

On Double Stage Shrinkage Estimator For the Variance of Normal Distribution With Unknown Mean

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Abstract

This paper is concerned with preliminary test double stage shrinkage estimators to estimate the variance (σ^2) of normal distribution when a prior estimate (σ_0^2) of the actual value (σ^2) is available when the mean is unknown, using specifying shrinkage weight factors $\psi(\cdot)$ in addition to pre-test region (R).

Expressions for the Bias, Mean squared error [MSE (\cdot)], Relative Efficiency [R.EFF (\cdot)], Expected sample size [E(n/ σ^2)] and percentage of overall sample saved of proposed estimator were derived. Numerical results (using MathCAD program) and conclusions are drawn about selection of different constants including in the mentioned expressions. Comparisons between the suggested estimator with the classical estimator in the sense of Bias and Relative Efficiency are given. Furthermore, comparisons with the earlier existing works are drawn.

Keywords: Normal Distribution, Double Stage Shrinkage Estimator, Bias Ratio, Mean Squared Error, Relative Efficiency, Expected sample size. Percentage of overall sample saved and probability of avoiding the second sample.

Introduction

" The normal distribution is the most widely known and used of all distributions. Because the normal distribution approximates many natural phenomena so well, it has developed into a standard of reference for many probability problems. The question, Why is the normal distribution useful?, and the answer is many things actually are normally distributed, or very close to it. For example, height and intelligence are approximately normally distributed; measurement errors also often have a normal distribution , the normal distribution is easy to work with mathematically. In many practical cases, the methods developed using normal theory work quite well, even when the distribution is not normal . The normal distribution plays a vital role in many applied problems of biology, economics, engineering, financial risk management, genetics, hydrology, mechanics, medicine, number theory, statistics, physics, psychology, reliability, etc., and has been extensively studied, both from theoretical and applications point of view, by many researchers, since its inception.

There is a very strong connection between the size of a sample N and the extent to which a sampling distribution approaches the normal form.

Many sampling distributions based on large sample of size can be approximated by the normal distribution even though the population distribution itself is definitely not normal"; [1],[12] .

The probability density of the normal distribution is:

$$f(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty \quad \dots(1)$$

Here, μ is the *mean* or *expectation* of the distribution (and also its median and mode). The parameter σ is its standard deviation with its variance then σ^2 . If $\mu = 0$ and $\sigma = 1$, the distribution is called the standard normal distribution or the unit normal distribution denoted by $N(0,1)$ and a random variable with that distribution is a standard normal deviate.

Assume that x_1, x_2, \dots, x_n be a random sample of size (n) from a normal population with unknown mean (μ) and unknown variance (σ^2). In conventional notation, we write $X \sim N(\mu, \sigma^2)$.

In this work, we suggest the problem of estimating the variance (σ^2) when some prior information (σ_0^2) regarding the variance (σ^2) is available. More specifically, we assume that the prior information regarding is due the following reasons, [10]:

1. We believe that (σ_0^2) is close to the true value of σ^2 , or
2. We fear that, (σ_0^2) may be near the true value of σ^2 , i.e.; something bad happens if (σ^2) approximately equals to (σ_0^2) and we do not known about it.

In such a situation it is natural to start with the MVUE ($\hat{\sigma}^2$) of σ^2 and modify it by moving it closure to (σ_0^2) using shrinkage weight factor [$\psi(\cdot)$], so that the resulting estimator though perhaps biased, has a smaller mean squared error [MSE] than that of ($\hat{\sigma}^2$) in some interval around (σ_0^2). This method of constructing an estimator that incorporates the prior value leads to what is known as a shrinkage estimator

$$\text{i.e.; } \psi_1(\hat{\sigma}^2)\hat{\sigma}^2 + [1 - \psi_1(\hat{\sigma}^2)]\sigma_0^2 ; 0 \leq \psi_1(\hat{\sigma}^2) \leq 1. \quad \dots(2)$$

where $\psi_i(\hat{\sigma}^2)$, $0 \leq \psi_i(\hat{\sigma}^2) \leq 1$ is shrinkage weight factor specifying the belief in $\hat{\sigma}^2$ and $(1 - \psi_i(\hat{\sigma}^2))$ specifying the belief in (σ_0^2) and $\psi_i(\hat{\sigma}^2)$ may be a function of $\hat{\sigma}^2$ or a constant.

Preliminary test double stage shrinkage estimator for the variance (σ^2) that utilize a prior estimate (σ_0^2) is represented as following steps:-

1. Select two positive integers (n_1) and (n_2) .
2. Obtain a random sample of size (n_1) on x [first stage sample]. Compute sample

variance $\hat{\sigma}_1^2(S_1^2)$ [MVUE], where $S_1^2 = \frac{\sum_{i=1}^{n_1} (x_i - \hat{\mu}_1)^2}{n_1 - 1}$ and $\hat{\mu}_1$ is the first sample mean.

3. Choose a suitable region (R) around σ_0^2 .

Noted that, this work concerns with pre-test region.

4. If $\hat{\sigma}_1^2 \in R$, take the shrinkage estimator of the form defined in (1).

Here, we put forward $\psi_1(\cdot) = k = n_1 \alpha^4$

However, if $\hat{\sigma}_1^2 \notin R$, obtain a second stage random sample of size n_2 on x and advise the estimator $\hat{\sigma}_p^2$ of σ^2 as polling estimator of two samples variance.

$$\text{i.e.; } \hat{\sigma}_p^2 = (n_1 \hat{\sigma}_1^2 + n_2 \hat{\sigma}_2^2) / n \quad \dots(3)$$

where $\hat{\sigma}_2^2$ is the variance of the second random sample and $n = n_1 + n_2$.

Thus, the suggested form of preliminary test double stage shrunken estimator (PDSE) has the following form:

$$\tilde{\sigma}_{DS}^2 = \begin{cases} k(\hat{\sigma}_1^2 - \hat{\sigma}_0^2) + \sigma_0^2 & , \quad \text{if } \hat{\sigma}_1^2 \in R, \\ \hat{\sigma}_p^2 & , \quad \text{if } \hat{\sigma}_1^2 \notin R, \end{cases} \quad \dots(4)$$

where R is the pre-test region for testing the hypothesis $H_0 : \hat{\sigma}^2 = \sigma_0^2$ against $H_A : \hat{\sigma}^2 \neq \sigma_0^2$ with level of significance (α) using test statistic $T(\hat{\sigma}^2 / \sigma_0^2) = \frac{(n_1 - 1)\hat{\sigma}^2}{\sigma_0^2}$.

$$\text{i.e.; } R = \left[X_{1-\alpha/2, n-1}^2 \frac{\sigma_0^2}{n-1}, X_{\alpha/2, n-1}^2 \frac{\sigma_0^2}{n-1} \right] \quad \dots(5)$$

$\hat{\sigma}_i^2$ ($i=1,2$) is the sample variance of stage (i), $X_{1-\alpha/2, n-1}^2$ and $X_{\alpha/2, n-1}^2$ are the lower and upper 100($\alpha/2$) percentile point of chi-square distribution with degree of freedom $(n - 1)$ and $n = n_1 + n_2$.

And for simplest we shall assume that $R=[a,b]$.

The aim of this paper is to show that the proposed estimators defined in (4) is better than the classical estimators and some estimators introduced by some authors in the sense of mean squared error and relative efficiency.

Several authors have studied (PDSE) for estimating the mean and variance of Normal distribution and also for estimating the parameters of different distributions, for example, Katti [7], Thompson [10], Al-Joboori [3,4], Al-Joboori et al [5,6], Waikar, Schuurmann and Raghunathan [11], Handa, Kambo and Al-Hemyari [8] and Maha [9].

Preliminary Test Double Stage Shrunken Estimator (PDSE)

In this section, recall (PDSE) defined in (3) for estimating the variance of normal distribution .

The expressions for Bias and Mean Squared Error [MSE(·)] of $\tilde{\sigma}_{DS}^2$ are respectively as follows:

$$\text{Bias}(\tilde{\sigma}_{DS}^2 | \hat{\sigma}^2, \mathbf{R}) = \int_{\hat{\sigma}_2^2=0}^{\infty} \int_{\hat{\sigma}_1^2 \in \mathbf{R}} [k(\hat{\sigma}_1^2 - \hat{\sigma}_0^2) + (\sigma_0^2 - \sigma^2)] f(\hat{\sigma}_1^2) f(\hat{\sigma}_2^2) d\hat{\sigma}_1^2 d\hat{\sigma}_2^2 +$$

$$\int_{\hat{\sigma}_2^2=0}^{\infty} \int_{\hat{\sigma}_1^2 \in \mathbf{R}} \left[\frac{1}{2} \left(\frac{n_1 \hat{\sigma}_1^2 + n_2 \hat{\sigma}_2^2}{n_1 + n_2} + \sigma_0^2 \right) - \sigma^2 \right] f(\hat{\sigma}_1^2) f(\hat{\sigma}_2^2) d\hat{\sigma}_1^2 d\hat{\sigma}_2^2$$

Where ,

$$f(\hat{\sigma}_i^2 | \sigma^2) = \begin{cases} \frac{\left[(n_i - 1) / \sigma^2 \right]^{\frac{n_i-1}{2}} \left[\hat{\sigma}_i^2 \right]^{\frac{n_i-3}{2}} \exp \left[-\frac{(n_i - 1) \hat{\sigma}_i^2}{2\sigma^2} \right]}{2^{\frac{n_i-1}{2}} \Gamma \left[\frac{(n_i - 1)}{2} \right]}, & \text{for } \hat{\sigma}_i^2 \geq 0, \sigma^2 \geq 0, i = 1, 2 \\ 0 & , \text{otherwise} \end{cases} \dots(6)$$

and by simple calculations, we get

$$\text{Bias}(\tilde{\sigma}_{DS}^2 | \hat{\sigma}^2, \mathbf{R}) = \frac{\sigma^2}{n_1 - 1} \left\{ k \{ J_1(a_i, b_i) - (n_1 - 1) \lambda J_0(a_i, b_i) \} \right.$$

$$\left. + (n_1 - 1) (\lambda - 1) J_0(a_i, b_i) - \left\{ \frac{1}{1 + u} [J_1(a_i, b_i) - (n_1 - 1) J_0(a_i, b_i)] \right\} \right\}$$

... (7)

The Bias Ratio of $\tilde{\sigma}_{DS}^2 = \text{Bias}(\tilde{\sigma}_{DS}^2 | \hat{\sigma}^2, \mathbf{R}) / \left(\frac{\sigma^2}{n_1 - 1} \right)$... (8)

And,

$$\text{MSE}(\tilde{\sigma}_{DS}^2 | \sigma^2, \mathbf{R}) = E[\tilde{\sigma}_{DS}^2 - \sigma^2]^2$$

$$= \int_{\hat{\sigma}_2^2=0}^{\infty} \int_{\mathbf{R}} [k(\hat{\sigma}_1^2 - \hat{\sigma}_0^2) + (\sigma_0^2 - \sigma^2)]^2 f(s_1^2 / \sigma^2) f(s_2^2 / \sigma^2) d\hat{\sigma}_1^2 d\hat{\sigma}_2^2 +$$

$$\int_{\hat{\sigma}_2^2=0}^{\infty} \int_{\mathbf{R}} [s_p^2 - \sigma^2]^2 f(s_1^2 / \sigma^2) f(s_2^2 / \sigma^2) d\hat{\sigma}_1^2 d\hat{\sigma}_2^2$$

We conclude,

$$\text{MSE}(\tilde{\sigma}_{DS}^2 | \sigma^2, \mathbf{R}) = \frac{\sigma^4}{(n_1 - 1)^2} \left\{ K^2 \{ J_2(a_i, b_i) - 2(n_1 - 1) \lambda J_1(a_i, b_i) + \right.$$

$$\left. (n_1 - 1)^2 \lambda^2 J_0(a_i, b_i) \} + 2k \{ (n_1 - 1) (\lambda - 1) [J_1(a_i, b_i) - (n_1 - 1) \lambda J_0(a_i, b_i)] \} \right\}$$

$$\begin{aligned}
 & + (n_1 - 1)^2 (\lambda - 1)^2 J_0(a_i, b_i) + 2 \left(\frac{1}{1+u} \right)^2 (n_1 - 1) + 2 \left(\frac{u}{1+u} \right)^2 \frac{(n_1 - 1)^2}{(un_1 - 1)} \\
 & - \left\{ \left(\frac{1}{1+u} \right)^2 [J_2(a_i, b_i) - 2(n_1 - 1)J_1(a_i, b_i) + (n_1 - 1)^2 J_0(a_i, b_i)] \right. \\
 & \left. + 2 \left(\frac{u}{1+u} \right)^2 \frac{(n_1 - 1)}{(un_1 - 1)} J_0(a_i, b_i) \right\} \quad \dots(9)
 \end{aligned}$$

$$\text{where } j_\ell(a_i, b_i) = \int_{a_i}^{b_i} y^\ell f(y) dy, \ell = 0, 1, 2. \quad \dots(10)$$

$$\lambda = \frac{\sigma_0^2}{\sigma^2}, y_i = \frac{(n_i - 1)s^2}{\sigma^2} \sim X_{n_i - 1}^2, \quad \dots(11)$$

$$a_1 = \lambda X_{1-\alpha/2, n_1-1}^2, b_1 = \lambda X_{\alpha/2, n_1-1}^2, \quad \dots(12)$$

$$\text{also } u = n_2/n_1, \text{ and } n = n_1 + n_2,$$

The Expected sample size can be obtained as:

$$E(n | \sigma^2, R) = n_1 [1 + u(1 - j_0(a_1, b_1))] \quad \dots(13)$$

The Efficiency of $\tilde{\sigma}_{DS}^2$ relative to $\hat{\sigma}^2(S^2)$ is given by :

$$R.Eff(\tilde{\sigma}_{DS}^2 | \sigma^2, R) = \frac{MSE(\hat{\sigma}^2 | \sigma^2)}{[MSE(\tilde{\sigma}_{DS}^2 | \sigma^2, R)][E(n | \sigma^2, R)/n]} \quad \dots(14)$$

Where $[E(n/\sigma^2, R)/n]$ is the Expected sample size proportion.

The probability of avoiding the second sample computing by $p(\hat{\sigma}_1^2 \in R)$.

Finally, the percentage of overall sample saved can be obtained by

$$(n_2/n) p(\hat{\sigma}_1^2 \in R) * 100 \quad \dots(15)$$

See for example [3],[4], [9], [10] and [11].

Numerical Results and Discussion

The computations (using MathCAD program) of Relative Efficiency [R.Eff(\cdot)], Bias ratio [B(\cdot)], Expected sample size [E(n | μ, R)], Expected sample size proportion, Percentage of the overall sample saved and probability of avoiding the second sample were used for the estimator $\tilde{\sigma}_{DS}^2$. These computations were performed for $n_1 = 5, 7, 9, 11, 13$; $u = 2, 6, 8, 10$; $\lambda = 0.0(0.1)1, 2$; $\alpha = 0.01, 0.05, 0.1$.

Some of these computations are given in annex tables (1)-(17).

The observations mentioned in the tables lead to the following results:

- i. R.Eff($\tilde{\sigma}_{DS}^2$) are adversely proportional with small value of α .
- ii. R.Eff($\tilde{\sigma}_{DS}^2$) are maximum when $\sigma^2 \approx \sigma_0^2$ and decreasing otherwise

- iii. The Bias Ratio of $\tilde{\sigma}_{DS}^2 [B(\tilde{\sigma}_{DS}^2) = \frac{n_1 - 1}{\sigma^2} \text{Bias}(\tilde{\sigma}_{DS})]$ are reasonably small when $\sigma^2 \approx \sigma_0^2$, otherwise $B(\tilde{\sigma}_{DS}^2)$ will be maximum.
- iv. $B(\tilde{\sigma}_{DS}^2)$ are reasonably small with small value of α .
- v. R.Eff($\tilde{\sigma}_{DS}^2$) and $B(\tilde{\sigma}_{DS}^2)$ are decreasing function with respect to first sample size n_1 .
- vi. The Expected values of sample size of $\tilde{\sigma}_{DS}^2$ is closer to n_1 specially when $\sigma^2 \approx \sigma_0^2$ and start faraway slowly with increasing of λ .
- vii. Percentage of the overall sample saved $[\frac{n_2}{n} j_0(a_1, b_1) \times 100]$ is a decreasing function of λ , and has a maximum value when $\sigma^2 \approx \sigma_0^2$.
- viii. R.Eff($\tilde{\sigma}_{DS}^2$) is an increasing function with respect to u ($u = n_2/n_1$).
- iv. The suggested estimator $\tilde{\sigma}_{DS}^2$ is more efficient than the estimators introduced by Al-Bermani [2] and AL-Joboori[4] .

Conclusions

From the above discussions, it is obvious that by using guess point value one can improve the usual estimator by using shrinkage technique. It can be noted that if the guess point is very close to the actual value of the parameter, the proposed estimator perform is better than the usual estimator. If one has no confidence in the guessed value, then proposed preliminary test shrunken estimators can be suggested. We can safely use the proposed estimators for small sample size at the usual level of significance and moderate value of shrunken weight factor $\Psi(\cdot)$. The difficulty of obtaining samples is because of the scarcity and high cost led researchers to use the double stage shrinkage estimators to reduce the sample size that we need and for achieving savings of the items in the sample and obtaining high efficiency estimators.

References

1. Ahsanullah, M. et al., (2014), "Normal and Student's t Distributions and Their Applications", Atlantis Press and the authors. DOI: 10.2991/978-94-6239-061-4_2.
2. Al-Bermani, M.H., (2008), "comparison between Bayesian Shrinkage Estimators and Shrinkage Estimators for the variance of normal distribution by using simulation", ph.D. thesis, college of Administration and economics, university of Baghdad.
3. Al-Joboori, A.N. (2010), "Pre-Test Single and Double Stage Shrunken Estimators for the Mean of Normal Distribution With Known Variance", Baghdad J. for Science, 7 (4), 1432-1442.
4. Al-Joboori, A.N., (2011), "Single and Double Stage Shrunken Estimators for the Variance of Normal Distribution". Journal of Education, AL-Mustansiriyah University, 2, 597-608.
5. Al-Joboori, A.N., et al., (2014), "Single and Double Stage Shrinkage Estimators for the Normal Mean with the Variance Cases", International Journal of Statistics, (38), 2, 1127 - 1134.

6. Al-Joboori, A.N. and Ameen, M. M.(2016), On Double Stage Minimax-Shrinkage Estimator For Generalized Rayleigh Model, International Journal of Applied Mathematical Research, 5 (1), 39-47.
7. Katti, S.K., (1962), Use of Some a Prior Knowledge in the Estimation of Means from Double Samples, Biometrics,.18,.139-147.
8. Handa, B.K., Kambo, N.S. and Al-Hemyari; Z.A., (1988), "On Double Stage Shrunken Estimator of the Mean of Exponential Distribution", IAPQR Trans.,(13),1,.19-33.
9. Maha.A. M.(2011) , Double Stage Shrinkage Estimator of Two Parameters Generalized Rayleigh Distribution, Education college journal AL-Mustansiriya University (2), 566-573
10. Thompson, J.R., (1968), Some Shrinkage Techniques for Estimating the Mean, J. Amer. Statist. Assoc.,63.113-122.
11. Waikar, V.B., Schuurmann, F.J. and Raghunathar, T.E., (1984), On a Two-Stage Shrinkage Testimator of the Mean of a Normal Distribution, Commum. Statist-Theor. Meth., A,13 (15),1901-1913.
12. <https://www3.nd.edu/~rwilliam/stats1/x21.pdf>.

Table (1): Shows Bias ratio [B(-)] and R.E.ff w.r.t. α , n_1 and λ when $u = 2$

α	n_1	R.Eff(-) B(-)	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
100	5	R.Eff(-)	3.4225875	6.921051	17.5040876	34.055869	19.9017239	8.5142224	4.335779	2.5633057
		B(-)	-1.3026559	-1.5453646	-0.8971937	0.0153905	0.9868869	1.9653436	2.9382656	3.902073
	7	R.Eff(-)	2.5218328	4.5657778	12.7316908	28.0764855	14.2199712	5.5730906	2.7507472	1.6013574
		B(-)	-1.4307066	-2.20102	-1.3479203	0.0157813	1.4734652	2.9296484	4.3617489	5.76015
	9	R.Eff(-)	2.2344079	3.4244664	10.2563163	25.4964189	11.3593618	4.1642427	2.0076996	1.1539179
		B(-)	-1.3689823	-2.7706791	-1.7927305	0.0159786	1.9578157	3.8804139	5.7438821	7.5241258
	11	R.Eff(-)	2.1910011	2.7529803	8.6693885	24.0612878	9.5571863	3.326823	1.574551	0.8951149
		B(-)	-1.2082129	-3.2566881	-2.2313379	0.0160965	2.4399322	4.8161439	7.0770414	9.1721113
	13	R.Eff(-)	2.2811736	2.3156926	7.5393297	23.1480114	8.2890208	2.7683441	1.2902951	0.7267732
		B(-)	-1.0097075	-3.6626852	-2.6635711	0.0161745	2.9197667	5.7353787	8.3540981	10.684783
050	5	R.Eff(-)	3.6194909	6.1808134	14.5712829	27.1849786	17.6374379	7.933749	4.0761615	2.406342
		B(-)	-0.8929279	-1.208987	-0.7460543	0.0480284	0.9402995	1.8412993	2.7191497	3.5619491
	7	R.Eff(-)	2.9246212	4.2077426	10.8292699	22.9410048	12.8134551	5.1866676	2.5615348	1.4850268
		B(-)	-0.8955581	-1.6696883	-1.1277614	0.0490329	1.3835582	2.6961091	3.9248409	5.0444625
	9	R.Eff(-)	2.7826467	3.25824	8.8296177	21.0500992	10.3076319	3.8608721	1.8563702	1.0656958
		B(-)	-0.7871207	-2.0348087	-1.4992136	0.0495215	1.8205121	3.5130704	5.0225485	6.3026994
	11	R.Eff(-)	2.8635758	2.7085502	7.5250407	19.9818656	8.7026587	3.0727202	1.4509899	0.8312489
		B(-)	-0.6412647	-2.3148738	-1.8601538	0.0498112	2.2514014	4.2914965	6.008853	7.3316557
	13	R.Eff(-)	3.0579131	2.3588939	6.5853355	19.2959357	7.561666	2.5489063	1.1900405	0.6854149
		B(-)	-0.4968093	-2.5203237	-2.2105946	0.049997	2.6762825	5.0307925	6.88241	8.1350713
10	5	R.Eff(-)	3.809139	5.8752143	12.8310536	22.523826	15.5288307	7.3579544	3.8501684	2.2911515
		B(-)	-0.7040529	-1.0102327	-0.6359713	0.0714824	0.8853387	1.7015398	2.4774536	3.1963249
	7	R.Eff(-)	3.2170698	4.1163008	9.6938813	19.242412	11.4827459	4.8503737	2.4332978	1.4289644
		B(-)	-0.6720805	-1.3703967	-0.967212	0.0727881	1.2837426	2.4483068	3.4831665	4.3586444
	9	R.Eff(-)	3.1448576	3.2715374	7.9930075	17.7562484	9.3307197	3.630191	1.7788781	1.0457584
		B(-)	-0.5642185	-1.6389512	-1.2865157	0.0733923	1.6732909	3.1433387	4.3485916	5.2481266
	11	R.Eff(-)	3.2748593	2.7871275	6.8728943	16.9099419	7.9327245	2.9040967	1.4075725	0.8379534
		B(-)	-0.4403664	-1.8297872	-1.5937468	0.0737264	2.054479	3.7879236	5.0795818	5.886642
	13	R.Eff(-)	3.4937754	2.4844107	6.0604952	16.3637492	6.9291647	2.4226542	1.1723785	0.7136349
		B(-)	-0.3276427	-1.9555015	-1.8890279	0.0739318	2.4275048	4.3832542	5.6833953	6.3014258

Table (2): shows Expected Sample Size of w.r.t. α , u , and λ when $u=2$

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.50	1.25	1.75	2
2	5	0.01	9.4626764	6.1612363	5.2784085	5.1	5.0863241	5.1104802	5.1458085	5.1868903
		0.05	10.9604355	7.4027053	5.9395857	5.5	5.4507682	5.541906	5.6868973	5.8569183
		0.1	11.7142558	8.286365	6.5968836	6	5.9220739	6.0697617	6.3146586	6.6033556
	7	0.01	15.2774575	9.2324937	7.4605309	7.14	7.1388037	7.2101349	7.3118377	7.4375958
		0.05	17.2132402	11.2628078	8.4749133	7.7	7.6964458	7.9650779	8.349908	8.8028357
		0.1	18.0780715	12.5900999	9.4439234	8.4	8.3949103	8.8294659	9.4531645	10.1659641
	9	0.01	21.677695	12.6609409	9.6835974	9.18	9.2034457	9.3521743	9.5716994	9.8578205
		0.05	23.7805868	15.5644753	11.0974452	9.9	9.9834929	10.5230634	11.2806411	12.1806478
		0.1	24.6328946	17.3373257	12.4021872	10.8	10.9315681	11.7892456	12.9614272	14.2785969
	11	0.01	28.3740477	16.445059	11.9491813	11.22	11.2808331	11.5426699	11.9456985	12.4917937
		0.05	30.4266966	20.2635052	13.8075215	12.1000215	12.3123412	13.2217745	14.4951456	16.0118761
		0.1	31.1907751	22.4627683	15.4703011	13.2	13.5316881	0.4529807	0.5100832	0.5730448
13	0.01	35.168353	20.576298	14.2587195	13.26	13.3715672	13.787624	14.4534252	15.380123	
	0.05	37.030099	25.314775	16.604957	14.300013	14.6835055	16.0662498	18.00335	20.2990627	
	0.1	37.6719203	27.9067068	18.6466128	15.6000225	16.1951122	18.3053181	21.0545513	24.0153376	

Table (3): shows Expected Sample Size Proportion w.r.t. α , u , n_1 and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
2	5	0.01	0.6308451	0.4107491	0.3518939	0.34	0.3390883	0.3406987	0.3430539	0.3457927
		0.05	0.7306957	0.4935137	0.3959724	0.3666667	0.3633845	0.3694604	0.3791265	0.3904612
		0.1	0.7809504	0.5524243	0.4397922	0.4	0.3948049	0.4046508	0.4209772	0.4402237
	7	0.01	0.727498	0.4396426	0.3552634	0.34	0.339943	0.3433398	0.3481827	0.3541712
		0.05	0.8196781	0.5363242	0.4035673	0.3666667	0.3664974	0.3792894	0.3976147	0.4191827
		0.1	0.8608605	0.5995286	0.4497106	0.4	0.3997576	0.4204508	0.4501507	0.4840935
	9	0.01	0.8028776	0.4689237	0.3586518	0.34	0.3408684	0.3463768	0.3545074	0.3651045
		0.05	0.8807625	0.576462	0.4110165	0.3666667	0.369759	0.3897431	0.4178015	0.4511351
		0.1	0.9123294	0.6421232	0.4593403	0.4	0.4048729	0.4366387	0.4800529	0.5288369
	11	0.01	0.8598196	0.4983351	0.3620964	0.34	0.3418434	0.3497779	0.3619909	0.3785392
		0.05	0.9220211	0.6140456	0.4184097	0.3666673	0.3731012	0.4006598	0.4392468	0.4852084
		0.1	0.945175	0.6806899	0.468797	0.4	0.4100512	0.4529807	0.5100832	0.5730448
13	0.01	0.9017526	0.5275974	0.3656082	0.34	0.3428607	0.3535288	0.3706006	0.3943621	
	0.05	0.9494897	0.6490968	0.4257681	0.3666667	0.3765001	0.4119551	0.4616244	0.5204888	
	0.1	0.9659467	0.7155566	0.4781183	0.4000006	0.4152593	0.4693671	0.5398603	0.6157779	

Table (4): shows Percentage of Overall Sample Saved w.r.t. α , u , n_1 and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
2	5	0.01	36.9154908	58.9250912	64.8106099	66	66.0911727	65.9301318	65.6946101	65.4207316
		0.05	26.93043	50.6486313	60.402762	63.3333333	63.6615452	63.0539602	62.0873511	60.9538781
		0.1	21.9049616	44.7575669	56.0207761	60	60.5195074	59.5349218	57.9022763	55.9776293
	7	0.01	27.2502024	56.035744	64.4736623	66	66.0056967	65.6660242	65.1817254	64.5828769
		0.05	18.0321895	46.3675819	59.6432699	63.3333333	63.350258	62.0710574	60.2385331	58.0817348
		0.1	13.9139454	40.0471431	55.0289363	60	60.0242367	57.9549245	54.9849309	51.5906472
	9	0.01	19.7122408	53.1076264	64.1348246	66	65.9131643	65.3623175	64.5492614	63.4895536
		0.05	11.9237526	42.3537951	58.898351	63.3333333	63.0241004	61.025691	58.2198476	54.8864896
		0.1	8.7670569	35.7876824	54.0659732	60	59.5127109	56.3361273	51.994714	47.1163077
	11	0.01	14.0180372	50.1664879	63.7903596	66	65.8156573	65.0222124	63.8009135	62.1460797
		0.05	7.797889	38.5954388	58.1590257	63.3332681	62.6898751	59.9340168	56.0753164	51.4791632
		0.1	5.4824996	31.9310052	53.1202997	60	58.9948844	54.7019278	48.9916837	42.6955182
13	0.01	9.824736	47.2402616	63.4391808	66	65.7139302	64.6471179	62.9399354	60.5637872	
	0.05	5.0510283	35.0903195	57.4231882	63.3332994	62.3499858	58.8044877	53.8375641	47.9511213	
	0.1	3.4053325	28.4443414	52.1881723	59.9999423	58.4740712	53.0632869	46.0139709	38.4222114	

Table (5): shows Bias ratio [B(.)] and R.E.ff w.r.t. α , n_1 and λ when $u = 6$

α	n_1	R.Eff(-) B(-)	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
100	5	R.Eff(-) B(-)	11.5006501 -1.5075366	33.8896946 -1.6724435	128.4109443 -0.9400311	367.1232704 6.5959247E-3	144.7705775 0.9894473	50.2690886 1.9725209	23.7539585 2.9485466	13.5001075 3.9153134
	7	R.Eff(-) B(-)	7.824044 -1.6642392	19.8793331 -2.3842134	85.9709005 -1.4066272	293.141886 6.7634277E-3	97.6234315 1.4801297	31.6149811 2.9441184	14.4828724 4.3834731	8.0291786 5.7900406
	9	R.Eff(-) B(-)	6.7130198 -1.600479	13.6628666 -3.0082667	65.1865558 -1.8677671	261.9441267 6.8479789E-3	74.7424065 1.9690037	22.8642487 3.9040281	10.1646016 5.7813398	5.482838 7.5781944
	11	R.Eff(-) B(-)	6.5571466 -1.418965	10.2569075 -3.5457157	52.5095898 -2.3232238	244.7941858 6.8984803E-3	60.8410979 2.4560206	17.7495447 4.8507276	7.6624893 7.1345046	4.0189104 9.2577111
	13	R.Eff(-) B(-)	6.9381658 -1.1906399	8.1644261 -3.9992212	43.8607296 -2.7728235	233.9596269 6.9319302E-3	51.3673362 2.9411151	14.3824044 5.782728	6.0326161 8.4356936	3.0809526 10.8086101
	15	R.Eff(-) B(-)	11.9571477 -1.0751756	26.3678738 -1.3863891	96.7325926 -0.8374679	279.9319496 0.020581	120.1103596 0.9486396	41.7774658 1.8700222	19.2261266 2.7618359	10.5928915 3.6163367
500	5	R.Eff(-) B(-)	9.2710531 -1.0793316	16.0497764 -1.9078691	64.8867595 -1.2501509	226.6866491 0.0210104	80.7341533 1.4074243	25.5180318 2.7515257	11.1806818 4.0054699	5.9511695 5.1488562
	7	R.Eff(-) B(-)	8.8439706 -0.9505521	11.5218048 -2.3241581	49.1298125 -1.6521752	203.8020931 0.0212187	61.3853871 1.8605786	17.9695621 3.5978076	7.5716586 5.1465369	3.9374258 6.4646175
	9	R.Eff(-) B(-)	9.371124 -0.7761139	9.0678239 -2.6461336	39.493593 -2.0434257	191.1080902 0.0213418	49.5700178 2.3081601	13.6334173 4.4076609	5.5762673 6.1798519	2.8671892 7.5543804
	11	R.Eff(-) B(-)	10.4950917 -0.6025631	7.579985 -2.8847259	32.9274671 -2.423926	183.0477624 0.0214203	41.5061007 2.7501513	10.8394335 5.1800754	4.3402899 7.102504	2.2319648 8.4182427
	13	R.Eff(-) B(-)	12.7626413 -0.8649671	23.7335622 -1.2001307	78.9331919 -0.7526722	215.928713 0.030574	99.0740976 0.897929	35.1681173 1.7493915	16.1459181 2.5500459	8.8558915 3.2882648
	15	R.Eff(-) B(-)	10.5600862 -0.82466	15.0114678 -1.6168933	53.6450635 -1.1218638	176.6596949 0.0311074	66.9926153 1.3214546	21.3915861 3.6124198	5.0146261 4.5197047	5.0146261 4.5197047
1000	5	R.Eff(-) B(-)	10.5459862 -0.692626	11.1664975 -1.929093	41.008249 -1.4778917	159.5533794 0.0313403	51.0635638 1.7365953	6.3899673 3.2772928	3.4077472 4.5357948	3.4077472 5.4777418
	7	R.Eff(-) B(-)	11.4988003 9.0868879	33.2356514 -2.1522249	33.2356514 -1.8208237	150.0039513 0.0314575	41.2888982 2.1435705	11.4400968 3.9657723	4.782322 5.3237704	2.578555 6.179403
	9	R.Eff(-) B(-)	13.0387903 -0.403068	7.8389809 -2.3003672	27.9187493 -2.1508414	143.9156124 0.0315197	34.6026821 2.5424719	9.1401254 4.604823	3.8043577 5.9819398	2.0999618 6.6488272
	11	R.Eff(-) B(-)								
	13	R.Eff(-) B(-)								
	15	R.Eff(-) B(-)								

Table (6): shows Expected Sample Size of w.r.t. α , u , and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.50	1.25	1.75	2
6	5	0.01	18.3880292	8.4837089	5.8352256	5.3	5.2589723	5.3314407	5.4374254	5.5606708
		0.05	22.8813065	12.2081159	7.8187571	6.5	6.3523047	6.6257179	7.060692	7.5707549
		0.1	25.1427673	14.8590949	9.7906508	8	7.7662217	8.2092852	8.9439757	9.8100668
	7	0.01	31.8323725	13.6974812	8.3815927	7.42	7.4164111	7.6304048	7.935513	8.3127875
		0.05	37.6397206	19.7884234	11.42474	9.1	9.0893375	9.8952338	11.0497241	12.4085071
		0.1	40.2342144	23.7702998	14.3317701	11.2	11.1847309	12.4883976	14.3594935	16.4978923
	9	0.01	47.033085	19.9828226	11.0507921	9.54	9.610337	10.0565228	10.7150982	11.5734615
		0.05	53.3417604	28.693426	15.2923357	11.7	11.9504787	13.5691903	15.8419234	18.5419434
		0.1	55.8986839	34.0119772	19.2065617	14.4	14.7947042	17.3677369	20.8842816	24.8357908
11	0.01	63.1221431	27.335177	13.847544	11.66	11.8424993	12.6280097	13.8370956	15.4753811	
	0.05	69.2800899	38.7905156	19.4225645	14.3000646	14.9370236	17.6653234	21.4854368	26.0356284	
	0.1	71.5723254	45.3883048	24.4109033	17.6	18.5950644	22.8450915	28.4982332	34.731437	
13	0.01	79.5050589	35.7288939	16.7761585	13.78	14.1147016	15.362872	17.3602755	20.140369	
	0.05	85.0902969	49.9443262	23.8148698	16.9000397	18.0505166	22.1987494	28.01005	34.8971881	
	0.1	87.015761	57.7201205	29.9398384	20.8000675	22.5853367	28.9159543	37.163654	46.0460127	

Table (7): shows Expected Sample Size Proportion w.r.t. α , u , n_1 and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
6	5	0.01	0.5253723	0.2423917	0.1667207	0.1514286	0.1502564	0.1523269	0.155355	0.1588763
		0.05	0.6537516	0.3488033	0.2233931	0.1857143	0.1814944	0.1893062	0.2017341	0.2163073
		0.1	0.7183648	0.4245456	0.2797329	0.2285714	0.221892	0.234551	0.2555422	0.2802876
	7	0.01	0.6496403	0.2795404	0.1710529	0.1514286	0.1513553	0.1557225	0.1619492	0.1696487
		0.05	0.7681576	0.4038454	0.233158	0.1857143	0.1854967	0.2019435	0.2255046	0.2532348
		0.1	0.8211064	0.4851082	0.2924851	0.2285714	0.2282598	0.2548653	0.2930509	0.3366917
	9	0.01	0.7465569	0.3171877	0.1754094	0.1514286	0.152545	0.1596273	0.1700809	0.1837057
		0.05	0.8466946	0.4554512	0.2427355	0.1857143	0.1896901	0.215384	0.2514591	0.2943166
		0.1	0.8872807	0.5398727	0.3048661	0.2285714	0.2348366	0.2756784	0.3314965	0.3942189
	11	0.01	0.8197681	0.3550023	0.1798382	0.1514286	0.1537987	0.1640001	0.1797025	0.200979
		0.05	0.8997414	0.5037729	0.2522411	0.1857151	0.1939873	0.2294198	0.2790316	0.338125
		0.1	0.9295107	0.5894585	0.3170247	0.2285714	0.2414943	0.2966895	0.3701069	0.4510576
	13	0.01	0.873682	0.3926252	0.1843534	0.1514286	0.1551066	0.1688228	0.1907723	0.2213227
		0.05	0.9350582	0.5488387	0.2617019	0.1857147	0.1983573	0.2439423	0.3078027	0.3834856
		0.1	0.9562172	0.634287	0.3290092	0.2285722	0.2481905	0.3177577	0.4083918	0.5060001

Table (8): shows Percentage of Overall Sample Saved w.r.t. α , u , n_1 and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
6	5	0.01	47.4627739	75.7608316	83.327927	84.8571429	84.9743648	84.7673123	84.4644987	84.1123692
		0.05	34.6248385	65.1196688	77.660694	81.4285714	81.8505581	81.0693774	79.8265943	78.3692718
		0.1	28.1635221	57.5454432	72.0267121	77.1428571	77.8107952	76.5448994	74.4457838	71.9712376
	7	0.01	35.0359746	72.0459566	82.8947087	84.8571429	84.8644672	84.4277453	83.8050755	83.0351275
		0.05	23.1842436	59.6154625	76.6842041	81.4285714	81.4503317	79.8056453	77.4495426	74.6765161
		0.1	17.8893584	51.489184	70.7514895	77.1428571	77.1740186	74.5134743	70.6949112	66.3308321
	9	0.01	25.3443096	68.281234	82.4590602	84.8571429	84.7454969	84.0372654	82.9919076	81.6294261
		0.05	15.330539	54.4548794	75.7264513	81.4285714	81.0309862	78.4616028	74.8540898	70.5683438
		0.1	11.2719303	46.0127345	69.5133941	77.1428571	76.5163426	72.4321637	66.8503466	60.5781099
	11	0.01	18.0231907	64.4997701	82.0161766	84.8571429	84.6201308	83.5999874	82.029746	79.9021024
		0.05	10.0258572	49.622707	74.7758902	81.4284876	80.601268	77.0580216	72.0968354	66.1874956
		0.1	7.0489281	41.0541495	68.2975282	77.1428571	75.8505657	70.3310501	62.9893076	54.8942377
	13	0.01	12.6318034	60.7374792	81.564661	84.8571429	84.4893389	83.1177231	80.9227741	77.8677263
		0.05	6.4941792	45.1161251	73.8298134	81.4285277	80.1642675	75.6057699	69.2197253	61.6514417
		0.1	4.3782846	36.5712961	67.0990787	77.1427829	75.1809487	68.224226	59.1608198	49.3999861

Table (9): shows Bias ratio [B(·)] and R.E.ff w.r.t. α , n_1 and λ when $u = 8$

α	n_1	R.Eff(-) B(-)	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
100	5	R.Eff(-) B(-)	15.7992081 -1.5416833	51.243284 -1.6936233	221.9193302 -0.9471706	750.5055682 5.1301631E-3	255.3246925 0.989874	83.6829607 1.9737171	38.7486865 2.9502601	21.7570903 3.9175201
	7	R.Eff(-) B(-)	10.5963984 -1.7031613	29.171536 -2.4147456	145.3474248 -1.4164116	596.7733005 5.260443E-3	169.5630469 1.4812404	51.9979693 2.9465301	23.2798893 4.3870937	12.68689 5.7950224
	9	R.Eff(-) B(-)	9.045174 -1.6390618	19.616913 -3.0478647	108.3130188 -1.8802732	532.1530326 5.3262048E-3	128.2543243 1.9708684	37.1936914 3.9079638	16.0918181 5.7875827	8.4817049 7.5872058
	11	R.Eff(-) B(-)	8.8373127 -1.4540904	14.486276 -3.5938869	86.0098203 -2.3385382	496.6870436 5.3654835E-3	103.3795428 2.458702	28.5774649 4.8564916	11.945746 7.1440818	6.0866627 9.2719778
	13	R.Eff(-) B(-)	9.3905281 -1.2207954	11.3847566 -4.0553105	70.9699266 -2.7910323	474.3030401 5.3914999E-3	86.5662921 2.9446732	22.9288293 5.7906195	9.2631688 8.4492929	4.5730995 10.8292479
500	5	R.Eff(-) B(-)	16.3784606 -1.1055502	38.1751751 -1.4159561	158.3777524 -0.8527035	548.2465274 0.0160065	202.6710004 0.9500296	65.8214265 1.8748094	29.3966741 2.7689502	15.8692206 3.6254012
	7	R.Eff(-) B(-)	12.6272074 -1.1099605	22.6975465 -1.9475659	103.5790628 -1.2705491	441.2897603 0.01634	133.5242593 1.411402	39.3949258 2.7607618	16.6710742 4.0189081	8.6530577 5.1662551
	9	R.Eff(-) B(-)	12.074578 -0.9777906	16.0400529 -2.372383	76.9340319 -1.6776688	395.5866532 0.0165016	99.9286385 1.8672564	27.2668572 3.6119305	11.0550078 5.1672016	5.5915848 6.4916038
	11	R.Eff(-) B(-)	12.8963957 -0.7985888	12.4842686 -2.7013436	60.9012672 -2.0739711	370.3070947 0.0165969	79.6540237 2.3176199	20.38264 4.4270216	8.0009247 6.2083517	3.9977397 7.5915012
	13	R.Eff(-) B(-)	14.6253932 -0.6201888	10.3513612 -2.9454596	50.1327912 -2.4594812	354.283013 0.0166575	65.9664258 2.7624628	15.9974695 5.2049559	6.1379857 7.1391863	3.0679499 8.465438
10	5	R.Eff(-) B(-)	17.5479115 -0.8917862	33.8859019 -1.2317803	125.6270148 -0.7721223	408.0886038 0.0237559	161.466766 0.9000274	53.2351603 1.7573668	23.6664404 2.5621447	12.711558 3.3035882
	7	R.Eff(-) B(-)	14.5043071 -0.8500899	21.0108329 -1.6579761	83.2637693 -1.1476391	331.625787 0.0241606	106.8234348 1.3277399	31.7118932 2.5537544	13.4014449 3.633962	7.0154928 4.5465481
	9	R.Eff(-) B(-)	14.5867575 -0.7140273	15.4296581 -1.97745	62.481054 -1.5097877	298.5410309 0.0243316	80.0743853 1.747146	21.9405279 3.2996185	8.9865301 4.5669953	4.682014 5.5160111
	11	R.Eff(-) B(-)	16.1093432 -0.5579119	12.4465036 -2.2059645	49.9081068 -1.8586699	280.1330665 0.0244127	63.8832813 2.1584191	16.4581892 3.9954137	6.6336712 5.3644685	3.4968988 6.2281965
	13	R.Eff(-) B(-)	18.5877299 -0.4156389	10.6714167 -2.3578448	41.4304458 -2.194477	268.4201894 0.024451	52.94306 2.561633	13.0032137 4.6417512	5.2203146 6.0316972	2.8206194 6.7067275



Table (10): shows Expected Sample Size w.r.t. α , u , and λ when $u=8$

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.50	1.25	1.75	2
5		0.01	22.8507055	9.6449453	6.1136341	5.4	5.3452964	5.4419209	5.5832339	5.7475611
		0.05	28.841742	14.6108212	8.7583428	7	6.8030729	7.1676239	7.7475893	8.4276732
		0.1	31.857023	18.1454599	11.3875344	9	8.6882956	9.2790469	10.2586342	11.4134224
7		0.01	40.1098299	15.929975	8.8421237	7.56	7.5552147	7.8405397	8.2473507	8.7503834
		0.05	47.8529608	24.0512312	12.8996533	9.8	9.7857833	10.8603118	12.3996322	14.2113428
		0.1	51.3122859	29.3603998	16.7756935	12.6	12.5796412	14.3178634	16.812658	19.6638564
8	9	0.01	40.1098299	15.929975	8.8421237	7.56	7.5552147	7.8405397	8.2473507	8.7503834
		0.05	68.1223472	35.2579013	17.3897809	12.6	12.9339716	15.0922537	18.1225646	21.7225912
		0.1	71.5315786	42.349303	22.608749	16.2	16.7262722	20.1569825	24.8457089	30.1143877
11		0.01	59.7107799	23.6437634	11.7343894	9.72	9.8137826	10.4086971	11.2867976	12.4312821
		0.05	88.7067866	48.0540208	22.230086	15.4000861	16.2493648	19.8870978	24.9805823	31.0475046
		0.1	91.7631005	56.8510731	28.8812044	19.8	21.1267526	26.7934553	34.3309775	42.641916
13		0.01	101.6734119	43.3051919	18.034878	14.04	14.4862688	16.150496	18.8137007	22.520492
		0.05	109.1203959	62.2591015	27.4198264	18.200053	19.7340221	25.2649992	33.0133999	42.1962508
		0.1	111.6876814	72.6268274	35.5864512	23.4000901	25.7804489	34.2212724	45.2182054	57.0613502

Table (11): shows Expected Sample Size Proportion w.r.t. α , u , n_1 and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
5		0.01	0.5077935	0.2143321	0.1358585	0.12	0.1187844	0.1209316	0.1240719	0.1277236
		0.05	0.6409276	0.3246849	0.1946298	0.1555556	0.1511794	0.1592805	0.1721687	0.1872816
		0.1	0.7079338	0.4032324	0.2530563	0.2	0.1930732	0.206201	0.2279696	0.2536316
7		0.01	0.636664	0.2528567	0.1403512	0.12	0.119924	0.124453	0.1309103	0.138895
		0.05	0.7595708	0.3817656	0.2047564	0.1555556	0.1553299	0.1723859	0.1968196	0.2255769
		0.1	0.8144807	0.4660381	0.2662808	0.2	0.1996768	0.2272677	0.2668676	0.3121247
8	9	0.01	0.636664	0.2528567	0.1403512	0.12	0.119924	0.124453	0.1309103	0.138895
		0.05	0.8410166	0.4352827	0.2146887	0.1555556	0.1596787	0.1863241	0.2237354	0.2681801
		0.1	0.8831059	0.5228309	0.2791204	0.2	0.2064972	0.2488516	0.3067371	0.3717826
11		0.01	0.7371701	0.2918983	0.144869	0.12	0.1211578	0.1285024	0.1393432	0.1534726
		0.05	0.8960281	0.4853941	0.2245463	0.1555564	0.164135	0.2008798	0.2523291	0.3136112
		0.1	0.9269	0.5742533	0.2917293	0.2	0.2134015	0.270641	0.3467776	0.4307264
13		0.01	0.8690035	0.3701298	0.1541443	0.12	0.1238143	0.1380384	0.1608009	0.1924828
		0.05	0.932653	0.5321291	0.2343575	0.155556	0.1686669	0.2159402	0.2821658	0.3606517
		0.1	0.9545956	0.6207421	0.3041577	0.2000008	0.2203457	0.2924895	0.3864804	0.4877038

Table (12): shows Percentage of Overall Sample Saved w.r.t. α , u , n_1 and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
5		0.01	49.2206544	78.5667883	86.4141465	88	88.1215635	87.9068424	87.5928135	87.2276421
		0.05	35.9072399	67.5315084	80.537016	84.4444444	84.8820603	84.0719469	82.7831348	81.2718374
		0.1	29.2066155	59.6767559	74.6943681	80	80.6926765	79.3798957	77.2030351	74.636839
7		0.01	36.3336033	74.7143254	85.9648831	88	88.0075957	87.5546989	86.9089672	86.1105026
		0.05	24.0429193	61.8234426	79.5243598	84.4444444	84.4670106	82.7614099	80.3180442	77.442313
		0.1	18.5519272	53.3961908	73.3719151	80	80.0323156	77.2732327	73.3132412	68.7875296
8	9	0.01	36.3336033	74.7143254	85.9648831	88	88.0075957	87.5546989	86.9089672	86.1105026
		0.05	15.8983367	56.4717268	78.5311347	84.4444444	84.0321339	81.3675881	77.6264635	73.1819862
		0.1	11.6894092	47.7169099	72.0879642	80	79.3502812	75.1148365	69.3262854	62.8217436
11		0.01	26.2829877	70.8101686	85.5130995	88	87.884219	87.1497567	86.0656819	84.6527382
		0.05	10.3971853	51.460585	77.5453677	84.4443575	83.5865002	79.9120224	74.7670885	68.6388843
		0.1	7.3099995	42.5746736	70.8270663	80	78.6598459	72.9359038	65.3222449	56.9273576
13		0.01	13.0996479	62.9870155	84.5855744	88	87.6185737	86.1961572	83.9199139	80.7517162
		0.05	6.7347044	46.7870927	76.5642509	84.4443991	83.1333144	78.4059836	71.7834189	63.9348284
		0.1	4.5404433	37.9257886	69.5842298	79.999923	77.9654283	70.7510492	61.3519612	51.2296152

Table (13): shows Bias ratio [B(·)] and R.E.ff w.r.t. α , n_1 and λ when $u = 10$

α	n_1	R.Eff(-) B(-)	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
100	5	R.Eff(-) B(-)	20.1608911 -1.5634131	70.0901591 -1.7071014	337.8975179 -0.951714	1.3283519E+3 4.1974058E-3	397.6598069 0.9901455	124.8418315 1.9744783	56.9138232 2.9513505	31.6400086 3.9189244
	7	R.Eff(-) B(-)	13.3973709 -1.7279299	39.0274664 -2.4341752	217.5781913 -1.4226381	1.0535918E+3 4.3039982E-3	261.0147471 1.4819473	76.7975902 2.9480647	33.7481223 4.3893978	18.1241341 5.7981926
	9	R.Eff(-) B(-)	11.3994496 -1.6636145	25.8338403 -3.0730633	159.9465795 -1.8882317	938.3261324 4.3578032E-3	195.5715793 1.972055	54.4183679 3.9104683	23.0147832 5.7915555	11.8933723 7.5929403
	11	R.Eff(-) B(-)	11.1412572 -1.4764429	18.8554033 -3.6245414	125.5839342 -2.3482836	875.1259032 4.38994E-3	156.4276203 2.4604083	41.440367 4.8601595	16.8569689 7.1501764	8.382714 9.2810565
	13	R.Eff(-) B(-)	11.8744639 -1.2399852	14.6867545 -4.0910038	102.6244565 -2.8026196	835.2613431 4.411226E-3	130.1326415 2.9469375	32.965176 5.7956414	12.9025777 8.4579469	6.1950229 10.8423811
500	5	R.Eff(-) B(-)	20.8613574 -1.1248795	50.5030161 -1.4347714	229.6000232 -0.8623989	929.6749514 0.0130954	302.3386783 0.9509142	93.3190785 1.8778557	40.7379243 2.7734775	21.6438516 3.6311696
	7	R.Eff(-) B(-)	16.029387 -1.1294517	29.5492404 -1.9728274	147.3846837 -1.2835298	745.5066524 0.0133679	196.1790652 1.4139333	54.9541643 2.7666393	22.6409719 4.0274596	11.5270072 5.1773272
	9	R.Eff(-) B(-)	15.3578356 -0.9951243	20.6607021 -2.4030716	107.9248367 -1.6938919	667.0983423 0.0134998	145.0392399 1.8715058	37.5186889 3.6209178	14.7696354 5.1803519	7.3170393 6.508777
	11	R.Eff(-) B(-)	16.497595 -0.812891	15.9609391 -2.7364772	84.4687135 -2.093409	623.805499 0.0135774	114.4561662 2.3236398	27.7215424 4.4393421	10.5487783 6.226488	5.1616786 7.6151236
	13	R.Eff(-) B(-)	18.8799178 -0.6314051	13.1624716 -2.9841083	68.8833075 -2.4821073	596.3929771 0.0136267	93.9822353 2.7702973	21.5408041 5.2207889	8.0061526 7.1625296	3.9211192 8.4954713
101	5	R.Eff(-) B(-)	22.4114451 -0.9088528	44.3832194 -1.251921	178.0034482 -0.7844997	670.6915364 0.0194171	233.5318898 0.9013627	72.9627158 1.7624419	31.6895138 2.5698439	16.7617537 3.3133394
	7	R.Eff(-) B(-)	18.521388 -0.8662726	27.157685 -1.6841197	115.868162 -1.1640416	542.716628 0.0197399	152.0394491 1.3317397	42.7926667 2.5633405	17.6322489 3.6476707	9.0783545 4.5636303
	9	R.Eff(-) B(-)	18.7244342 -0.7276463	19.7741578 -2.0082226	85.7923384 -1.5300851	487.5814411 0.0198716	112.5621374 1.7538601	29.2462296 3.3138258	11.6709842 4.5868502	5.9814853 5.5403642
	11	R.Eff(-) B(-)	20.8738002 -0.5685978	15.8588915 -2.2401624	67.8167002 -1.8827538	456.9695181 0.0199297	88.9104098 2.1678682	21.7230208 4.0142765	8.5311762 5.3903673	4.4272972 6.2592469
	13	R.Eff(-) B(-)	24.4004725 -0.4236386	13.5423265 -2.3944215	55.8224208 -2.2222451	437.5157543 0.0199527	73.0752811 2.5738265	17.0242182 4.6652509	6.6628313 6.063361	3.5477961 6.7435731

Table (14): shows Expected Sample Size of w.r.t. α , u , and λ

u	n ₁	α	λ								
			0.25	0.50	0.75	1	1.50	1.25	1.75	2	
10	5	0.01	27.3133819	10.8061816	6.3920426	5.5	5.4316205	5.5524012	5.7290424	5.9344513	
		0.05	34.8021775	17.0135265	9.6979285	7.5	7.2538411	7.7095298	8.4344867	9.2845914	
		0.1	38.5712788	21.4318248	12.984418	10	9.6103695	10.3488087	11.5732928	13.016778	
	7	0.01	48.3872874	18.1624687	9.3026546	7.7	7.6940184	8.0506746	8.5591883	9.1879792	
		0.05	58.0662011	28.314039	14.3745667	10.5	10.4822291	11.8253897	13.7495402	16.0141785	
		0.1	62.3903573	34.9504997	19.2196169	14	13.9745515	16.1473293	19.2658226	22.8298205	
	9	0.01	72.3884749	27.3047043	12.4179868	9.9	10.0172283	10.7608713	11.8584971	13.2891026	
		0.05	82.902934	41.8223766	19.4872261	13.5	13.9174644	16.6153171	20.4032057	24.903239	
		0.1	87.1644732	50.6866287	26.0109362	18	18.6578403	22.9462281	28.8071361	35.3929846	
	1	1	0.01	97.8702386	38.225295	15.7459067	12.1	12.4041654	13.7133495	15.7284927	18.4589686
			0.05	108.1334832	57.3175261	25.0376075	16.5001076	17.561706	22.1088723	28.4757279	36.0593807
			0.1	111.9538756	68.3138414	33.3515054	22	23.6584407	30.7418191	40.1637219	50.5523949
	1	3	0.01	123.8417649	50.8814899	19.2935975	14.3	14.857836	16.93812	20.2671259	24.900615
			0.05	133.1504949	74.5738769	31.024783	19.5000662	21.4175277	28.3312491	38.0167499	49.4953135
			0.1	136.3596017	87.5335342	41.2330639	26.0001126	28.9755611	39.5265905	53.2727567	68.0766878

Table (15): shows Expected Sample Size Proportion w.r.t. α , u , n_1 and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
10	5	0.01	0.4966069	0.196476	0.116219	0.1	0.0987567	0.1009527	0.1041644	0.1078991
		0.05	0.6327669	0.3093368	0.176326	0.1363636	0.131888	0.1401733	0.1533543	0.1688108
		0.1	0.701296	0.3896695	0.2360803	0.1818182	0.174734	0.1881602	0.2104235	0.2366687
	7	0.01	0.6284063	0.2358762	0.1208137	0.1	0.0999223	0.1045542	0.1111583	0.1193244
		0.05	0.7541065	0.3677148	0.1866827	0.1363636	0.1361328	0.1535765	0.1785655	0.2079763
		0.1	0.8102644	0.4539026	0.2496054	0.1818182	0.1814877	0.2097056	0.2502055	0.2964912
	9	0.01	0.7311967	0.2758051	0.1254342	0.1	0.1011841	0.1086957	0.1197828	0.1342334
		0.05	0.8374034	0.4224482	0.1968407	0.1363636	0.1405804	0.1678315	0.206093	0.2515479
		0.1	0.8804492	0.5119861	0.2627367	0.1818182	0.188463	0.2317801	0.2909812	0.3575049
	11	0.01	0.8088449	0.3159115	0.1301315	0.1	0.1025138	0.1133335	0.1299875	0.1525535
		0.05	0.8936652	0.4736986	0.2069224	0.1363645	0.1451381	0.182718	0.2353366	0.2980114
		0.1	0.9252386	0.5645772	0.2756323	0.1818182	0.1955243	0.2540646	0.3319316	0.4177884
	13	0.01	0.8660263	0.3558146	0.1349203	0.1	0.103901	0.1184484	0.1417282	0.1741302
		0.05	0.9311223	0.5214956	0.2169565	0.1363641	0.1497729	0.1981206	0.2658514	0.3461211
		0.1	0.9535636	0.6121226	0.2883431	0.181819	0.2026263	0.2764097	0.3725368	0.4760608

Table (16) :shows Percentage of Overall Sample Saved w.r.t. α , u , n_1 and λ

u	n ₁	α	λ							
			0.25	0.50	0.75	1	1.25	1.50	1.75	2
10	5	0.01	50.3393056	80.3523972	88.3781044	90	90.1243263	89.9047252	89.5835592	89.2100885
		0.05	36.7233136	69.0663154	82.3674027	86.3636364	86.811198	85.982673	84.6645697	83.1189246
		0.1	29.8704022	61.0330458	76.3919674	81.8181818	82.5266009	81.1839842	78.9576495	76.3331308
	7	0.01	37.159367	76.4123782	87.9186304	90	90.0077683	89.5445784	88.884171	88.0675594
		0.05	24.5893493	63.2285208	81.3317316	86.3636364	86.3867154	84.642351	82.1434543	79.2023656
		0.1	18.9735619	54.6097406	75.0394586	81.8181818	81.8512318	79.0294425	74.9794512	70.3508825
	9	0.01	26.8803284	72.4194906	87.456579	90	89.8815876	89.130433	88.0217202	86.5766641
		0.05	16.2596626	57.7551751	80.3159332	86.3636364	85.9419551	83.2168514	79.3907013	74.8452131
		0.1	11.9550776	48.8013851	73.7263271	81.8181818	81.1536967	76.8219918	70.9018828	64.2495105
	11	0.01	19.1155053	68.4088471	86.986854	90	89.7486236	88.6666533	87.0012457	84.7446541
		0.05	10.633485	52.6301438	79.3077624	86.3635474	85.4861934	81.7282047	76.4663405	70.1988589
		0.1	7.4761358	43.5422798	72.4367724	81.8181818	80.4475696	74.5935379	66.8068414	58.2211612
	13	0.01	13.3973672	64.4185385	86.5079738	90	89.6099049	88.1551608	85.8271847	82.5869825
		0.05	6.8877658	47.8504357	78.3043475	86.36359	85.0227079	80.1879377	73.4148602	65.3878927
		0.1	4.6436352	38.7877383	71.1656895	81.8181031	79.7373699	72.3590276	62.746324	52.3939246

Table (17): shows Probability of a Voiding Second Sample w.r.t. α , n_1 and λ

n_1	α	λ							
		0.25	0.50	0.75	1	1.25	1.50	1.75	2
5	0.01	0.5537324	0.8838764	0.9721591	0.99	0.9913676	0.988952	0.9854192	0.981311
	0.05	0.4039564	0.7597295	0.9060414	0.95	0.9549232	0.9458094	0.9313103	0.9143082
	0.1	0.3285744	0.6713635	0.8403116	0.9	0.9077926	0.8930238	0.8685341	0.8396644
7	0.01	0.408753	0.8405362	0.9671049	0.99	0.9900855	0.9849904	0.9777259	0.9687432
	0.05	0.2704828	0.6955137	0.894649	0.95	0.9502539	0.9310659	0.903578	0.871226
	0.1	0.2087092	0.6007071	0.825434	0.9	0.9003636	0.8693239	0.824774	0.7738597
9	0.01	0.2956836	0.7966144	0.9620224	0.99	0.9886975	0.9804348	0.9682389	0.9523433
	0.05	0.1788563	0.6353069	0.8834753	0.95	0.9453615	0.9153854	0.8732977	0.8232973
	0.1	0.1315059	0.5368152	0.8109896	0.9	0.8926907	0.8450419	0.7799207	0.7067446
11	0.01	0.2102706	0.7524973	0.9568554	0.99	0.9872349	0.9753332	0.9570137	0.9321912
	0.05	0.1169683	0.5789316	0.8723854	0.949999	0.9403481	0.8990103	0.8411297	0.7721874
	0.1	0.0822375	0.4789651	0.7968045	0.9	0.8849233	0.8205289	0.7348753	0.6404328
13	0.01	0.147371	0.7086039	0.9515877	0.99	0.985709	0.9697068	0.944099	0.9084568
	0.05	0.0757654	0.5263548	0.8613478	0.9499995	0.9352498	0.8820673	0.8075635	0.7192668
	0.1	0.05108	0.4266651	0.7828226	0.8999991	0.8771111	0.7959493	0.6902096	0.5763332

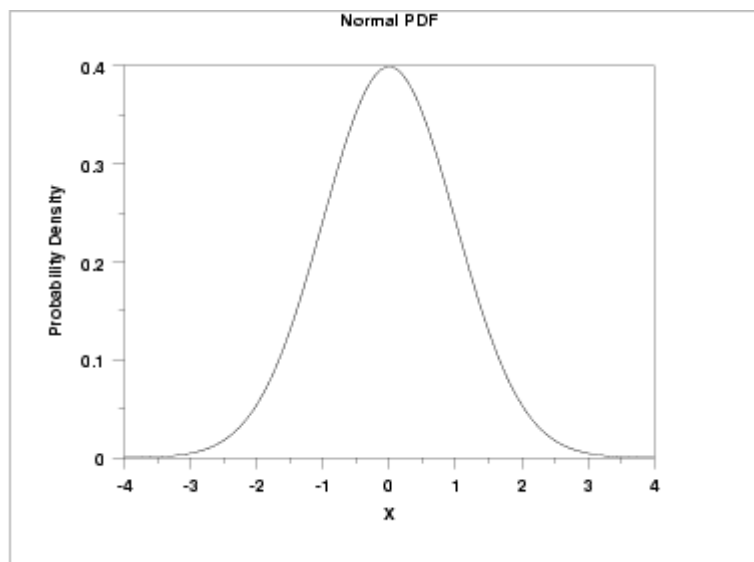


Figure (1): Graph for PDF of Normal Distribution