+y_{(j)}{}^{-\theta}\right)\right]^2} \end{array}\right\} \quad \dots.(21)$$

Where P_i and P_j as in (9).

Substitution the equations (21) in the equation (3), the RM estimator approximately for $R_{(s,k)}$, $\hat{R}_{BRg}$, can be obtained as:

$$\hat{R}_{BRg} = \frac{\hat{\lambda}_{Rg}}{\hat{\alpha}_{Rg}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\hat{\lambda}_{Rg}}{\hat{\alpha}_{Rg}} - j\right)\right]^{-1} \quad \dots.(22)$$

3-5 Moment method (MOM):-

The MOM, first introduced by Pearson (1894), was one of the first methods used to estimate the society parameter θ. [1]

To derive the method of moment estimators of the parameters of $Br3D$, let X_i strength random variable have $Br3(\alpha,\theta)$ distribution with sample size n, and let Y stress random sample have $Br3(\lambda,\theta)$ distribution with sample size m, first, we need the population mean, then since:

$$E(x) = \alpha\, B\left(1-\frac{1}{\theta}, \alpha+\frac{1}{\theta}\right), E(y) = \lambda\, B\left(1-\frac{1}{\theta}, \lambda+\frac{1}{\theta}\right); where\ \theta > 1$$

For θ is known, equating the sample mean with corresponding populations mean, we get the shape parameters moment estimators.

$$\frac{\sum_{i=1}^{n} x_i}{n} = \alpha\, B\left(1-\frac{1}{\theta}, \alpha+\frac{1}{\theta}\right), \qquad \frac{\sum_{j=1}^{m} y_j}{m} = \lambda\, B\left(1-\frac{1}{\theta}, \lambda+\frac{1}{\theta}\right)$$

Then the moment estimator of α and λ say $\hat{\alpha}_{MOM}$ and $\hat{\lambda}_{MOM}$ are:

$$\hat{\alpha}_{MOM} = \frac{\bar{x}}{B\left(1-\frac{1}{\theta}, \alpha_0+\frac{1}{\theta}\right)}, \hat{\lambda}_{MOM} = \frac{\bar{y}}{B\left(1-\frac{1}{\theta}, \lambda_0+\frac{1}{\theta}\right)} \qquad where\ \theta > 1 \qquad \text{....(23)}$$

Substitution the equations (23) in the equation (3), the MOM estimator approximately for $R_{(s,k)}$, $\hat{R}_{BMOM}$ we will obtain:

$$\hat{R}_{BMOM} = \frac{\hat{\lambda}_{MOM}}{\hat{\alpha}_{MOM}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i}\left(k+\frac{\hat{\lambda}_{MOM}}{\hat{\alpha}_{MOM}}-j\right)\right]^{-1} \qquad \text{....(24)}$$

4- Simulation study:

Results based on Monte Carlo simulations to compare the performance of the $R_{(s,k)}$ using different sample sizes are presented. 1500 random sample of size 10,15,20,25,35,50,75 and 100 each from stress population, strength population were generated (α,λ,θ) = (1.5, 0.8, 1.2), (α,λ,θ) = (1.5, 2, 1.2) for (s,k) = (2, 3) and (s,k) = (3, 4) for $R_{(s,k)}$. The Mean square error (MSE) and Mean Absolute Percentage Error (MAPE) of the reliability estimates over the 1500 replications are given in Tables 2 and 3.

The true value of reliability in multicomponent stress- strength with the given combinations of (α,λ,θ) = (1.5, 0.8, 1.2), (α,λ,θ) = (1.5, 2, 1.2) for (s,k) = (2, 3) are $R_{(s,k)}$ =0.6703 and $R_{(s,k)}$ =0.4154, and for (s,k) = (3, 4) are $R_{(s,k)}$ =0.5914 and $R_{(s,k)}$ =0.3115.

From the tables (2) and (3) below, we have observed that:-

1- *When* (s, k) = *(2, 3)***:-**

*The MSE value decreasing by increasing sample size for all estimator methods**.** The best MSE value is for MLE estimator, followed by the other methods**.**

***** The MAPE value decreasing by increasing sample size for all estimator methods. The best MAPE value is for MLE, followed by the other methods**.**

2- *When* (s, k) = *(3, 4)***:-**

***** The MSE value decreasing by increasing sample size for all estimator methods. The best MSE value is for MLE estimator, followed by the other methods**.**

***** The MAPE value decreasing by increasing sample size for all estimator methods. The best MAPE value is for MLE, followed by the other methods**.**

5- Conclusion:

- The MSE and MAPE value decreases by increasing sample size.
- The performance MLE was the best**,** as in the table below.

Table (1): The best estimation method of MSE and MAPE of Br3 for $R_{(s,k)}$.

Method / Sample size	MLE	LS	WLS	Rg	MOM	Best
From (10,10) to (35,35)	1	2	3	4	5	MLE
From (50,50) to (100,100)	1	2	4	3	5	MLE

Table (2): Results of Mean, MSE and MAPE values for Br3D ($R_{(s,k)}$ =0.6703 when (s,k) = (2, 3)) and ($R_{(s,k)}$ =0.5914 when (s,k) = (3, 4)) for (α,λ,θ) = (1.5, 0.8, 1.2).

Methods / (n,m)	S,k		MLE	LS	WLS	Rg	MOM	Best
(10,10)	2,3	Mean	0.6584	0.6572	0.6559	0.6528	0.6408	-
		MSE	0.0121	0.0138	0.0155	0.0192	0.0500	MLE
		MAPE	0.1310	0.1399	0.1478	0.1651	0.2630	MLE
(15,15)		Mean	0.6633	0.6604	0.6584	0.6551	0.6456	-
		MSE	0.0079	0.0098	0.0121	0.0147	0.0428	MLE
		MAPE	0.1053	0.1172	0.1300	0.1429	0.2407	MLE
(20,20)		Mean	0.6659	0.6650	0.6633	0.6621	0.6477	-
		MSE	0.0060	0.0072	0.0094	0.0108	0.0417	MLE
		MAPE	0.0918	0.1015	0.1157	0.1236	0.2370	MLE
(25,25)		Mean	0.6658	0.6641	0.6624	0.6609	0.6512	-
		MSE	0.0048	0.0057	0.0079	0.0085	0.0395	MLE
		MAPE	0.0820	0.0894	0.1052	0.1101	0.2328	MLE
(35,35)		Mean	0.6660	0.6659	0.6638	0.6643	0.6493	-
		MSE	0.0033	0.0041	0.0060	0.0061	0.0345	MLE
		MAPE	0.0680	0.0753	0.0909	0.0919	0.2120	MLE
(50,50)		Mean	0.6682	0.6680	0.6672	0.6669	0.6485	-
		MSE	0.0023	0.0029	0.0050	0.0046	0.0319	MLE
		MAPE	0.0562	0.0636	0.0831	0.0797	0.2007	MLE
(75,75)		Mean	0.6659	0.6649	0.6626	0.6632	0.6504	-
		MSE	0.0016	0.0021	0.0042	0.0033	0.0271	MLE
		MAPE	0.0474	0.0543	0.0759	0.0680	0.1845	MLE
(100,100)		Mean	0.6684	0.6684	0.6669	0.6679	0.6477	-
		MSE	0.0012	0.0014	0.0032	0.0022	0.0244	MLE,RSS
		MAPE	0.0404	0.0449	0.0668	0.0562	0.1740	MLE
(10,10)	3,4	Mean	0.5873	0.5867	0.5855	0.5829	0.5710	-
		MSE	0.0149	0.0170	0.0190	0.0238	0.0594	MLE
		MAPE	0.1656	0.1763	0.1865	0.2102	0.3395	MLE
(15,15)		Mean	0.5882	0.5879	0.5871	0.5859	0.5718	-
		MSE	0.0103	0.0122	0.0146	0.0176	0.0543	MLE
		MAPE	0.1367	0.1486	0.1629	0.1787	0.3205	MLE
(20,20)		Mean	0.5850	0.5825	0.5802	0.5782	0.5796	-
		MSE	0.0080	0.0095	0.0121	0.0137	0.0511	MLE
		MAPE	0.1204	0.1318	0.1486	0.1583	0.3120	MLE
(25,25)		Mean	0.5868	0.5854	0.5838	0.5825	0.5785	-
		MSE	0.0064	0.0076	0.0105	0.0114	0.0474	MLE
		MAPE	0.1071	0.1174	0.1382	0.1445	0.2969	MLE
(35,35)		Mean	0.5864	0.5865	0.5846	0.5856	0.5728	-
		MSE	0.0048	0.0057	0.0082	0.0082	0.0418	MLE
		MAPE	0.0936	0.1021	0.1237	0.1238	0.2728	MLE
(50,50)		Mean	0.5937	0.5924	0.5899	0.5906	0.5913	-
		MSE	0.0031	0.0038	0.0064	0.0060	0.0351	MLE
		MAPE	0.0748	0.0833	0.1082	0.1044	0.2497	MLE
(75,75)		Mean	0.5909	0.5911	0.5901	0.5905	0.5784	-
		MSE	0.0021	0.0025	0.0049	0.0038	0.0318	MLE
		MAPE	0.0615	0.0675	0.0948	0.0840	0.2340	MLE
(100,100)		Mean	0.5898	0.5897	0.5894	0.5891	0.5845	-
		MSE	0.0016	0.0021	0.0046	0.0034	0.0300	MLE
		MAPE	0.0549	0.0622	0.0907	0.0779	0.2270	MLE

Table (3): Results of Mean, MSE and MAPE values for Br3D ($R_{(s,k)}$ =0.4154 when (s,k) = (2, 3)) and ($R_{(s,k)}$ =0.3115 when (s,k) = (3,4)) for (α,λ,θ) = (1.5, 2, 1.2).

Methods / (n,m)	S,k		MLE	LS	WLS	Rg	MOM	Best
(10,10)	2,3	Mean	0.4159	0.4165	0.4173	0.4177	0.4134	-
		MSE	0.0161	0.0181	0.0199	0.0244	0.0551	MLE
		MAPE	0.2466	0.2631	0.2775	0.3088	0.4655	MLE
(15,15)		Mean	0.4188	0.4197	0.4200	0.4208	0.4228	-
		MSE	0.0116	0.0138	0.0163	0.0195	0.0493	MLE
		MAPE	0.2066	0.2271	0.2500	0.2763	0.4424	MLE
(20,20)		Mean	0.4169	0.4176	0.4172	0.4186	0.4141	-
		MSE	0.0079	0.0094	0.0120	0.0136	0.0456	MLE
		MAPE	0.1744	0.1891	0.2133	0.2272	0.4228	MLE
(25,25)		Mean	0.4175	0.4172	0.4168	0.4174	0.4238	-
		MSE	0.0063	0.0079	0.0107	0.0117	0.0435	MLE
		MAPE	0.1529	0.1708	0.2011	0.2092	0.4044	MLE
(35,35)		Mean	0.4153	0.4163	0.4171	0.4173	0.4197	-
		MSE	0.0049	0.0059	0.0084	0.0086	0.0403	MLE
		MAPE	0.1352	0.1498	0.1785	0.1803	0.3890	MLE
(50,50)		Mean	0.4178	0.4192	0.4208	0.4208	0.4166	-
		MSE	0.0034	0.0041	0.0068	0.0062	0.0347	MLE
		MAPE	0.1112	0.1242	0.1598	0.1530	0.3585	MLE
(75,75)		Mean	0.4172	0.4171	0.4165	0.4171	0.4211	-
		MSE	0.0024	0.0029	0.0055	0.0045	0.0315	MLE
		MAPE	0.0931	0.1035	0.1438	0.1289	0.3345	MLE
(100,100)		Mean	0.4169	0.4161	0.4161	0.4154	0.4223	-
		MSE	0.0018	0.0023	0.0049	0.0036	0.0295	MLE,RSS
		MAPE	0.0808	0.0911	0.1336	0.1153	0.3204	MLE
(10,10)	3,4	Mean	0.3245	0.3263	0.3272	0.3300	0.3399	-
		MSE	0.0161	0.0184	0.0203	0.0253	0.0554	MLE
		MAPE	0.3283	0.3493	0.3684	0.4137	0.6247	MLE
(15,15)		Mean	0.3134	0.3131	0.3150	0.3156	0.3319	-
		MSE	0.0115	0.0135	0.0162	0.0190	0.0521	MLE
		MAPE	0.2787	0.3011	0.3316	0.3607	0.5983	MLE
(20,20)		Mean	0.3172	0.3169	0.3176	0.3178	0.3462	-
		MSE	0.0088	0.0107	0.0134	0.0152	0.0491	MLE
		MAPE	0.2392	0.2662	0.3009	0.3217	0.5721	MLE
(25,25)		Mean	0.3155	0.3153	0.3151	0.3167	0.3332	-
		MSE	0.0066	0.0080	0.0107	0.0117	0.0432	MLE
		MAPE	0.2105	0.2305	0.2666	0.2795	0.5388	MLE
(35,35)		Mean	0.3154	0.3160	0.3173	0.3174	0.3274	-
		MSE	0.0047	0.0058	0.0085	0.0087	0.0374	MLE
		MAPE	0.1771	0.1961	0.2375	0.2391	0.4916	MLE
(50,50)		Mean	0.3113	0.3110	0.3122	0.3116	0.3300	-
		MSE	0.0038	0.0047	0.0076	0.0070	0.0357	MLE
		MAPE	0.1593	0.1771	0.2255	0.2163	0.4799	MLE
(75,75)		Mean	0.3131	0.3133	0.3143	0.3140	0.3314	-
		MSE	0.0023	0.0029	0.0056	0.0044	0.0332	MLE
		MAPE	0.1246	0.1384	0.1896	0.1694	0.4563	MLE
(100,100)		Mean	0.3128	0.3128	0.3136	0.3134	0.3171	-
		MSE	0.0017	0.0023	0.0048	0.0036	0.0282	MLE
		MAPE	0.1076	0.1219	0.1771	0.1529	0.4186	MLE

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