BER Performance of M-ary FSK Modulation over AWGN and Rayleigh Fading Channels

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Abstract

In a mobile communication environment, there is a rising demand of low Bit Error Rate (BER). The channel is not time invariant since the received signal depends on many fast changing factors, and thus it will lead to fading channels. Statistical models are typically used to simulate fading. In this paper, the BER versus E_b/N_o will simulate the non-faded and faded channels. Using Rayleigh statistical model, the BER of different channel fading types is studied based on time delay spread can have flat fading or frequency selective fading and also based on the Doppler shift can have slow fading or fast fading taking into consideration the M-ary FSK modulation scheme. The overall system is implemented by using MATLAB.

Keywords: Bit Error Rate (BER), FSK (Frequency Shift Keying), Additive White Gaussian Noise (AWGN), Rayleigh fading, Doppler Shift.

الخلاصة

في محيط الاتصال المتنقل هنالك طلب عالي لتوفير معدل خطأ الاشارة. القناة الناقلة لهذا النظام هي ليست قناة ثابتة مع الوقت بحيث الاشارة المستلمة تعتمد على عدة عوامل سريعة التغيير وهذا يؤدي الى بهت الاشارة. لذلك يتم استعمال موديل احصائي لمحاكاة القناة المبهتة للاشارة. في هذا البحث سيتم محاكاة معدل خطأ الاشارة مقابل نسبة الاشارة الى الضوضاء في القنوات المبهتة والغير مبهتة. بأستخدام الرايلي موديل الاحصائي سيتم دراسة معدل خطأ الاشارة لمختلف القنوات المبهتة والتي تصنف وفقا لانتشار تاخير الوقت الى بهت مستوي وبهت بتردد انتقائي وكذلك تصنف وفقا لتأثير دوبلار لبهت بطيء وبهت سريع، مع الاخذ بنظر الاعتبار تضمين ازاحة التردد متعدد المستويات. كل هذا النظام ينفذ بواسطة برنامج الماتلاب.

الكلمات المفتاحية: معدل خطأ الاشارة، تضمين ازاحة التردد، الضوضاء الكاوزيني الابيض المضاف، بهت الرايلي، ازاحة الدوبلر

1. Introduction

Wireless communication is one of the most active areas of challenges for the development which has experience massive growth and commercial success nowadays. These challenges are come due to random and time variant in nature simulation of wireless channels. The accurate simulation of fading channels is a crucial issue in the development and evaluation of wireless communication environments. The radio waves propagated from the transmitter to the receiver via multiple different paths due to the obstacles and reflectors existing in the wireless channel. These multipath fading are caused by propagation mechanisms such as reflection, diffraction, scattering from buildings, hills, ground terrain, structures, and other obstacles existing in the propagation environment around the transmitter and receiver.

The signal components are not reaching the receiver at the same time due to different path distances, also the phase difference between the received copies of transmitted signal leads a frequency shifts in case movement of transmitter/receiver or both which causes a frequency Doppler shift. The time varying nature of the channel caused by the movement is quantified by Doppler spread and Coherence time (Xiao *et.al.*, 2003). In the high mobility system, the relative motion between the transmitters and receivers results rapid time variation and significant Doppler shift. The Doppler shift will be positive if the mobile is moving towards the direction of the arrival of the wave whereas it will be negative if the mobile is moving away from the direction of arrival of the wave (Petrus *et.al.*, 2002). The channel may be classified either as a fast fading or slow fading channel depending on how rapidly the transmitted baseband signal changes as compared to the rate of change of the channel. When the channel impulse response changes rapidly within the symbol it is known as fast fading and when it changes at a rate much slower than the transmitted baseband signal then it is slow fading (Kharel and Shakya, 2014). Rayleigh model is highly optimistic in the mobile communication environment at high vehicular speeds. The Rayleigh fading model works on the assumption that the resultant fading arises from a large number of uncorrelated partial waves with identically distributed amplitudes and uniformly distributed random phases (Xiao *et.al.,* 2006). The objective of this paper is to find the low BER at low received Eb/No in a multipath conditions which is simulated in different fading types and fulfil a feasible agreement with the theoretical results as well as comparing the fading with AWGN channel in that communication system.

2. Fading Types

Fading is an upshot of time varying channel on transmitting signal to confront at the receiver with a signal fading which observed has a high amplitude variation; the different fading channel will undergo according to the relation between the transmitted signal parameters and channel parameters. Fig.1 shows a tree diagram with the different types of fading (Luengo and Martino, 2014). Fading can be divided into the following different types.

2.1 Due to effect of multipath

2.1.1 Large scale fading

Large scale fading refers to the average signal attenuation or path loss due to the motion over large distance between the transmitter and the receiver.

2.1.2 Small scale fading

Small scale fading refers to the dramatic rapid changes in amplitude, phase of multipath delays of the received signal over short period of time or distance. In Fig.1, the small scale fading may be divided based on multipath delay spread which is frequency characteristic (flat fading and frequency selective), or based on the mobile speed (fast fading and fast fading).

2.1.2.1 Flat fading

In flat fading, the coherence bandwidth of the channel is larger than the bandwidth of the signal or in other words the multipath delay spread is less than the transmitted symbol period. Therefore, all frequency components of the signal will experience the same magnitude of fading. This type of channels is also know amplitude varying channels and narrowband channels.

2.1.2.2 Frequency-selective fading

Frequency-selective fading caused by Inter-Symbol Interference (ISI), where received signal that consists of multiple delays of the signal components leads to attenuate the transmitted signal versions. In this type of fading, the coherence bandwidth of the channel is smaller than the bandwidth of the signal or in other words the multipath delay spread is greater than the transmitted symbol period. Different frequency components of the signal therefore experience uncorrelated fading.

2.2 Due to Doppler shift

2.2.1 Slow fading

Slow fading arises with low Doppler spread values, it is expected if the channel variation is slower than the baseband signal variation hence the Doppler spread is much less than the bandwidth of the baseband signal; i.e. slower moving transmitter/ receiver and the surrounding obstacles.

Slow fading can be imposed by events such as shadowing, where a large obstruction such as a hill or large building obscures the dominate signal path between the transmitter and the receiver.

2.2.2 Fast fading

The channel variation is faster than the baseband signal variation if channel coherence time is smaller than the symbol period or delay constraint of the channel, in that case fast fading occurs and hence the channel changes over the one period. In other words, the channel coherence time, T_c, is smaller than the symbol period. T_c is related to Doppler spread. In the fast fading the relative moving between the transmitter, receiver and surrounding objects are quite fast. A high Doppler shift value produced when the transmitter and receiver are being moving to each other, in vice versa low Doppler shift value is achieved when the transmitter and receiver are being far away from each other.

3. Channel Modeling

3.1 Rayleigh channel

In a wireless communication systems when a multipath existed between the transmitter and the receiver in heavily build up areas where non line of sight communication existed which means there is no direct path between the transmitter and the receiver. The objects attenuate, reflect, refract and scattered the signal before reaching the receiver. This propagation mechanism is known as Rayleigh fading. After propagation over N reflected and scattered paths, the received signal may be considered as the sum of these N components with random amplitude and phase for each component. Thus, when the receiving station is stationary, the received signal r(t) can be written as (Shankar, 2002):

$$r(t) = \sum_{i=1}^{N} a_i \cos(2\pi f_c t + \varphi_i) \quad (1)$$

Where a_i is a random variable corresponding to the amplitude of the i_{th} signal component, and φ_i is another uniformly distributed random variable corresponding to the phase angle of the i_{th} signal component.

Equation (1) can be re-written in the form:

$$r(t) = \cos(2\pi f_c t) \sum_{i=1}^{N} a_i \cos\varphi_i - \sin(2\pi f_c t) \sum_{i=1}^{N} a_i \sin\varphi_i \quad (2)$$

Equation (2) can be expressed as (Shankar, 2002):

$$r(t) = X\, cos(2\pi f_c t) - Y\, sin(2\pi f_c t) \quad (3)$$

Where

$$\mathrm{X} = \sum_{i=1}^{N} a_i \cos(2\pi f_c t) \quad (4)$$

$$\mathrm{Y} = \sum_{i=1}^{N} a_i \sin(2\pi f_c t) \quad (5)$$

X and *Y* can be considered as two identical, independent Gaussian random variables when N tends to a large value (Papoulis and Pillai, 2002).

Equation (3) represents the received RF signal when the receiver is stationary. If the mobile unit is moving with a speed of *v* meters per second relative to the base station, the received signal will acquire a Doppler shift in frequency. The maximum Doppler shift is given by (Al-Raie, 2010):

$$f_d = \frac{v}{c} f_c \quad (6)$$

The instantaneous Doppler shift in frequency is dependent on the angle of arrival of the incoming signal path component as shown in Fig. 2.

The instantaneous value of the Doppler shift f_{di} can be expressed as (Shankar, 2002):

$$f_{di} = f_d\, cos\, \alpha_i \quad (7)$$

Where α_i is the angle of arrival for the i_{th} path signal component.

On the other hand, the instantaneous frequency of the received RF signal becomes (Shankar, 2002):

$$f_i = f_c + f_d cos\, \alpha_i \quad (8)$$

Accordingly, the received signal can thereby expressed in the form:

$$r(t) = \sum_{i=1}^{N} a_i \cos(2\pi(f_c + f_{di})t + \varphi_i) \quad (9)$$

Equation (9) can alternatively be written in other form using the trigonometric identities:

$$r(t) = \cos(2\pi f_c t)\sum_{i=1}^{N} a_i \cos(2\pi f_{di} t + \varphi_i) - \sin(2\pi f_c t)\sum_{i=1}^{N} a_i (2\pi f_{di} t + \varphi_i) \quad (10)$$

The received signal can also be formulated as (Shankar, 2002):

$$r(t) = \cos(2\pi f_c t)X(t) - \sin(2\pi f_c t)Y(t) \quad (11)$$

Where

$$X(t) = \sum_{i=1}^{N} a_i \cos(2\pi f_{di} t + \varphi_i) \quad (12)$$

$$Y(t) = \sum_{i=1}^{N} a_i \sin(2\pi f_{di} t + \varphi_i) \quad (13)$$

And $f_{di} = f_d \cos \alpha_i$ (14)

X(t) and *Y(t)* are also known as the in-phase and quadrature components of the received signal respectively. It is seen from (11) that the received signal is like a quadrature modulated carrier. The probability density function (PDF) of the received signal envelope, f(r), can be shown to be Rayleigh given by (Luengo and Martino, 2014):

$$f(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \quad (15)$$

Where σ^2 is the time-average power of the received signal before the envelope detection.

3.2 Additive White Gaussian Noise (AWGN) Channel

AWGN channel is straight forward and very commonly used to add the noise to transmitted signal while signals travel from the channel. The Gaussian channel is important for providing an upper bound on system performance. The mathematical expression in received signal r(t) = s(t) + n(t) that passed through the AWGN channel where s(t) is transmitted signal and n(t) is background noise.

4. M-ARY FSK

Frequency Shift Keying (FSK) is a modulation technique in which data is transmitted through discrete frequencies of a carrier wave with fixed amplitude.

This technique can use multiple carriers to transmit k > 1 bits. In order to guarantee that the symbols are orthogonal, there should be a $\Delta f = n/2T$ frequency separation among the $M = 2^k$ carrier frequencies, where n is an integer and T is the bit period. The complex baseband signaling waveforms for M-ary FSK (Madhow, 2008) are given in the following equation:

$$s_m(t) = e^{j2\pi f_m} I_{[0,T]}, m = 1,2,\dots,M \quad (16)$$

Coherent and non-coherent demodulation methods exist and depend on whether the phase of the sinusoids φ are equal or not. FSK systems have one of the highest power efficiencies among the available modulation schemes, but they suffer from low bandwidth efficiency. The main advantages of this modulating scheme are in-sensitiveness to amplitude variations in the channel, its compatibility with non-linear transmitters and receivers and the fact that it is not required to have absolute frequency accuracy for correct demodulation. The signal detection is also less complex than for PSK. One of the drawbacks is the fact that it is less bandwidth efficient when compared with other schemes such as ASK or PSK.

The M-ary FSK modulated signals can be written as follows:

$$s_m(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_m t + \varphi_m), 0 \le t \le T_b \quad (17)$$

Where E_b is the transmitted signal energy per bit, T_b is the bit period of the binary data stream φ_m is the phase.

The analytical probability of bit error (BER) expressions for Multi-Level Frequency Shift Keying (M-FSK) using non coherent detection in AWGN channel and Rayleigh fading channel can be approximated respectively given by, (Proakis, 2001)

$$P_{b,AWGN} = \left(\frac{M/2}{M-1}\right)\sum_{n=1}^{M-1}\frac{(-1)^{n+1}}{n+1}\frac{(M-1)!}{(M-1-n)!n!}e^{-\left(\frac{n(\log_2 M)E_b}{(n+1)N_o}\right)} \quad (18)$$

$$P_{b,Rayleigh} = \frac{1}{2}\frac{M}{M-1}\sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n}\frac{1}{m+1}\exp\left[-\frac{n}{n+1}\frac{kE_b}{N_o}\right] \quad (19)$$

5. SIMULATION RESULTS

5.1 Description of simulation process

The description of simulation process is shown in a block diagram of Fig.3

5.2 Simulation setup

The simulation parameters are produced in Table 1.

Table 1. Simulation parameters

Parameter	Specification
Symbol period	2 msec
Sampling frequency	40000 Hz
No. of samples	80 samples
Transmitted symbols	20000 symbols
Doppler shift	0Hz- 200Hz
Modulation technique	FSK (noncoherent detection)

5.3 Simulation of AWGN Channel

It is recommended to use modulation with M<16 to avoid increasing the complexity of the system (Suthar *et.al.*, 2009), and also the changing in BER will reduces for increasing the M-levels (Sackey, 2006).

Fig. 4 shows the results of simulation at several different values of E_b/N_o and how it compares with the theory. It is seen that at a particular error probability, at a constant E_b/N_o, 8-level FSK has the lowest error probability, and binary FSK the largest. Hence, the curves in the figure confirm that M-ary FSK is a power efficient modulation scheme whose power efficiency increases as the number of frequencies employed increases.

5.4 Simulation of Rayleigh Fading Channel

The comparison of theoretical and simulated results for M- levels (2, 4, and 8). As it is clear in Fig. 5, it is observed that the BER monotonically decreases as E_b/N_o increases.

Fig. 6 demonstrates a single path (flat fading) and multipath (frequency selective fading) and shows roughly similar performance due to the simulation that was carried out with FSK modulation technique. Thus, the performance is not affected by the rapid fluctuation of the amplitude resulted by increasing the number of the received paths. In the above simulation figure, four received paths have been used in multipath delay spread greater than symbol period of the transmitted signal with different path gains. On the other hand, the single path used a channel delay less than the symbol period.

Fig. 7 simulates the different Doppler shifts in BFSK. So, when the carrier frequency is assumed 900MHz for f_d=0Hz there is no mobility which leads to the slow fading channel, for f_d=21.15Hz with the mobile velocity is 25.38 Km/h where at this considered Doppler shift the threshold between the fast and slow fading channels is achieved i.e. the theoretical and simulated results in Rayleigh fading channel would be identical the same as in Fig. 5, and for f_d=200Hz the mobile velocity is 240Km/h which deemed to be fast fading channel. Therefore, any f_d value greater than 21.5Hz in this system is considered working in fast fading.

It can be noticed that the increase in the Doppler shift increases the BER for the constant signal to noise ratio (SNR), whereas the BER decreases with the increasing in SNR for the same Doppler Shift. Generally in low Doppler shift (slow fading) the BER decreases but conversely by increasing Doppler shift (fast fading) the BER increases.

In Fig. 8, it can be seen that Raleigh fading channel has higher BER than AWGN channel. For example, when BER = 10^{-1}, AWGN is about 4 dB better than Rayleigh Channel in terms of E_b/N_o due to that Rayleigh fading is one of the worst case fading scenarios.

We have used the following MATLAB syntaxes to calculate the theoretical results for AWGN and Rayleigh channel respectively:

```
berawgn(EbNo,'fsk',M, 'noncoherent')
berfading(EbNo,'fsk',M,divorder,'noncoherent')
```

6. Conclusion

The BER performance can be found over AWGN and Rayleigh fading channel using M-FSK modulation technique. It is obvious from analyzing the representation of Eb/No vs BER for both channels that the performance improves as the number of frequencies employed increases. The bit error probability for lower value of M is lower in case of AWGN environment and increases in fading scenario. Given the simulation results of multipath scenario when the speed increasing initiated from the stationary status, the channel fading becomes gradually faster continuing to exceed the threshold between slow and fast fading which leads reducing the system performance. Moreover, there is a Doppler shift value for every mobile communication system which specify that threshold. Also, the variation in Doppler frequency shifts is the dominant effect on the BER performance rather than changing the number of received paths due to the fact that the system was considered the FSK modulation scheme since it carries the transmitted information on the channel frequency which is not affected by the rapid amplitude fluctuation. In this system, the simulated results comparison revealed a close agreement to the theoretical results. It will be of a great interest for potential researchers to implement more functionality support for channel fading.

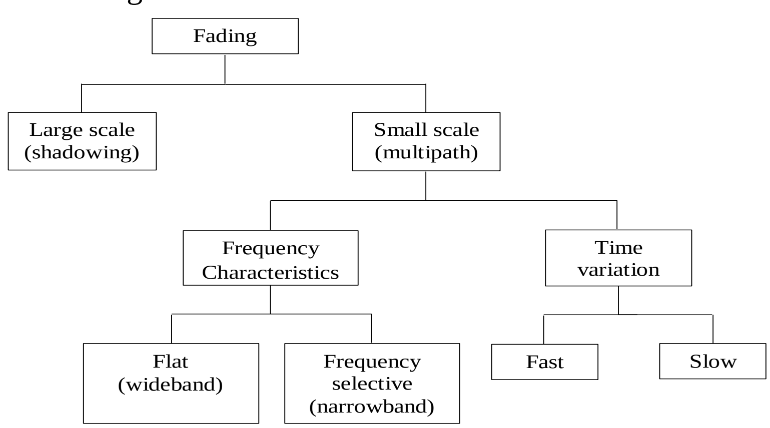

Figure 1. Block diagram summarizing the different types of fading

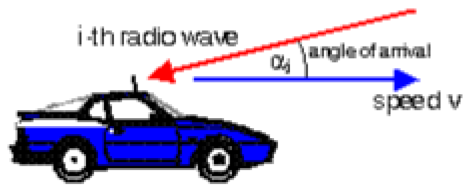

Figure 2. A mobile unit moving at speed v (Al-Raie, 2010)

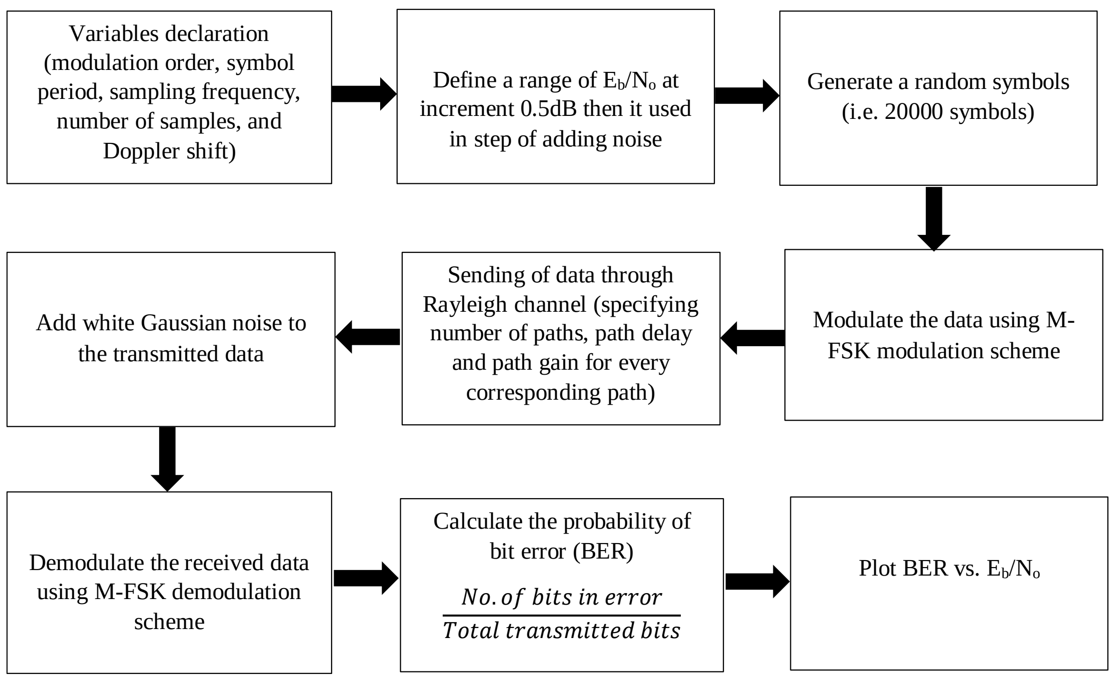

Figure 3. Block diagram of simulation process

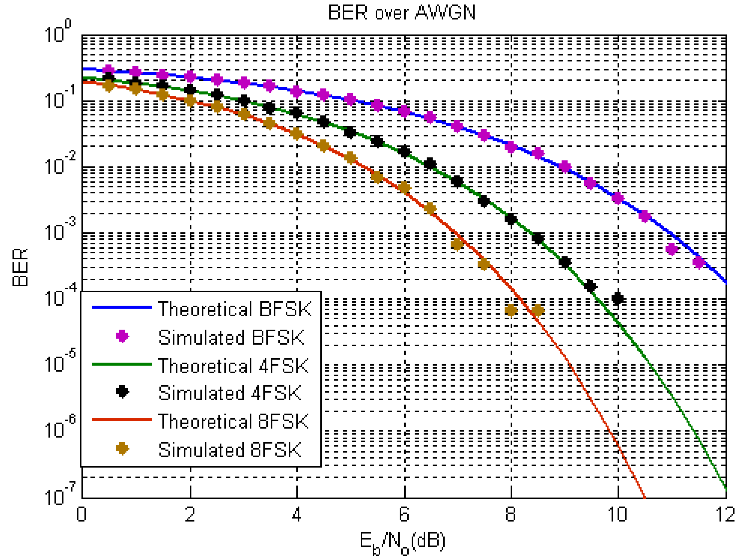

Figure 4. Comparison of simulated and theoretical BER, M=2, 4, 8 for AWGN channel

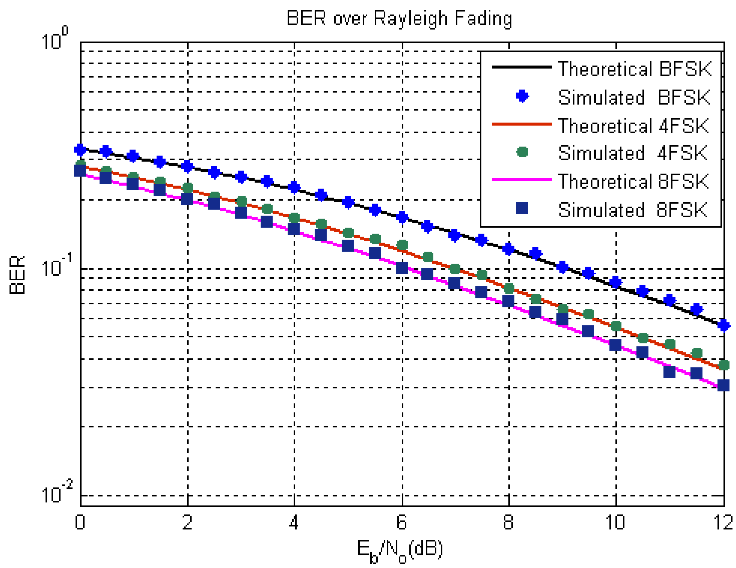

Figure 5. Comparison of simulated and theoretical BER, M=2, 4, 8 for Rayleigh channel

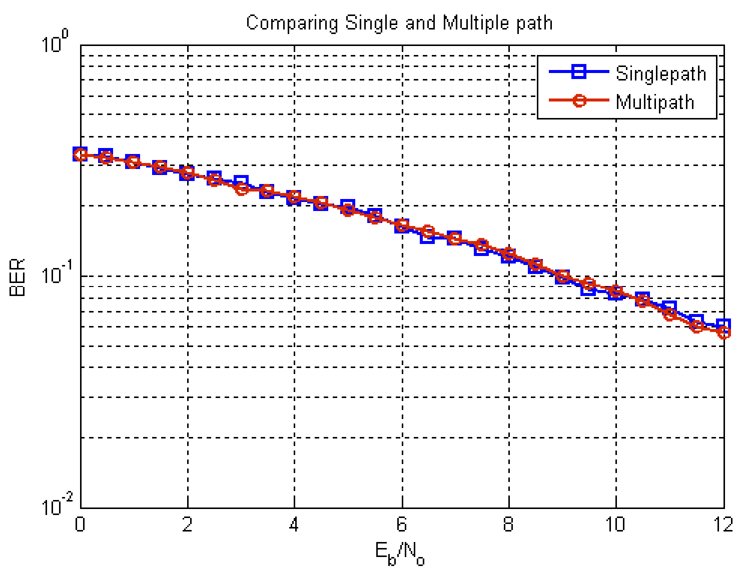

Figure 6. Comparison of single and multiple path of BER for Rayleigh channel

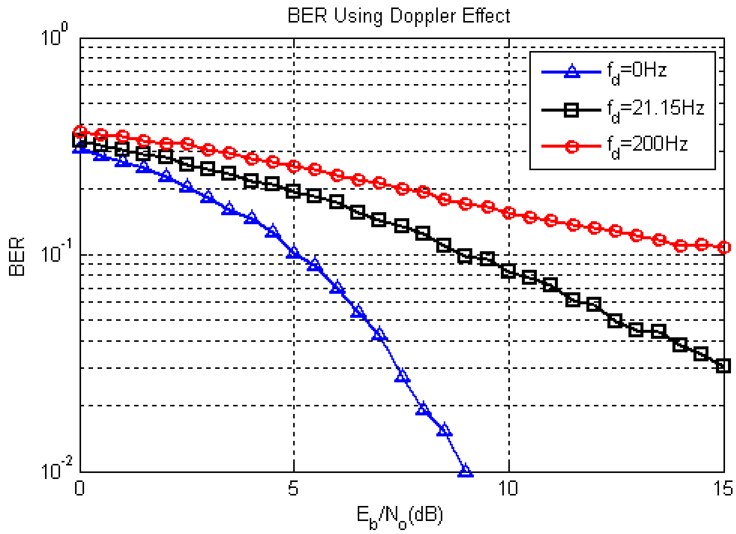

Figure 7. BER performance for different Doppler shift values

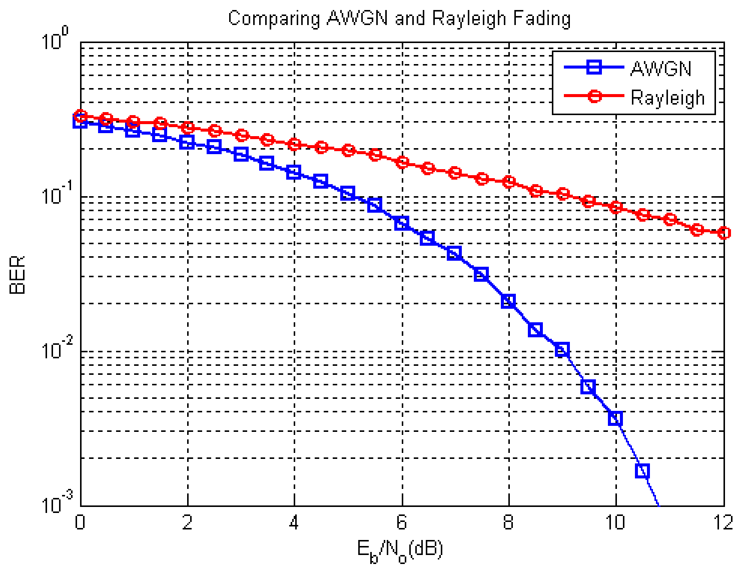

Figure 8. Comparison of BER over AWGN and Rayleigh channel

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