Comparing Between Some Methods for Estimating Reliability Function of Gamma Distribution

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مقارنة بين بعض طرائق تقدير المعولية لأنموذج كاما

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المستخلص:

في هذا البحث تم تقدير معلمة القياس ودالة المعولية لتوزيع كاما ذي المعلمتين وهو أحد نماذج الفشل الشائعة في حقل المعولية واختبارات الحياة، تَركّزَ هذا البحث على المقارنة ما بين بعض طرائق التقدير وطريقتين الخلط (P)، طريقة بيتمان (SB)وطريقة البيز القياسية (MLE)المعروفة (طريقة الاحتمالالأعظم بين طريقة الاحتمال الأعظم وطريقة بيتمان، والطريقة الثانية (MIX I)).الطريقة الأولى سُميَت (mixture لمعلمة القياس ودالة المعولية له..بين طريقة البيز القياسية وطريقة بيتمان(MIX II)سُميَت

إن منهجية البحث تعتمد على دراسة نظرية فقد تم اشتقاق طرائق التقدير الاعتيادية والبيزية وبشكل تفصيلي للتوصل إلى صيغ مقدّرات هذه الطرائق وصيغ مقدّرات دالة المعولية لها.

(Simulation) كذلك اعتمد البحث على دراسة تجريبية عن طريق تصميم عدد من تجارب المحاكاة باستخدام تنويعة من قيم المعلمة وحجوم العينات وكررت التجربة للحصول على تجانسٍ عالٍ وذلك لأغراض المقارنة ما بين طرائق التقدير.

والإمكانالأعظم) كأفضل طريقتين بين طرائق التقدير لتقدير دالة (MIX II)وقد تمَّ التوصل إلى طريقتي (المعولية باستخدام المقياسيين الإحصائيين التاليين:-

1-(IMSE)Integral Mean Squared Error متوسط مربعات الخطأ التكاملي

2-(IMAPE)Integral Mean Absolute Percentage Error متوسط الخطأ النسبي المطلق التكاملي

الكلمات الاستدلالية: توزيع كاما, دالة المعولية, دالة الفشل, مقدر بيتمان, دالة الخسارة, متوسط مربعات الخطأ التكاملي, متوسط الخطأ النسبي المطلق.

Abstract:

This research has studied some different estimation methods for estimation the reliability function for the two parameter Gamma distribution. The methods are: Maximum likelihood Estimator Method (MLE), Standard Bayesian Method (SB), Pitman Method (P), and two suggested mixture methods for estimation; the first Mixture Method between Maximum Likelihood Estimator Method and Pitman Method (MIX I), and the second Mixture Method between Standard Bayesian Method and Pitman Method (MIX II).

Comparison between the estimation methods of estimating the reliability function has been made using two important statistical measures: Integral Mean Square Error (IMSE) and the Integral Mean Absolute Percentage Error (IMAPE), to find the best method through Monte Carlo simulation. Simulation examples are worked out, where the generation of random data, depending on the different sample sizes and run size (L=1000). The second suggested method (Mix II) and (MLE) method were found to be the best methods for estimation the reliability function.

Key Words:Gamma Distribution, Reliability Function, Failure Function, Pitman Estimator, Loss Function, Integral Mean Square Error (IMSE) , Integral Mean Absolute Percentage Error (IMAPE).

1. Introduction:

Researches and modern studies on reliability subject have given a great attention as a result of the role played by this field of statistics in dealing with ages, whether equipment or living organisms, both of reliability theory and survival theory share in measuring the length of life period, whether it is machine, system or living organism. The differences occur in the optimization of reliability system in multiple parts of the regulations because such optimization is the number and the locations of parts of the system and easily find a replacement for these parts and fast processing to make it optimum, while in the survival theory, there is no such optimization because the system here is a living organism where the difficulty and rarity lies in its arrangement to reach its optimum.

Attention to the issue of reliability has increased after a wideexpansion of industry and the increasing complexity of mechanical, electrical and electronic parts in equipments in the last century. The equipments and researches before 1940 or pre-World War II were gave a take care to quality control and maintenance of the machines and the reliability was not recognized as separate field. In the beginning of World War II and the complexity of the machinery and military equipment, the reliability field has become as an independent entity,rapid technological developments and the use of machinery and complex systems in various areas of life, such as medicine, communications fields, space researches, military operations and others caused a significant attention in studying the causes of engine troubles, faults and sudden stops of devices or machines that may occur in the work which lead to material losses due to increasing costs and decreasing the production.(Dhillon,1999) & (Mishra & Ankit,2009)

The main aims for this research are:

To use some estimation methods and two suggested methods (which are funded by a new formula to obtain an estimator which have good characteristics in terms of small mean square error; through mixing the Pittman estimator with Bayesian and maximum likelihood estimators), and comparing them using simulations to arrive at the best estimation method, by utilizing two statistical measures, namely Integral Mean Squared Error (IMSE) and Integral Mean absolute percent Error (IMAPE).

2. Related Work

In 1969, (Choi & Wette) examined the numerical technique of the maximum likelihood method to estimate the parameters of Gamma distribution. (Choi & Wette, 1969)

In 1980, (Miller) presented a Bayesian analysis of shape, scale, and mean of the two-parameter gamma distribution. Attention is given to conjugate and "non-informative" priors, to simplifications of the numerical analysis of posterior distributions. (Miller, 1980)

In 2000, (Coit & Jin) developed Maximum likelihood estimators have been for the gamma distribution when there is missing time-to-failure information. (Coit & Jin, 2000)

In 2007, (Freue) used The Pitman estimator of the Cauchy location parameter when the scale parameter is known. Using the squared error loss function, a closed form of the minimum risk equivariant (MRE) estimator. (Freue, 2007)

In 2007, (Akahira, Ohyauchi and Takeuchi) are used the Pitman estimator to obtain the asymptotic expansion of the Pitman estimator and its asymptotic variance. In a nonregular case when the density has an unbounded support. (Akahira, Ohyauchi, & Takeuchi, 2007)

In 2010, (Jasim) derived Bayes' estimator for the Scale parameter in Gamma distribution when the shape parameter is known, depending on squared error and LINEX loss function, then comparisons of risks for scale parameter under squared and LINEX loss function have been made. (Jasim, 2010)

In the same year, (Kishan) compared between maximum likelihood estimator(MLE) and Bayes estimator of scale parameter of Generalized gamma distribution under Squared error loss function when shape parameters are known. (Kishan, 2014).

3. Background Information:

This section studied some basic concepts of reliability; two parameter gamma distributions; several estimation methods to estimate the reliability function.

3.1: Failure Function

Failure function is a basic (logistic) reliability measure and is defined as the probability that an item will fail before or at the moment of operating time t. Here time t is used in a generic sense and it can have units such as miles, number of landings, flying hours, number of cycles, etc., depending on the operational profile and the utilization of the system. That is, Failure function is equal to the probability that the time-to-failure random variable will be less than or equal a particular value t. The failure function is usually represented as F(t). (Kumar, Crocker, Knezevic & El-Haram, 2000)

$$F(t) = \Pr[T \le t] = \int_0^t f(s)\,ds \ldots (1)$$

Where;

f(s): probability density function

3.2: Reliability Function

Reliability can be defined as the probability of non-failure. If $F(t)$ is the failure probability; then $[1 - F(t)]$ gives the non-failure probability. Thus, the reliability of device for time T = t (i.e., the device functions satisfactorily for $T \ge t$) is

$$R(t) = \Pr[T > t] = \int_t^{\infty} f(s)ds \ldots (2)$$

$$R(t) = 1 - F(t) \ldots (3)$$

Corresponding to reliability function R(t), F(t) is the unreliability function and represented by Q(t). The probability density f(t) was defined as the derivative of the failure distribution function F(t). Since F(t) = 1 – R(t). (Mishra & Sandilya, 2009)

3.2.1: Properties of Reliability Function:

1. Reliability is a decreasing function with time t. That is, for $t_1 < t_2$; $R(t_1) \ge R(t_2)$.

2. It is usually assumed that $R(0) = 1$. As t becomes larger and larger R(t)approaches zero, that is, $R(\infty) = 0$. (Kumar, Crocker, Knezevic & El-Haram, 2000)

3.3: Hazard Function

A useful concept in reliability theory to describe failures in a system and its components is the failure rate. It is defined as the probability that a failure per unit time occurs in the interval, say, $[t, t + \Delta t]$, given that a failure has not occurred before t. In other words, the failure rate is the rate at which failures occur in $[t, t + \Delta t]$. Or is defined as the limit of the failure rate as the interval approaches zero, that is, $\Delta t \rightarrow 0$. Thus, we obtain the hazard rate at time t as:(Lyu, 1995)

$$h(t) = \lim_{\Delta t \to 0}\left[\frac{P_r(t \le T < t + \Delta t | T > t)}{\Delta t}\right]$$

$$= \lim_{\Delta t \to 0}\left[\frac{P_r(t \le T < t + \Delta t)}{\Delta t}\right] * \frac{1}{P_r(T > t)}$$

$$= \frac{1}{R(t)} * \lim_{\Delta t \to 0}\left[\frac{F(t + \Delta t) - F(t)}{\Delta t}\right] = \frac{1}{R(t)} * \frac{dF(t)}{dt}$$

$$h(t) = \frac{f(t)}{R(t)} \ldots (4)$$

3.4: Gamma Distribution

The two-parameter gamma distribution has been used quite extensively in reliability and survival analysis particularly when the data are not censored. Gamma distribution is an exciting extension of exponential distribution. It is of limited use in survival analysis because the gamma models do not have closed-form expressions for survival and hazard functions. Both include the incomplete gamma integral

$$I_\alpha(x) = \frac{1}{\Gamma\alpha}\int_0^x y^{\alpha-1}e^{-y}\,dy$$

. If variable T follows a gamma distribution with shape parameter α and scale parameter $\theta(T \sim \Gamma(\alpha,\theta))$, then the following relations hold

Probability density function:

$$f(t;\alpha,\theta) = \begin{cases} \frac{1}{\Gamma\alpha\theta^\alpha}t^{\alpha-1}e^{-\frac{t}{\theta}} & ; 0 < t < \infty \\ & \alpha,\theta > 0 \\ 0 & elsewhere \end{cases} \quad \dots (5)$$

Reliability function:

$$R(t) = \sum_{j=0}^{\alpha-1}\frac{\left(\frac{t}{\theta}\right)^j e^{-\frac{t}{\theta}}}{j!} \dots (6)$$

Failure function:

$$F(t) = \sum_{j=\alpha}^{\infty}\frac{\left(\frac{t}{\theta}\right)^j e^{-\frac{t}{\theta}}}{j!} \dots (7)$$

Hazard function:

$$h(t) = \frac{\frac{1}{\Gamma\alpha\theta^\alpha}t^{\alpha-1}}{\sum_{j=0}^{\alpha-1}\frac{(\frac{t}{\theta})^j}{j!}} \dots (8)$$

Here $\Gamma(\alpha)$ is the gamma function and it is expressed as

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\,dx = (\alpha-1)!$$

4. Estimation methods for the reliability function

4.1: Maximum Likelihood Estimation Method

Let $(t_1,\dots,t_n)$ be the set of n random lifetime from Gamma distribution with parameters α and θ.

The probability density function of Gamma distribution is given by:

$$f(t;\alpha,\theta) = \frac{1}{\Gamma\alpha\theta^{\alpha}} t^{\alpha-1} e^{-\frac{t}{\theta}}$$

Hence, the MLE of θ is:

$$\hat{\theta}_{MLE} = \frac{\bar{t}}{\alpha} \dots (9)$$

The MLE of the reliability function:

$$\hat{R}(t) = \sum_{j=0}^{\alpha-1} \frac{\left(\frac{t}{\hat{\theta}_{MLE}}\right)^{j} e^{-\frac{t}{\hat{\theta}_{MLE}}}}{j!} \dots (10)$$

4.2: Standard Bayes Estimation Method

Consider the two parameter gamma distribution

$$f(t;\alpha,\theta) = \frac{1}{\Gamma\alpha\theta^{\alpha}} t^{\alpha-1} e^{-\frac{t}{\theta}}$$

We find Jeffery prior by taking $P(\theta) \propto \sqrt{I_x(\theta)}$, where

$$I_x(\theta) = -E\left(\frac{\partial^2 \ln L(\theta)}{\partial\theta^2}\right) = \frac{n\alpha}{\theta^2}$$

Taking $P(\theta) = \frac{k\sqrt{n\alpha}}{\theta}$, with k a constant

The joint probability density function $f(t_1, t_2, \dots, t_n; \alpha, \theta)$ is given by

$$H(t_1, t_2, \dots, t_n; \alpha, \theta) == \frac{K\sqrt{n}\alpha}{(\Gamma\alpha)^n} \theta^{-(n\alpha+1)} \prod_{i=1}^{n} t_i^{\alpha-1} e^{-\frac{\sum_{i=1}^{n} t_i}{\theta}} \dots (11)$$

The marginal probability density function of $(t_1, t_2, \dots, t_n; \alpha, \theta)$ is given by:

$$P(t_1, t_2, \dots, t_n) = \int_0^{\infty} H(t_1, t_2, \dots, t_n; \alpha, \theta)\, d\theta$$

$$= \frac{K\sqrt{n}\alpha}{(\Gamma\alpha)^n} \prod_{i=1}^{n} t_i^{\alpha-1} \frac{\Gamma n\alpha}{(\sum_{i=1}^{n} t_i)^{n\alpha}} \dots (12)$$

The conditional probability density function of θ given the data $(t_1, t_2, \dots, t_n; \alpha, \theta)$ is called posterior distribution of θ, given by

$$h(\theta | t_1, t_2, \dots, t_n) = \frac{H(t_1, t_2, \dots, t_n; \alpha, \theta)}{P(t_1, t_2, \dots, t_n)} \cdots (13)$$

By substituting equation (11) and (12) in equation (13) we have:

$$h(\theta | t_1, t_2, \dots, t_n) = \begin{cases} \frac{(\sum_{i=1}^{n} t_i)^{n\alpha}}{\Gamma n\alpha} \theta^{-(n\alpha+1)} e^{-\frac{\sum_{i=1}^{n} t_i}{\theta}} & t > 0 \dots (14) \\ 0 & o \cdot w \end{cases}$$

By using quadratic error loss function $L(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2$, we can obtain the Risk function, such that

$$R_s(\hat{\theta},\theta) = E[L(\hat{\theta},\theta)] = E\left[c(\hat{\theta}-\theta)^2\right]$$

$$= \int_0^\infty c(\hat{\theta}-\theta)^2\, h(\theta|t_1,t_2,\dots,t_n)d\theta$$

$$= c\hat{\theta}^2 - 2c\hat{\theta}E(\theta|t_1,t_2,\dots,t_n) + E(\theta^2|t_1,t_2,\dots,t_n)$$

$$\frac{\partial R_s(\hat{\theta},\theta)}{\partial\hat{\theta}} = 2c\hat{\theta} - 2cE(\theta|t_1,t_2,\dots,t_n)$$

Let $\frac{\partial R_s(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, then the Bayes estimator is

$$\hat{\theta}_{SB} = E(\theta|t_1,t_2,\dots,t_n) = \int_0^\infty \theta\, h(\theta|t_1,t_2,\dots,t_n)\, d\theta$$

Substituting Equation (14):

$$\hat{\theta}_{SB} = \frac{\sum_{i=1}^n t_i}{n\alpha-1} = \frac{n}{n\alpha-1}\bar{t} \cdots (15)$$

The Standard Bayes estimator of the Reliability function:

$$\hat{R}_{SB}(t) = E(R(t)|t_1,t_2,\dots,t_n) = \int_0^\infty R(t)\, h(\theta|t_1,t_2,\dots,t_n)d\theta$$

$$= \int_0^\infty \sum_{j=0}^{\alpha-1} \frac{(\frac{t}{\theta})^j\, e^{-\frac{t}{\theta}}}{j!} \frac{(\sum_{i=1}^n t_i)^{n\alpha}}{\Gamma n\alpha} \theta^{-(n\alpha+1)} e^{-\frac{\sum_{i=1}^n t_i}{\theta}} d\theta$$

$$= \frac{(\sum_{i=1}^n t_i)^{n\alpha}}{\Gamma(n\alpha)} \sum_{j=0}^{\alpha-1} \frac{(t)^j}{j!} \int_0^\infty \theta^{-(n\alpha+j+1)} e^{-\frac{(t+\sum_{i=1}^n t_i)}{\theta}}\, d\theta$$

set $y = \frac{(t+\sum_{i=1}^n t_i)}{\theta}$, $\Rightarrow \theta = \frac{(t+\sum_{i=1}^n t_i)}{y}$ and $d\theta = \frac{t+\sum_{i=1}^n t_i}{y^2}dy$

Then, the Standard Bayes estimator of the Reliability function is:

$$\hat{R}_{SB}(t) = \frac{(\sum_{i=1}^n t_i)^{n\alpha}}{\Gamma(n\alpha)} \sum_{j=0}^{\alpha-1} \frac{(t)^j\Gamma(n\alpha+j)}{j!\ (t+\sum_{i=1}^n t_i)^{n\alpha+j}} \cdots (16)$$

4.3: Pitman Estimator for the Scale Parameter of the Gamma Distribution

Let t has Gamma(α,θ) distribution then:

$$f(t;\alpha,\theta) = \frac{1}{\Gamma\alpha\theta^\alpha} t^{\alpha-1} e^{-\frac{t}{\theta}}$$

The pitman estimator $\hat{\theta}_p$ of the scale parameter θ is given by:

$$\hat{\theta}_P = \frac{\int_0^\infty \frac{1}{\theta^2} L(\theta) d\theta}{\int_0^\infty \frac{1}{\theta^3} L(\theta) d\theta}$$

$$= \frac{\int_0^\infty \frac{1}{\theta^2} \frac{1}{\theta^{n\alpha}} e^{-\frac{\sum_{i=1}^n t_i}{\theta}} d\theta}{\int_0^\infty \frac{1}{\theta^3} \frac{1}{\theta^{n\alpha}} e^{-\frac{\sum_{i=1}^n t_i}{\theta}} d\theta}$$

$$= \frac{\sum_{i=1}^n t_i}{n\alpha + 1} = \frac{n}{n\alpha + 1} \bar{t}$$

Then, the pitman estimator for the parameter θ is:

$$\hat{\theta}_P = \frac{n}{n\alpha + 1} \bar{t} \cdots (17)$$

The Pitman estimator of the Reliability function:

$$\hat{R}_P(t) = \sum_{j=0}^{\alpha-1} \frac{\left(\frac{t}{\hat{\theta}_P}\right)^j e^{-\frac{t}{\hat{\theta}_P}}}{j!} \cdots (18)$$

4.4: First Mixture Method of MLE and Pitman (Mix I)

This method is obtained from mixing two estimators which are maximum likelihood estimator and pitman estimator, the aim of this estimator is to find an estimator that minimizes the MSE.

$$\hat{\theta}_{Mix\ I} = P\hat{\theta}_{MLE} + (1-P)\hat{\theta}_P$$

We have to find the value of P which makes the MSE has minimum value According to the following steps:

Subtracting θ from both sides

$$\hat{\theta}_{Mix\ I} - \theta = \left[P\hat{\theta}_{MLE} + (1-P)\hat{\theta}_P\right] - \theta$$

Squaring the both sides and taking the expectation

$$E\left(\hat{\theta}_{Mix\ I} - \theta\right)^2 = E\{\left[P\hat{\theta}_{MLE} + (1-P)\hat{\theta}_P\right] - \theta\}^2$$

$$= P^2 E\left(\hat{\theta}_{MLE}\right)^2 + 2P(1-P)E\left(\hat{\theta}_{MLE}\right)E\left(\hat{\theta}_P\right) + (1-P)^2 E\left(\hat{\theta}_P\right)^2$$
$$-2PE\left(\hat{\theta}_{MLE}\right)E(\theta) - 2(1-P)E\left(\hat{\theta}_P\right)E(\theta) + E(\theta)^2$$

To find the minimizing value

$$\frac{dE\left(\hat{\theta}_{Mix\ I} - \theta\right)^2}{dP} = 2PE\left(\hat{\theta}_{MLE}\right)^2 + (2-4P)E\left(\hat{\theta}_{MLE}\right)E\left(\hat{\theta}_P\right)$$
$$-2(1-P)E\left(\hat{\theta}_P\right)^2 - 2E\left(\hat{\theta}_{MLE}\right)E(\theta) + 2E\left(\hat{\theta}_P\right)E(\theta)$$

setting $\frac{dE\left(\hat{\theta}_{Mix\ I}-\theta\right)^2}{dP}=0,$

$E\left(\hat{\theta}_{MLE}\right)=E\left(\frac{\sum_{i=1}^{n}t_i}{n\alpha}\right)=\theta,\qquad E\left(\hat{\theta}_P\right)=E\left(\frac{\sum_{i=1}^{n}t_i}{n\alpha+1}\right)=\frac{n\alpha\theta}{n\alpha+1},$

$E(\theta)=\theta,\qquad E\left(\hat{\theta}_{MLE}\right)^2=\frac{n\alpha+1}{n\alpha},\ E\left(\hat{\theta}_P\right)^2=\frac{n\alpha}{n\alpha+1}$

By simplifying the value of P: $P=\frac{n\alpha}{2n\alpha+1}$

Then the Mix I estimator for the parameter θ is:

$$\hat{\theta}_{Mix\ I}=\frac{2\sum_{i=1}^{n}t_i}{2n\alpha+1}\cdots(19)$$

Then, the Mix I estimator of the Reliability function is given by:

$$\hat{R}_{Mix\ I}(t)=\sum_{j=0}^{\alpha-1}\frac{\left(\frac{t}{\hat{\theta}_{Mix\ I}}\right)^j e^{-\frac{t}{\hat{\theta}_{Mix\ I}}}}{j!}\cdots(20)$$

4.5: Second Mixture Method of SB and Pitman (Mix II)

This method is obtained from mixing two estimators which are Standard Bayesian estimator and pitman estimator to find an estimator that minimizes the MSE

$\hat{\theta}_{Mix\ II}=P\hat{\theta}_{SB}+(1-P)\hat{\theta}_P$

We have to find the value of P which makes the MSE has minimum value According to the following steps:

Subtracting θ from both sides

$\hat{\theta}_{Mix\ II}-\theta=\left[P\hat{\theta}_{SB}+(1-P)\hat{\theta}_P\right]-\theta$

Squaring the both sides and taking the expectation

$$E\left(\hat{\theta}_{Mix\ II}-\theta\right)^2=E\{\left[P\hat{\theta}_{SB}+(1-P)\hat{\theta}_P\right]-\theta\}^2$$
$$=P^2E\left(\hat{\theta}_{SB}\right)^2+2P(1-P)E\left(\hat{\theta}_{SB}\right)E\left(\hat{\theta}_P\right)+(1-P)^2E\left(\hat{\theta}_P\right)^2-$$
$$2PE(B)E(\theta)-2(1-P)E\left(\hat{\theta}_P\right)E(\theta)+E(\theta)^2$$

To find the minimizing value

$$\frac{dE\left(\hat{\theta}_{Mix\ II}-\theta\right)^2}{dP}=2PE\left(\hat{\theta}_{SB}\right)^2+(2-4P)E\left(\hat{\theta}_{SB}\right)E\left(\hat{\theta}_P\right)$$
$$-2(1-P)E\left(\hat{\theta}_P\right)^2-2E\left(\hat{\theta}_{SB}\right)E(\theta)+2E\left(\hat{\theta}_P\right)E(\theta)=0$$

And setting $\frac{dE\left(\hat{\theta}_{Mix\ I}-\theta\right)^2}{dP}=0$

By simplifying the value of P: $P=\frac{n\alpha-1}{2(n\alpha+1)}$

Then the Mix II estimator for the parameter θ is:

$$\hat{\theta}_{Mix\ II} = \frac{(n\alpha + 2)\sum_{i=1}^{n} t_i}{(n\alpha + 1)^2} \cdots (21)$$

Then, the Mix II estimator of the Reliability function is given by:

$$\hat{R}_{Mix\ II}(t) = \sum_{j=0}^{\alpha-1} \frac{\left(\frac{t}{\hat{\theta}_{Mix\ II}}\right)^j e^{-\frac{t}{\hat{\theta}_{Mix\ II}}}}{j!} \cdots (22)$$

5: Statistical Measures For Evaluating The Reliability Function

5.1: Integral Mean Square Error (IMSE)

The fact that (MSE) will be calculated for each (t_i) of time, the (IMSE) will represents the integration of the area's for (t_i) and its reduce to a single value is calculated for general time, or it will express to total time, and the formula of this measure will be as follow:

$$\mathrm{IMSE}\left[\hat{R}(t)\right] = \frac{1}{L}\sum_{j=1}^{L}\left\{\frac{1}{n_t}\sum_{i=1}^{n_t}\left[\hat{R}_j(t_i) - R(t_i)\right]^2\right\} = \frac{1}{n_t}\sum_{i=1}^{n_t}\mathrm{MSE}\left[\hat{R}(t_i)\right] \quad ,j = 1,2,\dots,L \quad \cdots (24)$$

5.2: Integral Mean Absolute Percentage Error (IMAPE)

This measure is calculated according to the following formula:

$$\mathrm{IMAPE}\left[\hat{R}(t)\right] = \frac{1}{L}\sum_{j=1}^{L}\left\{\frac{1}{n_t}\sum_{i=1}^{n_t}\left|\frac{\hat{R}_j(t_i) - R(t_i)}{R(t_i)}\right|\right\} = \frac{1}{n_t}\sum_{i=1}^{n_t}\mathrm{MAPE}\left[\hat{R}(t_i)\right] \quad ,j = 1,2,\dots,L \quad \cdots (25)$$

where: L: no. of replications, n_t: the limits of variable (t_i)

6: Application Part

6.1: Simulation

Simulation is one of the important means to solve problems (Problem Solving techniques), and is the only and the last method to solve any problems when the problems cannot be solved in analytical methods or numerical methods or in the case of facing difficulties to obtain real data for a certain phenomenon, simulation depend on re-sampling methods, generate numbers and random variables that have certain characteristic.

6.2: Stages of Building Simulation Experiment

The stages for estimate reliability function of the gamma distribution (α,θ) are as follows:

a. First Stage (set default values):

This is the most important stage of the basicstages; the other stages depend on it inthe program of building simulation experience; it sets to true, the default values, namely:

1. Specify default values for the parameters (α,θ). In this research six models have been considered, which are arranged as follows:

Table number (1)

The default value of the scale parameter

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1	1	2	2	3	3
θ	0.5	1.5	0.5	1.5	0.5	1.5

2. Choosing the sample size (n): choosing different sizes of the sample to determine the effect of sample size in deciding the accuracy and bitterness of the results obtained from the estimation methods used in this study. The samples have taken volumes characterized by being small (n = 30), medium (n = 50) and large sizes (n = 75, 100).
3. Choosing the number of sample replicatedsize (L): the number ofsample replicated size (L = 1000).
4. Set times of estimating the reliability function: take (10) times of each case of the six models to assess the reliability function and the times are arranged as follows:

Table number (2)

Times of estimating the reliability function

t_i	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
t_1	0.1	0.1	0.2	0.4	0.3	1.3
t_2	0.15	0.3	0.3	0.7	0.5	1.8
t_3	0.2	0.5	0.4	1.0	0.7	2.3
t_4	0.25	0.7	0.5	1.3	0.9	2.8

t_5	0.3	0.9	0.6	1.6	1.1	3.3
t_6	0.35	1.1	0.7	1.9	1.3	3.8
t_7	0.4	1.3	0.8	2.2	1.5	4.3
t_8	0.45	1.5	0.9	2.5	1.7	4.8
t_9	0.5	1.7	1.0	2.8	1.9	5.3
t_{10}	0.55	1.9	1.1	3.1	2.1	5.8

b. The Second Stage (data generation):

Generating random numbers for the Gamma distribution with two parameters (α, θ), According to the available generation function in (Matlab-R2011a) language:

t = gamrnd (α, θ,[n 1])

c. The Third Stage (find estimators):

At this stage, estimating the scale parameter and reliability function of the Gamma distribution through estimation methods are dealt with in the theoretical part of this thesis, according to the methods: (MLE), (SB), (P), (Mix I) and (Mix II).

d. The Fourth Stage (comparison):

After finding estimators of reliability function two criteria were used to evaluate the accuracy of estimation methods, which are: Integral Mean squares error (IMSE) and Integral Mean Absolute Percentage Error (IMAPE).

The method that yields the smallest value of IMSE and IMAPE is considered of the best fitted one.

6.3: Analysis of Simulation Results

In this section the results of the simulation and analysis to get the best methods for estimating the scale parameters and reliability function for the gamma distribution with two parameters are presented.

The results were obtained utilizing a program written in a Matlab language (Matlab-R2011a) by researchers.

6.3.1: Methods of Estimating the Scale Parameter θ:

The results are explained in tables (3) as follow:

Tablenumber (3)

Estimates of scale parameter of various estimation methods, for different sample size and all models

Model	n	$\hat{\theta}_{MLE}$	$\hat{\theta}_{SB}$	$\hat{\theta}_{Pitman}$	$\hat{\theta}_{MixI}$	$\hat{\theta}_{MixII}$
1	30	0.49451	0.51157	0.47856	0.48641	0.494
	50	0.49755	0.5077	0.48779	0.49262	0.49736

	75	0.50003	0.50679	0.49346	0.49672	0.49995
	100	0.50032	0.50537	0.49537	0.49783	0.50027
2	30	1.48354	1.5347	1.43569	1.45922	1.482
	50	1.49264	1.52311	1.46338	1.47787	1.49207
	75	1.5001	1.52038	1.48037	1.49017	1.49984
	100	1.50096	1.51612	1.4861	1.49349	1.50081
3	30	0.49814	0.50658	0.48997	0.49402	0.498
	50	0.49976	0.50481	0.49481	0.49727	0.49971
	75	0.50087	0.50423	0.49756	0.49921	0.50085
	100	0.50061	0.50312	0.49812	0.49936	0.50059
4	30	1.49441	1.51974	1.46991	1.48206	1.49401
	50	1.49928	1.51442	1.48443	1.49182	1.49913
	75	1.50262	1.5127	1.49267	1.49763	1.50255
	100	1.50182	1.50936	1.49435	1.49807	1.50178
5	30	0.49879	0.50439	0.49331	0.49603	0.49873
	50	0.50009	0.50345	0.49678	0.49843	0.50007
	75	0.50086	0.5031	0.49865	0.49975	0.50085
	100	0.50052	0.50219	0.49885	0.49968	0.50051
6	30	1.49636	1.51317	1.47992	1.48809	1.49618
	50	1.50027	1.51034	1.49034	1.49529	1.5002
	75	1.50259	1.5093	1.49595	1.49926	1.50256
	100	1.50155	1.50657	1.49656	1.49905	1.50153

6.3.2: Methods of Estimating the Reliability Function:

The results are shown in tables (4), (5), …, (9) as follow:

Tablenumber (4)

Estimates of reliability function of various estimation methods for different sample sizes in model 1

n	t_i	Real	MLE	SB	Pitman	Mix I	Mix II
30	0.1	0.81873	0.81196	0.73856	0.80636	0.80915	0.81178
	0.15	0.74082	0.73205	0.63686	0.72452	0.72827	0.73181
	0.2	0.67032	0.66024	0.55035	0.65123	0.65572	0.65996
	0.25	0.60653	0.59569	0.4766	0.58558	0.59062	0.59537
	0.3	0.54881	0.53764	0.41357	0.52675	0.53217	0.5373
	0.35	0.49659	0.48542	0.35959	0.474	0.47968	0.48506
	0.4	0.44933	0.43842	0.31326	0.4267	0.43252	0.43805
	0.45	0.40657	0.3961	0.27341	0.38425	0.39013	0.39573
	0.5	0.36788	0.35799	0.23907	0.34615	0.35202	0.35762
	0.55	0.33287	0.32366	0.20941	0.31194	0.31774	0.32328
50	0.1	0.81873	0.81492	0.73856	0.8116	0.81325	0.81485
	0.15	0.74082	0.73588	0.63686	0.7314	0.73364	0.7358
	0.2	0.67032	0.66465	0.55035	0.65927	0.66196	0.66455
	0.25	0.60653	0.60045	0.4766	0.59438	0.59741	0.60033
	0.3	0.54881	0.54255	0.41357	0.536	0.53927	0.54243
	0.35	0.49659	0.49035	0.35959	0.48346	0.48689	0.49021
	0.4	0.44933	0.44325	0.31326	0.43616	0.43969	0.44311
	0.45	0.40657	0.40076	0.27341	0.39357	0.39715	0.40062
	0.5	0.36788	0.36242	0.23907	0.35521	0.3588	0.36228
	0.55	0.33287	0.32781	0.20941	0.32066	0.32422	0.32767
75	0.1	0.81873	0.81672	0.73856	0.81452	0.81562	0.81669
	0.15	0.74082	0.73825	0.63686	0.73527	0.73676	0.73821
	0.2	0.67032	0.66741	0.55035	0.66383	0.66562	0.66737
	0.25	0.60653	0.60346	0.4766	0.59942	0.60144	0.60341
	0.3	0.54881	0.54571	0.41357	0.54134	0.54352	0.54565
	0.35	0.49659	0.49356	0.35959	0.48895	0.49125	0.4935
	0.4	0.44933	0.44645	0.31326	0.4417	0.44407	0.44639
	0.45	0.40657	0.40389	0.27341	0.39907	0.40148	0.40383
	0.5	0.36788	0.36545	0.23907	0.36061	0.36302	0.36538
	0.55	0.33287	0.3307	0.20941	0.3259	0.32829	0.33064
100	0.1	0.81873	0.81743	0.73856	0.81578	0.8166	0.81741
	0.15	0.74082	0.73916	0.63686	0.73693	0.73804	0.73914
	0.2	0.67032	0.66845	0.55035	0.66577	0.66711	0.66842
	0.25	0.60653	0.60457	0.4766	0.60154	0.60305	0.60454
	0.3	0.54881	0.54684	0.41357	0.54356	0.5452	0.54681
	0.35	0.49659	0.49467	0.35959	0.49121	0.49294	0.49464
	0.4	0.44933	0.44753	0.31326	0.44396	0.44574	0.44749
	0.45	0.40657	0.40491	0.27341	0.40128	0.40309	0.40488
	0.5	0.36788	0.36639	0.23907	0.36275	0.36456	0.36635
	0.55	0.33287	0.33156	0.20941	0.32794	0.32975	0.33153

Tablenumber (5)

Estimates of reliability function of various estimation methods for different sample sizes in model 2

n	t_i	Real	MLE	SB	Pitman	Mix I	Mix II
30	0.1	0.93551	0.93277	0.90529	0.93061	0.93169	0.9327
	0.3	0.81873	0.81196	0.74409	0.80636	0.80915	0.81178
	0.5	0.71653	0.70726	0.61391	0.69919	0.70321	0.707
	0.7	0.62709	0.61645	0.50835	0.60667	0.61154	0.61614
	0.9	0.54881	0.53764	0.42241	0.52675	0.53217	0.5373
	1.1	0.48031	0.4692	0.35218	0.45766	0.46339	0.46883
	1.3	0.42035	0.40972	0.29459	0.39789	0.40376	0.40934
	1.5	0.36788	0.35799	0.24718	0.34615	0.35202	0.35762
	1.7	0.32196	0.31298	0.20804	0.30132	0.3071	0.31261
	1.9	0.28177	0.27379	0.1756	0.26247	0.26807	0.27343
50	0.1	0.93551	0.93396	0.90529	0.93269	0.93333	0.93394
	0.3	0.81873	0.81492	0.74409	0.8116	0.81325	0.81485
	0.5	0.71653	0.71131	0.61391	0.7065	0.7089	0.71122
	0.7	0.62709	0.62111	0.50835	0.61526	0.61818	0.621
	0.9	0.54881	0.54255	0.42241	0.536	0.53927	0.54243
	1.1	0.48031	0.47411	0.35218	0.46713	0.47061	0.47397
	1.3	0.42035	0.41444	0.29459	0.40727	0.41084	0.4143
	1.5	0.36788	0.36242	0.24718	0.35521	0.3588	0.36228
	1.7	0.32196	0.31704	0.20804	0.30993	0.31346	0.3169
	1.9	0.28177	0.27745	0.1756	0.27051	0.27396	0.27731
75	0.1	0.93551	0.93468	0.90529	0.93384	0.93426	0.93467
	0.3	0.81873	0.81672	0.74409	0.81452	0.81562	0.81669
	0.5	0.71653	0.71383	0.61391	0.71063	0.71223	0.71378
	0.7	0.62709	0.62405	0.50835	0.62016	0.6221	0.624
	0.9	0.54881	0.54571	0.42241	0.54134	0.54352	0.54565
	1.1	0.48031	0.47732	0.35218	0.47266	0.47498	0.47726
	1.3	0.42035	0.4176	0.29459	0.4128	0.41519	0.41754
	1.5	0.36788	0.36545	0.24718	0.36061	0.36302	0.36538
	1.7	0.32196	0.31988	0.20804	0.31509	0.31748	0.31982
	1.9	0.28177	0.28007	0.1756	0.27539	0.27772	0.28
	0.1	0.93551	0.93497	0.90529	0.93434	0.93465	0.93496
	0.3	0.81873	0.81743	0.74409	0.81578	0.8166	0.81741
	0.5	0.71653	0.71479	0.61391	0.7124	0.71359	0.71476
	0.7	0.62709	0.62514	0.50835	0.62222	0.62368	0.62512
	0.9	0.54881	0.54684	0.42241	0.54356	0.5452	0.54681

100	1.1	0.48031	0.47842	0.35218	0.47492	0.47667	0.47839
	1.3	0.42035	0.41864	0.29459	0.41503	0.41683	0.41861
	1.5	0.36788	0.36639	0.24718	0.36275	0.36456	0.36635
	1.7	0.32196	0.32071	0.20804	0.31711	0.31891	0.32068
	1.9	0.28177	0.28078	0.1756	0.27726	0.27901	0.28075

Table number (6)

Estimates of reliability function of various estimation methods for different sample sizes in model 3

n	t_i	Real	MLE	SB	Pitman	Mix I	Mix II
30	0.2	0.93845	0.93564	0.87112	0.93377	0.93471	0.93561
	0.3	0.8781	0.87329	0.76347	0.8699	0.8716	0.87324
	0.4	0.80879	0.80233	0.65407	0.79743	0.79988	0.80225
	0.5	0.73576	0.72816	0.55164	0.72195	0.72505	0.72806
	0.6	0.66263	0.65444	0.46012	0.64719	0.65081	0.65432
	0.7	0.59183	0.58356	0.38073	0.57555	0.57954	0.58343
	0.8	0.52493	0.51698	0.31322	0.50848	0.51272	0.51684
	0.9	0.46284	0.45552	0.25659	0.44676	0.45112	0.45537
	1.0	0.40601	0.39953	0.20956	0.39073	0.39511	0.39938
	1.1	0.35457	0.34905	0.17079	0.34039	0.3447	0.34891
50	0.2	0.93845	0.93702	0.87112	0.93592	0.93647	0.93701
	0.3	0.8781	0.87567	0.76347	0.87366	0.87467	0.87565
	0.4	0.80879	0.80556	0.65407	0.80265	0.80411	0.80553
	0.5	0.73576	0.732	0.55164	0.7283	0.73015	0.73196
	0.6	0.66263	0.65864	0.46012	0.6543	0.65647	0.65859
	0.7	0.59183	0.58787	0.38073	0.58306	0.58546	0.58783
	0.8	0.52493	0.52121	0.31322	0.51609	0.51865	0.52116
	0.9	0.46284	0.45951	0.25659	0.45422	0.45686	0.45946
	1.0	0.40601	0.40318	0.20956	0.39785	0.40051	0.40313
	1.1	0.35457	0.35229	0.17079	0.34702	0.34965	0.35224
75	0.2	0.93845	0.93765	0.87112	0.93693	0.93729	0.93765
	0.3	0.8781	0.87678	0.76347	0.87545	0.87612	0.87677
	0.4	0.80879	0.80708	0.65407	0.80515	0.80612	0.80707
	0.5	0.73576	0.73383	0.55164	0.73137	0.7326	0.73381
	0.6	0.66263	0.66066	0.46012	0.65778	0.65922	0.66064
	0.7	0.59183	0.58999	0.38073	0.58678	0.58838	0.58996
	0.8	0.52493	0.52332	0.31322	0.5199	0.52161	0.5233
	0.9	0.46284	0.46154	0.25659	0.458	0.45977	0.46152
	1.0	0.40601	0.40508	0.20956	0.40151	0.40329	0.40505

	1.1	0.35457	0.35402	0.17079	0.35049	0.35225	0.354
100	0.2	0.93845	0.9379	0.87112	0.93736	0.93763	0.9379
	0.3	0.8781	0.87719	0.76347	0.8762	0.8767	0.87719
	0.4	0.80879	0.80762	0.65407	0.80617	0.80689	0.80761
	0.5	0.73576	0.73443	0.55164	0.73259	0.73351	0.73442
	0.6	0.66263	0.66128	0.46012	0.65911	0.66019	0.66127
	0.7	0.59183	0.59056	0.38073	0.58815	0.58936	0.59055
	0.8	0.52493	0.52382	0.31322	0.52125	0.52254	0.52381
	0.9	0.46284	0.46195	0.25659	0.45929	0.46061	0.46193
	1.0	0.40601	0.40537	0.20956	0.40268	0.40402	0.40535
	1.1	0.35457	0.35419	0.17079	0.35154	0.35286	0.35418

Table number (7)

Estimates of reliability function of various estimation methods for different sample sizes in model 4

n	t_i	Real	MLE	SB	Pitman	Mix I	Mix II
30	0.4	0.97018	0.96868	0.92076	0.96773	0.9682	0.96867
	0.7	0.91973	0.91624	0.80745	0.91387	0.91506	0.9162
	1.0	0.8557	0.85029	0.68488	0.84638	0.84833	0.85022
	1.3	0.78465	0.77775	0.56849	0.77239	0.77507	0.77766
	1.6	0.71125	0.70339	0.46503	0.69681	0.7001	0.70329
	1.9	0.63868	0.63041	0.37656	0.62287	0.62664	0.63028
	2.2	0.56904	0.56083	0.30277	0.55263	0.55672	0.5607
	2.5	0.50367	0.4959	0.24223	0.48729	0.49158	0.49576
	2.8	0.4433	0.43624	0.19314	0.42745	0.43183	0.43609
	3.1	0.38826	0.38209	0.15365	0.37333	0.37769	0.38195
50	0.4	0.97018	0.96941	0.92076	0.96885	0.96913	0.96941
	0.7	0.91973	0.91796	0.80745	0.91656	0.91726	0.91794
	1.0	0.8557	0.85297	0.68488	0.85066	0.85182	0.85295
	1.3	0.78465	0.78121	0.56849	0.77803	0.77962	0.78118
	1.6	0.71125	0.70738	0.46503	0.70345	0.70542	0.70735
	1.9	0.63868	0.63467	0.37656	0.63016	0.63241	0.63463
	2.2	0.56904	0.56514	0.30277	0.56021	0.56267	0.5651
	2.5	0.50367	0.50007	0.24223	0.49487	0.49746	0.50002
	2.8	0.4433	0.44013	0.19314	0.43481	0.43747	0.44008
	3.1	0.38826	0.38561	0.15365	0.38029	0.38295	0.38556
	0.4	0.97018	0.96975	0.92076	0.96938	0.96956	0.96974
	0.7	0.91973	0.91875	0.80745	0.91783	0.91829	0.91875
	1.0	0.8557	0.85423	0.68488	0.85269	0.85346	0.85422

75	1.3	0.78465	0.78285	0.56849	0.78074	0.78179	0.78283
	1.6	0.71125	0.70929	0.46503	0.70668	0.70799	0.70927
	1.9	0.63868	0.63674	0.37656	0.63373	0.63523	0.63672
	2.2	0.56904	0.56726	0.30277	0.56397	0.56562	0.56724
	2.5	0.50367	0.50216	0.24223	0.49868	0.50042	0.50213
	2.8	0.4433	0.44212	0.19314	0.43856	0.44034	0.4421
	3.1	0.38826	0.38746	0.15365	0.38389	0.38567	0.38744
100	0.4	0.97018	0.96988	0.92076	0.96961	0.96974	0.96988
	0.7	0.91973	0.91906	0.80745	0.91837	0.91872	0.91906
	1.0	0.8557	0.85469	0.68488	0.85354	0.85411	0.85468
	1.3	0.78465	0.78341	0.56849	0.78183	0.78262	0.78341
	1.6	0.71125	0.7099	0.46503	0.70795	0.70893	0.70989
	1.9	0.63868	0.63734	0.37656	0.63509	0.63621	0.63733
	2.2	0.56904	0.56782	0.30277	0.56535	0.56658	0.56781
	2.5	0.50367	0.50263	0.24223	0.50002	0.50132	0.50261
	2.8	0.4433	0.44249	0.19314	0.43981	0.44115	0.44247
	3.1	0.38826	0.38771	0.15365	0.38503	0.38637	0.3877

Table number (8)

Estimates of reliability function of various estimation methods for different sample sizes in model 5

n	t_i	Real	MLE	SB	Pitman	Mix I	Mix II
30	0.3	0.97688	0.97561	0.95659	0.97491	0.97526	0.9756
	0.5	0.9197	0.9162	0.86276	0.91408	0.91514	0.91618
	0.7	0.8335	0.82787	0.73837	0.82403	0.82596	0.82783
	0.9	0.73062	0.72374	0.60697	0.71834	0.72104	0.72368
	1.1	0.62271	0.61563	0.48409	0.6091	0.61236	0.61556
	1.3	0.56971	0.56288	0.42848	0.55598	0.55942	0.56281
	1.5	0.42319	0.41808	0.28922	0.41078	0.41442	0.418
	1.7	0.33974	0.33616	0.21877	0.32908	0.33261	0.33608
	1.9	0.2689	0.26685	0.16385	0.26025	0.26353	0.26677
	2.1	0.21024	0.20955	0.12181	0.20359	0.20655	0.20948
50	0.3	0.97688	0.97622	0.95659	0.97581	0.97602	0.97622
	0.5	0.9197	0.91789	0.86276	0.91664	0.91726	0.91788
	0.7	0.8335	0.83063	0.73837	0.82835	0.82949	0.83062
	0.9	0.73062	0.72719	0.60697	0.72397	0.72558	0.72717
	1.1	0.62271	0.61931	0.48409	0.61539	0.61735	0.61929
	1.3	0.56971	0.56651	0.42848	0.56236	0.56444	0.56649
	1.5	0.42319	0.4211	0.28922	0.41668	0.41889	0.42107

	1.7	0.33974	0.33857	0.21877	0.33427	0.33641	0.33854
	1.9	0.2689	0.2686	0.16385	0.26459	0.26659	0.26858
	2.1	0.21024	0.2107	0.12181	0.20707	0.20888	0.21067
75	0.3	0.97688	0.97655	0.95659	0.97628	0.97641	0.97654
	0.5	0.9197	0.91881	0.86276	0.91798	0.9184	0.91881
	0.7	0.8335	0.83216	0.73837	0.83065	0.83141	0.83216
	0.9	0.73062	0.72914	0.60697	0.727	0.72807	0.72913
	1.1	0.62271	0.62142	0.48409	0.61881	0.62011	0.62141
	1.3	0.56971	0.56861	0.42848	0.56583	0.56722	0.56859
	1.5	0.42319	0.42288	0.28922	0.41992	0.4214	0.42287
	1.7	0.33974	0.34001	0.21877	0.33712	0.33856	0.34
	1.9	0.2689	0.26968	0.16385	0.26698	0.26833	0.26967
	2.1	0.21024	0.21143	0.12181	0.20899	0.2102	0.21142
100	0.3	0.97688	0.9766	0.95659	0.9764	0.9765	0.9766
	0.5	0.9197	0.91896	0.86276	0.91834	0.91865	0.91896
	0.7	0.8335	0.83237	0.73837	0.83124	0.8318	0.83236
	0.9	0.73062	0.72934	0.60697	0.72773	0.72854	0.72934
	1.1	0.62271	0.62155	0.48409	0.61959	0.62057	0.62154
	1.3	0.56971	0.56868	0.42848	0.5666	0.56764	0.56867
	1.5	0.42319	0.42277	0.28922	0.42054	0.42165	0.42276
	1.7	0.33974	0.33977	0.21877	0.3376	0.33868	0.33976
	1.9	0.2689	0.26933	0.16385	0.2673	0.26832	0.26932
	2.1	0.21024	0.211	0.12181	0.20917	0.21008	0.21099

Table number (9)

Estimates of reliability function of various estimation methods for different sample sizes in model 6

n	t_i	Real	MLE	SB	Pitman	Mix I	Mix II
30	1.3	0.94252	0.9398	0.89749	0.93822	0.93901	0.93979
	1.8	0.87949	0.87485	0.80063	0.87188	0.87337	0.87482
	2.3	0.80043	0.79427	0.69154	0.78988	0.79208	0.79423
	2.8	0.71271	0.70572	0.58226	0.7001	0.70291	0.70566
	3.3	0.62271	0.61563	0.48051	0.6091	0.61236	0.61556
	3.8	0.53529	0.52874	0.39033	0.52166	0.52519	0.52866
	4.3	0.45372	0.44814	0.31314	0.44084	0.44448	0.44806
	4.8	0.3799	0.37555	0.24874	0.36831	0.37192	0.37547
	5.3	0.31473	0.31167	0.19604	0.30472	0.30818	0.31159
	5.8	0.2583	0.25649	0.15354	0.24999	0.25322	0.25642
	1.3	0.94252	0.94111	0.89749	0.94018	0.94064	0.9411

50	1.8	0.87949	0.8771	0.80063	0.87534	0.87622	0.87709
	2.3	0.80043	0.79732	0.69154	0.7947	0.79601	0.7973
	2.8	0.71271	0.70925	0.58226	0.70589	0.70757	0.70923
	3.3	0.62271	0.61931	0.48051	0.61539	0.61735	0.61929
	3.8	0.53529	0.53229	0.39033	0.52802	0.53015	0.53226
	4.3	0.45372	0.45134	0.31314	0.44693	0.44913	0.45131
	4.8	0.3799	0.37827	0.24874	0.37388	0.37607	0.37824
	5.3	0.31473	0.31385	0.19604	0.30963	0.31174	0.31383
	5.8	0.2583	0.25814	0.15354	0.25418	0.25616	0.25811
75	1.3	0.94252	0.94182	0.89749	0.9412	0.94151	0.94181
	1.8	0.87949	0.87834	0.80063	0.87718	0.87776	0.87834
	2.3	0.80043	0.79901	0.69154	0.79728	0.79815	0.79901
	2.8	0.71271	0.71124	0.58226	0.70901	0.71012	0.71123
	3.3	0.62271	0.62142	0.48051	0.61881	0.62011	0.62141
	3.8	0.53529	0.53434	0.39033	0.53149	0.53292	0.53433
	4.3	0.45372	0.45322	0.31314	0.45026	0.45174	0.4532
	4.8	0.3799	0.37989	0.24874	0.37695	0.37842	0.37988
	5.3	0.31473	0.31518	0.19604	0.31234	0.31376	0.31516
	5.8	0.2583	0.25916	0.15354	0.2565	0.25782	0.25915
100	1.3	0.94252	0.94194	0.89749	0.94148	0.94171	0.94193
	1.8	0.87949	0.87853	0.80063	0.87766	0.87809	0.87853
	2.3	0.80043	0.79922	0.69154	0.79792	0.79857	0.79922
	2.8	0.71271	0.71143	0.58226	0.70976	0.71059	0.71143
	3.3	0.62271	0.62155	0.48051	0.61959	0.62057	0.62154
	3.8	0.53529	0.53438	0.39033	0.53224	0.53331	0.53437
	4.3	0.45372	0.45314	0.31314	0.45092	0.45203	0.45314
	4.8	0.3799	0.37971	0.24874	0.3775	0.3786	0.3797
	5.3	0.31473	0.3149	0.19604	0.31277	0.31383	0.31489
	5.8	0.2583	0.25879	0.15354	0.25679	0.25779	0.25879

From the tables (4), (5), …, (9) we observe that the estimators of the reliability function by using all studied estimation methods revealed that the average is close to the real value of reliability function to all models and samples sizes, in addition we notice the followings:

1. For all models we noticed that most of the averages of estimates of reliability function for all methods are close to the real value of the reliability function when the sample size increases, except for the SB method.
2. The estimation of reliability function using both the methods (MLE & MIX II) were very close and in some times they were equal, also the both (P & MIX I) methods were close for all models and samples sizes.

3. It shows that the estimated and real value of reliability function decreases by increasing of time t_i and it is always lies within the period [0,1].
4. The increase in the value of scale and shape parameter leads to increase estimated reliability function for all estimation methods.

To reach the best estimator through preference between different studied estimated methods, in this research, it has been generally depended on thefollowing two statistical measures for comparison:

1. Integral Mean Square Error (IMSE)
2. Integral Mean Absolute Percentage Error (IMAPE)

Table number (10)

IMSE of different estimation methods of R(t), for different sample size

Model	n	$\hat{R}_{MLE}$	$\hat{R}_{SB}$	$\hat{R}_{Pitman}$	$\hat{R}_{MixI}$	$\hat{R}_{MixII}$
1	30	0.00342	0.01536	0.00378	0.00357	0.00343
	50	0.00205	0.01536	0.00218	0.00211	0.00205
	75	0.00139	0.01536	0.00143	0.00141	0.00139
	100	0.00096	0.01536	0.00099	0.00097	0.00096

2	30	0.00315	0.01181	0.00346	0.00328	0.00316
	50	0.0019	0.01181	0.00201	0.00195	0.00191
	75	0.00129	0.01181	0.00133	0.0013	0.00129
	100	0.00089	0.01181	0.00092	0.0009	0.00089
3	30	0.00311	0.03208	0.00328	0.00318	0.00311
	50	0.00179	0.03208	0.00185	0.00182	0.00179
	75	0.00128	0.03208	0.0013	0.00129	0.00128
	100	0.00088	0.03208	0.00089	0.00088	0.00088
4	30	0.0028	0.04772	0.00297	0.00287	0.0028
	50	0.00161	0.04772	0.00167	0.00164	0.00161
	75	0.00115	0.04772	0.00117	0.00115	0.00115
	100	0.00079	0.04772	0.0008	0.00079	0.00079
5	30	0.0029	0.01186	0.00298	0.00293	0.0029
	50	0.00178	0.01186	0.0018	0.00179	0.00178
	75	0.00117	0.01186	0.00118	0.00117	0.00117
	100	0.0009	0.01186	0.00091	0.0009	0.0009
6	30	0.00317	0.01404	0.00327	0.00321	0.00317
	50	0.00193	0.01404	0.00197	0.00195	0.00193
	75	0.00127	0.01404	0.00128	0.00127	0.00127
	100	0.00098	0.01404	0.00099	0.00098	0.00098

Tablenumber (11)
IMAPE of different estimation methods of R(t), for different sample size

Model	n	$\hat{R}_{MLE}$	$\hat{R}_{SB}$	$\hat{R}_{Pitman}$	$\hat{R}_{P\&MLE}$	$\hat{R}_{P\&B}$
1	30	0.09556	0.25051	0.0997	0.09731	0.09565
	50	0.07455	0.25051	0.07672	0.07549	0.07458
	75	0.06001	0.25051	0.0609	0.06032	0.06001
	100	0.05101	0.25051	0.05136	0.05112	0.05101
2	30	0.09758	0.2311	0.10146	0.09918	0.09765
	50	0.07628	0.2311	0.07834	0.07716	0.07631
	75	0.06144	0.2311	0.06226	0.06172	0.06145
	100	0.05225	0.2311	0.05256	0.05234	0.05226
3	30	0.07919	0.31572	0.08122	0.08	0.07921
	50	0.05973	0.31572	0.06054	0.06005	0.05973
	75	0.05071	0.31572	0.05107	0.05085	0.05072
	100	0.04177	0.31572	0.04192	0.04182	0.04177
4	30	0.06899	0.35603	0.07087	0.06975	0.069
	50	0.05199	0.35603	0.05275	0.0523	0.052
	75	0.04412	0.35603	0.04445	0.04425	0.04412
	100	0.03633	0.35603	0.03648	0.03639	0.03633
5	30	0.09604	0.23205	0.09693	0.09632	0.09604
	50	0.0768	0.23205	0.07714	0.07693	0.0768
	75	0.0619	0.23205	0.06204	0.06194	0.0619
	100	0.05395	0.23205	0.05393	0.05392	0.05395
6	30	0.09293	0.23934	0.09403	0.09332	0.09293
	50	0.0742	0.23934	0.07464	0.07438	0.0742
	75	0.05976	0.23934	0.05996	0.05983	0.05976
	100	0.05212	0.23934	0.05214	0.0521	0.05211

From tables (10) & (11) we noticed s the followings (IMSE) & (IMAPE):

1. By Increasing the sample size, the values of each (IMSE) & (IMAPE) decrease for estimating the reliability function for all models and all estimating methods except (SB) method is constant by changing sample sizes.
2. The values of statistical measures (IMSE) & (IMAPE) for estimating the reliability function, by using of each of the MLE and MIX II methods were very close or in most cases are equal.
3. The values of statistical measures (IMSE) & (IMAPE) for estimating the reliability function by using each of the Pitman & MIX I methods, the results

were close by increasing the values of the scale parameter θ and shape parameter α.

4. In some models the (MLE) method was the best when the sample size (n=30), with very few differences compared with the results of (MIX II) method, but in the other models both methods were equal.
5. For all models the two methods of (MLE & MIX II) reached the first priority in estimating for the sample sizes (n=50, 75, 100).
6. Increasing the value of each of the shape parameter α and scale parameter θwill not lead to certain style of effect on (IMSE) & (IMAPE) values.

7 : Conclusions:

During conducting the simulation experiments and according to the analyses of the results from the practical part the following conclusions have been drawn:

1. It was shown that the real value of the reliability function and estimated reliability function decrease with the increase of time t_i ; and it is always between (0-1), and this is in coincide with the theoretical aspect of characteristics of the reliability function.
2. The values of the two statistical measures: the (IMSE) and (IMAPE) in estimating the reliability function was decreased by increasing the sample sizes and to all estimation methods, and this is in line with statistical theory.
3. The results of (IMSE) at estimating the reliability function for each of (MEL) & (MIX II) methods were close and in most cases they were equal.
4. The results of (IMAPE) at estimating the reliability function for each of the (MLE & MIX II) methods were close by increasing the values of scale Parameter θ and shape parameter α.
5. In general, the researchers noticed during conducting the simulation experiments, the preferences of MLE and MIX II methods on other used methods in estimating the reliability function by using two statistical measures (IMSE) & (IMAPE) for comparison between the preferences of parameters for all the sample sizes and all models.

References:

1. Agarwal, B. (2003) Programmed Statistics (Question-Answers), Second Edition. New Age International.
2. Akahira, M., Ohyauchi, N. & Takeuchi, K. (2007) On the Pitman Estimator for a Family of Non-Regular Distributions. METRON-International Journal of Statistics. 65 (1). p.113-127.
3. Al-Nasser, A. H. (2009) An Introduction to Statistical Reliability. ITHRAA Publishing and Distribution.

4. Choi, S. C. & Wette, R. (1969) Maximum Likelihood Estimation of the Parameters of the Gamma Distribution and Their Bias. Journal of American Statistical Association and American Society for Quality. 11 (4). p.683-690.
5. Coit, D. W. & Jin, T. (2000) Gamma Distribution Parameter Estimation for Field Reliability Data with Missing Failure Times. IIE Transaction. 32 (12). p.1161-1166.
6. Dhillon, B. S., (1999) Design Reliability. CRC Press LLC, USA.
7. Forbes, C., Evans, M., Hastings, N. & Peacock, B. (2011) Statistical Distributions.4th Edition. John Wiley & Sons.
8. Freue, G. C. (2007) The Pitman Estimator of the Cauchy Location Parameter. Journal of Statistical Planning and Inference, 137 (6). p.1900-1913.
9. Gentle, J. E. (2003) Random Number Generation & Monte Carlo Methods. Springer Sciences Business Media. Inc., New York, USA.
10. Jasim, W. A. (2010) Bayes Estimator of One Parameter Gamma Distribution Under Quadratic and LINEX Loss Function. Iraqi Journal of Statistical Science. 9 (16). p.13-28.
11. Kishan, R. (2014) Comparison Between MLE and Bayes Estimation of Scale Parameter of Generalized Gamma Distribution with Known Shape Parameters Under Squared Error Loss Function. Journal of Reliability and Statistical Studies. 7 (1). p.43-50.
12. Kumar, U. D., Crocker, J., Chitra, T. & Saranga, H. (2006) Reliability and Six Sigma. Springer Science & Business Media.
13. Kumar, U., Crocker, J., Knezevic, J. & El-Haram, M. (2000). Reliability, Maintenance and Logistic Support. kluwer Academic.
14. Lyu, M. R. (1995) Handbook of Software Reliability Engineering. IEEE computer society press.
15. Miller, R. B. (1980) Bayesian Analysis of the Two-Parameter Gamma Distribution. Journal of American Statistical Association and American Society for Quality, 22 (1). p.65-69.

Mishra, R. C., & Sandilya, A. (2009) Reliability and Quality management. New Age International.