# Root systems of the complex reflection arrangements $(\mathcal{A}(G_{24})\text{and}\,\mathcal{A}(G_{27}))$

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## ABSTRACT

The purpose of this paper is to study root systems of complex reflection arrangements $\mathcal{A}(G_{24})$ *and* $\mathcal{A}(G_{27})$.Where we found coxeter graph and coxtere matrix through the derivation of the angles between the simple roots of the complex reflection groups $G_{24}$ *and* $G_{27}$

## 1. Introduction

We review necessary definitions, notations and theorems of root systems. Bourbaki [2] discussed the relationship between the reflection and the root system, and we discussed the same relationship with more explicate and clear way. In [1] Rabeaa found the lattices of $\mathcal{A}(G_{24})$ *and* $\mathcal{A}(G_{27})$. In this paper we construct the simple root systems of $G_{24}$ and $G_{27}$ depending on the defining polynomial of the complex reflection arrangements of $G_{24}$ and $G_{27}$ also we find the angle between simple root system by using matlab and then we find coxeter matrix, coxeter graph. Throughout this paper $V$ is a finite dimensional complex vector space. A hyperplane H in $V$ is an affine subspace of dimension $\ell$-1.A hyperplane arrangement $\mathcal{A} = (\mathcal{A}, V)$ is a finite set of hyperplanes in $V$. The arrangement is called centerless if $\cap_{H\in\mathcal{A}} H = \emptyset$, centered with center $T(\mathcal{A})$ if $T(\mathcal{A}) = \cap_{H\in\mathcal{A}} H = \emptyset$ If $\mathcal{A}$ is centered, then coordinate may be chosen so that hyperplane contains the origin and hence $\mathcal{A}$ is called central. A projective arrangement is a finite set of projective hyperplane in projective space.

The product $Q(\mathcal{A})=\prod_{H\in\mathcal{A}} \alpha_H$ $(\textit{where } \alpha_H \textit{ is a linear form and } H = \ker \alpha_H)$ is called a defining polynomial of $\mathcal{A}$.we agree that $Q(\emptyset_\ell) = 1$ is the defining polynomial of $\emptyset_\ell$, where $\emptyset_\ell$ is empty l-arrangement. A reflection on $V$ is a linear transformation on $V$ of finite order with exactly n-1 eigenvalues equal to 1. A reflection group G on $V$ is a finite group generated by reflections on $V$. The group G is reducible if it is a direct product of two proper reflection subgroups and irreducible otherwise. A finite subgroup $G$ of $O(V)$ is generated by a set of reflections $S$ will be called ***Coxeter group***. Take $\Phi$ to be a finite set of nonzero vectors in $V$ satisfying: $(R_1)\Phi$ $spans\, V, (R_2)\, s_\alpha \Phi = \Phi \;\forall \alpha \in \Phi$. Then defined W to be the group generated by all reflections $s_\alpha, \alpha \in \Phi$ .Call $\Phi$ a root system with associated reflection group W.A root system $\Phi$ is crystallographic if it satisfies the additional condition $\frac{2(\alpha,\beta)}{(\beta,\beta)} \in \mathbb{Z}$ for all $\alpha, \beta \in \Phi$ ; (see Humphreys[6] ).

**Definition** (1.1)**:** [7, 9]

A finite simple graph $\Gamma$= (A, B) is an ordered pair consisting of the set A of vertices and the set B of edges with the following two conditions:

1- A is a finite set
2- B is a collection of 2-element subset of A

The mapping $\Psi : B \to A \times A$ is called an incidence map which maps an edge into a pair of vertices called end-vertices of the edge.

**Definition** (1.2)**:** [2]

Let $a_{ij} = <\alpha_i, \alpha_j>$. The matrix $(a_{ij})$ is called the ***cartan matrix*** of its root system $\Phi$.

A generalized cartan matrix is a square matrix M= $(a_{ij})$ such that:

1- For diagonal entries, $a_{ij}$=0. 2- For non-diagonal entries, $a_{ij} \leq 0$.

3-$a_{ij}$=0 iff $a_{ji}$=0. Where $a_{ij} = 2\frac{<\alpha_i,\alpha_j>}{<\alpha_j,\alpha_j>}$ , $\alpha_i, \alpha_j$ are simple root

**Definition** (1.3)**:** [5]

A ***dynkin diagram*** for $\Phi$ which is denoted by $D_G$ is a graph has vertices $\{\alpha_1, \dots, \alpha_n\}$, between any two vertices; we place no edge, one edge, two edges or three edges obtained by the following two methods:

***First method***: $n\beta_\alpha\, n\alpha_\beta$

Two vertices connected by the $n\beta_\alpha\, n\alpha_\beta$ edges, an edge is drawn between each non- orthogonal pair of vectors such that:

1- An undirected one edge if they make an angle of $\frac{2\pi}{3}$ radians.

2- A directed two edge if they make an angle of $\frac{3\pi}{4}$ radians.

3- A directed three edge if they make an angle of $\frac{5\pi}{6}$ radians.

## ***Second method*: Cartan matrix**

The generalized cartan matrices are equivalent to dynking diagrams. A multi-edged diagram corresponds to the non-diagonal cartan matrix elements - $a_{12}$, - $a_{21}$, with the number of edges drawn equal to max ($-a_{21}$, - $a_{12}$), and an arrow pointing towards non unity elements.

**Definition (1.4): [2]**

(1) A ***coxeter system*** (W,S) is a pair consisting of group W together with a finite subset S⊂W satisfying the following conditions :

1- For s, $s'$ ∈S, let m(s, $s'$) be the order of s$s'$ and let A be the set of pairs $(s, s')$ such that m$(s, s')$ is finite.

2- The generating set S and the relations $(ss')^{m(s,s')} = 1$ for $(s, s')$ in A form a presentation of the group W.

(2) Let A be a set of vertices. A ***coxeter matrix*** of type A is a symmetric matrix M=$(m_{ij})_{ij\in A}$ whose entries are integers or +∞ and with 1's on the diagonal such that all non-diagonal entries are greater than 1

i.e. $m_{ii}$=1 for i∈S

$m_{ij}$≥2 for i, j ∈S with i≠ j

(3) A ***coxeter graph*** (Γ, *f*) is a pair consisting of graph Γ together with a map *f* from the set of edges of this graph to the set consisting of +∞ and the set of integers ≥3.

A coxeter graph is associated to any coxeter matrix N of type A as follows:

i. The graph Γ has set of vertices A and set of edges the set pairs {i, j} of elements of A.
ii. Vertices i and j are connected iff $m_{ij}$≥ 3.
iii. The map *f* associates to the edge {i, j} the corresponding element $m_{ij}$ of M.

**Proposition (1.5): [4]**

If $\alpha_i, \alpha_j \in \Delta$, then there is an integer $m_{ij}$≥ 1 such that

$(\alpha_i, \alpha_j)/\|\alpha_i\|\,\|\alpha_j\| = -\cos(\pi/m_{ij})$, $m_{ij}$ is the order $\alpha_i\alpha_j$.

## 2. The complex Reflection Group $G_{24}$:

The defining polynomial of $\mathcal{A}(G_{24})$ defined by

$$Q(\mathcal{A}(G_{24})) = x_1 x_2 x_3 \prod_{i,j,k=1,2,3} (x_1 \mp x_2)(\beta x_i \mp x_j \mp x_k).$$

**The hyperplane arrangement of** $G_{24}$ [1]

The hyperplanes of $\mathcal{A}(G_{24})$ where $Hi = \ker \alpha_{H_i}$ , $1\leq i\leq 21$ are:

| | | |
|---|---|---|
| $H_1$: $x_1=0$ | $H_2$: $x_2=0$ | $H_3$: $x_3=0$ |
| $H_4$: $x_1+x_2=0$ | $H_5$: $x_1+x_3=0$ | $H_6$: $x_2+x_3=0$ |
| $H_7$: $x_1-x_2=0$ | $H_8$: $x_1-x_3=0$ | $H_9$: $x_2-x_3=0$ |
| $H_{10}$: $\beta x_1+x_2+x_3=0$ | $H_{11}$: $\beta x_1-x_2+x_3=0$ | $H_{12}$: $\beta x_1+x_2-x_3=0$ |
| $H_{13}$: $\beta x_1-x_2-x_3=0$ | $H_{14}$: $\beta x_2+x_1+x_3=0$ | $H_{15}$: $\beta x_2-x_1-x_3=0$ |
| $H_{16}$: $\beta x_2-x_1+x_3=0$ | $H_{17}$: $\beta x_2+x_1-x_3=0$ | $H_{18}$: $\beta x_3+x_1+x_2=0$ |
| $H_{19}$: $\beta x_3-x_1-x_2=0$ | $H_{20}$: $\beta x_3-x_1+x_2=0$ | $H_{21}$: $\beta x_3+x_1-x_2=0$ |

Table (1) the hyperplanes of $\mathcal{A}(G_{24})$

Therefore the set $S_\Phi(G_{24})$ is the simple root system for the group $G_{24}$ i.e. $S_\Phi(G_{24})$ consists of:

| | |
|---|---|
| $e_1$=(1, 0, 0) | $e_2$=(0, 1, 0) |
| $e_3$=(0, 0, 1) | $e_4$=(1,1, 0) |
| $e_5$=(1, 0, 1) | $e_6$=(0, 1, 1) |
| $e_7$=(1, -1, 0) | $e_8$=(1,0, -1) |
| $e_9$=(0, 1, -1) | $e_{10}$=(0.5-1.3229i ,1, 1) |
| $e_{11}$=(0.5-1.3229i, -1, 1) | $e_{12}$=(0.5-1.3229i , 1, -1) |
| $e_{13}$=(0.5-1.3229i, -1, -1) | $e_{14}$=(1, 0.5-1.3229i , 1) |
| $e_{15}$=(-1, 0.5-1.3229i, -1) | $e_{16}$=(-1, 0.5-1.3229i, 1) |
| $e_{17}$=(1, 0.5-1.3229i, -1) | $e_{18}$=(1, 1, 0.5-1.3229i) |
| $e_{19}$=(-1, -1, 0.5-1.3229i) | $e_{20}$=(-1, 1, 0.5-1.3229i) |
| $e_{21}$=(1, -1, 0.5-1.329i) | |

Table (2) the simple root of $G_{24}$

| | F1 | F2 | F3 | F4 | F5 | F6 | F7 | F8 | F9 | F10 | F11 | F12 | F13 | F14 | F15 | F16 | F17 | F18 | F19 | F20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F1 | | 90 | 90 | 45 | 45 | 90 | 45 | 45 | 90 | 75 | 75 | 75 | 75 | 60 | 120 | 120 | 60 | 60 | 120 | 120 |
| F2 | 90 | | 90 | 45 | 90 | 45 | 135 | 90 | 45 | 60 | 120 | 60 | 120 | 75 | 75 | 75 | 75 | 60 | 120 | 60 |
| F3 | 90 | 90 | | 90 | 45 | 45 | 90 | 135 | 135 | 60 | 60 | 120 | 120 | 60 | 120 | 60 | 120 | 75 | 75 | 75 |
| F4 | 45 | 45 | 90 | | 60 | 60 | 90 | 60 | 60 | 60 | 105 | 60 | 105 | 60 | 105 | 105 | 60 | 45 | 135 | 90 |
| F5 | 45 | 90 | 45 | 60 | | 60 | 60 | 90 | 120 | 60 | 60 | 105 | 105 | 45 | 135 | 90 | 90 | 60 | 105 | 105 |
| F6 | 90 | 45 | 45 | 60 | 60 | | 120 | 120 | 90 | 45 | 60 | 90 | 135 | 60 | 105 | 60 | 105 | 60 | 105 | 60 |
| F7 | 45 | 135 | 90 | 90 | 60 | 120 | | 60 | 120 | 105 | 60 | 105 | 60 | 75 | 120 | 120 | 75 | 90 | 90 | 135 |
| F8 | 45 | 90 | 135 | 60 | 90 | 120 | 60 | | 60 | 105 | 105 | 60 | 60 | 90 | 90 | 135 | 45 | 75 | 120 | 120 |
| F9 | 90 | 45 | 135 | 60 | 120 | 90 | 120 | 60 | | 90 | 135 | 45 | 90 | 105 | 60 | 105 | 60 | 75 | 120 | 75 |
| F10 | 75 | 60 | 60 | 60 | 60 | 45 | 105 | 105 | 90 | | 60 | 60 | 90 | 60 | 105 | 75 | 90 | 60 | 105 | 75 |
| F11 | 75 | 120 | 60 | 105 | 60 | 60 | 60 | 105 | 135 | 60 | | 90 | 60 | 75 | 120 | 90 | 105 | 90 | 75 | 105 |
| F12 | 75 | 60 | 120 | 60 | 105 | 90 | 105 | 60 | 45 | 60 | 90 | | 60 | 90 | 75 | 105 | 60 | 75 | 120 | 90 |
| F13 | 75 | 120 | 120 | 105 | 105 | 135 | 60 | 60 | 90 | 90 | 60 | 60 | | 105 | 90 | 120 | 75 | 105 | 90 | 120 |
| F14 | 60 | 75 | 60 | 60 | 45 | 60 | 75 | 90 | 105 | 60 | 75 | 90 | 105 | | 90 | 60 | 60 | 60 | 105 | 90 |
| F15 | 120 | 75 | 120 | 105 | 135 | 105 | 120 | 90 | 60 | 105 | 120 | 75 | 90 | 90 | | 60 | 60 | 105 | 90 | 75 |
| F16 | 120 | 75 | 60 | 105 | 90 | 60 | 120 | 135 | 105 | 75 | 90 | 105 | 120 | 60 | 60 | | 90 | 90 | 75 | 60 |
| F17 | 60 | 75 | 120 | 60 | 90 | 105 | 75 | 45 | 60 | 90 | 105 | 60 | 75 | 60 | 60 | 90 | | 75 | 120 | 105 |
| F18 | 60 | 60 | 75 | 45 | 60 | 60 | 90 | 75 | 75 | 60 | 90 | 75 | 105 | 60 | 105 | 90 | 75 | | 90 | 60 |
| F19 | 120 | 120 | 75 | 135 | 105 | 105 | 90 | 120 | 120 | 105 | 75 | 120 | 90 | 105 | 90 | 75 | 120 | 90 | | 60 |
| F20 | 120 | 60 | 75 | 90 | 105 | 60 | 135 | 120 | 75 | 75 | 105 | 90 | 120 | 90 | 75 | 60 | 105 | 60 | 60 | |
| F21 | 60 | 120 | 75 | 90 | 60 | 105 | 45 | 75 | 120 | 90 | 60 | 105 | 75 | 75 | 120 | 105 | 90 | 60 | 60 | 90 |

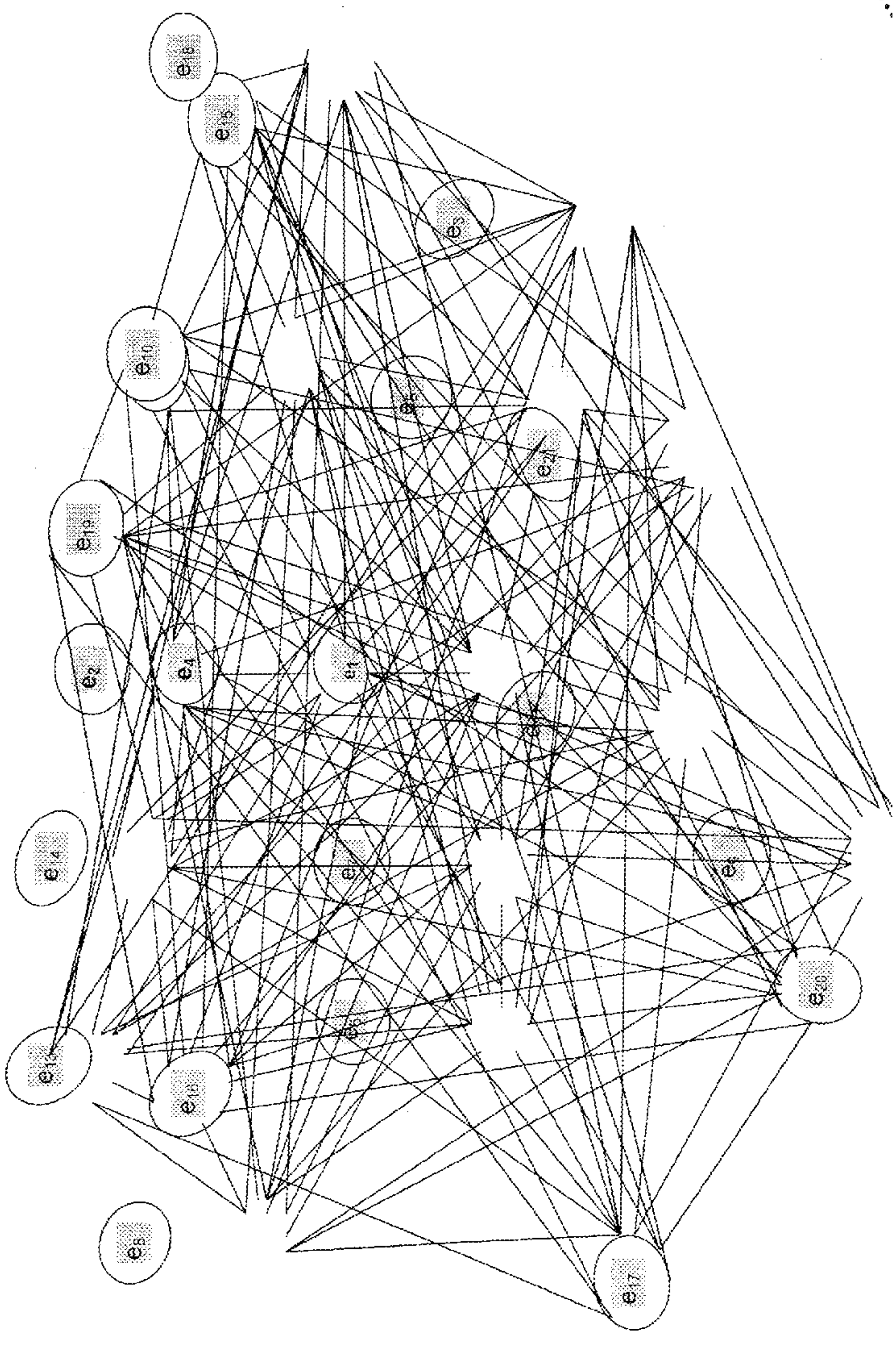
Table (3) The angle between simple root ofG24

The coxeter graph of the complex group $G_{24}$ is denoted by $C_{G_{24}}$
The cardinality of vertices is 21

| Root | Degree of root | Root | Degree of root | Root | Degree of root |
|---|---|---|---|---|---|
| $e_1$ | 16 | $e_8$ | 16 | $e_{15}$ | 17 |
| $e_2$ | 16 | $e_9$ | 16 | $e_{16}$ | 16 |
| $e_3$ | 16 | $e_{10}$ | 16 | $e_{17}$ | 16 |
| $e_4$ | 16 | $e_{11}$ | 16 | $e_{18}$ | 16 |
| $e_5$ | 16 | $e_{12}$ | 16 | $e_{19}$ | 16 |
| $e_6$ | 16 | $e_{13}$ | 16 | $e_{20}$ | 16 |
| $e_7$ | 16 | $e_{14}$ | 16 | $e_{21}$ | 17 |

**The Coxeter matrix of the complex group $G_{24}$**

$$\begin{bmatrix}
1 & 2 & 2 & 4 & 4 & 2 & 4 & 4 & 2 & 2.4 & 2.4 & 2.4 & 2.4 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
2 & 1 & 2 & 4 & 2 & 4 & 4 & 2 & 4 & 3 & 3 & 3 & 3 & 2.4 & 2.4 & 2.4 & 2.4 & 3 & 3 & 3 & 3 \\
2 & 2 & 1 & 2 & 4 & 4 & 2 & 4 & 4 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 2.4 & 2.4 & 2.4 & 2.4 \\
4 & 4 & 2 & 1 & 3 & 3 & 2 & 3 & 3 & 3 & 2.4 & 3 & 2.4 & 3 & 2.4 & 2.4 & 3 & 4 & 4 & 2 & 2 \\
4 & 2 & 4 & 3 & 1 & 3 & 3 & 2 & 3 & 3 & 3 & 2.4 & 2.4 & 4 & 4 & 2 & 2 & 3 & 2.4 & 2.4 & 3 \\
2 & 4 & 4 & 3 & 3 & 1 & 3 & 3 & 2 & 4 & 2 & 2 & 4 & 3 & 2.4 & 3 & 2.4 & 3 & 2.4 & 3 & 2.4 \\
4 & 4 & 2 & 2 & 3 & 3 & 1 & 3 & 3 & 2.4 & 3 & 2.4 & 3 & 2.4 & 3 & 3 & 2.4 & 2 & 2 & 4 & 4 \\
4 & 2 & 4 & 3 & 2 & 3 & 3 & 1 & 3 & 2.4 & 2.4 & 3 & 3 & 2 & 2 & 4 & 4 & 2.4 & 3 & 3 & 2.4 \\
2 & 4 & 4 & 3 & 3 & 2 & 3 & 3 & 1 & 2 & 4 & 4 & 2 & 2.4 & 3 & 2.4 & 3 & 2.4 & 3 & 2.4 & 3 \\
2.4 & 3 & 3 & 3 & 3 & 4 & 2.4 & 2.4 & 2 & 1 & 3 & 3 & 2 & 3 & 2.4 & 2.4 & 2 & 3 & 2.4 & 2.4 & 2 \\
2.4 & 3 & 3 & 2.4 & 3 & 2 & 3 & 2.4 & 4 & 3 & 1 & 2 & 3 & 2.4 & 3 & 2 & 2.4 & 2 & 2.4 & 2.4 & 3 \\
2.4 & 3 & 3 & 3 & 2.4 & 2 & 2.4 & 3 & 4 & 3 & 2 & 1 & 3 & 2 & 2.4 & 2.4 & 3 & 2.4 & 3 & 2 & 2.4 \\
2.4 & 3 & 3 & 2.4 & 2.4 & 4 & 3 & 3 & 2 & 2 & 3 & 3 & 1 & 2.4 & 2 & 3 & 2.4 & 2.4 & 2 & 3 & 2.4 \\
3 & 2.4 & 3 & 3 & 4 & 3 & 2.4 & 2 & 2.4 & 3 & 2.4 & 3 & 2.4 & 1 & 2 & 3 & 3 & 3 & 2.4 & 2 & 2.4 \\
3 & 2.4 & 3 & 2.4 & 4 & 2.4 & 3 & 2 & 3 & 2.4 & 3 & 2.4 & 2 & 2 & 1 & 3 & 3 & 2.4 & 2 & 2.4 & 3 \\
3 & 2.4 & 3 & 2.4 & 2 & 3 & 3 & 4 & 2.4 & 2.4 & 2 & 2.4 & 3 & 3 & 3 & 1 & 2 & 2 & 2.4 & 3 & 2.4 \\
3 & 2.4 & 3 & 3 & 2 & 2.4 & 2.4 & 4 & 3 & 2 & 2.4 & 3 & 2.4 & 3 & 3 & 2 & 1 & 2.4 & 3 & 2.4 & 2 \\
3 & 3 & 2.4 & 4 & 3 & 3 & 2 & 2.4 & 2.4 & 3 & 2 & 2.4 & 2.4 & 3 & 2.4 & 2 & 2.4 & 1 & 2 & 3 & 3 \\
3 & 3 & 2.4 & 4 & 2.4 & 2.4 & 2 & 3 & 3 & 2.4 & 2.4 & 3 & 2 & 2.4 & 2 & 2.4 & 3 & 2 & 1 & 3 & 3 \\
3 & 3 & 2.4 & 2 & 2.4 & 3 & 4 & 3 & 2.4 & 2.4 & 2.4 & 2 & 3 & 2 & 2.4 & 3 & 2.4 & 3 & 3 & 1 & 2 \\
3 & 3 & 2.4 & 2 & 3 & 2.4 & 4 & 2.4 & 3 & 2 & 3 & 2.4 & 2.4 & 2.4 & 3 & 2.4 & 2 & 3 & 3 & 2 & 1
\end{bmatrix}$$




3. **The complex Reflection Group** $G_{27}$:

The defining polynomial of $\mathcal{A}(G_{27})$ defined by

$$\Phi_0\mathcal{A}(G_{27})) = x_1x_2x_3\Pi(x_i \pm \omega x_j)(x_i \pm \gamma x_j \pm \gamma 2 x_k)(x_i \pm \omega\gamma 2 x_j \pm \omega 2\gamma (1 - \omega 2\gamma)x_i \pm \omega x_j)$$

The hyperplanes of $\mathcal{A}(G_{27})$ where $Hi = ker\, \alpha_{H_i}$ , $1 \le i \le 45$ are:

| | |
|---|---|
| $H_1$ : $x_1 = 0$ | $H_2$ : $x_2 = 0$ |
| $H_3$ : $x_3 = 0$ | $H_4$ : $x_2 + \omega x_1 = 0$ |
| $H_5$ : $x_2 - \omega x_1 = 0$ | $H_6$ : $x_3 + \omega x_2 = 0$ |
| $H_7$ : $x_3 - \omega x_2 = 0$ | $H_8$ : $x_1 + \omega x_3 = 0$ |
| $H_9$ : $x_1 - \omega x_3 = 0$ | $H_{10}$ : $x_1 + \gamma x_2 + \gamma^2 x_3 = 0$ |
| $H_{11}$ : $x_1 - \gamma x_2 - \gamma^2 x_3 = 0$ | $H_{12}$ : $x_1 + \gamma x_2 - \gamma^2 x_3 = 0$ |
| $H_{13}$ : $x_1 - \gamma x_2 + \gamma^2 x_3 = 0$ | $H_{14}$ : $x_2 + \gamma x_3 + \gamma^2 x_1 = 0$ |
| $H_{15}$ : $x_2 - \gamma x_3 - \gamma^2 x_1 = 0$ | $H_{16}$ : $x_2 + \gamma x_3 - \gamma^2 x_1 = 0$ |
| $H_{17}$ : $x_2 - \gamma x_3 + \gamma^2 x_1 = 0$ | $H_{18}$ : $x_3 + \gamma x_1 + \gamma^2 x_2 = 0$ |
| $H_{19}$ : $x_3 - \gamma x_1 - \gamma^2 x_2 = 0$ | $H_{20}$ : $x_3 + \gamma x_1 - \gamma^2 x_2 = 0$ |
| $H_{21}$ : $x_3 - \gamma x_1 + \gamma^2 x_2 = 0$ | $H_{22}$ : $x_1 + (1 - \omega^2 \gamma) x_2 + \omega x_3 = 0$ |
| $H_{23}$ : $x_1 - (1 - \omega^2 \gamma) x_2 - \omega x_3 = 0$ | $H_{24}$ : $x_1 + (1 - \omega^2 \gamma) x_2 - \omega x_3 = 0$ |
| $H_{25}$ : $x_1 - (1 - \omega^2 \gamma) x_2 + \omega x_3 = 0$ | $H_{26}$ : $x_2 + (1 - \omega^2 \gamma) x_3 + \omega x_1 = 0$ |
| $H_{27}$ : $x_2 - (1 - \omega^2 \gamma) x_3 - \omega x_1 = 0$ | $H_{28}$ : $x_2 + (1 - \omega^2 \gamma) x_3 - \omega x_1 = 0$ |
| $H_{29}$ : $x_2 - (1 - \omega^2 \gamma) x_3 + \omega x_1 = 0$ | $H_{30}$ : $x_3 + (1 - \omega^2 \gamma) x_1 + \omega x_2 = 0$ |
| $H_{31}$ : $x_3 - (1 - \omega^2 \gamma) x_1 - \omega x_2 = 0$ | $H_{32}$ : $x_3 + (1 - \omega^2 \gamma) x_1 - \omega x_2 = 0$ |
| $H_{33}$ : $x_3 - (1 - \omega^2 \gamma) x_1 + \omega x_2 = 0$ | $H_{34}$ : $x_1 + \omega \gamma^2 x_2 + \omega^2 \gamma x_3 = 0$ |
| $H_{35}$ : $x_1 - \omega \gamma^2 x_2 - \omega^2 \gamma x_3 = 0$ | $H_{36}$ : $x_1 + \omega \gamma^2 x_2 - \omega^2 \gamma x_3 = 0$ |
| $H_{37}$ : $x_1 - \omega \gamma^2 x_2 + \omega^2 \gamma x_3 = 0$ | $H_{38}$ : $x_2 + \omega \gamma^2 x_3 + \omega^2 \gamma x_1 = 0$ |
| $H_{39}$ : $x_2 - \omega \gamma^2 x_3 - \omega^2 \gamma x_1 = 0$ | $H_{40}$ : $x_2 + \omega \gamma^2 x_3 - \omega^2 \gamma x_1 = 0$ |
| $H_{41}$ : $x_2 - \omega \gamma^2 x_3 + \omega^2 \gamma x_1 = 0$ | $H_{42}$ : $x_3 + \omega \gamma^2 x_1 + \omega^2 \gamma x_2 = 0$ |
| $H_{43}$ : $x_3 - \omega \gamma^2 x_1 - \omega^2 \gamma x_2 = 0$ | $H_{44}$ : $x_3 + \omega \gamma^2 x_1 - \omega^2 \gamma x_2 = 0$ |
| $H_{45}$ : $x_3 - \omega \gamma^2 x_1 + \omega^2 \gamma x_2 = 0$ | |

Table (4) the hyperplanes of $\mathcal{A}(G_{27})$

Therefore the set $S_\Phi(G_{27})$ is the simple root system for the group $G_{27}$ i.e.

| | |
|---|---|
| $e_1$= (1, 0, 0) | $e_2$= (0, 1, 0) |
| $e_3$= (0, 0, 1) | $e_4$= (-0.5+0.8660i, 1, 0) |
| $e_5$= (0.5-0.8660i, 1, 0) | $e_6$= (0, -0.5+0.8660i, 1) |
| $e_7$= (0, 0.5-0.8660i, 1) | $e_8$= (1, 0, -0.5+0.8660i) |
| $e_9$= (1, 0, 0.5-0.8660i) | $e_{10}$= (1, 0.6180, 0.3819) |
| $e_{11}$= (1, -0.6180, -0.3819) | $e_{12}$= (1, 0.6180, -0.3819) |
| $e_{13}$= (1, -0.6180, 0.3819) | $e_{14}$= (0.3819, 1, 0.6180) |
| $e_{15}$= (-0.3819, 1, -0.6180) | $e_{16}$= (-0.3819, 1, 0.6180) |
| $e_{17}$= (0.3819, 1, -0.6180) | $e_{18}$= (0.6180, 0.3819, 1) |
| $e_{19}$= (-0.6180, -0.3819, 1) | $e_{20}$= (0.6180, -0.3819, 1) |
| $e_{21}$= (-0.6180, 0.3819, 1) | $e_{22}$= (1, 1.309+0.5352i,-0.5+0.8660i) |
| $e_{23}$= (1, -1.309-0.5352i, 0.5-0.8660i) | $e_{24}$= (1, 1.309+0.5352i, 0.5-0.8660i) |
| $e_{25}$= (1, -1.309-0.5352i, -0.5+0.8660i) | $e_{26}$= (-0.5+0.8660i, 1, 1.309+0.5352i) |
| $e_{27}$= (0.5-0.8660i, 1, -1.309-0.5352i) | $e_{28}$= (0.5-0.8660i, 1, 1.309+0.5352i) |
| $e_{29}$= (-0.5+0.8660i ,1, -1.309-0.5352i) | $e_{30}$= (1.309+0.5352i, -0.5+0.8660i, 1) |
| $e_{31}$= (-1.309-0.5352i, 0.5-0.8660i, 1) | $e_{32}$= (1.309+0.5352i, 0.5-0.8660i, 1) |
| $e_{33}$= (-1.309-0.5352i, -0.5+0.8660i, 1) | $e_{34}$=(1,-0.19095+0.33073i,-0.309-0.5352i) |
| $e_{35}$=(1, 0.19095-0.33073i,0.309+0.5352i) | $e_{36}$=(1,-0.19095+0.33073i,0.309+0.5352i) |
| $e_{37}$=(1, 0.19095-0.33073i,-0.309-0.5352i) | $e_{38}$=(-0.309-0.5352i,1,-0.19095+0.33073i) |
| $e_{39}$= (0.309+0.5352i, 1, 0.19095-0.33073i) | $e_{40}$= (0.309+0.5352i, 1, -0.19095+0.33073i) |
| $e_{41}$= (-0.309-0.5352i, 1, 0.19095-0.33073i) | $e_{42}$= (-0.19095+0.33073i, -0.309-0.5352i, 1) |
| $e_{43}$= (0.19095-0.33073i, 0.309+0.5352i, 1) | $e_{44}$= (-0.19095+0.33073i, 0.309+0.5352i, 1) |
| $e_{45}$= (0.19095-0.33073i, -0.309-0.5352i, 1) | |

$S_\Phi(G_{27})$ consists of The following is

1- The set is the simple root system for the group $G_{27}$

2- The Coxeter matrix of the complex group $G_{27}$

| | F1 | F2 | F3 | F4 | F5 | F6 | F7 | F8 | F9 | F10 | F11 | F12 | F13 | F14 | F15 | F16 | F17 | F18 | F19 | F20 | F21 | F22 | F23 | F24 | F25 | F26 | F27 | F28 | F29 | F30 | F31 | F32 | F33 | F34 | F35 | F36 | F37 | F38 | F39 | F40 | F41 | F42 | F43 | F44 | F45 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | 90 | 90 | 105 | 75 | 90 | 90 | 45 | 45 | 30 | 30 | 30 | 30 | 75 | 105 | 105 | 75 | 60 | 120 | 60 | 120 | 60 | 60 | 60 | 60 | 105 | 75 | 75 | 105 | 45 | 135 | 45 | 135 | 30 | 30 | 30 | 30 | 105 | 75 | 75 | 105 | 105 | 75 | 105 | 75 |
| 2 | 90 | | 90 | 45 | 45 | 105 | 75 | 90 | 90 | 60 | 120 | 60 | 120 | 30 | 30 | 30 | 30 | 75 | 105 | 105 | 75 | 45 | 135 | 45 | 135 | 60 | 60 | 60 | 60 | 105 | 75 | 75 | 105 | 105 | 75 | 105 | 75 | 30 | 30 | 30 | 30 | 105 | 75 | 75 | 105 |
| 3 | 90 | 90 | | 90 | 90 | 45 | 45 | 105 | 75 | 75 | 105 | 105 | 75 | 60 | 120 | 60 | 120 | 30 | 30 | 30 | 30 | 105 | 75 | 75 | 105 | 45 | 135 | 45 | 135 | 60 | 60 | 60 | 60 | 105 | 75 | 75 | 105 | 105 | 75 | 105 | 75 | 30 | 30 | 30 | 30 |
| 4 | 105 | 45 | 90 | | 90 | 105 | 75 | 105 | 105 | 90 | 135 | 90 | 135 | 60 | 45 | 45 | 60 | 90 | 90 | 120 | 60 | 75 | 135 | 75 | 135 | 45 | 90 | 90 | 45 | 105 | 75 | 90 | 90 | 120 | 105 | 120 | 105 | 60 | 45 | 45 | 60 | 90 | 90 | 60 | 120 |
| 5 | 75 | 45 | 90 | 90 | | 105 | 75 | 75 | 75 | 45 | 90 | 45 | 90 | 45 | 60 | 60 | 45 | 60 | 120 | 90 | 90 | 45 | 105 | 45 | 105 | 90 | 45 | 45 | 90 | 90 | 90 | 75 | 105 | 75 | 60 | 75 | 60 | 45 | 60 | 60 | 45 | 120 | 60 | 90 | 90 |
| 6 | 90 | 105 | 45 | 105 | 105 | | 90 | 105 | 75 | 90 | 90 | 120 | 60 | 90 | 135 | 90 | 135 | 60 | 45 | 45 | 60 | 105 | 75 | 90 | 90 | 75 | 135 | 75 | 135 | 45 | 90 | 90 | 45 | 90 | 90 | 60 | 120 | 120 | 105 | 120 | 105 | 60 | 45 | 45 | 60 |
| 7 | 90 | 75 | 45 | 75 | 75 | 90 | | 105 | 75 | 60 | 120 | 90 | 90 | 45 | 90 | 45 | 90 | 45 | 60 | 60 | 45 | 90 | 90 | 75 | 105 | 45 | 105 | 45 | 105 | 90 | 45 | 45 | 90 | 120 | 60 | 90 | 90 | 75 | 60 | 75 | 60 | 45 | 60 | 60 | 45 |
| 8 | 45 | 90 | 105 | 105 | 75 | 105 | 105 | | 90 | 60 | 45 | 45 | 60 | 90 | 90 | 120 | 60 | 90 | 135 | 90 | 135 | 45 | 90 | 90 | 45 | 105 | 75 | 90 | 90 | 75 | 135 | 75 | 135 | 60 | 45 | 45 | 60 | 90 | 90 | 60 | 120 | 120 | 105 | 120 | 105 |
| 9 | 45 | 90 | 75 | 105 | 75 | 75 | 90 | 90 | | 45 | 60 | 60 | 45 | 60 | 120 | 90 | 90 | 45 | 90 | 45 | 90 | 90 | 45 | 45 | 90 | 90 | 90 | 75 | 105 | 45 | 105 | 45 | 105 | 45 | 60 | 60 | 45 | 120 | 60 | 90 | 90 | 75 | 60 | 75 | 60 |
| 10 | 30 | 60 | 75 | 90 | 45 | 90 | 60 | 60 | 45 | | 75 | 30 | 60 | 30 | 90 | 75 | 60 | 30 | 105 | 60 | 90 | 45 | 75 | 30 | 90 | 75 | 75 | 45 | 105 | 60 | 105 | 30 | 120 | 60 | 30 | 45 | 45 | 75 | 45 | 60 | 75 | 90 | 60 | 75 | 75 |
| 11 | 30 | 120 | 105 | 135 | 90 | 90 | 120 | 45 | 60 | 75 | | 60 | 30 | 105 | 120 | 150 | 90 | 90 | 120 | 75 | 150 | 75 | 45 | 90 | 30 | 135 | 75 | 105 | 105 | 60 | 150 | 75 | 120 | 30 | 60 | 45 | 45 | 120 | 105 | 105 | 135 | 105 | 105 | 120 | 90 |
| 12 | 30 | 60 | 105 | 90 | 45 | 120 | 90 | 45 | 60 | 30 | 60 | | 75 | 60 | 75 | 90 | 30 | 75 | 105 | 90 | 120 | 30 | 90 | 45 | 75 | 105 | 45 | 75 | 75 | 75 | 120 | 60 | 150 | 45 | 45 | 60 | 30 | 75 | 60 | 45 | 75 | 120 | 90 | 105 | 105 |
| 13 | 30 | 120 | 75 | 135 | 90 | 60 | 90 | 60 | 45 | 60 | 30 | 75 | | 90 | 150 | 150 | 120 | 60 | 90 | 30 | 105 | 90 | 30 | 75 | 45 | 105 | 105 | 75 | 135 | 30 | 120 | 60 | 105 | 45 | 45 | 30 | 60 | 135 | 105 | 105 | 120 | 75 | 75 | 90 | 60 |
| 14 | 75 | 30 | 60 | 60 | 45 | 90 | 45 | 90 | 60 | 30 | 105 | 60 | 90 | | 75 | 30 | 60 | 30 | 90 | 75 | 60 | 60 | 105 | 30 | 120 | 45 | 75 | 30 | 90 | 75 | 75 | 45 | 105 | 90 | 60 | 75 | 75 | 60 | 30 | 45 | 45 | 75 | 75 | 60 | 75 |
| 15 | 105 | 30 | 120 | 45 | 60 | 135 | 90 | 90 | 120 | 90 | 120 | 75 | 150 | 75 | | 60 | 30 | 105 | 120 | 150 | 90 | 60 | 150 | 75 | 120 | 75 | 45 | 90 | 30 | 135 | 75 | 105 | 105 | 105 | 105 | 120 | 90 | 30 | 60 | 45 | 45 | 120 | 105 | 105 | 135 |
| 16 | 105 | 30 | 60 | 45 | 60 | 90 | 45 | 120 | 90 | 75 | 150 | 90 | 150 | 30 | 60 | | 75 | 60 | 75 | 90 | 30 | 75 | 120 | 60 | 150 | 30 | 90 | 45 | 75 | 105 | 45 | 75 | 75 | 120 | 90 | 105 | 105 | 45 | 45 | 60 | 30 | 75 | 60 | 45 | 75 |
| 17 | 75 | 30 | 120 | 60 | 45 | 135 | 90 | 60 | 90 | 60 | 90 | 30 | 120 | 60 | 30 | 75 | | 90 | 150 | 120 | 105 | 30 | 120 | 60 | 105 | 90 | 30 | 75 | 45 | 105 | 105 | 75 | 135 | 75 | 75 | 90 | 60 | 45 | 45 | 30 | 60 | 135 | 105 | 105 | 120 |
| 18 | 60 | 75 | 30 | 90 | 60 | 60 | 45 | 90 | 45 | 30 | 90 | 75 | 60 | 30 | 105 | 60 | 90 | | 75 | 30 | 60 | 75 | 75 | 45 | 105 | 60 | 105 | 30 | 120 | 45 | 75 | 30 | 90 | 75 | 45 | 60 | 75 | 90 | 60 | 75 | 75 | 60 | 30 | 45 | 45 |
| 19 | 120 | 105 | 30 | 90 | 120 | 45 | 60 | 135 | 90 | 105 | 120 | 105 | 90 | 90 | 120 | 75 | 150 | 75 | | 60 | 30 | 135 | 75 | 105 | 105 | 60 | 150 | 75 | 120 | 75 | 45 | 90 | 30 | 120 | 105 | 105 | 135 | 105 | 105 | 120 | 90 | 30 | 60 | 45 | 45 |
| 20 | 60 | 105 | 30 | 120 | 90 | 45 | 60 | 90 | 45 | 60 | 75 | 90 | 30 | 75 | 150 | 90 | 120 | 30 | 60 | | 75 | 105 | 45 | 75 | 75 | 75 | 120 | 60 | 150 | 30 | 90 | 45 | 75 | 75 | 60 | 45 | 75 | 120 | 90 | 105 | 105 | 45 | 45 | 60 | 30 |
| 21 | 120 | 75 | 30 | 60 | 90 | 60 | 45 | 135 | 90 | 90 | 150 | 120 | 105 | 60 | 90 | 30 | 105 | 60 | 30 | 75 | | 105 | 105 | 75 | 135 | 30 | 120 | 60 | 105 | 90 | 30 | 75 | 45 | 135 | 105 | 105 | 120 | 75 | 75 | 90 | 60 | 45 | 45 | 30 | 60 |
| 22 | 60 | 45 | 105 | 75 | 45 | 105 | 90 | 45 | 90 | 45 | 75 | 30 | 90 | 60 | 60 | 75 | 30 | 75 | 135 | 105 | 105 | | 120 | 60 | 90 | 75 | 60 | 60 | 75 | 75 | 120 | 75 | 120 | 75 | 60 | 60 | 75 | 60 | 60 | 30 | 75 | 120 | 75 | 90 | 120 |
| 23 | 60 | 135 | 75 | 135 | 105 | 75 | 90 | 90 | 45 | 75 | 45 | 90 | 30 | 105 | 150 | 120 | 120 | 75 | 75 | 45 | 105 | 120 | | 90 | 60 | 120 | 105 | 105 | 120 | 60 | 105 | 60 | 105 | 60 | 75 | 75 | 60 | 105 | 105 | 120 | 120 | 60 | 90 | 105 | 60 |
| 24 | 60 | 45 | 75 | 75 | 45 | 90 | 75 | 90 | 45 | 30 | 90 | 45 | 75 | 30 | 75 | 60 | 60 | 45 | 105 | 75 | 75 | 60 | 90 | | 120 | 75 | 60 | 60 | 75 | 60 | 105 | 60 | 105 | 60 | 75 | 75 | 60 | 75 | 30 | 60 | 60 | 105 | 60 | 60 | 90 |
| 25 | 60 | 135 | 105 | 135 | 105 | 90 | 105 | 45 | 90 | 90 | 30 | 75 | 45 | 120 | 120 | 150 | 105 | 105 | 105 | 75 | 135 | 90 | 60 | 120 | | 120 | 105 | 105 | 120 | 75 | 120 | 75 | 120 | 75 | 60 | 60 | 75 | 120 | 120 | 105 | 150 | 90 | 120 | 120 | 75 |
| 26 | 105 | 60 | 45 | 45 | 90 | 75 | 45 | 105 | 90 | 75 | 135 | 105 | 105 | 45 | 75 | 30 | 90 | 60 | 60 | 75 | 30 | 75 | 120 | 75 | 120 | | 120 | 60 | 90 | 75 | 60 | 60 | 75 | 120 | 75 | 90 | 120 | 75 | 60 | 60 | 75 | 60 | 60 | 30 | 75 |
| 27 | 75 | 60 | 135 | 90 | 45 | 135 | 105 | 75 | 90 | 75 | 75 | 45 | 105 | 75 | 45 | 90 | 30 | 105 | 150 | 120 | 120 | 60 | 105 | 60 | 105 | 120 | | 90 | 60 | 120 | 105 | 105 | 120 | 60 | 90 | 105 | 60 | 60 | 75 | 75 | 60 | 150 | 105 | 120 | 120 |
| 28 | 75 | 60 | 45 | 90 | 45 | 75 | 45 | 90 | 75 | 45 | 105 | 75 | 75 | 30 | 90 | 45 | 75 | 30 | 75 | 60 | 60 | 60 | 105 | 60 | 105 | 60 | 90 | | 120 | 75 | 60 | 60 | 75 | 105 | 60 | 60 | 90 | 60 | 75 | 75 | 60 | 75 | 30 | 60 | 60 |
| 29 | 105 | 60 | 135 | 45 | 90 | 135 | 105 | 90 | 105 | 105 | 105 | 75 | 135 | 90 | 30 | 75 | 45 | 120 | 120 | 150 | 105 | 75 | 120 | 75 | 120 | 90 | 60 | 120 | | 120 | 105 | 105 | 120 | 90 | 120 | 120 | 75 | 75 | 60 | 60 | 75 | 120 | 120 | 105 | 150 |
| 30 | 45 | 105 | 60 | 105 | 90 | 45 | 90 | 75 | 45 | 60 | 60 | 75 | 30 | 75 | 135 | 105 | 105 | 45 | 75 | 30 | 90 | 75 | 60 | 60 | 75 | 75 | 120 | 75 | 120 | | 120 | 60 | 90 | 60 | 60 | 30 | 75 | 120 | 75 | 90 | 120 | 75 | 60 | 60 | 75 |
| 31 | 135 | 75 | 60 | 75 | 90 | 90 | 45 | 135 | 105 | 105 | 150 | 120 | 120 | 75 | 75 | 45 | 105 | 75 | 45 | 90 | 30 | 120 | 105 | 105 | 120 | 60 | 105 | 60 | 105 | 120 | | 90 | 60 | 150 | 105 | 120 | 120 | 60 | 90 | 105 | 60 | 60 | 75 | 75 | 60 |
| 32 | 45 | 75 | 60 | 90 | 75 | 90 | 45 | 75 | 45 | 30 | 75 | 60 | 60 | 45 | 105 | 75 | 75 | 30 | 90 | 45 | 75 | 75 | 60 | 60 | 75 | 60 | 105 | 60 | 105 | 60 | 90 | | 120 | 75 | 30 | 60 | 60 | 105 | 60 | 60 | 90 | 60 | 75 | 75 | 60 |
| 33 | 135 | 105 | 60 | 90 | 105 | 45 | 90 | 135 | 105 | 120 | 120 | 150 | 105 | 105 | 105 | 75 | 135 | 90 | 30 | 75 | 45 | 120 | 105 | 105 | 120 | 75 | 120 | 75 | 120 | 90 | 60 | 120 | | 120 | 120 | 105 | 150 | 90 | 120 | 120 | 75 | 75 | 60 | 60 | 75 |
| 34 | 30 | 105 | 105 | 120 | 75 | 90 | 120 | 60 | 45 | 60 | 30 | 45 | 45 | 90 | 105 | 120 | 75 | 75 | 120 | 75 | 135 | 75 | 60 | 60 | 75 | 120 | 60 | 105 | 90 | 60 | 150 | 75 | 120 | | 75 | 60 | 30 | 120 | 75 | 90 | 105 | 120 | 90 | 105 | 105 |
| 35 | 30 | 75 | 75 | 105 | 60 | 90 | 60 | 45 | 60 | 30 | 60 | 45 | 45 | 60 | 105 | 90 | 75 | 45 | 105 | 60 | 105 | 60 | 75 | 75 | 60 | 75 | 90 | 60 | 120 | 60 | 105 | 30 | 120 | 75 | | 30 | 60 | 90 | 75 | 60 | 105 | 75 | 75 | 90 | 60 |
| 36 | 30 | 105 | 75 | 120 | 75 | 60 | 90 | 45 | 90 | 45 | 45 | 60 | 30 | 75 | 120 | 105 | 90 | 60 | 105 | 45 | 105 | 60 | 75 | 75 | 60 | 90 | 106 | 60 | 120 | 30 | 120 | 60 | 105 | 60 | 30 | | 75 | 105 | 90 | 75 | 120 | 90 | 60 | 75 | 75 |
| 37 | 30 | 75 | 105 | 105 | 60 | 120 | 90 | 60 | 45 | 45 | 45 | 30 | 60 | 75 | 90 | 105 | 60 | 75 | 135 | 75 | 120 | 75 | 60 | 60 | 75 | 120 | 60 | 90 | 75 | 75 | 120 | 60 | 150 | 30 | 60 | 75 | | 105 | 60 | 75 | 90 | 105 | 105 | 120 | 90 |
| 38 | 105 | 30 | 105 | 60 | 45 | 120 | 75 | 90 | 120 | 75 | 120 | 75 | 135 | 60 | 30 | 45 | 45 | 90 | 105 | 120 | 75 | 60 | 105 | 75 | 120 | 75 | 60 | 60 | 75 | 120 | 60 | 105 | 90 | 120 | 90 | 105 | 105 | | 75 | 60 | 30 | 120 | 75 | 90 | 105 |
| 39 | 75 | 30 | 75 | 45 | 60 | 105 | 60 | 90 | 60 | 45 | 105 | 60 | 105 | 30 | 60 | 45 | 45 | 60 | 105 | 90 | 75 | 60 | 105 | 30 | 120 | 60 | 75 | 75 | 60 | 75 | 90 | 60 | 120 | 75 | 75 | 90 | 60 | 75 | | 30 | 60 | 90 | 75 | 60 | 105 |
| 40 | 75 | 30 | 105 | 45 | 60 | 120 | 75 | 60 | 90 | 60 | 105 | 45 | 105 | 45 | 45 | 60 | 30 | 75 | 120 | 105 | 90 | 30 | 120 | 60 | 105 | 60 | 75 | 75 | 60 | 90 | 105 | 60 | 120 | 90 | 60 | 75 | 75 | 60 | 30 | | 75 | 105 | 90 | 75 | 120 |
| 41 | 105 | 30 | 75 | 60 | 45 | 105 | 60 | 120 | 90 | 75 | 135 | 75 | 120 | 45 | 45 | 30 | 60 | 75 | 90 | 105 | 60 | 75 | 120 | 60 | 150 | 75 | 60 | 60 | 75 | 120 | 60 | 90 | 75 | 105 | 105 | 120 | 90 | 30 | 60 | 75 | | 105 | 60 | 75 | 90 |
| 42 | 105 | 105 | 30 | 90 | 120 | 60 | 45 | 120 | 75 | 90 | 105 | 120 | 75 | 75 | 120 | 75 | 135 | 60 | 30 | 45 | 45 | 120 | 60 | 105 | 90 | 60 | 150 | 75 | 120 | 75 | 60 | 60 | 75 | 120 | 75 | 90 | 105 | 120 | 90 | 105 | 105 | | 75 | 60 | 30 |
| 43 | 75 | 75 | 30 | 90 | 60 | 45 | 60 | 105 | 60 | 60 | 105 | 90 | 75 | 75 | 105 | 60 | 105 | 30 | 60 | 45 | 45 | 75 | 90 | 60 | 120 | 60 | 105 | 30 | 120 | 60 | 75 | 75 | 60 | 90 | 75 | 60 | 105 | 75 | 75 | 90 | 60 | 75 | | 30 | 60 |
| 44 | 105 | 75 | 30 | 60 | 90 | 45 | 60 | 120 | 75 | 75 | 120 | 105 | 90 | 60 | 105 | 45 | 105 | 45 | 45 | 60 | 30 | 90 | 105 | 60 | 120 | 30 | 120 | 60 | 105 | 60 | 75 | 75 | 60 | 105 | 90 | 75 | 120 | 90 | 60 | 75 | 75 | 60 | 30 | | 75 |
| 45 | 75 | 105 | 30 | 120 | 90 | 60 | 45 | 105 | 60 | 75 | 90 | 105 | 60 | 75 | 135 | 75 | 120 | 45 | 45 | 30 | 60 | 120 | 60 | 90 | 75 | 75 | 120 | 60 | 150 | 75 | 60 | 60 | 75 | 105 | 60 | 75 | 90 | 105 | 105 | 120 | 90 | 30 | 60 | 75 | |

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 2.4 | 2.4 | 2 | 2 | 4 | 4 | 6 | 6 | 6 | 6 | 2.4 | 2.4 | 2.4 | 2.4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.4 | 2.4 | 2.4 | 2.4 | 4 | 4 | 4 | 4 | 6 | 6 | 6 | 6 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 |
| 2 | 1 | 2 | 4 | 4 | 2.4 | 2.4 | 2 | 2 | 3 | 3 | 3 | 3 | 6 | 6 | 6 | 6 | 2.4 | 2.4 | 2.4 | 2.4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 6 | 6 | 6 | 6 | 2.4 | 2.4 | 2.4 | 2.4 |
| 2 | 2 | 1 | 2 | 2 | 4 | 4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 3 | 3 | 3 | 3 | 6 | 6 | 6 | 6 | 2.4 | 2.4 | 2.4 | 2.4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 6 | 6 | 6 | 6 |
| 2.4 | 4 | 2 | 1 | 2 | 2.4 | 2.4 | 2.4 | 2.4 | 2 | 4 | 2 | 4 | 3 | 4 | 4 | 3 | 2 | 2 | 3 | 3 | 2.4 | 4 | 2.4 | 4 | 4 | 2 | 2 | 4 | 2.4 | 2.4 | 2 | 2 | 3 | 2.4 | 3 | 2.4 | 3 | 4 | 4 | 3 | 2 | 2 | 3 | 3 |
| 2.4 | 4 | 2 | 2 | 1 | 2.4 | 2.4 | 2.4 | 2.4 | 4 | 2 | 4 | 2 | 4 | 3 | 3 | 4 | 3 | 3 | 2 | 2 | 4 | 2.4 | 4 | 2.4 | 2 | 4 | 4 | 2 | 2 | 2 | 2.4 | 2.4 | 2.4 | 3 | 2.4 | 3 | 4 | 3 | 3 | 4 | 3 | 3 | 2 | 2 |
| 2 | 2.4 | 4 | 2.4 | 2.4 | 1 | 2 | 2.4 | 2.4 | 2 | 2 | 3 | 3 | 2 | 4 | 2 | 4 | 3 | 4 | 4 | 3 | 2.4 | 2.4 | 2 | 2 | 2.4 | 2 | 2.4 | 4 | 4 | 2 | 2 | 4 | 2 | 2 | 3 | 3 | 3 | 2.4 | 3 | 2.4 | 3 | 4 | 4 | 3 |
| 2 | 2.4 | 4 | 2.4 | 2.4 | 2 | 1 | 2.4 | 2.4 | 3 | 3 | 2 | 2 | 4 | 2 | 4 | 2 | 4 | 3 | 3 | 4 | 2 | 2 | 2.4 | 2.4 | 4 | 2.4 | 4 | 2.4 | 2 | 4 | 4 | 2 | 3 | 3 | 2 | 2 | 2.4 | 3 | 2.4 | 3 | 4 | 3 | 3 | 4 |
| 4 | 2 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 1 | 2 | 3 | 4 | 4 | 3 | 2 | 2 | 3 | 3 | 2 | 4 | 2 | 4 | 4 | 2 | 2 | 4 | 2.4 | 2.4 | 2 | 2 | 2.4 | 4 | 2.4 | 4 | 3 | 4 | 4 | 3 | 2 | 2 | 3 | 3 | 3 | 2.4 | 4 | 2.4 |
| 4 | 2 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2 | 1 | 4 | 3 | 3 | 4 | 3 | 3 | 2 | 2 | 4 | 2 | 4 | 2 | 2 | 4 | 4 | 2 | 2 | 2 | 2.4 | 2.4 | 4 | 2.4 | 4 | 2.4 | 4 | 3 | 3 | 4 | 3 | 3 | 2 | 2 | 2.4 | 3 | 2.4 | 3 |
| 6 | 3 | 2.4 | 2 | 4 | 2 | 3 | 3 | 4 | 1 | 2.4 | 6 | 3 | 6 | 2 | 2.4 | 3 | 6 | 2.4 | 3 | 2 | 4 | 2.4 | 6 | 2 | 2.4 | 2.4 | 4 | 2.4 | 3 | 2.4 | 6 | 3 | 3 | 6 | 4 | 4 | 2.4 | 4 | 3 | 2.4 | 2 | 3 | 2.4 | 2.4 |
| 6 | 3 | 2.4 | 4 | 2 | 2 | 3 | 4 | 3 | 2.4 | 1 | 3 | 6 | 2.4 | 3 | 6 | 2 | 2 | 3 | 2.4 | 6 | 2.4 | 4 | 2 | 6 | 4 | 2.4 | 2.4 | 2.4 | 3 | 6 | 2.4 | 3 | 6 | 3 | 4 | 4 | 3 | 2.4 | 2.4 | 4 | 2.4 | 2.4 | 3 | 2 |
| 6 | 3 | 2.4 | 2 | 4 | 3 | 2 | 4 | 3 | 6 | 3 | 1 | 2.4 | 3 | 2.4 | 2 | 6 | 2.4 | 6 | 2 | 3 | 6 | 2 | 4 | 2.4 | 2.4 | 4 | 2.4 | 2.4 | 2.4 | 3 | 3 | 6 | 4 | 4 | 3 | 6 | 2.4 | 3 | 4 | 4 | 3 | 2 | 2.4 | 2.4 |
| 6 | 3 | 2.4 | 4 | 2 | 3 | 2 | 3 | 4 | 3 | 6 | 2.4 | 1 | 2 | 6 | 3 | 2.4 | 3 | 2 | 6 | 2.4 | 2 | 6 | 2.4 | 4 | 2.4 | 2.4 | 2.4 | 4 | 6 | 3 | 3 | 2.4 | 4 | 4 | 6 | 3 | 4 | 2.4 | 2.4 | 3 | 2.4 | 2.4 | 2 | 3 |
| 2.4 | 6 | 3 | 3 | 4 | 2 | 4 | 2 | 3 | 6 | 2.4 | 3 | 2 | 1 | 2.4 | 6 | 3 | 6 | 2 | 2.4 | 3 | 3 | 2.4 | 6 | 3 | 4 | 2.4 | 6 | 2 | 2.4 | 2.4 | 4 | 2.4 | 2 | 3 | 2.4 | 2.4 | 3 | 6 | 4 | 4 | 2.4 | 4 | 3 | 2.4 |
| 2.4 | 6 | 3 | 4 | 3 | 4 | 2 | 2 | 3 | 2 | 3 | 2.4 | 6 | 2.4 | 1 | 3 | 6 | 2.4 | 3 | 6 | 2 | 3 | 6 | 2.4 | 3 | 2.4 | 4 | 2 | 6 | 4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 3 | 2 | 6 | 3 | 4 | 4 | 3 | 2.4 | 4 | 4 |
| 2.4 | 6 | 3 | 4 | 3 | 2 | 4 | 3 | 2 | 2.4 | 6 | 2 | 3 | 6 | 3 | 1 | 2.4 | 3 | 2.4 | 2 | 6 | 2.4 | 3 | 3 | 6 | 6 | 2 | 4 | 2.4 | 2.4 | 4 | 2.4 | 2.4 | 3 | 2 | 2.4 | 2.4 | 4 | 4 | 3 | 6 | 2.4 | 3 | 4 | 2.4 |
| 2.4 | 6 | 3 | 3 | 4 | 4 | 2 | 3 | 2 | 3 | 2 | 6 | 2.4 | 3 | 6 | 2.4 | 1 | 2 | 6 | 3 | 2.4 | 6 | 3 | 3 | 2.4 | 2 | 6 | 2.4 | 4 | 2.4 | 2.4 | 2.4 | 4 | 2.4 | 2.4 | 2 | 3 | 4 | 4 | 6 | 3 | 4 | 2.4 | 2.4 | 3 |
| 3 | 2.4 | 6 | 2 | 3 | 3 | 4 | 2 | 4 | 6 | 2 | 2.4 | 3 | 6 | 2.4 | 3 | 2 | 1 | 2.4 | 6 | 3 | 2.4 | 2.4 | 4 | 2.4 | 3 | 2.4 | 6 | 3 | 4 | 2.4 | 6 | 2 | 2.4 | 4 | 3 | 2.4 | 2 | 3 | 2.4 | 2.4 | 3 | 6 | 4 | 4 |
| 3 | 2.4 | 6 | 2 | 3 | 4 | 3 | 4 | 2 | 2.4 | 3 | 6 | 2 | 2 | 3 | 2.4 | 6 | 2.4 | 1 | 3 | 6 | 4 | 2.4 | 2.4 | 2.4 | 3 | 6 | 2.4 | 3 | 2.4 | 4 | 2 | 6 | 3 | 2.4 | 2.4 | 4 | 2.4 | 2.4 | 3 | 2 | 6 | 3 | 4 | 4 |
| 3 | 2.4 | 6 | 3 | 2 | 4 | 3 | 2 | 4 | 3 | 2.4 | 2 | 6 | 2.4 | 6 | 2 | 3 | 6 | 3 | 1 | 2.4 | 2.4 | 4 | 2.4 | 2.4 | 2.4 | 3 | 3 | 6 | 6 | 2 | 4 | 2.4 | 2.4 | 3 | 4 | 2.4 | 3 | 2 | 2.4 | 2.4 | 4 | 4 | 3 | 6 |
| 3 | 2.4 | 6 | 3 | 2 | 3 | 4 | 4 | 2 | 2 | 6 | 3 | 2.4 | 3 | 2 | 6 | 2.4 | 3 | 6 | 2.4 | 1 | 2.4 | 2.4 | 2.4 | 4 | 6 | 3 | 3 | 2.4 | 2 | 6 | 2.4 | 4 | 4 | 2.4 | 2.4 | 3 | 2.4 | 2.4 | 2 | 3 | 4 | 4 | 6 | 3 |
| 3 | 4 | 2.4 | 2.4 | 4 | 2.4 | 2 | 4 | 2 | 4 | 2.4 | 6 | 2 | 3 | 3 | 2.4 | 6 | 2.4 | 4 | 2.4 | 2.4 | 1 | 3 | 3 | 2 | 2.4 | 3 | 3 | 2.4 | 2.4 | 2 | 2.4 | 3 | 2.4 | 3 | 3 | 2.4 | 3 | 3 | 6 | 2.4 | 3 | 2.4 | 2 | 3 |
| 3 | 4 | 2.4 | 4 | 2.4 | 2.4 | 2 | 2 | 4 | 2.4 | 4 | 2 | 6 | 2.4 | 6 | 3 | 3 | 2.4 | 2.4 | 4 | 2.4 | 3 | 1 | 2 | 3 | 3 | 2.4 | 2.4 | 3 | 3 | 2.4 | 3 | 2.4 | 3 | 2.4 | 2.4 | 3 | 6 | 2.4 | 3 | 3 | 3 | 2 | 2.4 | 3 |
| 3 | 4 | 2.4 | 2.4 | 4 | 2 | 2.4 | 2 | 4 | 6 | 2 | 4 | 2.4 | 6 | 2.4 | 3 | 3 | 4 | 2.4 | 2.4 | 2.4 | 3 | 2 | 1 | 3 | 2.4 | 3 | 3 | 2.4 | 3 | 2.4 | 3 | 2.4 | 3 | 2.4 | 2.4 | 3 | 2.4 | 6 | 3 | 3 | 2.4 | 3 | 3 | 2 |
| 3 | 4 | 2.4 | 4 | 2.4 | 2 | 2.4 | 4 | 2 | 2 | 6 | 2.4 | 4 | 3 | 3 | 6 | 2.4 | 2.4 | 2.4 | 2.4 | 4 | 2 | 3 | 3 | 1 | 3 | 2.4 | 2.4 | 3 | 2.4 | 3 | 2.4 | 3 | 2.4 | 3 | 3 | 2.4 | 3 | 3 | 2.4 | 6 | 2 | 3 | 3 | 2.4 |
| 2.4 | 3 | 4 | 4 | 2 | 2.4 | 4 | 2.4 | 2 | 2.4 | 4 | 2.4 | 2.4 | 4 | 2.4 | 6 | 2 | 3 | 3 | 2.4 | 6 | 2.4 | 3 | 2.4 | 3 | 1 | 3 | 3 | 2 | 2.4 | 6 | 3 | 2.4 | 3 | 2.4 | 2 | 3 | 2.4 | 3 | 3 | 2.4 | 3 | 3 | 6 | 2.4 |
| 2.4 | 3 | 4 | 2 | 4 | 2 | 2.4 | 2.4 | 2 | 2.4 | 2.4 | 4 | 2.4 | 2.4 | 4 | 2 | 6 | 2.4 | 6 | 3 | 3 | 3 | 2.4 | 3 | 2.4 | 3 | 1 | 2 | 3 | 3 | 2.4 | 2.4 | 2 | 3 | 2 | 2.4 | 3 | 3 | 2.4 | 2.4 | 3 | 6 | 2.4 | 3 | 3 |
| 2.4 | 3 | 4 | 2 | 4 | 2.4 | 4 | 2 | 2.4 | 4 | 2.4 | 2.4 | 2.4 | 6 | 2 | 4 | 2.4 | 6 | 2.4 | 3 | 3 | 3 | 2.4 | 3 | 2.4 | 3 | 2 | 1 | 3 | 2.4 | 3 | 3 | 2.4 | 2.4 | 3 | 3 | 2 | 3 | 2.4 | 2.4 | 3 | 2.4 | 6 | 3 | 3 |
| 2.4 | 3 | 4 | 4 | 2 | 4 | 2.4 | 2 | 2.4 | 2.4 | 2.4 | 2.4 | 4 | 2 | 6 | 2.4 | 4 | 3 | 3 | 6 | 2.4 | 2.4 | 3 | 2.4 | 3 | 2 | 3 | 3 | 1 | 3 | 2.4 | 2.4 | 3 | 2 | 3 | 3 | 2.4 | 2.4 | 3 | 3 | 2.4 | 3 | 3 | 2.4 | 6 |
| 4 | 2.4 | 3 | 2.4 | 2 | 4 | 2 | 2.4 | 4 | 3 | 3 | 2.4 | 6 | 2.4 | 4 | 2.4 | 2.4 | 4 | 2.4 | 6 | 2 | 2.4 | 3 | 3 | 2.4 | 2.4 | 3 | 2.4 | 3 | 1 | 3 | 3 | 2 | 3 | 3 | 6 | 2.4 | 3 | 2.4 | 2 | 3 | 2.4 | 3 | 3 | 2.4 |
| 4 | 2.4 | 3 | 2.4 | 2 | 2 | 4 | 4 | 2.4 | 2.4 | 6 | 3 | 3 | 2.4 | 2.4 | 2.4 | 4 | 2.4 | 2.4 | 4 | 2 | 6 | 2 | 2.4 | 2.4 | 3 | 6 | 2.4 | 3 | 2.4 | 1 | 2 | 3 | 6 | 2.4 | 3 | 3 | 3 | 2 | 2.4 | 3 | 3 | 2.4 | 2.4 | 3 |
| 4 | 2.4 | 3 | 2 | 2.4 | 2 | 4 | 2.4 | 4 | 6 | 2.4 | 3 | 3 | 4 | 2.4 | 2.4 | 2.4 | 6 | 2 | 4 | 2.4 | 2.4 | 3 | 3 | 2.4 | 3 | 2.4 | 3 | 2.4 | 3 | 2 | 1 | 3 | 2.4 | 6 | 3 | 3 | 2.4 | 3 | 3 | 2 | 3 | 2.4 | 2.4 | 3 |
| 4 | 2.4 | 3 | 2 | 2.4 | 4 | 2 | 4 | 2.4 | 3 | 3 | 6 | 2.4 | 2.4 | 2.4 | 2.4 | 4 | 2 | 6 | 2.4 | 4 | 3 | 2.4 | 2.4 | 3 | 2.4 | 2 | 2.4 | 3 | 2 | 3 | 3 | 1 | 3 | 3 | 2.4 | 6 | 2 | 3 | 3 | 2.4 | 2.4 | 3 | 3 | 2.4 |
| 6 | 2.4 | 2.4 | 3 | 2.4 | 2 | 3 | 3 | 4 | 3 | 6 | 4 | 4 | 2 | 2.4 | 3 | 2.4 | 2.4 | 3 | 2.4 | 4 | 2.4 | 3 | 3 | 2.4 | 3 | 3 | 2.4 | 2 | 3 | 6 | 2.4 | 3 | 1 | 2.4 | 3 | 6 | 3 | 2.4 | 2 | 2.4 | 3 | 2 | 2.4 | 2.4 |
| 6 | 2.4 | 2.4 | 2.4 | 3 | 2 | 3 | 4 | 3 | 6 | 3 | 4 | 4 | 3 | 2.4 | 2 | 2.4 | 4 | 2.4 | 3 | 2.4 | 3 | 2.4 | 2.4 | 3 | 2.4 | 2 | 3 | 3 | 3 | 2.4 | 6 | 3 | 2.4 | 1 | 6 | 3 | 2 | 2.4 | 3 | 2.4 | 2.4 | 2.4 | 2 | 3 |
| 6 | 2.4 | 2.4 | 3 | 2.4 | 3 | 2 | 4 | 3 | 4 | 4 | 3 | 6 | 2.4 | 3 | 2.4 | 2 | 3 | 2.4 | 4 | 2.4 | 3 | 2.4 | 2.4 | 3 | 2 | 2.4 | 3 | 3 | 6 | 3 | 3 | 2.4 | 3 | 6 | 1 | 2.4 | 2.4 | 2 | 2.4 | 3 | 2 | 3 | 2.4 | 2.4 |
| 6 | 2.4 | 2.4 | 2.4 | 3 | 3 | 2 | 3 | 4 | 4 | 4 | 6 | 3 | 2.4 | 2 | 2.4 | 3 | 2.4 | 4 | 2.4 | 3 | 2.4 | 3 | 3 | 2.4 | 3 | 3 | 2 | 2.4 | 2.4 | 3 | 3 | 6 | 6 | 3 | 2.4 | 1 | 2.4 | 3 | 2.4 | 2 | 2.4 | 2.4 | 3 | 2 |
| 2.4 | 6 | 2.4 | 3 | 4 | 3 | 2.4 | 2 | 3 | 2.4 | 3 | 2.4 | 4 | 3 | 6 | 4 | 4 | 2 | 2.4 | 3 | 2.4 | 3 | 6 | 2.4 | 3 | 2.4 | 3 | 3 | 2.4 | 3 | 3 | 2.4 | 2 | 3 | 2 | 2.4 | 2.4 | 1 | 2.4 | 3 | 6 | 3 | 2.4 | 2 | 2.4 |
| 2.4 | 6 | 2.4 | 4 | 3 | 2.4 | 3 | 2 | 3 | 4 | 2.4 | 3 | 2.4 | 6 | 3 | 4 | 4 | 3 | 2.4 | 2 | 2.4 | 3 | 2.4 | 6 | 3 | 3 | 2.4 | 2.4 | 3 | 2.4 | 2 | 3 | 3 | 2.4 | 2.4 | 2 | 3 | 2.4 | 1 | 6 | 3 | 2 | 2.4 | 3 | 2.4 |
| 2.4 | 6 | 2.4 | 4 | 3 | 3 | 2.4 | 3 | 2 | 3 | 2.4 | 4 | 2.4 | 4 | 4 | 3 | 6 | 2.4 | 3 | 2.4 | 2 | 6 | 3 | 3 | 2.4 | 3 | 2.4 | 2.4 | 3 | 2 | 2.4 | 3 | 3 | 2 | 3 | 2.4 | 2.4 | 3 | 6 | 1 | 2.4 | 2.4 | 2 | 2.4 | 3 |
| 2.4 | 6 | 2.4 | 3 | 4 | 2.4 | 3 | 3 | 2 | 2.4 | 4 | 4 | 3 | 4 | 4 | 6 | 3 | 2.4 | 2 | 2.4 | 3 | 2.4 | 3 | 3 | 6 | 2.4 | 3 | 3 | 2.4 | 3 | 3 | 2 | 2.4 | 2.4 | 2.4 | 3 | 2 | 6 | 3 | 2.4 | 1 | 2.4 | 3 | 2.4 | 2 |
| 2.4 | 2.4 | 6 | 2 | 3 | 3 | 4 | 3 | 2.4 | 2 | 2.4 | 3 | 2.4 | 2.4 | 3 | 2.4 | 4 | 3 | 6 | 4 | 4 | 3 | 3 | 2.4 | 2 | 3 | 6 | 2.4 | 3 | 2.4 | 3 | 3 | 2.4 | 3 | 2.4 | 2 | 2.4 | 3 | 2 | 2.4 | 2.4 | 1 | 2.4 | 3 | 6 |
| 2.4 | 2.4 | 6 | 2 | 3 | 4 | 3 | 2.4 | 3 | 3 | 2.4 | 2 | 2.4 | 4 | 2.4 | 3 | 2.4 | 6 | 3 | 4 | 4 | 2.4 | 2 | 3 | 3 | 3 | 2.4 | 6 | 3 | 3 | 2.4 | 2.4 | 3 | 2 | 2.4 | 3 | 2.4 | 2.4 | 2.4 | 2 | 3 | 2.4 | 1 | 6 | 3 |
| 2.4 | 2.4 | 6 | 3 | 2 | 4 | 3 | 4 | 2.4 | 2.4 | 3 | 2.4 | 2 | 3 | 4 | 4 | 2.4 | 4 | 4 | 3 | 6 | 2 | 2.4 | 3 | 3 | 6 | 3 | 3 | 2.4 | 3 | 2.4 | 2.4 | 3 | 2.4 | 2 | 2.4 | 3 | 2 | 3 | 2.4 | 2.4 | 3 | 6 | 1 | 2.4 |
| 2.4 | 2.4 | 6 | 3 | 2 | 3 | 4 | 2.4 | 3 | 2.4 | 2 | 2.4 | 3 | 2.4 | 4 | 2.4 | 3 | 4 | 4 | 6 | 3 | 3 | 3 | 2 | 2.4 | 2.4 | 3 | 3 | 6 | 2.4 | 3 | 3 | 2.4 | 2.4 | 3 | 2.4 | 2 | 2.4 | 2.4 | 3 | 2 | 6 | 3 | 2.4 | 1 |

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