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On Bayes Estimation

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Abstract

The main aim of this paper is to give a comprehensive presentation of estimating method namely standard Bayes and proposed Bayes estimator for exponential and weibull parameter. To have the goal that is the best estimation method, we used simulation with different samples sizes. Some new results are obtained and some of our modifications are much better than the classical methods of estimation.

1. Introduction

Mathematical statistics introduces abstraction forms, each of which provides valuable tools for application in very wide areas. The study and research of survival or reliability or life time belong to the same area of study but they may belong to a different area of applications.

In survival analysis one can use several life time distributions, the negative exponential distribution NE with mean θ is one of them, it was the first widely discussed life time distribution model. The probability density function of a random variable $T \sim NE$ with mean θ, which represents the life time variable of interest, is given by:

$$f(t;\theta)=\frac{1}{\theta}e^{\left(-\frac{t}{\theta}\right)}, \qquad t \geq 0, \quad \theta > 0.$$

The development of theoretical probability distributions is useful in analyzing failure processes with discussions on the weibull. This distribution has hazard rate function that is not constant over time, thus providing a necessary alternative to the exponential failure law. The probability Weibull function is given by:

$$f(t;\alpha,\beta)=\frac{\alpha}{\beta}t^{\alpha-1}e^{\left(-\frac{t^{\alpha}}{\beta}\right)}, \qquad t,\alpha,\beta > 0.$$

In this paper, we present exponential and Weibull distribution in general and then some proposed new estimators using Bayes technique .

2. Bayes Estimation

Let $t_1,t_2,...,t_n$ be a random sample of size n with distribution function $F(t;\beta,\theta)$ and the probability density function $f(t;\beta,\theta)$, there are several steps to find the Bayes estimation of the parameters β,θ, So we take two distribution the exponential distribution with one parameter and Weibull distribution with two parameters.

2.1 Exponential Distribution

One of the most useful and widely exploited model is the exponential distribution with one parameter with probability density function

$$f(t;\theta)=\frac{1}{\theta}e^{\left(-\frac{t}{\theta}\right)}, \qquad t\geq 0, \quad \theta>0.$$

2.1.1 Jefferys' Prior information

Let $g(\theta)\propto\sqrt{I(\theta)}$**, where**

$\sqrt{I(\theta)}=\sqrt{\text{Fisher Information}}$

, and

$$I(\theta)=-nE\left(\frac{\partial^2 \ln f(t,\theta)}{\partial\theta^2}\right),$$

$$f(t,\theta)=\frac{1}{\theta}e^{\left(-\frac{t}{\theta}\right)}, \qquad t\geq 0, \quad \theta>0,$$

$$\ln f(t,\theta)=-\ln\theta-\frac{t}{\theta},$$

$$\frac{\partial \ln f(t,\theta)}{\partial\theta}=-\frac{1}{\theta}+\frac{t}{\theta^2},$$

$$\frac{\partial^2 \ln f(t,\theta)}{\partial\theta^2}=\frac{1}{\theta^2}-\frac{2t}{\theta^3},$$

$$E\left(\frac{\partial^2 \ln f(t,\theta)}{\partial\theta^2}\right)=E\left(\frac{1}{\theta^2}-\frac{2t}{\theta^3}\right)=\left(\frac{1}{\theta^2}-\frac{2E(t)}{\theta^3}\right)=\frac{1}{\theta^2}-\frac{2}{\theta^2}=-\frac{1}{\theta^2},$$

$$I(\theta)=-nE\left(\frac{\partial^2 \ln f(t,\theta)}{\partial\theta^2}\right)=\frac{-nt}{\theta^2}.$$

$\because\ g(\theta)\propto\sqrt{I(\theta)}\Rightarrow g(\theta)\propto\frac{\sqrt{n}}{\theta}$**, let** $g(\theta)=k\frac{\sqrt{n}}{\theta}$**, so**

$$L(t_1,\ldots,t_n|\theta)=\prod_{i=1}^{n} f(t_i|\theta)$$

$$=\prod_{i=1}^{n}\frac{1}{\theta}e^{\left(\frac{t_i}{\theta}\right)}=\frac{1}{\theta^n}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}.$$

The joint probability density function $f(t_1,\ldots,t_n,\theta)$ **is given by:**

$$H(t_1,\ldots,t_n,\theta)=\prod_{i=1}^{n} f(t_i|\theta)g(\theta)$$

$$=L(\theta)k\frac{\sqrt{n}}{\theta}=\frac{k\sqrt{n}}{\theta^{n+1}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}.$$

The marginal probability density function of $(t_1,\ldots,t_n)$ **is given by:**

$$p(t_1,...,t_n)=\int_0^\infty H(t_1,...,t_n,\theta)d\theta$$

$$=\int_0^\infty \frac{k\sqrt{n}}{\theta^{n+1}}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}d\theta$$

Let $y=\frac{\sum_{i=1}^n t_i}{\theta} \Rightarrow \theta=\frac{\sum_{i=1}^n t_i}{y} \Rightarrow d\theta=-\frac{\sum_{i=1}^n t_i}{y^2}dy,$

$$\therefore \int_0^\infty \frac{k\sqrt{n}}{\theta^{n+1}}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}d\theta=\int_0^\infty \frac{k\sqrt{n}}{\left(\frac{\sum_{i=1}^n t_i}{y}\right)^{n+1}}e^{(-y)}\left(-\frac{\sum_{i=1}^n t_i}{y^2}\right)dy=k\sqrt{n}\int_0^\infty\left(\frac{\sum_{i=1}^n t_i}{y}\right)^{-n-1}e^{(-y)}\left(-\frac{\sum_{i=1}^n t_i}{y^2}\right)dy$$

$$=-k\sqrt{n}\int_0^\infty \frac{\left(\sum_{i=1}^n t_i\right)^{-n-1+1}}{y^{-n-1+2}}e^{(-y)}dy=\frac{-k\sqrt{n}}{\left(\sum_{i=1}^n t_i\right)^n}\int_0^\infty y^{n-1}e^{(-y)}dy=\frac{-k\sqrt{n}\overline{)n}}{\left(\sum_{i=1}^n t_i\right)^n}$$

$$=\frac{-k\sqrt{n}(n-1)!}{\left(\sum_{i=1}^n t_i\right)^n}.$$

And the conditional probability density function of θ given the data $(t_1,...,t_n)$ is given by :

$$h(\theta|t_1,...,t_n)=\frac{H(t_1,...,t_n,\theta)}{p(t_1,...,t_n)}=\frac{\frac{k\sqrt{n}}{\theta^{n+1}}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}}{\frac{-k\sqrt{n}(n-1)!}{\left(\sum_{i=1}^n t_i\right)^n}}=\frac{e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}\left(\sum_{i=1}^n t_i\right)^n}{\theta^{n+1}(n-1)!}.$$

Using squared error loss function $\ell(\hat{\theta},\theta)=c(\hat{\theta}-\theta)^2$**, the risk function is given by:**

$$R(\hat{\theta},\theta)=E\ell(\hat{\theta},\theta)=\int_0^\infty \ell(\hat{\theta},\theta)h(\theta|t_1,...,t_n)d\theta$$

$$=\int_0^\infty c(\hat{\theta}-\theta)^2\frac{e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}\left(\sum_{i=1}^n t_i\right)^n}{\theta^{n+1}(n-1)!}d\theta=\int_0^\infty c\left((\hat{\theta})^2-2\theta\hat{\theta}+\theta^2\right)\frac{e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}\left(\sum_{i=1}^n t_i\right)^n}{\theta^{n+1}(n-1)!}d\theta$$

$$=\int_0^\infty c\left(\hat{\theta}\right)^2\left(\frac{e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}\left(\sum_{i=1}^n t_i\right)^n}{\theta^{n+1}(n-1)!}\,d\theta\right)-2c\hat{\theta}\int_0^\infty\left(\frac{\theta e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}\left(\sum_{i=1}^n t_i\right)^n}{\theta^{n+1}(n-1)!}\,d\theta\right)+c\int_0^\infty\left(\frac{\theta^2 e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}\left(\sum_{i=1}^n t_i\right)^n}{\theta^{n+1}(n-1)!}\,d\theta\right),$$

to solve the above integration let $y=\frac{\sum_{i=1}^n t_i}{\theta}\Rightarrow\theta=\frac{\sum_{i=1}^n t_i}{y}\Rightarrow d\theta=-\frac{\sum_{i=1}^n t_i}{y^2}dy,$**, we denote the first integral by** I_1**, the second integral by** I_2**, and the third integral by** I_3**,**

$$\therefore\ I_1=c\left(\hat{\theta}\right)^2\frac{\left(\sum_{i=1}^n t_i\right)^n}{(n-1)!}\int_0^\infty\left(\frac{\left(-\frac{\sum_{i=1}^n t_i}{y^2}\right)e^{(-y)}}{\left(-\frac{\sum_{i=1}^n t_i}{y}\right)^{n+1}}\,dy\right)$$

$$=-c\left(\hat{\theta}\right)^2\frac{\left(\sum_{i=1}^n t_i\right)^n}{(n-1)!}\int_0^\infty\left(\frac{\left(\sum_{i=1}^n t_i\right)^{-n}e^{(-y)}}{y^{-n+1}}\,dy\right)=\frac{-c\left(\hat{\theta}\right)^2}{(n-1)!}\int_0^\infty\left(y^{n-1}e^{(-y)}dy\right)=-c\left(\hat{\theta}\right)^2.$$

$$I_2=2c\hat{\theta}\frac{\left(\sum_{i=1}^n t_i\right)}{(n-1)!}\int_0^\infty\left(y^{n-2}e^{(-y)}dy\right)$$

$$=\frac{2c\hat{\theta}\left(\sum_{i=1}^n t_i\right)\Gamma(n-1)}{(n-1)!}=\frac{2c\hat{\theta}\left(\sum_{i=1}^n t_i\right)}{(n-1)}.$$

$$I_3=-\frac{c\left(\sum_{i=1}^n t_i\right)^2}{(n-1)!}\int_0^\infty\left(y^{n-3}e^{(-y)}dy\right)=-\frac{c\left(\sum_{i=1}^n t_i\right)^2\Gamma(n-2)}{(n-1)!}=-\frac{c\left(\sum_{i=1}^n t_i\right)^2}{(n-1)(n-2)}.$$

$$\therefore\ R\left(\hat{\theta},\theta\right)=-c\left(\hat{\theta}\right)^2+\frac{2c\hat{\theta}\left(\sum_{i=1}^n t_i\right)}{(n-1)}-\frac{c\left(\sum_{i=1}^n t_i\right)^2}{(n-1)(n-2)}$$

$$\therefore\ \frac{\partial R\left(\hat{\theta},\theta\right)}{\partial\hat{\theta}}=-2c\hat{\theta}+\frac{2c\left(\sum_{i=1}^n t_i\right)}{(n-1)}.$$

Let $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$

$$\therefore \ \hat{\theta}_B = \frac{\sum_{i=1}^{n} t_i}{(n-1)}.$$

We can find the estimator of survival function by:

$\hat{s}_B(t) = \int_0^\infty e^{\left(\frac{-t_i}{\theta}\right)} \Pi(\theta|t_1,...,t_n) d\theta = \frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!} \int_0^\infty \theta^{-n-1} e^{-\left(\frac{t_i + \sum_{i=1}^{n} t_i}{\theta}\right)} d\theta$, **now, to evaluate the above integral, let** $y = \frac{t_i + \sum_{i=1}^{n} t_i}{\theta} \Rightarrow d\theta = -\frac{t_i + \sum_{i=1}^{n} t_i}{y^2} dy$

$$\therefore \ \hat{s}_B(t) = -\frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!} \int_0^\infty \frac{\left(t_i + \sum_{i=1}^{n} t_i\right)^{-n}}{y^{-n+1}} e^{(-y)} dy$$

$$\hat{s}_B(t) = -\frac{\left(\sum_{i=1}^{n} t_i\right)^n (n-1)!}{(n-1)! \left(t_i + \sum_{i=1}^{n} t_i\right)^n} = \left(\frac{t_i + \sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} t_i}\right)^{-n} = \left(\frac{t_i}{\sum_{i=1}^{n} t_i} + 1\right)^{-n}.$$

2.1.2 Extension Of Jeffery's Prior

The extension of Jeffery's prior is $g(\theta) \propto [I(\theta)]^{c_1}, c_1 \in R^+$, **so** $g(\theta) \propto \left[\frac{n}{\theta^2}\right]^{c_1}$**, let** $g(\theta) = k\frac{n^{c_1}}{\theta^{2c_1}}$

$$L(t_1,...,t_n|\theta) = \prod_{i=1}^{n} f(t_i;\theta) = \prod_{i=1}^{n} \frac{1}{\theta} e^{\left(\frac{t_i}{\theta}\right)} = \frac{1}{\theta^n} e^{\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)}.$$

The joint probability density function $f(t_1,...,t_n;\theta)$ **is given by:**

$$H(t_1,...,t_n,\theta) = \prod_{i=1}^{n} f(t_i|\theta) g(\theta) = \frac{kn^{c_1}}{\theta^{n+2c_1}} e^{\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)}.$$

The marginal probability density function of $(t_1,...,t_n)$ **is given by:**

$$p(t_1,...,t_n) = \int_0^\infty H(t_1,...,t_n,\theta) d\theta = kn^{c_1} \int_0^\infty \frac{e^{\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)}}{\theta^{n+2c_1}} d\theta$$

let $y = \frac{\sum_{i=1}^{n} t_i}{\theta} \Rightarrow \theta = \frac{\sum_{i=1}^{n} t_i}{y} \Rightarrow d\theta = -\frac{\sum_{i=1}^{n} t_i}{y^2} dy$

$$\therefore\ kn^{c_1}\int_0^\infty \frac{exp\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}{\theta^{n+2c_1}}d\theta = -kn^{c_1}\int_0^\infty \frac{\left(\sum_{i=1}^n t_i\right)^{-n-2c_1+1}}{y^{-n-2c_1+2}}e^{(-y)}dy = \frac{-kn^{c_1}}{\left(\sum_{i=1}^n t_i\right)^{n+2c_1-1}}\int_0^\infty y^{n+2c_1-2}e^{(-y)}dy$$

$$= \frac{-kn^{c_1}}{\left(\sum_{i=1}^n t_i\right)^{n+2c_1-1}}\overline{)n+2c_1-1} = \frac{-kn^{c_1}(n+2c_1-2)!}{\left(\sum_{i=1}^n t_i\right)^{n+2c_1-1}}.$$

And the conditional probability density function of θ given the data $(t_1,...,t_n)$ is given by:

$$h(\theta|t_1,...,t_n) = \frac{H(t_1,...,t_n,\theta)}{p(t_1,...,t_n)} = \frac{\frac{kn^{c_1}}{\theta^{n+2c_1}}exp\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}{\frac{-kn^{c_1}(n+2c_1-2)!}{\left(\sum_{i=1}^n t_i\right)^{n+2c_1-1}}} = \frac{\theta^{-n-2c_1}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}}{\left(\sum_{i=1}^n t_i\right)^{1-n-2c_1}(n+2c_1-2)!}.$$

Using squared error loss function $\ell(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2$, The risk function is given by:

$$R(\hat{\theta},\theta) = E\ell(\hat{\theta},\theta)$$

$$= \int_0^\infty \ell(\hat{\theta},\theta)\Pi(\theta|t_1,...,t_n)d\theta = \int_0^\infty c(\hat{\theta}-\theta)^2\frac{\theta^{-n-2c_1}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}}{\left(\sum_{i=1}^n t_i\right)^{1-n-2c_1}(n+2c_1-2)!}d\theta$$

$$= \int_0^\infty c\left((\hat{\theta})^2 - 2\theta\hat{\theta}+\theta^2\right)\frac{\theta^{-n-2c_1}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}}{\left(\sum_{i=1}^n t_i\right)^{1-n-2c_1}(n+2c_1-2)!}d\theta$$

$$= \int_0^\infty c(\hat{\theta})^2\frac{\theta^{-n-2c_1}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}}{\left(\sum_{i=1}^n t_i\right)^{1-n-2c_1}(n+2c_1-2)!}d\theta - 2c\hat{\theta}\int_0^\infty \frac{\theta^{-n-2c_1}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}}{\left(\sum_{i=1}^n t_i\right)^{1-n-2c_1}(n+2c_1-2)!}d\theta + c\int_0^\infty \frac{\theta^{-n-2c_1}e^{\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)}}{\left(\sum_{i=1}^n t_i\right)^{1-n-2c_1}(n+2c_1-2)!}d\theta$$

to solve the above integral, let $y=\frac{\sum_{i=1}^{n}t_i}{\theta}\Rightarrow\theta=\frac{\sum_{i=1}^{n}t_i}{y}\Rightarrow d\theta=-\frac{\sum_{i=1}^{n}t_i}{y^2}dy$ **, we denote the first integral by** I_1**, to the second integral by** I_2**, and to the third integral by** I_3**,**

$$\therefore\ I_1=\frac{c(\hat{\theta})^2}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}(n+2c_1-2)!}\int_0^\infty\left(\frac{\left(-\frac{\sum_{i=1}^{n}t_i}{y^2}\right)exp(-y)}{\left(-\frac{\sum_{i=1}^{n}t_i}{y}\right)^{n+2c_1}}\right)dy$$

$$=-\frac{c(\hat{\theta})^2}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}(n+2c_1-2)!}\int_0^\infty\left(\frac{\left(\sum_{i=1}^{n}t_i\right)^{-n-2c_1+1}e^{(-y)}}{(y)^{-n-2c_1+2}}\right)dy$$

$$=-\frac{c(\hat{\theta})^2\left(\sum_{i=1}^{n}t_i\right)^{-n-2c_1+1}}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}(n+2c_1-2)!}\int_0^\infty\left((y)^{n+2c_1-2}e^{(-y)}\right)dy=-\frac{c(\hat{\theta})^2}{(n+2c_1-2)!}\overline{)n+2c_1-1}=-c(\hat{\theta})^2.$$

$$I_2=-\frac{2c\hat{\theta}\left(\sum_{i=1}^{n}t_i\right)^{-n-2c_1+2}}{(n+2c_1-2)!\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}}\int_0^\infty\left(y^{n+2c_1-3}e^{(-y)}\right)dy$$

$$=\frac{2c\hat{\theta}\left(\sum_{i=1}^{n}t_i\right)}{(n+2c_1-2)!}\overline{)n+2c_1-2}=\frac{2c\hat{\theta}\left(\sum_{i=1}^{n}t_i\right)}{(n+2c_1-2)}.$$

$$I_3=-\frac{c\left(\sum_{i=1}^{n}t_i\right)^{-n-2c_1+3}}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}(n+2c_1-2)!}\int_0^\infty\left(y^{n+2c_1-4}e^{(-y)}\right)dy$$

$$=-\frac{c\left(\sum_{i=1}^{n}t_i\right)^2}{(n+2c_1-2)!}\overline{)n+2c_1-3}=-\frac{c\left(\sum_{i=1}^{n}t_i\right)^2}{(n+2c_1-2)(n+2c_1-3)}.$$

$$\therefore\ R(\hat{\theta},\theta)=c(\hat{\theta})^2+\frac{2c\hat{\theta}\left(\sum_{i=1}^{n}t_i\right)}{(n+2c_1-2)}-\frac{c\left(\sum_{i=1}^{n}t_i\right)^2}{(n+2c_1-2)(n+2c_1-3)}$$

$$\therefore\ \frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=2c\hat{\theta}+\frac{2c\left(\sum_{i=1}^{n}t_i\right)}{(n+2c_1-2)}.$$

To find the maximum risk ,

$$\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$$

$$\therefore\ \hat{\theta}_B=\frac{\sum_{i=1}^{n}t_i}{(n+2c_1-2)}.$$

We can find the estimator of survival function by:

$$\hat{s}_B(t)=\int_0^{\infty}exp\left(\frac{-t_i}{\theta}\right)\Pi(\theta|t_1,...,t_n)d\theta$$

$$=\frac{1}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}(n+2c_1-2)!}\int_0^{\infty}\theta^{-n-2c_1}e^{-\left(\frac{t_i+\sum_{i=1}^{n}t_i}{\theta}\right)}d\theta$$

now, to evaluate the above integral, let $y=\frac{t_i+\sum_{i=1}^{n}t_i}{\theta}\Rightarrow d\theta=-\frac{t_i+\sum_{i=1}^{n}t_i}{y^2}dy$ **,**

$$\therefore\ \hat{s}_B(t)=-\frac{1}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}(n+2c_1-2)!}\int_0^{\infty}\frac{\left(t_i+\sum_{i=1}^{n}t_i\right)^{-n-2c_1+1}}{y^{-n-2c_1+2}}e^{(-y)}dy$$

$$=-\frac{\left(t_i+\sum_{i=1}^{n}t_i\right)^{-n-2c_1+1}}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}(n+2c_1-2)!}\int_0^{\infty}y^{n+2c_1-2}e^{(-y)}dy$$

$$=\left(\frac{t_i+\sum_{i=1}^{n}t_i}{\sum_{i=1}^{n}t_i}\right)^{-n-2c_1+1}=\left(\frac{t_i}{\sum_{i=1}^{n}t_i}+1\right)^{-n-2c_1+1}.$$

2.1.3 The Proposed Estimator

The extension of Jeffery's prior is $g(\theta)\propto[I(\theta)]^{\frac{1}{c_1}}, c_1\in R^+$, **so** $g(\theta)\propto\left[\frac{n}{\theta^2}\right]^{\frac{1}{c_1}}$, **let** $g(\theta)=k\frac{n^{\frac{1}{c_1}}}{\theta^{\frac{2}{c_1}}}$,

$$L(t_1,...,t_n|\theta)=\prod_{i=1}^{n} f(t_i|\theta)=\prod_{i=1}^{n}\frac{1}{\theta}exp(\frac{t_i}{\theta})=\frac{1}{\theta^n}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}.$$

The joint probability density function $f(t_1,...,t_n;\theta)$ **is given by:**

$$H(t_1,...,t_n;\theta)=\prod_{i=1}^{n} f(t_i|\theta)g(\theta)=L(t_1,...,t_n|\theta)k\frac{n^{c_1}}{\theta^{2c_1}}=\frac{kn^{\frac{1}{c_1}}}{\theta^{n+\frac{2}{c_1}}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}.$$

The marginal probability density function of $(t_1,...,t_n)$ **is given by:**

$$p(t_1,...,t_n)=\int_0^\infty H(t_1,...,t_n,\theta)d\theta=\int_0^\infty\frac{kn^{\frac{1}{c_1}}}{\theta^{n+\frac{2}{c_1}}}exp\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)d\theta=kn^{\frac{1}{c_1}}\int_0^\infty\frac{exp\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}{\theta^{n+\frac{2}{c_1}}}d\theta,$$

let $y=\frac{\sum_{i=1}^{n}t_i}{\theta}\Rightarrow\theta=\frac{\sum_{i=1}^{n}t_i}{y}\Rightarrow d\theta=-\frac{\sum_{i=1}^{n}t_i}{y^2}dy,$

$$\therefore\ kn^{\frac{1}{c_1}}\int_0^\infty\frac{exp\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}{\theta^{n+\frac{2}{c_1}}}d\theta=-kn^{\frac{1}{c_1}}\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\int_0^\infty y^{n+\frac{2}{c_1}-2}exp(-y)dy$$

$$=-kn^{\frac{1}{c_1}}\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\overline{\left)n+\frac{2}{c_1}-1\right.}=-kn^{\frac{1}{c_1}}\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\left(n+\frac{2}{c_1}-2\right)!.$$

And the conditional probability density function of θ **given the data** $(t_1,...,t_n)$ **is given by :**

$$h(\theta|t_1,...,t_n)=\frac{H(t_1,...,t_n,\theta)}{p(t_1,...,t_n)}$$

$$=\frac{\dfrac{kn^{\frac{1}{c_1}}}{\theta^{n+\frac{2}{c_1}}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}}{-kn^{\frac{1}{c_1}}\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\left(n+\frac{2}{c_1}-2\right)!}=\frac{\theta^{-n-\frac{2}{c_1}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}}{\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\left(n+\frac{2}{c_1}-2\right)!}.$$

Using squared error loss function $\ell(\hat{\theta},\theta)=c(\hat{\theta}-\theta)^2$**, and the risk function is given by:**

$$R(\hat{\theta},\theta)=E\ell(\hat{\theta},\theta)=\int_0^\infty \ell(\hat{\theta},\theta)h(\theta|t_1,...,t_n)d\theta$$

$$=\int_0^\infty c(\hat{\theta}-\theta)^2\frac{\theta^{-n-\frac{2}{c_1}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}}{\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\left(n+\frac{2}{c_1}-2\right)!}d\theta$$

$$=\int_0^\infty c\left((\hat{\theta})^2-2\theta\hat{\theta}+\theta^2\right)\frac{\theta^{-n-\frac{2}{c_1}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}}{\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\left(n+\frac{2}{c_1}-2\right)!}d\theta$$

$$=\int_0^\infty c(\hat{\theta})^2\frac{\theta^{-n-\frac{2}{c_1}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}}{\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\left(n+\frac{2}{c_1}-2\right)!}d\theta-2c\hat{\theta}\int_0^\infty\frac{\theta^{-n-\frac{2}{c_1}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}}{\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\left(n+\frac{2}{c_1}-2\right)!}d\theta+c\int_0^\infty\frac{\theta^{-n-\frac{2}{c_1}}e^{\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}}{\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}\left(n+\frac{2}{c_1}-2\right)!}d\theta$$

To solve the above integration let $y=\frac{\sum_{i=1}^{n}t_i}{\theta}\Rightarrow\theta=\frac{\sum_{i=1}^{n}t_i}{y}\Rightarrow d\theta=-\frac{\sum_{i=1}^{n}t_i}{y^2}dy$**, we denote the first integral by** I_1**, the second integral by** I_2**, and the third integral by** I_3**,**

$$\therefore\ I_1=\frac{c\left(\hat{\theta}\right)^2}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-\frac{2}{c_1}}\left(n+\frac{2}{c_1}-2\right)!}\int_0^\infty\left(\frac{\left(-\frac{\sum_{i=1}^{n}t_i}{y^2}\right)e^{(-y)}}{\left(-\frac{\sum_{i=1}^{n}t_i}{y}\right)^{n+\frac{2}{c_1}}}dy\right)$$

$$=-\frac{c\left(\hat{\theta}\right)^2}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-\frac{2}{c_1}}\left(n+\frac{2}{c_1}-2\right)!}\int_0^\infty\left(\frac{\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}e^{(-y)}}{(y)^{-n-\frac{2}{c_1}+2}}dy\right)$$

$$=-\frac{c\left(\hat{\theta}\right)^2\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-\frac{2}{c_1}}\left(n+\frac{2}{c_1}-2\right)!}\int_0^\infty\left((y)^{n+\frac{2}{c_1}-2}e^{(-y)}dy\right)=-\frac{c\left(\hat{\theta}\right)^2}{\left(n+\frac{2}{c_1}-2\right)!}\Gamma\left(n+\frac{2}{c_1}-1\right)=-c\left(\hat{\theta}\right)^2.$$

$$I_2=-\frac{2c\hat{\theta}\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+2}}{\left(n+\frac{2}{c_1}-2\right)!\left(\sum_{i=1}^{n}t_i\right)^{1-n-\frac{2}{c_1}}}\int_0^\infty\left(y^{n+\frac{2}{c_1}-3}e^{(-y)}dy\right)$$

$$=\frac{2c\hat{\theta}\left(\sum_{i=1}^{n}t_i\right)}{\left(n+\frac{2}{c_1}-2\right)!}\Gamma\left(n+\frac{2}{c_1}-2\right)=\frac{2c\hat{\theta}\left(\sum_{i=1}^{n}t_i\right)}{\left(n+\frac{2}{c_1}-2\right)}.$$

$$I_3=-\frac{c\left(\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+3}}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-\frac{2}{c_1}}\left(n+\frac{2}{c_1}-2\right)!}\int_0^\infty\left(y^{n+\frac{2}{c_1}-4}e^{(-y)}dy\right)$$

$$=-\frac{c\left(\sum_{i=1}^{n}t_i\right)^2}{\left(n+\frac{2}{c_1}-2\right)!}\Gamma\left(n+\frac{2}{c_1}-3\right)=-\frac{c\left(\sum_{i=1}^{n}t_i\right)^2}{\left(n+\frac{2}{c_1}-2\right)\left(n+\frac{2}{c_1}-3\right)}.$$

$$\therefore\ R(\hat{\theta},\theta)=c(\hat{\theta})^2+\frac{2c\hat{\theta}\left(\sum_{i=1}^{n}t_i\right)}{\left(n+\frac{2}{c_1}-2\right)}-\frac{c\left(\sum_{i=1}^{n}t_i\right)^2}{\left(n+\frac{2}{c_1}-2\right)\left(n+\frac{2}{c_1}-3\right)}$$

$$\therefore\ \frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=2c\hat{\theta}+\frac{2c\left(\sum_{i=1}^{n}t_i\right)}{\left(n+\frac{2}{c_1}-2\right)}.$$

Let $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0,$

$$\therefore\ \hat{\theta}_B=\frac{\sum_{i=1}^{n}t_i}{\left(n+\frac{2}{c_1}-2\right)}.$$

We can find the estimator of survival function by:

$$\hat{s}_B(t)=\int_0^\infty exp\left(\frac{-t_i}{\theta}\right)\Pi(\theta|t_1,\ldots,t_n)d\theta$$

$$=\frac{1}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-\frac{2}{c_1}}\left(n+\frac{2}{c_1}-2\right)!}\int_0^\infty\theta^{-n-\frac{2}{c_1}}e^{-\left(\frac{t_i+\sum_{i=1}^{n}t_i}{\theta}\right)}d\theta$$

now, to evaluate the above integral, let $y=\frac{t_i+\sum_{i=1}^{n}t_i}{\theta}\Rightarrow d\theta=-\frac{t_i+\sum_{i=1}^{n}t_i}{y^2}dy$ **,**

$$\therefore\ \hat{s}_B(t)=-\frac{1}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-\frac{2}{c_1}}\left(n+\frac{2}{c_1}-2\right)!}\int_0^\infty\frac{\left(t_i+\sum_{i=1}^{n}t_i\right)^{-n-2c_1+1}}{y^{-n-\frac{2}{c_1}+2}}e^{(-y)}dy$$

$$\hat{s}_B(t)=-\frac{\left(t_i+\sum_{i=1}^{n}t_i\right)^{-n-\frac{2}{c_1}+1}}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-\frac{2}{c_1}}\left(n+\frac{2}{c_1}-2\right)!}\int_0^\infty y^{n+\frac{2}{c_1}-2}e^{(-y)}dy$$

$$\hat{s}_B(t)=\left(\frac{t_i+\sum_{i=1}^{n}t_i}{\sum_{i=1}^{n}t_i}\right)^{-n-\frac{2}{c_1}+1}=\left(\frac{t_i}{\sum_{i=1}^{n}t_i}+1\right)^{-n-\frac{2}{c_1}+1}.$$

2.2 Weibull Distribution

One of the most useful probability distribution in survival time is the Weibull distribution. The Weibull failure distribution may be used to model both increasing and decreasing failure rates. It is characterized by a hazard rate function of the form $\lambda(t)=\alpha t^{\beta}$ which is a power function. The function $\lambda(t)$ is increasing for $\alpha>0$, $\beta>0$ and is decreasing for $\alpha>0$, $\beta<0$. For mathematical convenience it is better to express $\lambda(t)$ in the following manner

$\lambda(t)=\frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1}$, $\alpha>0,\beta>0,t\geq 0$,

$R(t)=e^{\left[-\int_0^t \frac{\alpha}{\beta}\left(\frac{t'}{\beta}\right)^{\alpha-1}dt'\right]}=e^{-\left(\frac{t}{\beta}\right)^{\alpha}}$ **, and** $f(t)=\frac{-d\,R(t)}{dt}=\frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1}e^{-\left(\frac{t}{\beta}\right)^{\alpha}}$ **,** α **is referred to as the shape parameter and** β **is a scale parameter.**

Consider the Weibull probability density function which is given by:

$f(t|\alpha,\beta)=\frac{\alpha}{\beta}t^{\alpha-1}e^{\left(-\frac{t^{\alpha}}{\beta}\right)}$, $\quad t,\alpha,\beta>0$**, and the prior distribution of** (β,α) **is** $g(\beta)h(\alpha)$ **where** $g(\beta)\propto\frac{1}{\beta}$ **and** $h(\alpha)\propto\frac{1}{a}$, $\quad 0<\alpha<a$ **with these priors obtain the posterior distribution of** (β,α)**:**

$\prod(\beta,\alpha|t)=\frac{k\alpha^{n}}{\beta^{n+c}}\gamma^{\alpha-1}e^{\left(\frac{\sum_{i=1}^{n}t_i^{\alpha}}{\beta}\right)}$ $\quad \ni \gamma=\prod_{i=1}^{n}x_i$, **and**

$$k^{-1}=\int_0^a\int_0^\infty \prod(\beta,\alpha|t)d\beta d\alpha=\overline{)(n+c-1)}\int_0^a\frac{\alpha^{n}\gamma^{\alpha-1}}{\left(\sum_{i=1}^{n}t_i^{\alpha}\right)^{(n+c-1)}}d\alpha.$$

The marginal posterior of α **:**

$\prod(\alpha|t)=\frac{\frac{\alpha^{n}\gamma^{\alpha-1}}{\left(\sum_{i=1}^{n}t_i^{\alpha}\right)^{(n+c-1)}}}{\int_0^a\frac{\alpha^{n}\gamma^{\alpha-1}}{\left(\sum_{i=1}^{n}t_i^{\alpha}\right)^{(n+c-1)}}d\alpha}$**, and the marginal posterior of** β **:**

$\prod(\beta|t)=\frac{\frac{1}{\beta^{(n+c)}}\int_0^a\alpha^{n}\gamma^{\alpha-1}e^{\left(\sum_{i=1}^{n}t_i^{\alpha}\right)}d\alpha}{\overline{)(n+c-1)}\int_0^a\frac{\alpha^{n}\gamma^{\alpha-1}}{\left(\sum_{i=1}^{n}t_i^{\alpha}\right)^{(n+c-1)}}d\alpha}$**, then Bayes estimators are:**

$$\hat{\alpha} = E(\alpha|t) = \int_0^a \alpha f(\alpha|t)d\alpha = \frac{\int_0^a \frac{\alpha^{n+1}\gamma^{\alpha-1}}{\left(\sum_{i=1}^n t_i^\alpha\right)^{(n+c-1)}}d\alpha}{\int_0^a \frac{\alpha^{n}\gamma^{\alpha-1}}{\left(\sum_{i=1}^n t_i^\alpha\right)^{(n+c-1)}}d\alpha}, \text{ and } \hat{\beta} = \frac{1}{(n+c-2)}\frac{\int_0^a \frac{\alpha^{n+1}\gamma^{\alpha-1}}{\left(\sum_{i=1}^n t_i^\alpha\right)^{(n+c-1)}}d\alpha}{\int_0^a \frac{\alpha^{n}\gamma^{\alpha-1}}{\left(\sum_{i=1}^n t_i^\alpha\right)^{(n+c-1)}}d\alpha}.$$

2.2.1Proposed Estimator

Now, we introduce the new estimator using the loss function $\ell(\hat{\alpha},\alpha) = \sqrt{\alpha}(\hat{\alpha}-\alpha)^2$ which was introduce by Al-kutubi in 2005 [3].

$$\ell(\hat{\alpha},\alpha) = \sqrt{\alpha}(\hat{\alpha}-\alpha)^2$$

$$E\ell(\hat{\alpha},\alpha) = \int_0^\infty \ell(\hat{\alpha},\alpha)\Pi(\alpha|t)d\alpha$$

$$= \int_0^\infty \sqrt{\alpha}(\hat{\alpha}-\alpha)^2\Pi(\alpha|t)d\alpha$$

$$= \hat{\alpha}^2\int_0^\infty \sqrt{\alpha}\Pi(\alpha|t)d\alpha - 2\hat{\alpha}^2\int_0^\infty \alpha^{\frac{3}{2}}\Pi(\alpha|t)d\alpha + \int_0^\infty \alpha^2\Pi(\alpha|t)d\alpha$$

$$= \hat{\alpha}^2 E\left(\alpha^{\frac{1}{2}}|t\right) - 2\hat{\alpha}E\left(\alpha^{\frac{3}{2}}|t\right) + E\left(\alpha^{\frac{5}{2}}|t\right),$$

$$\frac{\partial E\ell(\hat{\alpha},\alpha)}{\partial\hat{\alpha}} = 2\hat{\alpha}E\left(\alpha^{\frac{1}{2}}|t\right) - 2E\left(\alpha^{\frac{3}{2}}|t\right) = 0$$

$$\therefore\ 2\hat{\alpha}E\left(\alpha^{\frac{1}{2}}|t\right) = 2E\left(\alpha^{\frac{3}{2}}|t\right),$$

$$\therefore\ \hat{\alpha} = \frac{E\left(\alpha^{\frac{3}{2}}|t\right)}{E\left(\alpha^{\frac{1}{2}}|t\right)} = \frac{\int_0^a \frac{\alpha^{n+\frac{3}{2}}\gamma^{\alpha-1}}{\left(\sum_{i=1}^n t_i^\alpha\right)^{(n+c-1)}}d\alpha}{\int_0^a \frac{\alpha^{n+\frac{1}{2}}\lambda^{\alpha-1}}{\left(\sum_{i=1}^n x_i^\alpha\right)^{(n+c-1)}}d\alpha}.$$

Similarly, we can find $\hat{\beta}$ as follows:

$$\hat{\beta} = \frac{E\left(\beta^{\frac{3}{2}}|t\right)}{E\left(\beta^{\frac{1}{2}}|t\right)} = \frac{\int_0^a \frac{\alpha^{n}\gamma^{\alpha-1}}{\left(\sum_{i=1}^n t_i^\alpha\right)^{\left(n+c-\frac{3}{2}\right)}}d\alpha}{\left(n+c-\frac{3}{2}\right)\int_0^a \frac{\alpha^{n}\gamma^{\alpha-1}}{\left(\sum_{i=1}^n t_i^\alpha\right)^{\left(n+c-\frac{3}{2}\right)}}d\alpha}.$$

3. Simulation

In this section, we will compare between the standard Bayes estimation for one parameter with our proposition, also we will compare between the standard estimator of survival function with our proposition for exponential distribution using MSE and $|MPE|$ criterions to compare the methods of estimator, where: $MSE(\hat{\theta}) = \frac{\sum_{i=1}^{R}(\hat{\theta}_i - \theta)^2}{R}$, $|MPE|(\hat{\theta}) = \frac{\sum_{i=1}^{R}\frac{|\hat{\theta}_i - \theta|}{\theta}}{R}$ **.**

In this experiment, we chooses the size of the samples $n = 25,50,100,150$**, with parameter value namely** $\theta = .5,1,1.5,2$**, with constants** $c_1 = .2,.4,1,1.4$ **and the size of replication is** 1000**.**

Table 1: MSE and MPE of estimated parameter exponential distribution

Size	θ	C1	mseθk	Mseθp	Best	mpeθk	mpeθp	Best
25	0.5	0.2	0.0128	0.0197	Mseθk	0.175	0.2488	Mpeθk
		0.4	0.0114	0.0108	mseθp	0.1651	0.1694	Mpeθk
		1	0.0093	0.0093	Same	0.154	0.154	Same
		1.4	0.01	0.0112	mseθk	0.1598	0.1657	Mpeθk
25	1	0.2	0.0489	0.08	mseθk	0.1761	0.2491	Mpeθk
		0.4	0.045	0.0432	mseθp	0.1697	0.1693	Mpeθp
		1	0.0389	0.0389	Same	0.1567	0.1567	Same
		1.4	0.0367	0.0401	mseθk	0.1517	0.1567	Mpeθk
25	1.5	0.2	0.1176	0.1838	mseθk	0.1782	0.2524	Mpeθk
		0.4	0.1103	0.0995	mseθp	0.1736	0.1736	Same
		1	0.0861	0.0861	Same	0.155	0.155	Same
		1.4	0.0844	0.0945	mseθk	0.1554	0.1609	Mpeθk
25	2	0.2	0.2016	0.3439	mseθk	0.1729	0.2621	Mpeθk
		0.4	0.1835	0.166	mseθp	0.169	0.1672	Mpeθp
		1	0.1609	0.1609	Same	0.1614	0.1614	Same
		1.4	0.1525	0.1694	mseθk	0.1558	0.1608	Mpeθk
50	0.5	0.2	0.0055	0.0081	mseθk	0.1166	0.1524	Mpeθk
		0.4	0.005	0.0052	mseθk	0.1116	0.1173	Mpeθk
		1	0.0051	0.0051	same	0.1146	0.1146	Same
		1.4	0.0051	0.0052	mseθk	0.1137	0.1133	Mpeθp
50	1	0.2	0.0214	0.0347	mseθk	0.1136	0.1593	Mpeθk
		0.4	0.0217	0.022	mseθk	0.1177	0.1217	Mpeθk
		1	0.0182	0.0182	same	0.106	0.106	Same
		1.4	0.0191	0.0198	mseθk	0.111	0.1106	Mpeθp
50	1.5	0.2	0.0512	0.078	mseθk	0.1171	0.16	Mpeθk

Size	θ	C_1	mseθk	Mseθp	Best	mpeθk	mpeθp	Best
		0.4	0.0496	0.0474	mseθp	0.1159	0.1165	Mpeθk
		1	0.0425	0.0425	Same	0.1084	0.1084	Same
		1.4	0.0455	0.048	mseθk	0.1143	0.116	Mpeθk
50	2	0.2	0.0845	0.1306	mseθk	0.1149	0.1537	Mpeθk
		0.4	0.0867	0.0873	mseθk	0.1172	0.12	Mpeθk
		1	0.0809	0.0809	same	0.1129	0.1129	Same
		1.4	0.0775	0.0824	mseθk	0.1115	0.1139	Mpeθk
100	0.5	0.2	0.0025	0.0032	mseθk	0.0796	0.0938	Mpeθk
		0.4	0.0025	0.0025	same	0.0787	0.0809	Mpeθk
		1	0.0026	0.0026	same	0.0802	0.0802	Same
		1.4	0.0026	0.0027	mseθk	0.082	0.0825	Mpeθk
100	1	0.2	0.0108	0.0149	mseθk	0.0819	0.1012	Mpeθk
		0.4	0.0104	0.0103	mseθp	0.0815	0.0825	Mpeθk
		1	0.0098	0.0098	Same	0.078	0.078	Same
		1.4	0.0099	0.0103	mseθk	0.0782	0.0794	Mpeθk
100	1.5	0.2	0.0253	0.032	mseθk	0.0827	0.099	Mpeθk
		0.4	0.0223	0.0219	mseθp	0.0778	0.0796	Mpeθk
		1	0.0226	0.0226	Same	0.0803	0.0803	Same
		1.4	0.021	0.0215	mseθp	0.0776	0.078	Mpeθk
100	2	0.2	0.0397	0.0513	mseθk	0.0788	0.0936	Mpeθk
		0.4	0.0396	0.0409	mseθk	0.0784	0.0804	Mpeθk
		1	0.0412	0.0412	same	0.0808	0.0808	Same
		1.4	0.0391	0.0431	mseθk	0.078	0.0785	Mpeθk
150	0.5	0.2	0.0017	0.0022	mseθk	0.0666	0.0776	Mpeθk
		0.4	0.0017	0.0017	same	0.0647	0.0668	Mpeθk
		1	0.0018	0.0018	same	0.0684	0.0684	Same
		1.4	0.0016	0.0016	same	0.0626	0.0622	Mpeθp
150	1	0.2	0.0070	0.0087	mseθk	0.0659	0.0766	Mpeθk
		0.4	0.0076	0.0073	mseθp	0.0698	0.0683	Mpeθp
		1	0.0067	0.0067	Same	0.0655	0.0655	Same
		1.4	0.0070	0.0072	mseθk	0.0675	0.0680	Mpeθk
150	1.5	0.2	0.0150	0.0193	mseθk	0.0645	0.0756	Mpeθk
		0.4	0.0164	0.0163	mseθp	0.0666	0.0677	Mpeθk
		1	0.0158	0.0158	Same	0.0659	0.0659	Same
		1.4	0.0147	0.0151	mseθk	0.0643	0.0645	Mpeθk
150	2	0.2	0.0249	0.0353	mseθk	0.0623	0.0772	Mpeθk
		0.4	0.0277	0.0277	same	0.0641	0.0659	Mpeθk
		1	0.0267	0.0267	same	0.0659	0.0659	Same

Size	θ	C_1	mseθk	Mseθp	Best	mpeθk	mpeθp	Best
		1.4	0.0252	0.0256	mseθk	0.0633	0.0634	Mpeθk

θp: proposed parameter
θk: Al-kutubi parameter

Table 2: MSE and MPE of estimated survival function

Size	θ	C_1	mseSk	mseSp	Best	mpeSk	mpeSp	Best
25	0.5	0.2	0.0031	0.0104	mseSk	0.2308	0.2529	mpeSk
		0.4	0.0032	0.0050	mseSk	0.2146	0.1847	mpeSk
		1	0.0031	0.0031	same	0.2016	0.2016	Same
		1.4	0.0034	0.0030	mseSp	0.1785	0.1953	mpeSp
25	1	0.2	0.0033	0.0103	mseSk	0.2458	0.2532	mpeSk
		0.4	0.0031	0.0049	mseSk	0.2089	0.1779	mpeSk
		1	0.0032	0.0032	Same	0.1920	0.1920	Same
		1.4	0.0034	0.0030	mseSp	0.1839	0.2028	mpeSp
25	1.5	0.2	0.0031	0.0109	mseSk	0.2176	0.2615	mpeSk
		0.4	0.0028	0.0045	mseSk	0.2048	0.1767	mpeSk
		1	0.0034	0.0034	Same	0.1943	0.1943	Same
		1.4	0.0036	0.0032	mseSp	0.1975	0.2185	mpeSp
25	2	0.2	0.0031	0102	mseSk	0.2310	0.2528	mpeSk
		0.4	0.0031	0.0047	mseSk	0.2142	0.1799	mpeSk
		1	0.0034	0.0034	Same	0.2011	0.2011	Same
		1.4	0.0034	0.0030	mseSp	0.1934	0.2172	mpeSp
50	0.5	0.2	0.0016	0.0037	mseSk	0.1386	0.1552	mpeSk
		0.4	0.0015	0.0019	mseSk	0.1309	0.1202	mpeSk
		1	0.0015	0.0015	Same	0.1192	0.1192	Same
		1.4	0.0016	0.0015	mseSp	0.1240	0.1309	mpeSp
50	1	0.2	0.0016	0.0039	mseSk	0.1310	0.1596	mpeSk
		0.4	0.0015	0.0020	mseSk	0.1261	0.1216	mpeSk
		1	0.0015	0.0015	Same	0.1221	0.1221	Same
		1.4	0.0015	0.0014	mseSp	0.1135	0.1178	mpeSk
50	1.5	0.2	0.0015	0.0037	mseSk	0.1352	0.1584	mpeSk
		0.4	0.0016	0.0020	mseSk	0.1326	0.1216	mpeSk
		1	0.0015	0.0015	Same	0.1196	0.1196	Same
		1.4	0.0017	0.0016	mseSp	0.1243	0.1299	mpeSp
50	2	0.2	0.0014	0.0035	mseSk	0.1310	0.1516	mpeSk
		0.4	0.0016	0.0021	mseSk	0.1302	0.1227	mpeSk
		1	0.0015	0.0015	Same	0.1274	0.1274	Same
		1.4	0.0016	0.0015	mseSp	0.1219	0.1278	mpeSp
100	0.5	0.2	7.5390e-004	0.0013	mseSk	0.0885	0.0950	mpeSk

Size	θ	C_1	mseSk	mseSp	Best	mpeSk	mpeSp	Best
		0.4	7.2514e-004	8.6717e-004	mseSk	0.0830	0.0820	mpeSk
		1	7.6902e-004	7.6902e-004	Same	0.0849	0.0849	Same
		1.4	7.3790e-004	7.1335e-004	mseSp	0.0810	0.0825	mpeSp
100	1	0.2	7.5800e-004	0.0013	mseSk	0.0870	0.0979	mpeSk
		0.4	7.8889e-004	9.3861e-004	mseSk	0.0861	0.0855	mpeSk
		1	7.5897e-004	7.5897e-004	Same	0.0822	0.0822	Same
		1.4	7.7367e-004	7.5221e-004	mseSp	0.0816	0.0836	mpeSp
100	1.5	0.2	7.5533e-004	0.0013	mseSk	0.0881	0.0956	mpeSk
		0.4	7.0392e-004	8.1393e-004	mseSk	0.0840	0.0818	mpeSk
		1	6.8772e-004	6.8772e-004	Same	0.0811	0.0811	Same
		1.4	7.7824e-004	7.4507e-004	mseSp	0.0798	0.0811	mpeSp
100	2	0.2	7.7312e-004	0.0014	mseSk	0.0871	0.0994	mpeSk
		0.4	7.4939e-004	8.7409e-004	mseSk	0.0869	0.0843	mpeSk
		1	7.2724e-004	7.2724e-004	Same	0.0823	0.0823	Same
		1.4	7.6207e-004	7.4832e-004	mseSp	0.0826	0.0851	mpeSp
150	0.5	0.2	5.1095e-004	7.5489e-004	mseSk	0.0698	0.0752	mpeSk
		0.4	5.0716e-004	5.6984e-004	mseSk	0.0684	0.0680	mpeSk
		1	5.1261e-004	5.1261e-004	Same	0.0688	0.0688	Same
		1.4	5.4210e-004	5.2933e-004	mseSp	0.0696	0.0703	mpeSp
150	1	0.2	4.9525e-004	7.6907e-004	mseSk	0.0693	0.0767	mpeSk
		0.4	4.4602e-004	4.9157e-004	mseSk	0.0651	0.0639	mpeSk
		1	4.8461e-004	4.8461e-004	Same	0.0665	0.0665	Same
		1.4	4.9518e-004	4.8830e-004	mseSp	0.0668	0.0677	mpeSp
150	1.5	0.2	5.1199e-004	7.6929e-004	mseSk	0.0712	0.0777	mpeSk
		0.4	5.3044e-004	5.8176e-004	mseSp	0.0705	0.0686	mpeSk
		1	4.9185e-004	4.9185e-004	Same	0.0671	0.0671	Same
		1.4	4.9504e-004	4.8535e-004	mseSp	0.0658	0.0668	mpeSp
150	2	0.2	4.8331e-004	7.5766e-004	mseSk	0.0678	0.0740	mpeSk
		0.4	4.7058e-004	5.2155e-004	mseSk	0.0675	0.0658	mpeSk
		1	5.4808e-004	5.4808e-004	Same	0.0709	0.0709	Same
		1.4	4.9632e-004	4.8401e-004	mseSp	0.0665	0.0670	mpeSp

Sp: proposed survival function
Sk: Al- kutubi survival function

4. Conclusions

1- When the survival function of exponential distribution is estimated, we obtain:

- **θk is the best when we used MSE**
- **θk is the best when we used MPE**

2- When the parameter of exponential distribution is estimated ,we obtain

- **Sk is the best when we used MSE**

- **Sk is the best when we used MPE**

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حول تقدير بيز

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الخلاصة

الهدف الرئيسي من بحثنا هذا هو تقديم مقارنات بين طريقة بيز القياسية وطرق مقترحة لبيز باستخدام التوزيع الآسي وتوزيع ويبل لإيجاد مقدرات جديدة. تم استخدام المحاكاة للحصول على الهدف المطلوب وهو إيجاد الطريقة الأنسب من بين الطرق المطروحة وبأحجام عينات متنوعة، تم الحصول على مقدرات جديدة فضلاً عن إجراء بعض التعديلات على الطرق القديمة.