New Robust Estimation in Compound Exponential Weibull-Poisson Distribution for both contaminated and non-contaminated Data

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Abstract

The research Compared two methods for estimating fourparametersof the compound exponential Weibull - Poisson distribution which are the maximum likelihood method and the Downhill Simplex algorithm. Depending on two data cases, the first one assumed the original data (Non-polluting), while the second one assumeddata contamination. Simulation experimentswere conducted for different sample sizes and initial values of parameters and under different levels of contamination. Downhill Simplex algorithm was found to be the best method for in the estimation of the parameters, the probability function and the reliability function of the compound distribution in cases of natural and contaminateddata.

Key Words :Compound distributions, Exponential Weibull Poisson distribution, Maximum Likelihood method, EM algorithm, Downhill Simplex algorithm, Data contamination.

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1-Introduction

The process of compounding two or more distributions to obtain a new one is more flexible and useful statistically it has several Procedures, One of them include compounding of discrete distribution with one of the common life time's distributions. As the compound distribution resulting from this method represents the distribution of a new life time, It can be used to model complex phenomena and situations thatcannotbe modeled by the common and individual life time's distribution (Such as exponential distribution, weibell, gamma, logarithmic, etc. from life time distributions). The resulting compound distributions are often used in medical, engineering and biological studies.

Many researchers have taken this procedure in compounding, Loukas & Adamidis, (1998) was the first to use this procedure for compounding the geometric distribution with exponential distribution. This procedure was followed by a large number of researchers like,Kus,(2007) which introduced the compounding of Poisson distribution with exponential distribution, Souza & Neto, (2009) presented the generalized exponential poisson distribution, Morais & Barreto (2011) showedthe concept of compounding weibull distribution with power Series distributions (WPS), Neto et al.,(2011)also introduced a new two-parameter distribution resulting from compounding zero truncated Poisson distribution (which is the distribution to the random variable that represent the number of complementary risk (CR)) with a life-time event that related to JTH (CR) and that these variables are assumed to be independent and identicallydistributed according to exponential distribution, Lu & Shi,(2012) also touched on a new distribution resulting from compounding Poisson distribution with Weibull distribution. This distribution is flexible and has the advantage that its failure rate function can take a number of forms. It may be increasing, decreasing, taking the shape of the bathtube or having a uniform shape (unimodal) This distribution can therefore be used to model data on life-time or biological studies that rely on the property of having many forms of failure rate function Alkarni & Oraby,(2012) found a new class of distributions that had a decreasing failure rate by compounding zero truncated Poisson distribution with life time distribution, Both Mahmoudi & Sepahdar,(2013) proposed exponential Weibull –poisson distribution that have four parameters which result from compounding exponential weibull distribution with poisson distribution, The resulting distribution is the general statusfor anather six compound distributions, this distribution is characterized by the fact that the failure rate function takes several forms (increasing, decreasing, bathtube shape, uniform shape),Ristic & Nadarajah (2014) introduced a new three parameter life-time distribution called the exponentiated exponential Poisson distribution, Sanjay et al. (2014)also estimated the shape and scale parameters of the exponential Poisson distribution byusing two methods, maximum likelihood method and Bayes method.

The mcmc algorithm was used to calculate the Bayes properties under a symmetric and asymmetric loss function. Results show that the Bayes estimaters were better than the maximum likelihood estimators.
maximum likelihood estimaters, Oluyede et al.,(2015) introduced a new class of compound distributions (life-time distributions), which is called the Log-Logistic Weibull-Poisson Distribution which has five parameters resulting from compounding Log-Logistic Weibull distribution with Poissondistribution. This new distribution is considered as a general case for many other compound and individual distributions as well as the failure rate function has many forms that exhibit distribution elasticity.

Most of the researches and studies above dealt with finding new life time distributions and estimating their parameters for non-contaminated data.So the state of contaminated dataof the compound distributions was not addressed Hence, the importance of this research is emerged to compare between two methods for estimating the parameters of compound exponential Weibull -poisson distribution, The first is the maximum likelihood method using the EM algorithm which was first derived by Sepahdar &Mahmoudi,(2011), Andthe second one is the Downhill Simplex engineering algorithm. The comparison was made using the status of non-polluting data and the status of polluting data (at different rates), And to determine the preference for estimation methods when there is no contamination status in the sample data, the simulation experiment was carried out for different sample sizes and three percentages of contamination, The standard comparison mean square error (MSE) and mean absolute average error (MAPE) were adopted.

2-Compound Exponential Weibull –Poisson Distribution

This distribution was first found by Mahmoudi & Sepahdar, (2013). They used the same compounding procedure used by Loukas & Adamidis, (1998). This distribution resulted from compounding exponential weibull distribution with poisson distribution.
Suppose that there are N independent and identical random variables which are distributed according to exponential weibull distribution with three parameters (One of common life time distributions) with the following probability function:

$$f(x;\alpha,\beta,\lambda) = \alpha\,\lambda\,\beta^{\lambda}x^{\lambda-1}e^{-(\beta x)^{\lambda}}\left(1-e^{-(\beta x)^{\lambda}}\right)^{\alpha-1} \qquad (1)$$

Assuming that N represents the number of random variables that are distributed according to exponential weibull distribution as above shown and also represent a random variable distributed according to the zero- truncated Poisson distribution with the following probability function.

$$f(N=n) = \frac{e^{-\theta}\theta^{n}}{n!}\left(1-e^{-\theta}\right)^{-1} \qquad z = 1, 2, \ldots\ldots, \ \theta>0 \qquad (2)$$

Assuming that $Y = \max(x_1, x_2, \ldots, x_N)$

Then the probability function for Y can be obtained by applying the order statistics formula for the exponential Weibull distribution

$f(Y|\ N=n) = n\ [\ F\ (y)]^{n-1} f\ (y)$

$f(Y|\ N=n) = n\ [\left(1 - e^{-(\beta y)^{\lambda}}\right)^{\alpha}]^{n-1} \alpha\ \lambda\ \beta^{\lambda} y^{\lambda-1} e^{-(\beta y)^{\lambda}} (1 - e^{-(\beta y)^{\lambda}})^{\alpha-1}$

$f(Y|\ N=n) = \frac{n\ \alpha\ \lambda\ \beta^{\lambda} y^{\lambda-1} e^{-(\beta y)^{\lambda}}}{1 - e^{-(\beta y)^{\lambda}}} \left[\left(1 - e^{-(\beta y)^{\lambda}}\right)^{\alpha}\right]^{n}$

$$f(Y|\ N=n) = \frac{n\ \alpha\ \lambda\ \beta^{\lambda} y^{\lambda-1} e^{-(\beta y)^{\lambda}}}{1 - e^{-(\beta y)^{\lambda}}} \left[1 - e^{-(\beta y)^{\lambda}}\right]^{n\alpha} \quad (3)$$

The probability function of the exponential Weibull – Poisson distribution can be found as

$f(y\ ;\ \alpha, \beta, \lambda, \theta) = \sum_{\forall N} f(Y|\ N = n)\ .f(N)$

$f(y;\ \alpha, \beta, \lambda, \theta\ ) = \sum_{N=1}^{\infty} \frac{n\ \alpha\ \lambda\ \beta^{\lambda} y^{\lambda-1} e^{-(\beta y)^{\lambda}}}{1 - e^{-(\beta y)^{\lambda}}} \left[1 - e^{-(\beta y)^{\lambda}}\right]^{n\alpha} . \frac{e^{-\theta} \theta^{n}}{n!} \left(1 - e^{-\theta}\right)^{-1}$

$$f(y;\ \alpha, \beta, \lambda, \theta\ ) = \frac{\alpha\ \theta\ \lambda\ \beta^{\lambda} y^{\lambda-1} e^{-(\beta y)^{\lambda}}}{e^{\theta} - 1} e^{-(\beta y)^{\lambda}} (1 - e^{-(\beta y)^{\lambda}})^{\alpha-1} e^{\theta(1-e^{-(\beta y)^{\lambda}})^{\alpha}} \quad (4)$$

where $\alpha,\ \beta,\ \lambda, \theta > 0$, $y > 0$

3-Methods Of Estimating The Parameters Of Compound Exponential Weibull – Poisson Distribution

3-1Maximum Likelihood Method

The Likelihood function of the compound Exponential Weibull –Poisson Distribution
can be expressed as follows by Mahmoudi & Sepahdar, (2013)

$L\ (y_1, y_2, \ldots, y_n;\ \alpha,\ \beta,\ \lambda, \theta) = (\alpha\ \theta\ \lambda\ \beta^{\lambda})^{n} (e^{\theta} - 1)^{-n} \prod_{i=1}^{n} y_i^{\lambda-1}$.

$e^{-\sum_{i=1}^{n} (\beta y_i)^{\lambda}} \prod_{i=1}^{n} (1 - e^{-(\beta y_i)^{\lambda}})^{\alpha-1} e^{\theta \sum_{i=1}^{n} (1-e^{-(\beta y_i)^{\lambda}})^{\alpha}}$

Since the behavior of the Likelihood function is approximated to the behavior of the logarithmic Likelihood function so the logarithm will be taken to the ends of the formula above so that the formula of the logarithmic function of the exponential Weibull –Poisson distribution will be as follows

$$\text{Log } L\ (y_1, y_2, \ldots, y_n;\ \alpha,\ \beta,\ \lambda, \theta) = n\ [\ \text{Log } \alpha + \text{Log } \lambda + \text{Log } \theta + \lambda \text{ Log } \beta\ ] - n\ \text{Log}\ (e^{\theta} - 1) + (\lambda-1) \sum_{i=1}^{n} \text{Log}\ (y_i) - \sum_{i=1}^{n} (\beta y_i)^{\lambda} + (\alpha-1) \sum_{i=1}^{n} \text{Log}(1 - e^{-(\beta y_i)^{\lambda}}) + \theta \sum_{i=1}^{n} (1 - e^{-(\beta y_i)^{\lambda}})^{\alpha} \quad (5)$$

By derivation of the logarithmic function as above for the four distribution parameters α, β, λ, θ, we obtain:

$\frac{\partial \text{Log } L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \text{Log}(1 - e^{-(\beta y_i)^{\lambda}}) + \theta \sum_{i=1}^{n} (1 - e^{-(\beta y_i)^{\lambda}})^{\alpha} \text{ Log } (1 - e^{-(\beta y_i)^{\lambda}})$

$$\frac{\partial \text{Log L}}{\partial \beta} = \frac{n\lambda}{\beta} - \lambda\, \beta^{\lambda-1} \sum_{i=1}^{n} y_i^{\lambda} + (\alpha\text{-}1)\, \lambda\, \beta^{\lambda-1} \sum_{i=1}^{n} \frac{y_i^{\lambda}\, e^{-(\beta y_i)^{\lambda}}}{1-e^{-(\beta y_i)^{\lambda}}}$$
$$+ \alpha\, \theta\, \lambda\, \beta^{\lambda-1} \sum_{i=1}^{n} \left(1-e^{-(\beta y_i)^{\lambda}}\right)^{\alpha-1} e^{-(\beta y_i)^{\lambda}}\, y_i^{\lambda}$$
$$\frac{\partial \text{Log L}}{\partial \lambda} = \frac{n}{\lambda} + n \text{ Log } \beta + \sum_{i=1}^{n} \text{Log}\,(y_i) - \sum_{i=1}^{n} (\beta y_i)^{\lambda}\, \text{Log}\,(\beta y_i) + \qquad (\alpha\text{-}1)$$
$$\sum_{i=1}^{n} \frac{e^{-(\beta y_i)^{\lambda}} (\beta y_i)^{\lambda}\, \text{Log}\,(\beta y_i)}{1-e^{-(\beta y_i)^{\lambda}}} + \alpha\theta \sum_{i=1}^{n} \left(1-e^{-(\beta y_i)^{\lambda}}\right)^{\alpha-1} e^{-(\beta y_i)^{\lambda}} (\beta y_i)^{\lambda}\, \text{Log}\,(\beta y_i)$$
$$\frac{\partial \text{Log L}}{\partial \theta} = \frac{n}{\theta} - \frac{n}{1-e^{-\theta}} + \sum_{i=1}^{n} \left(1-e^{-(\beta y_i)^{\lambda}}\right)^{\alpha}$$

Since the equations above represent nonlinear equations, it is not possible to obtain a closed form of the parameters estimators Therefore, the expectation maximization algorithm will be used because it considered as a numerical technique used in the case of missing values in the sample data under study. Thisalgorithm was first used by Dempster et al. This iterative method replaces the missing values in the sample data with the conditional expectation of the missing values by giving observed values and initial values of the distribution parameters. Implementation of this algorithm involves two-steps.The first step represent the expectation step in which the conditional expectation of the missing data is calculated by giving the observed values the and the initial values of the distribution parameters. The second step: represent the maximization step in which each value of the missing values is replaced by the conditional expectation calculated in the first step and each time the parameters are updated.

For the application of the maximization algorithm,we assume that there are missing values in the sample data and that $Z_1, \dots, Z_2, Z_n$And the observed values of the sample data are represented by $y_1, y_2, \dots, y_n$and therefore, the complete data represent the observed values with the missing values and let it be $Y_1, Y_2, \dots, Y_n$The theoretical distribution of the complete data can be formulatedby Mahmoudi & Sepahdar, (2013):

$$f(y, z; \alpha, \beta, \lambda, \theta) = f(y \mid z) \,.\, f(z)$$
$$f(y, z; \alpha, \beta, \lambda, \theta) = \frac{z\,\alpha\,\lambda\,\beta^{\lambda} y^{\lambda-1} e^{-(\beta y)^{\lambda}}}{1-e^{-(\beta y)^{\lambda}}} \left[1-e^{-(\beta y)^{\lambda}}\right]^{z\alpha} . \frac{e^{-\theta}\,\theta^{z}}{z!} \left(1-e^{-\theta}\right)^{-1}$$
$$f(y, z; \alpha, \beta, \lambda, \theta) = \frac{\theta^{z} z\,\alpha\,\lambda\,\beta^{\lambda} y^{\lambda-1} e^{-(\beta y)^{\lambda}}}{z!\,(e^{\theta}-1)} \left[1-e^{-(\beta y)^{\lambda}}\right]^{z\alpha-1} \qquad (6)$$

The conditional expectation which is necessary for the first step of the expectation maximization algorithm is to be found after finding the conditional function to z by giving y and the initial values of the parameters

$$f(z \mid y) = \frac{f(y, z; \alpha, \beta, \lambda, \theta)}{f(y; \alpha, \beta, \lambda, \theta)}$$

$$f(z \mid y) = \frac{\left[\theta(1-e^{-(\beta y_i)^{\lambda}})^{\alpha}\right]^{z-1} e^{-\theta(1-e^{-(\beta y_i)^{\lambda}})^{\alpha}}}{(z-1)!} \quad (7)$$

$$E(z \mid y) = \sum_{i=1}^{n} z \; f(z \mid y)$$

$$E(z \mid y) = 1+ \theta(1-e^{-(\beta y_i)^{\lambda}})^{\alpha} \quad (8)$$

After finding the conditional expectation, the second step of the algorithm is to be applied by using the maximum potential estimates for the four parameters of the distribution, replacing the missing values with the conditional expectation. Loglikelihood function of the complete data Yi can be formulated as follows

$$\text{Log } L^{*}(y_1,\dots,y_n,\ z_1,\dots,z;\alpha,\ \beta,\ \lambda,\theta) \propto \sum_{i=1}^{n} z_i \text{Log } \theta - n \text{ Log}(e^{\theta}-1) + n \text{ Log } \alpha + n \text{ Log } \lambda + n\lambda \text{ Log } \beta + (\lambda-1)\sum_{i=1}^{n} \text{Log }(y_i) - \sum_{i=1}^{n}(\beta y_i)^{\lambda} + \sum_{i=1}^{n}(z_i\alpha-1)\text{Log}(1-e^{-(\beta y_i)^{\lambda}}) \quad (9)$$

By deriving the formula (9) above for the four distribution parameters α, β, λ, θ we obtain the following formulas

$$\frac{\partial \text{Log } L^{*}}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} z_i \text{ Log }(1-e^{-(\beta y_i)^{\lambda}}) \quad (10)$$

$$\frac{\partial \text{Log } L^{*}}{\partial \beta} = \frac{n\lambda}{\beta} - \lambda\,\beta^{\lambda-1}\sum_{i=1}^{n} y_i^{\lambda} + \lambda\,\beta^{\lambda-1}\sum_{i=1}^{n}\frac{(z_i\alpha-1)\,y_i^{\lambda}\,e^{-(\beta y_i)^{\lambda}}}{1-e^{-(\beta y_i)^{\lambda}}} \quad (11)$$

$$\frac{\partial \text{Log } L^{*}}{\partial \lambda} = \frac{n}{\lambda} + n \text{ Log } \beta + \sum_{i=1}^{n}\text{Log }(y_i) - \sum_{i=1}^{n}(\beta y_i)^{\lambda}\text{ Log }(\beta y_i) + \sum_{i=1}^{n}\frac{(z_i\alpha-1)\,e^{-(\beta y_i)^{\lambda}}(\beta y_i)^{\lambda}\text{ Log}(\beta y_i)}{1-e^{-(\beta y_i)^{\lambda}}} \quad (12)$$

$$\frac{\partial \text{Log } L^{*}}{\partial \theta} = \frac{\sum_{i=1}^{n} z_i}{\theta} - \frac{n}{1-e^{-\theta}} \quad (13)$$

By equaling equations (10)-(13) to zero we get the following formulas, which solved numerically,we could obtain parameters values

$$\hat{\alpha}^{(t+1)} = \frac{-n}{\sum_{i=1}^{n} z_i^{(t)} \text{ Log }\left(1-e^{-(\hat{\beta}^{(t)} y_i)^{\hat{\lambda}^{(t)}}}\right)} \quad (14)$$

$$\frac{n\hat{\lambda}^{(t)}}{\hat{\beta}^{(t+1)}} - \hat{\lambda}^{(t)}\hat{\beta}^{(t+1)\,\hat{\lambda}^{(t)}-1}\sum_{i=1}^{n} y_i^{\hat{\lambda}^{(t)}} + \hat{\lambda}^{(t)}\hat{\beta}^{(t+1)\,\hat{\lambda}^{(t)}-1}\sum_{i=1}^{n}\frac{(z_i^{(t)}\hat{\alpha}-1)\,y_i^{\hat{\lambda}^{(t)}}\,e^{-(\hat{\beta}^{(t+1)}y_i)^{\hat{\lambda}^{(t)}}}}{1-e^{-(\hat{\beta}^{(t+1)}y_i)^{\hat{\lambda}^{(t)}}}} = 0 \quad (15)$$

$$\frac{n}{\hat{\lambda}^{(t+1)}} + n \text{ Log } \hat{\beta}^{(t)} + \sum_{i=1}^{n}\text{Log }(y_i) - \sum_{i=1}^{n}(\hat{\beta}^{(t)} y_i)^{\hat{\lambda}^{(t+1)}}\text{ Log }(\hat{\beta}^{(t)} y_i) + \sum_{i=1}^{n}\frac{(z_i^{(t)}\hat{\alpha}^{(t)}-1)\,e^{-(\hat{\beta}^{(t)}y_i)^{\hat{\lambda}^{(t+1)}}}(\hat{\beta}^{(t)}y_i)^{\hat{\lambda}^{(t+1)}}\text{ Log}(\hat{\beta}^{(t)}y_i)}{1-e^{-(\hat{\beta}^{(t)}y_i)^{\hat{\lambda}^{(t+1)}}}} \quad (16)$$

$$\hat{\theta}^{(t+1)} = \frac{\sum_{i=1}^{n} z_i^{(t)}}{\hat{\theta}^{(t+1)}} - \frac{n}{1-e^{-\hat{\theta}^{(t+1)}}} \quad (17)$$

Where $z_i^{(t)} = 1 + \hat{\theta}^{(t)}\left(1 - e^{-\left(\hat{\beta}^{(t)} y_i\right)^{\hat{\lambda}^{(t)}}}\right)^{\hat{\alpha}^{(t)}}$

<u>3-2 Downhill SimplexAlgorithm</u>

It is a numerical method used to reduce the objective function (mathematical problem under study) in a multi-dimensional space , This numerical technique was first used by John Nelder & Roger Mead, 1965, This method applies to nonlinear optimization problems when we cannot find its derivative, This algorithm is based on comparing the values of the function to (n + 1) of the points (the points of the multi-dimensional geometry) Then the point that has the highest value in the objective function is replaced by another point and the geometric shape changes from one state to another until it reaches the optimal form (i.e.the values that make the objective function as low as possible) anddepending on the number of operations, the initial geometry is converted in each process to a different shape until it reaches the optimum shape These include Reflection, Expansion, Contraction, Shrinkage,This technique was used in estimating the Hussian matrix in the lowest neighborhood required for statistical estimation problems introduced byGao, F. & Han, L., (2012).

The following are the steps for estimating compound exponential weibull – poisson distribution parameters using the Downhill Simplex algorithm Gao, F. & Han, L., (2012).

1- **Determine the objective function of the algorithm [The mathematical formula that cannot be solved] and it's type and here,it represent chi-square formula that we aim to make it as low as possible.**

2- **Enter the values of the four parameters of the algorithm (σ: reflection parameter), (γ: expansion parameter) , (ρ: contraction parameter), (τ: shrink parameter), where the parameter defined as follows**

$$0 < \tau < 1 \ , \ 0 < \rho < 1 \ , \ \gamma > 1 \ , \ \sigma > 0$$

In most of the researches on this algorithm, each value of the algorithm parameter values was equal to σ = 1 ، γ = 2 ، ρ = 0.5 ، τ = 0.5and these are the values adopted when applying the algorithm

3- **Generate a matrix consisting of (n + 1) of the test points for each variable or parameter in the function and this matrix is called initial solutions matrix, and it has the initial dimension [m × (n+1)]**

Where m: represents the number of columns and the number of parameters in the distribution

n+1: The number of test values for each parameter

$$W = \begin{vmatrix} \alpha_1 & \beta_1 & \lambda_1 & \theta_1 \\ \alpha_2 & \beta_2 & \lambda_2 & \theta_2 \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{n+1} & \beta_{n+1} & \lambda_{n+1} & \theta_{n+1} \end{vmatrix}$$

4- **The values of each row of the above matrix are offset in the objective function and its value calculation The resulting objective functions (n + 1) are then put in order from the lowest to the highest value where the least value is the best solution and the highest value is the worst solution.**

5- **Find the mean of the solutions matrix using the following formula**

$$x_m = \frac{1}{n+1} \sum_{i=1}^{n+1} w\,(i) \quad (18)$$

6-Create a new testpoint called Reflection point (x_r) and this can be found according to the following formula

$$x_r = x_m + \sigma\,(x_m - x_{n+1}) \quad (19)$$

And then we calculate the objective function for this point (the f (x_r)). if f $_1$<f x_r<f$_{n+1}$ i.e. the point of reflection lies between the best point and the worst point, then the worst point will be replaced with the point x_r, i.e. We make $x_{n+1} = x_r$

And If f x_r<f $x_1$this will mean that the reflection point is better than the best point, then move on to the next step.

7-Creating a new test point which is called Expansion Point: x_e calculated as follows:

$$x_e = x_m + \gamma\,(x_r - x_m) \quad (20)$$

Then find the objective function for the expansion point f (x_e) if it was f (x_e) <f (x_r) (i.e. the expansion point is better than the best point),then replace the worst point with the expansion point $xn_{+1} = x_e$, If f (x_r) <f (x_{n+1}<f (x_n)) then move on to the next step.

8- Create a new test point of contract (Contraction Point:x_c), which can befound in two cases

a) If f (x_n) <f (x_r) <f (x_{n+1}) then the following formula will be used to find the outside contraction point (x_{OC})

$$x_{oc} = x_m + \rho\,(x_r - x_m) \quad (21)$$

Then the function of the contraction point above f (x_{oc}) can be found. If f (x_{oc}) ≤ f (x_r) we replace the worst point (x_{n+1}) with the outside contraction point (x_{oc}) otherwise move to step 9.

b) If f (x_r) ≥ f (x_{n+1}) [means that the reflection point is worse than the worst point]

The inside contraction (x_{ic}) will then be found in the following formula

$$x_{ic} = x_m - \rho\,(x_m - x_{n+1}) \quad (22)$$

The function is then calculated to the inside contraction point f (x_{ic}). If f (x_{ic}) < f (x_{n+1}), then replaces the worst point x_{n+1} with the insidex_{ic} contraction point, and if not then move to step 11.

9- Create a new test point that represent the (Shrink Point: x_{sh}) at n points and this can be calculated by the following formula

$$x_{sh} = x_i + \tau (x_i - x_1) \qquad (23)$$

10-When a stop condition is met, it will stop and the best solution is printed. This condition is achieved when the minimum value of the objective function is reached

$$\left|\frac{\max(f) - \min(f)}{\max(f)}\right|$$

And that ∈ is a very small number and when this condition is met we move on to step 11

11- Print the optimal solution that makes the objective function as low as possible (i.e. the values of the parameters that achieved the least value for the objective function)

12- Stop algorithm work.

4-Stages of the simulation experiment and the results of its implementation

The stages of the simulation experience can be summarized in the following points

1- Generation of random variable that follow the uniformdistribution U (0,1)

2- Determination of sample sizes and the values of the default parameters in addition to the percentage of contamination used and as follow:

n = 100,150,200,250

$\alpha_1 = 0.5$, $\beta_1 = 1$, $\lambda_1 = 1.2$, $\theta_1 = 0.5$ $\alpha_2 = 0.2$, $\beta_2 = 0.8$, $\lambda_2 = 1.1$, $\theta_2 = 0.3$

$\tau = 0\%$, 10% , 20%

3- GenerateContaminated compound exponential weibull –poisson data according to the following formula

$$x_U = [(1-\tau)\frac{1}{\beta_1}[-\text{Log}\left(1-(\frac{1}{\theta_1}\text{Log}\left(U\left(e^{\theta_1}-1\right)+1\right)^{\frac{1}{\alpha_1}}\right)]]^{\frac{1}{\lambda_1}} + [\tau\frac{1}{\beta_2}[-\text{Log}\left(1-(\frac{1}{\theta_2}\text{Log}\left(U\left(e^{\theta_2}-1\right)+1\right)^{\frac{1}{\alpha_2}}\right)]]^{\frac{1}{\lambda_2}} \qquad (24)$$

4- Estimation of the four distribution parameters using the maximum likelihood method and Downhill Simplex algorithm

5- To determine the priority between parameter estimation methods, probability function and reliability function, two comparison criteria (mean error squares) (MSE) and mean absolute percentage error (MAPE) had been used as shown in the following formulas

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(e_i)^2 \qquad (25)$$

$$e_i = \text{obs}(i) - \text{pre}(i)$$

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\text{obs}(i) - \text{pre}(i)}{\text{obs}(i)}\right| . 100\% \qquad (26)$$

Table (1):Shows estimators of the parameters, the probability function and the reliability function of the estimation methods when contamination ratio (0%) and for all selected sample sizes

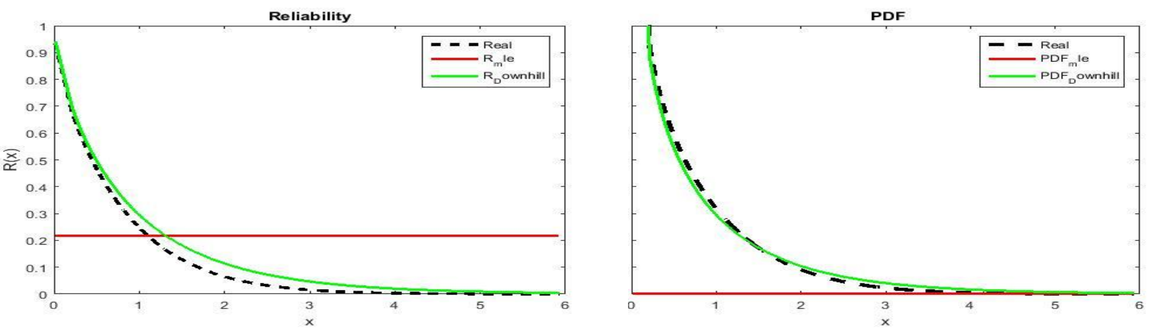
n	Method		α = 0.5	β = 1	θ = 0.5	γ = 1.2	pdf	R
100	MLE	Est.	0.429478	0.928034	0.456240	1.12334		
		MSE	0.16812	0.321079	0.4337741	0.00134	0.253324	0.267968
		MAPE	76.1044	47.19664	4157.249	100	100	3153.2
	D.S	Est.	0.505029	1.001008	0.552988	1.048137		
		MSE	0.199278	0.693469	0.002819	0.000405	0.005991	0.001345
		MAPE	84.81723	71.2124	10.59763	39.1167	92.2545	478.269
	Best		mle	mle	D.S	D.S	D.S	D.S
150	MLE	Est.	0.499228	0.951075	0.491843	1.19223		
		MSE	0.113377	0.315551	0.012400	0.00044	0.20349	0.228253
		MAPE	70.15444	46.92496	4013.685	98.543	96	3080.1
	D.S	Est.	0.480411	1.0892964	0.553175	1.1953777		
		MSE	0.127099	0.523233	0.002734	0.0002246	0.000612	0.00126
		MAPE	77.6232	64.25563	10.53505	35.85885	89.4516	438.878
	Best		mle	mle	D.S	D.S	D.S	D.S
200	MLE	Est.	0.503552	1.00055	0.503264	1.20054		
		MSE	0.100511	0.306125	0.012100	0.00031	0.201564	0.218359
		MAPE	61.28953	36.16347	3566.528	95.526	85	3001.6
	D.S	Est.	0.500969	1.000667	0.555197	1.200024		
		MSE	0.111116	0.057307	0.002052	0.00021206	0.000302	0.00113
		MAPE	69.823	62.76613	10.03942	32.66526	79.4382	317.539
	Best		mle	mle	D.S	D.S	D.S	D.S
250	MLE	Est.	0.500041	1.00045	0.502153	1.20045		
		MSE	0.100411	0.300014	0.011345	0.00021	0.101453	0.107248
		MAPE	55.17841	30.05236	2423.528	90.415	80.1090	279.040
	D.S	Est.	0.500969	1.000046	0.423100	1.200011		
		MSE	0.100104	0.046200	0.002041	0.0001010	0.000103	0.00024
		MAPE	49.411	20.65502	9.02831	30.55415	70.3271	306.428
	Best		D.S	D.S	D.S	D.S	D.S	D.S

Table (2):Shows estimators of the parameters, the probability function and the reliability function of the estimation methods when contamination ratio (10%) and for all selected sample sizes

n	Method		α = 0.5	β = 1	θ = 0.5	γ = 1.2	pdf	R
100	MLE	Est.	0. 48684	0.983308	0.4826	1..94533		
		MSE	0.157036	0.309144	0.3157559	0.4300	0.222496	0.189078
		MAPE	73.06327	41.66923	439.52	98.001	95.00556	842.41
	D.S	Est.	0.513406	1.004121	0.545157	1.955806		
		MSE	0.296405	0.318883	0.002057	0.188209	0.002279	0.00125
		MAPE	90.73951	62.53341	9.031341	31.23352	13.63218	40.56515
	Best		mle	mle	D.S	D.S	D.S	D.S
150	MLE	Est.	0.506869	1.003211	0.49427	1.957866		
		MSE	0.131282	0.214168	0.22687	0.40110	0.210244	0.151448
		MAPE	64.62628	33.02082	431.853	90.2533	90.78665	807.3744
	D.S	Est.	0.506816	1.00051	0.504572	1.965831		
		MSE	0.225405	0.286154	0.001994	0.159812	0.001591	0.001216
		MAPE	73.63203	49.10506	8.91432	31.18076	12.13143	30.30231
	Best		mle	mle	D.S	D.S	D.S	D.S
200	MLE	Est.	0.50021	1.00011	0.500431	1.987210		
		MSE	0.120171	0.193057	0.20576	0.30001	0.155653	0.141402
		MAPE	53.51517	22.01071	320.741	80.1422	82.2344	671.743
	D.S	Est.	0.505705	1.00040	0.503461	1.974720		
		MSE	0.214304	0.275043	0.001882	0.148701	0.001480	0.001105
		MAPE	62.52102	38.10405	7.80321	21.070656	10.02032	20.20120
	Best		mle	mle	D.S	D.S	D.S	D.S
250	MLE	Est.	0.50011	1.00002	0.500320	1.998311		
		MSE	0.110160	0.182046	0.19465	0.20001	0.134542	0.131301
		MAPE	42.41416	21.00060	210.630	70.0311	71.1233	560.632
	D.S	Est.	0.504604	1.00021	0.502350	1.985830		
		MSE	0.203203	0.254032	0.001771	0.137601	0.001370	0.001004
		MAPE	51.41001	27.10304	7.70211	20.060554	10.01021	20.00110
	Best		mle	mle	D.S	D.S	D.S	D.S

Table (3):Shows estimators of the parameters, the probability function and the reliability function of the estimation methods when contamination ratio (20%) and for all selected sample sizes

n	Method		α = 0.5	β = 1	θ = 0.5	γ = 1.2	pdf	R
100	MLE	Est.	0.443043	0.989715	0.49178	1.18989		
		MSE	0.148553	0.205452	0.10848	0.20033	0.430603	0.150703
		MAPE	67.86815	31.00610	43.02800	87.4023	97.0122	237.4437
	D.S	Est.	0.491422	1.099163	0.502312	1.197968		
		MSE	0.201042	0.376682	0.001125	0.01144	0.002100	0.001642
		MAPE	68.82028	32.71546	6.575758	20.00872	9.581124	21.5304
	Best		mle	mle	D.S	D.S	D.S	D.S
150	MLE	Est.	0.474154	0.999827	0.49989	1.19979		
		MSE	0.137442	0.194341	0.01736	0.11021	0.320501	0.130501
		MAPE	32.75602	20.00420	21.01600	56.3012	86.0011	125.2225
	D.S	Est.	0.502311	1.009274	0.503423	1.199976		
		MSE	0.190031	0.244461	0.000112	0.00122	0.001244	0.001431
		MAPE	57.71017	21.60433	6.453634	20.00051	9.320013	20.3202
	Best		mle	mle	D.S	D.S	D.S	D.S
200	MLE	Est.	0.501100	1.00412	0.50122	1.20012		
		MSE	0.114331	0.103320	0.01024	0.10023	0.120300	0.010300
		MAPE	23.53101	19.00312	20.02311	45.1011	73.00230	112.3455
	D.S	Est.	0.501200	1.001052	0.500212	1.20006		
		MSE	0.170021	0.200301	0.000011	0.00111	0.000356	0.000211
		MAPE	43.51012	20.10211	6.1324310	19.00023	9.110010	20.0103
	Best		mle	mle	D.S	D.S	D.S	D.S
250	MLE	Est.	0.500211	1.003010	0.50011	1.20001		
		MSE	0.1013221	0.102201	0.00113	0.10011	0.100200	0.010022
		MAPE	23.32001	18.00201	19.01211	34.0022	63.00120	110.2322
	D.S	Est.	0.50001	1.001041	0.500011	1.20002		
		MSE	0.140021	0.190201	0.000011	0.00022	0.000245	0.000100
		MAPE	33.40011	19.00322	5.021331	17.00011	9.010010	19.0002
	Best		mle	mle	D.S	D.S	D.S	D.S

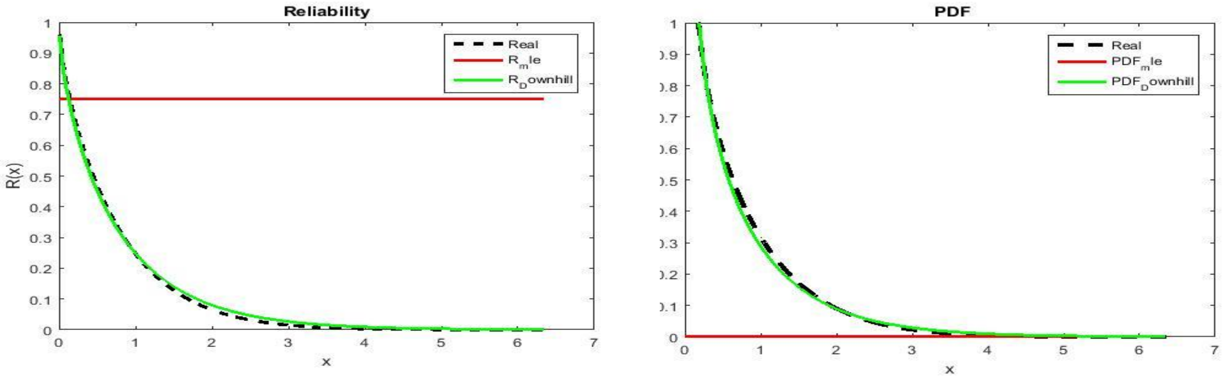


$\tau = 0\%$ $\tau = 0\%$

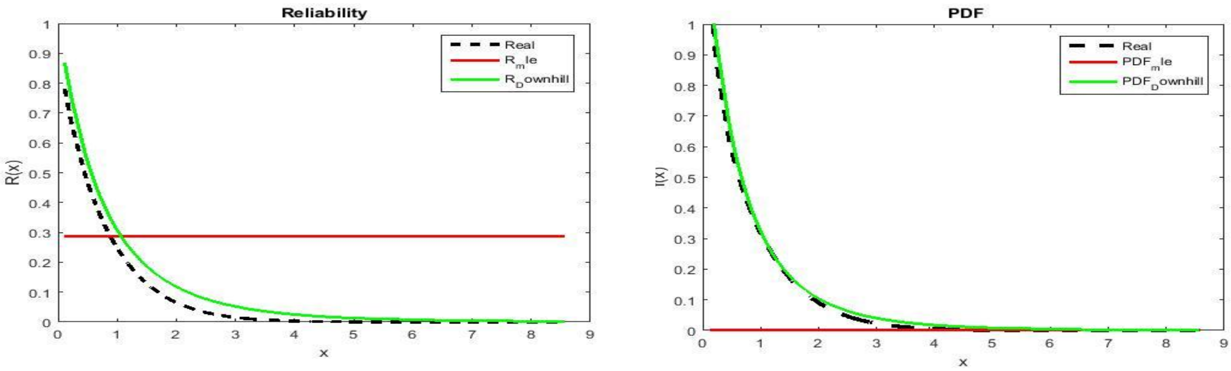
$\tau = 10\%$ $\tau = 10\%$

$\tau = 20\%$ $\tau = 20\%$

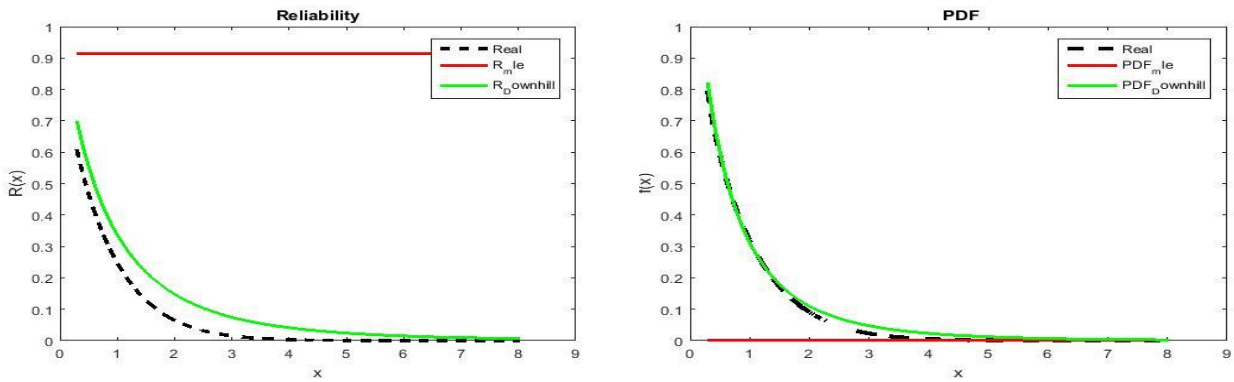
Figure (1): Shows the plot of probability function and reliability functionfor the two estimation methods at sample size 100 and for all contamination ratios



τ = 0%0% τ =

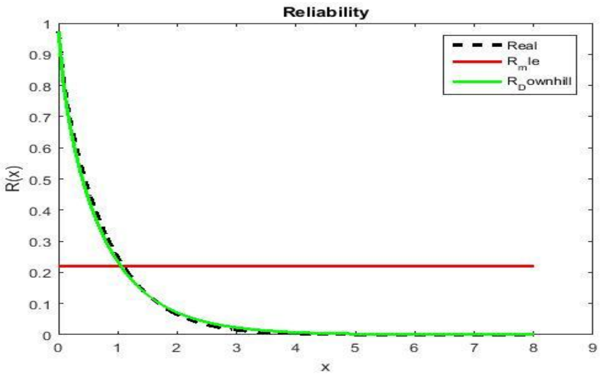


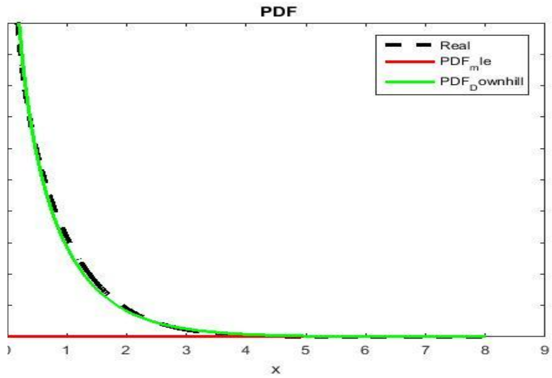
τ = 10%10%τ =



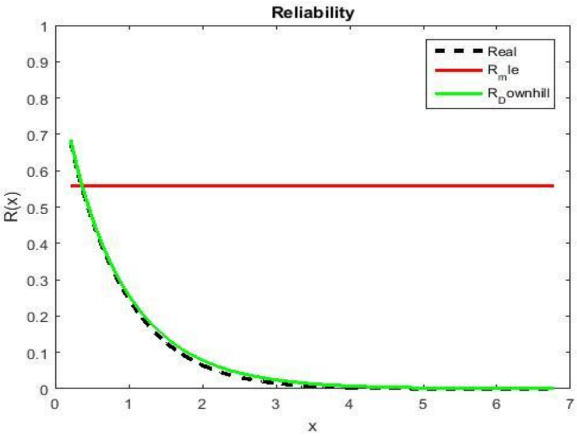
τ = 20%20%τ =

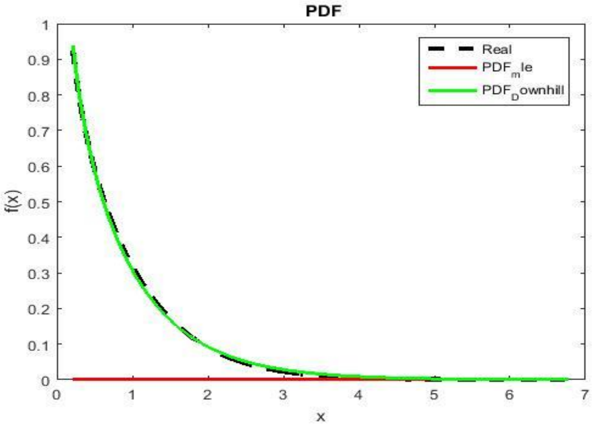
Figure (2): Shows the plot of probability function and reliability functionfor the two estimation methods at sample size 150 and for all contamination ratios



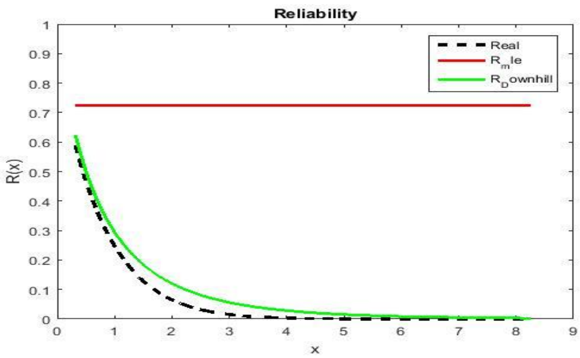


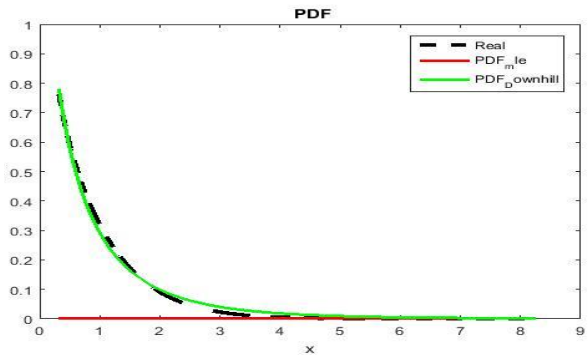
τ = 0% 0% τ =





τ = 10% τ =10%





τ = 20% τ =20%

Figure (3): Shows the plot of probability function and reliability functionfor the two estimation methods at sample size 200 and for all contamination ratios

τ = 0% 0%τ =

τ = 10% τ = 10%

τ = 20% τ =20%

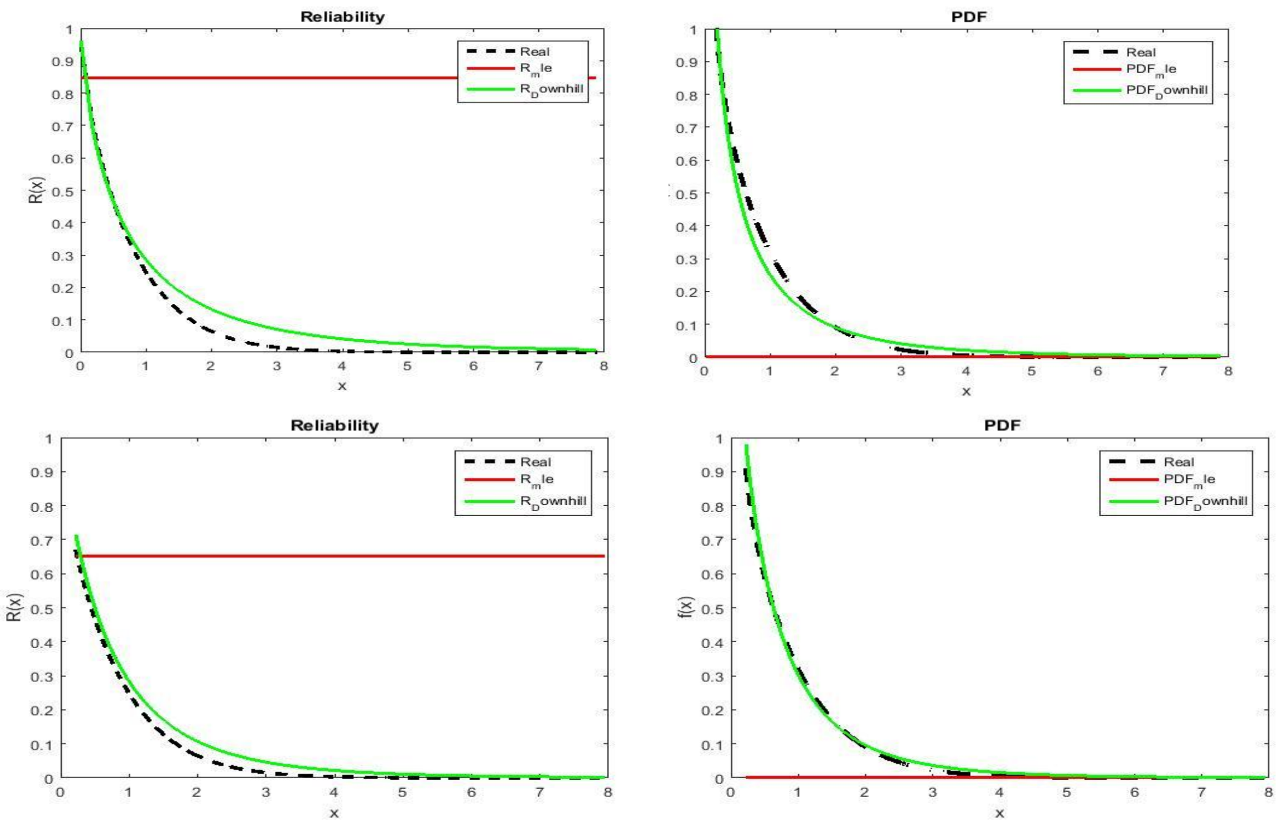


Figure (4): Shows the plot of probability function and reliability functionfor the two estimation methods at sample size 250 and for all contamination ratios

5-Analysing the results of simulation experimant

According to the results shown in Table (1) - Table (4) and Figure (1) - Figure (4) it is clear that:

1- The Downhill Simplex algorithm is superior to the maximum likelihood method for estimating the parameters λ, θ in addition its superiority in the probability function and the reliability function of the compound exponential Weibull–Poissondistribution according to two comparison criterion: mean error squares (MSE) and mean absolute percentage error (MAPE).

2- By increasing the sample size, we notice a decrease in the comparison criterion (MSE) and (MAPE) for all sample sizes and different contamination ratioswhich isconsistent with the statistical theory.

6-conclutions

Although the Downhill Simplex algorithm is an engineering algorithm and it is not one of the robust methodsthat often used to estimate parameters of models whose data contain pollutant or abnormal values in their observations However, it proved to be superior to the normal maximum likelihood method by using the EM algorithm in estimating the parameters, the probability function, and the reliability function of the compound exponential Weibull–Poissondistribution in the cases of presence and absence of the contaminated data, Accordingly, we can depend on this algorithm in the estimation other models experiencing the problem of extreme values in the sample data.

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مقدر حصين جديد لتوزيع بواسون- ويبل المركب في حالة البيانات الملوثة والغير ملوثة

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المستخلص

تناول البحث مقارنة طريقتين لتقدير معلمات توزيع بواسون–ويبل الاسي المركب ذو الاربع معلمات هي طريقة الامكان الاعظم الاعتيادية وخوارزمية Downhill Simplex وذلك بالاعتماد على حالتين للبيانات الاولى تفترض حالة البيانات الطبيعية (غير الملوثة) والثانية تفترض حالة تلوث البيانات. ومن خلال تنفيذ تجربة المحاكاة لحجوم عينات مختلفة وقيم اولية للمعلمات وتحت مستويات مختلفة للتلويث تبين ان خوارزمية Downhill Simplexكانت هي الافضل في تقدير المعلمات والدالة الاحتمالية ودالة المعولية للتوزيع المركب في حالتي البيانات الطبيعية والملوثة.

المصطلحات الرئيسة للبحث/ التوزيعات المركبة، توزيع ويبل الاسي، توزيع بواسون–ويبل الاسي، طريقة الامكان الاعظم، خوارزمية EM، خوارزمية Downhill Simplex، تلوث البيانات.