Evaluating Fuzzy Reliability System using Intuitionistic Fuzzy Set

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Abstract
In this paper a fuzzy reliability of a different types of a systems is calculated by using a reduction method to series system and applying Intuitionistic rules of fuzzy Sets which deals with uncertainty and incomplete informations to calculate the fuzzy reliability via illustrative example is presented with conclusions.

Keywords : Fuzzy logic , reliability , mixed system , membership function , Intuitionistic fuzzy set .

الخلاصة
في هذه البحث، يتم حساب المعولية الضبابية لنوع مختلف من الأنظمة باستخدام طريقة الاختزال لنظم السلسلة وتطبيق قواعد مجموعة الحدس الضبابي كما في المثال التوضيحي مع الاستنتاجات.

الكلمات الدالة : المنطق الضبابي , المعولية , النظام المختلط , الدالة العضوية ، مجموعة الحدس الضبابي .

1. Introduction
The concept of Intuitionistic Fuzzy Set (IFS) is introduced by [9,10] which permit to incorporate simultaneously the membership degree and the non- membership degree of each element, Fuzzy logic is becoming more popular and resulting in many applications in several fields of real world including reliability theory. Reliability of a device is the probability that the device performs as a specified function under some specified condition during specified time period, while in real life situation it is impossible having accurate and complete information about the system. Therefore , in many cases it is very difficult to calculate the system reliability. To handle the complete information, the approach of Fuzzy Set theory can be used to estimate the system reliability.The idea of Fuzzy reliability has been proposed and developed by several authors like [2,9,10]

When based on possibility assumption or fuzzy state assumption. Recently the researchers [10]

Have proposed that the reliability of every component is a fuzzy variable and evaluating system reliability.

In this paper an intuitionistic fuzzy set with reduction method is presented to calculate the reliability of systems of different types, some illustrative examples are also presented.

2. Some definitions and concepts
In this section some definitions and concepts are presented .Fuzzy set theory is first introduced by [10]

Definition 2.1 [7]

The membership function of a classical fuzzy set assigns a number from [0 , 1] to each element of the universe of discourse to indicate the degree of belonging ness to the set under study .

Let X be the universe of discourse defined by $X = \{x_1, x_2, \dots \dots \dots . x_n\}$. The grades of membership in a fuzzy set defined on X indicates the evidence for $x_i \in$ X but do not indicates the evidence against $x_i \in$ X .
[9] introduces the concept of an IFS which is characterized by a membership function μ_A (X) and a non-membership function V_X(X) which seems to be useful in many life situations . An ***IFS*** $\tilde{A}$ on X is given by:
$= \{< x , \mu_{\tilde{A}}(x), V_{\tilde{A}}(x) >: x \in X\}$ which$\tilde{A}$
and $V_{\tilde{A}}(x): X \to [0 , 1]$ such that $\mu_{\tilde{A}}(x): X \to [0 , 1]$
$(x) \leq 1 , \; \forall \; x \in X \; 0 \leq \mu_{\tilde{A}}(x) + V_{\tilde{A}}$

Definition 2.2 [8]

The intuitionistic index $\pi_{\tilde{A}}(x)$ of X is the hesitancy of X in $\tilde{A}$ where

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - V_{\tilde{A}}(x) , x \in X$$

3. Reduction to series elements methods

In this article a reduction to series elements methods is applied to calculate the system reliability systematically and replace each parallel path by an equivalent single path and ultimately reduce the given system into one system consisting of only series elements .

The following example illustrate the above method for a question which is taken from [6] as shown in fig.(1) below .To calculate the system reliability , suppose that the components have the same reliabilities p = 0.8

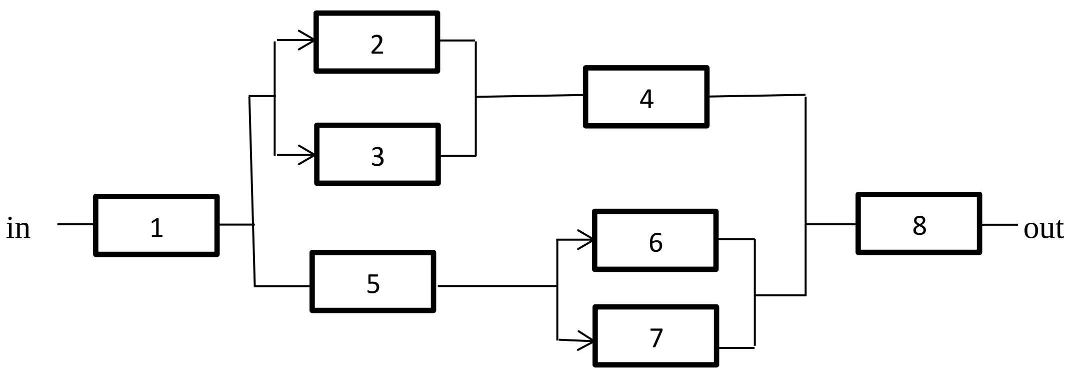

Fig (1) Block diagram of a system consists of 8 components

Solution :

The solution is explained by the following steps

Let the component (9) is equivalent to the component 2 and 3

$$R_9 = 1 - (1 - R_2)(1 - R_3)$$
$$= 1 - (1 - p)(1 - p)$$
$$= 1 - (1 - p)^2$$
$$= 1 - (1 - 0.8)^2 = 1 - (0.2)^2 = 0.96$$

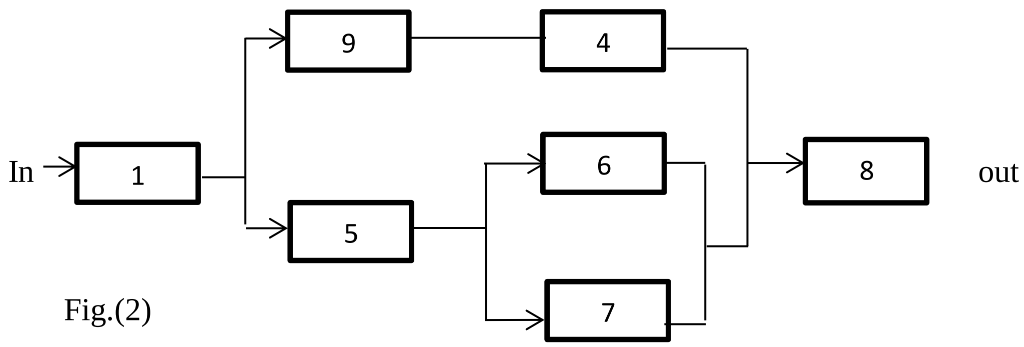

Fig.(2)

Let the component (10) is equivalent to the components 6 and 7

$$R_{10} = 1 - (1 - R_6)(1 - R_7)$$
$$= 1 - (1 - p)(1 - p)$$
$$= 1 - (1 - p)^2$$
$$= 1 - (1 - 0.8)^2$$
$$= 1 - (0.2)^2$$
$$= 0.96$$

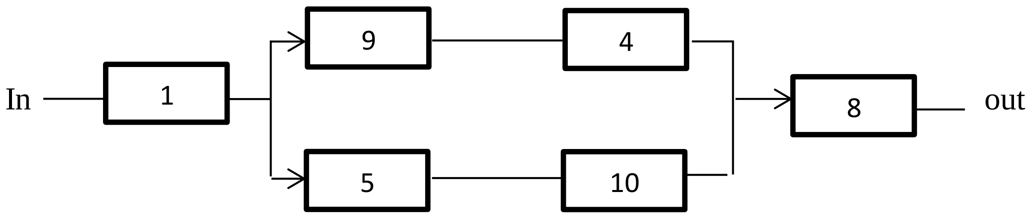

Fig (3)

Let the components (11) equivalent to the components 9 , 4 , 5 and 10 .

$$R_{11} = 1 - (1 - \prod_{i=1}^{2} R_i)^2$$
$$= 1 - (1 - R^2)^2$$
$$= 1 - \left(1 - (0.8)(0.8)\right)^2$$

$= 1 - (1 - 0.64)^2$

$= 1 - (0.36)^2$

$= 1 - 0.1296$

$= 0.8704$

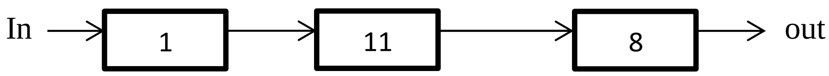

Fig (4) shows the final step

$R_S = R_1 * R_{11} * R_8$

$= 0.8 * 0.8704 * 0.8$

$= 0.556$

Definition 3.1 [7,8]

Let A be a non-empty set . A fuzzy set A drawn from X is defined as $A = \{< x , \mu_A(x) >: x \in X \}$ where $\mu_A(x): X \longrightarrow [0,1]$ is the membership function of the fuzzy set A .

Fuzzy set is a collection of objects with graded membership i.e having degrees of membership .

Definition 3.2 [8,9]

Let X be a non-empty set . An intuitionistic fuzzy set A in X is an object having the from $= \{< x , \mu_A(x), V_A(x) >: x \in X \}$, where the functions $\mu_A(x), V_A(x): X \longrightarrow [0,1]$ defined respectively , the degree of membership and degree of non-membership of the element $x \in X$ to the set A . which is a subset of X . and for every element $x \in X$

$. 0 \leq \mu_A(x) + V_A(x) \leq 1$

Furthermore , we have $\prod A(x) = 1 - \mu_A(x) - V_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of X in A .

is the degree of indeterminacy of $x \in X$ to the IFS A and $\prod A(x) \in [0,1]$ i.e , $\prod A(x)$ $\prod A(x): X \longrightarrow [0,1]$ and $0 \leq \prod A \leq 1$ for every $x \in X$, $\prod A\,(x)$ expresses the lack of knowledge of whether X belongs to IFS A or not .

For example , let A is an intuitionistic fuzzy set with :

and $V_A(x) = 0.2$ then , $\mu_A(x) = 0.4$

$\prod A\,(x) = 1 - (0.4 + 0.2) = 0.4$. It can be interpreted as " the degree that the object X belongs to IFS A is 0.4 the degree that the object does not belong to IFS A is 0.2 and the degree of hesitancy is 0.4 " .

If A, B be IFS in X then ,

a) [inclusion] $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $V_A(x) \geq V_B(x)$, $\forall\, x \in X$.
b) [equality] $A = B \Leftrightarrow \mu_A(x) = \mu_B(x)$ and $V_A(x) = V_B(x)$, $\forall\, x \in X$.
c) [complement] $A^c = \{< x , V_A(x), \mu_A(x) >: x \in X \}$.
d) [union]$A \cup B = \{x , \max(\mu_A(x), \mu_B(x)) , \min(V_A(x), V_B(x)) >: x \in X \}$
e) [intersection]$A \cap B = \{< x , \min(\mu_A(x), \mu_B(x)), \max(V_A(x), V_B(x)) >: xX \}$
f) [addition]$A \oplus B \{x , \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x) , V_A(x)V_B(x): xX \}$
g) [multiplication]$A \otimes B = \{< x , \mu_A(x)\mu_B(x), V_A(x) + V_B(x) - V_A(x)V_B(x): x \in X \}$.
h) [difference]$A - B = \{< x , \min(\mu_A(x), V_B(x)) , \max(V_A(x), \mu_B(x)) >: x \in X \}$
i) [Cartesian product] $A \times B = \{< \mu_A(x)\, \mu_B(x) , V_A(x)V_B(x) >: x \in X \}$

Definition 3.3 [2,7]

Triangle intuitionistic fuzzy number (TIFN)

An IFS denoted by $\tilde{A} =< [(a, b, c);\ \mu , \nu\ >$, where $a , b , c \in R$ is said to be $TIFN$ if its membership function is given by :

$$\mu_{\tilde{A}(x)} = \begin{cases} \mu\left(\dfrac{x-a}{b-a}\right) & , a \leq x \leq b \\ \mu & , x = b \\ \mu\left(\dfrac{c-x}{c-b}\right) & , b \leq x \leq c \\ 0 & , other\ wise \end{cases}$$

$$1 - V_{\tilde{A}(x)} = \begin{cases} (1-v)\left(\frac{x-a}{b-a}\right) & , a \le x \le b \\ 1-v & , x = b \\ (1-v)\left(\frac{c-x}{c-b}\right) & , b \le x \le c \\ 0 & , other\ wise \end{cases}$$

Where the parameter b gives the modal value of A that is $\mu_{\tilde{A}}(b) = 1$, and a, c are the lower and upper bounds of available area for the evaluation data . A triangular IFS is defined by the triple (a, b, c) with α −cuts is defined below and shown graphically in fig. (5) :

$$A^{(\alpha)} = \left[a^{(\alpha)}, c^{(\alpha)}\right]$$

$$A^{(1-\alpha)} = \left[a^{(1-\alpha)}, c^{(1-\alpha)}\right]$$

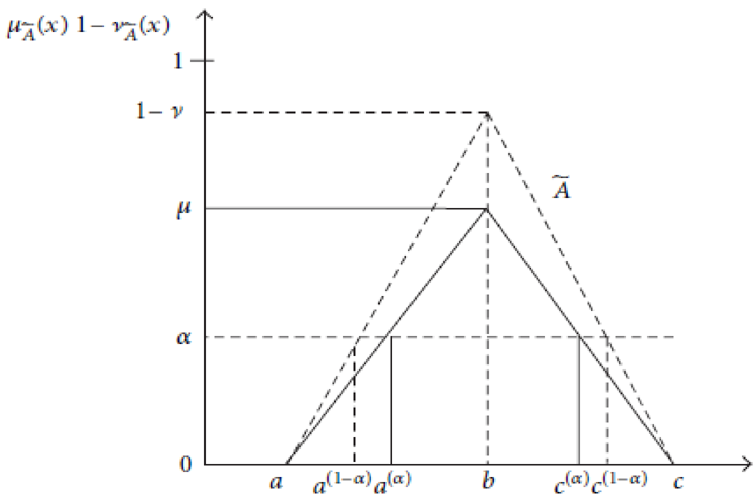

Fig . (5) α −cut of IFS $\tilde{A}$

Where

$$a^{(\alpha)} = a + \frac{\alpha}{\mu_i}(b-a)$$

$$a^{(1-\alpha)} = a + \frac{\alpha}{\mu_i}(b-a)$$

$$c^{(\alpha)} = c + \frac{\alpha}{1-v_i}(c-b)$$

$$c^{(1-\alpha)} = c - \frac{\alpha}{1-v_i}(c-b)$$

The four basic arithmetic operations that is addition subtraction multiplication and division on two triangular vague sets $\tilde{A} =< (a_1, b_1, c_1); \mu_1, v_1 >$ and $\tilde{B} =< (a_2, b_2, c_2); \mu_2, v_2 >$ with $\mu = \min(\mu_1, \mu_2)$ and $v = \min(v_1, v_2)$ are given in the above reference .

Now we return to solve the above example to calculate the fuzzy reliability of the system below for values of α, β as in table below

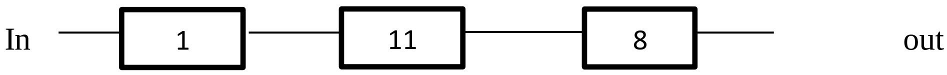

Fig.(6) The final reduction of Fig.(1)

Definition 3.4 [11]

Let components are connected in series configuration (fig.7)

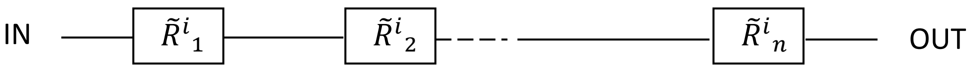

Figure (7) shows series system

The fuzzy Reliability of series system is :

$$\tilde{R}^i{}_s = \tilde{\varphi}\left(\tilde{R}^i{}_1, \tilde{R}^i{}_2, \ldots \tilde{R}^i{}_n\right) = \prod_{j=1}^{n} \tilde{R}^i{}_j$$

For membership function α −cut of $\tilde{R}^i{}_s$ is

$$\tilde{R}^i{}_{s\alpha} = [\varphi_{s\alpha}{}^L, \varphi_{s\alpha}{}^R] \qquad \forall \alpha \in (0,1]$$

Where

$$, \varphi_{s\alpha}{}^R = max \prod_{j=1}^{n} x_j \ \varphi_{s\alpha}{}^L = min \prod_{j=1}^{n} x_j$$

$$\text{s.t} \begin{cases} r^L{}_{1\alpha} \le x_1 \le r^R{}_{1\alpha} \\ r^L{}_{2\alpha} \le x_2 \le r^R{}_{2\alpha} \\ \quad\quad . \\ \quad\quad . \\ \quad\quad . \\ r^L{}_{n\alpha} \le x_n \le r^R{}_{n\alpha} \end{cases}$$

For non-membership function β −cut of $\tilde{R}^i{}_s$ is $\tilde{R}^i{}_{s\beta} = \left[\varphi_{s\beta}{}^L, \varphi_{s\beta}{}^R\right] \qquad \forall \beta \in [0,1)$

Where

$$, \varphi_{s\beta}{}^{R} = max \sum_{j=1}^{n} x_j \varphi_{s\beta}{}^{L} = min \sum_{j=1}^{n} x_j$$

$$\text{s.t} \begin{cases} r^{L}{}_{1\beta} \le x_1 \le r^{R}{}_{1\beta} \\ r^{L}{}_{2\beta} \le x_2 \le r^{R}{}_{2\beta} \\ \quad\quad . \\ \quad\quad . \\ \quad\quad . \\ r^{L}{}_{n\beta} \le x_n \le r^{R}{}_{n\beta} \end{cases}$$

α, β	$\widetilde{\boldsymbol{R}}^{\boldsymbol{i}}{}_{\boldsymbol{\alpha}} = [\boldsymbol{\varphi_\alpha}{}^{\boldsymbol{L}}, \boldsymbol{\varphi_\alpha}{}^{\boldsymbol{R}}]$	$\widetilde{\boldsymbol{R}}^{\boldsymbol{i}}{}_{\beta} = [\boldsymbol{\varphi}_{\beta}{}^{\boldsymbol{L}}, \boldsymbol{\varphi}_{\beta}{}^{\boldsymbol{R}}]$
0	[0.00204, 0.018532]	[0.005226, 0.010118]
0.1	[0.002399, 0.01747]	[0.004543, 0.011687]
0.2	[0.00278, 0.016438]	[0.003904, 0.013366]
0.3	[0.003183, 0.015436]	[0.00331, 0.015155]
0.4	[0.003609, 0.014463]	[0.002762, 0.017055]
0.5	[0.004057, 0.013521]	[0.002259, 0.019063]
0.6	[0.004527, 0.012609]	[0.001804, 0.02118]
0.7	[0.005018, 0.011728]	[0.001397, 0.023406]
0.8	[0.005531, 0.010877]	[0.001039, 0.025739]

Table (1)α *and* β cuts of system reliability$\widetilde{\boldsymbol{R}}^{\boldsymbol{i}}$

The fuzzy system reliability

$\tilde{R}^{i}{}_{1} = \{0.02, 0.03, 0.04, 0.03, 0.05, 0.07\}$

$\tilde{R}^{i}{}_{11} = \{0.03, 0.05, 0.07, 0.04, 0.09, 0.12\}$

$\tilde{R}^{i}{}_{8} = \{0.02, 0.04, 0.06, 0.01, 0.04, 0.08\}$

and $\tilde{R}^{i}{}_{s} = \tilde{R}^{i}{}_{1} \otimes \tilde{R}^{i}{}_{11} \otimes \tilde{R}^{i}{}_{8}$

Conclusion

In this paper a reduction method is applied to calculate the fuzzy reliability for a complex system using intuitionistic fuzzy sets.An illustrative examples is solved with two cases ,the first is the classical reliability and the fuzzy reliability by applying the operations an intuitionistic fuzzy sets to evaluate the reliability of the system .

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