

## Modified Bohm Diffusion Equation in Q-Machine

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### Abstract:

The modified Bohm diffusion equation is studied in the inner region of plasma column in Q-machine. The instability factor (C-factor) and modified Bohm diffusion efficient are calculated in that region of plasma column. The results are compared with those calculated according to Bohm equation and with experimental results. The comparison with experimental shows a better agreement when the instability factor (c) is taken into the account.

The diffusion coefficient and the instability factor are calculated for strong and weak turbulence case. The results of both cases are examined with Bohm diffusion equation as well as with experimental results. A good fitting has been obtained for strong turbulence results with experimental results. Whereas, an approximate agreement are found between weak turbulence results in the region near from a center and edge of plasma.

**Key words: Bohm Diffusion, Q-Machine, and Plasma Diffusion**

### Introduction:

The diffusion of ions and electrons in plasma across magnetic field occur in the presence of collisions. But this diffusion (classical diffusion) cannot regarded as a serious case of charge leakage at high temperature experiment. That's due to the rate of collisions between ions and electrons are very low [1,2]. In all almost pervious experiment, the diffusion of plasma across a magnetic field scaled as  $B^{-1}$  rather than  $B^{-2}$ , and the decay of plasma with time is found to be exponential rather than reciprocal [3,4].

Bohm first notes this anomalous diffusion of plasma across magnetic field in 1949. Bohm found that, the plasma created by electric arc leaked across magnetic field with unexpected fast and large amplitude oscillations of electric field, which is observed inside arc. So that, Bohm surmised that, these

fluctuated electric field are causes of anomalous diffusion [1]. Bohm gave a semi-empirical formula of anomalous diffusion in the form [4,5]:

$$D_B = \frac{1}{16} \frac{KT}{eB} \quad (1)$$

where K, T, e, and B are Boltzmann constant, plasma temperature, electronic charge, and magnetic field, respectively. The value 1/16 has no theoretical justification but number agreeing with most experiments by a factor of two or three.

### Diffusion in Q-Machine:

From theoretical, numerical, and experimental studies of low- $\beta$  plasma (where  $\beta$  is the ratio of particle pressure of magnetic field pressure) have demonstrated the important of low frequency electrostatic fluctuation

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for anomalous plasma diffusion across magnetic field [6,7].

The anomalous conduct of plasma diffusion will relate to low frequency instability, because of high frequency instability have influence only on electron density and such instability cannot change the plasma density [4,8].

In this research, the experimental work is done on the princeton design [see ref (9)]. The Q-Machine is operated in double-ended operation. Potassium ions are produced by ionization of atomic beam incident on hot tungsten plate ( $T_{\text{plate}}=T_{\text{plasma}}=2760^{\circ}\text{K}$ ) located at both ends of plasma column has diameters is equal to 3 cm and long is equal to 128 cm. The magnetic field which generated by solenoid is constant and uniform with deviation of less than 0.3%.

From this experiment, Chu et,al [10,11] found that, at low magnetic field, he plasma confinement increases with B. When  $B_c$  is reached, the wave destabilizes to large amplitude and the plasma decreases in the central part and increases in the outer part and

outside of plasma, coincident with abrupt onset of  $m=1$  azimuthal mode of collisional drift wave instability. This instability was identified as density gradient-driven collisional drift wave instability.

Measurements of radial plasma density profiles ( $n_{os}$ ,  $n_{ow}$ ), for stable and drift-wave regimes, taken adjacent to onset of the  $m=1$  mode is shown in figure (1). Measurements shown extend to  $r=15$  mm, equal to the radius of end plate. Figure (2), and (3) shows the radial amplitude distribution of density ( $n_1$ ) and potential ( $\phi_1$ ) oscillations, and the local radial diffusive flux due to electron-ion collision ( $\Gamma_{ei}$ ) and radial flux ( $\Gamma_w$ ) due to drift wave of azimuthal mode number.

Based on the local density-gradient scale length, a “diffusion coefficient”  $D_{\text{wave}}$  for the flux associated with the drift wave can be calculated. In figure (4) this diffusion coefficient is compared with vale of  $D_{\text{classical}}$  and  $D_{\text{Bohm}}$ .

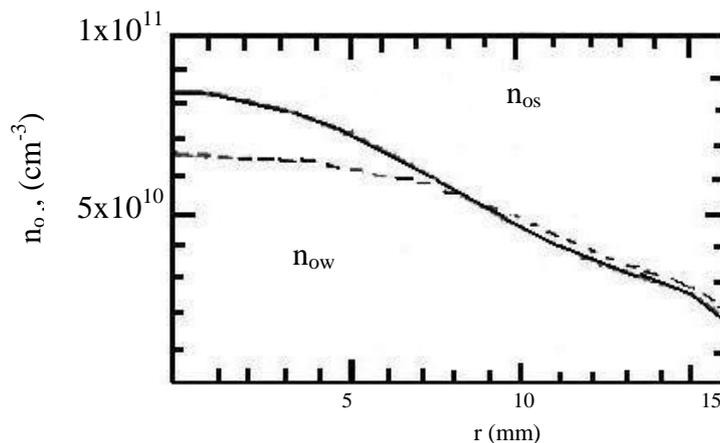


Fig. (1): Density profile in stable ( $n_{os}$ ,  $B=0.196$  T) and drift wave regimes ( $n_{ow}$ ,  $B=0.205$  T) versus radial position [10,11].

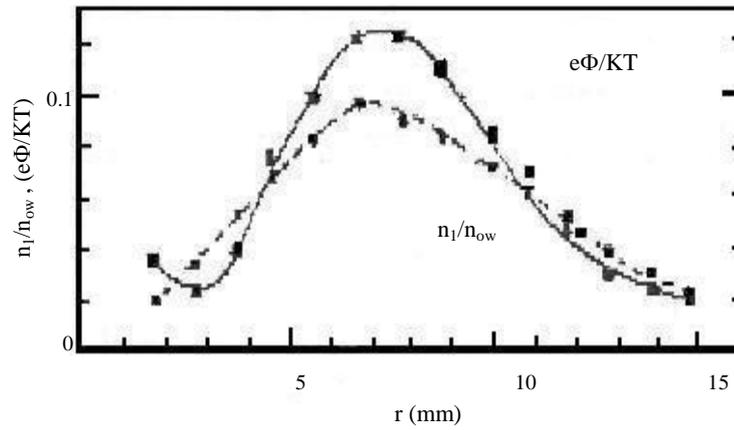


Fig. (2):Relative amplitude of density ( $n_1/n_0$ ) and potential ( $e\Phi/KT$ ) oscillation versus radial position [10,11].

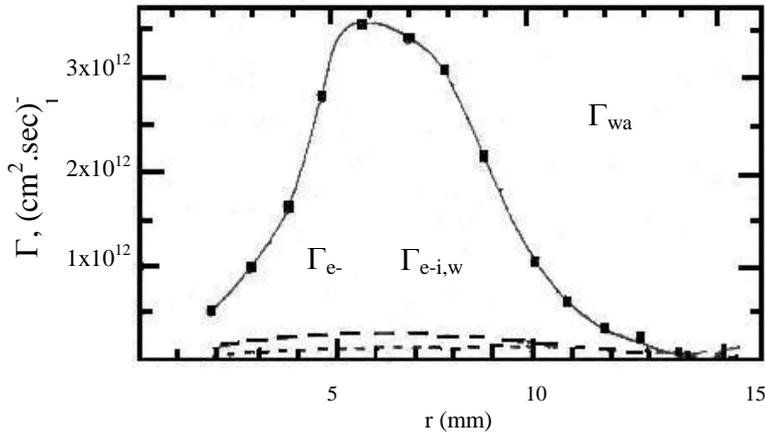


Fig. (3): Radial particle flux due to drift wave ( $\Gamma_w$ ) and electron-ion collision ( $\Gamma_{e-i,s}, \Gamma_{e-i,w}$ ) versus radial position [10,11].

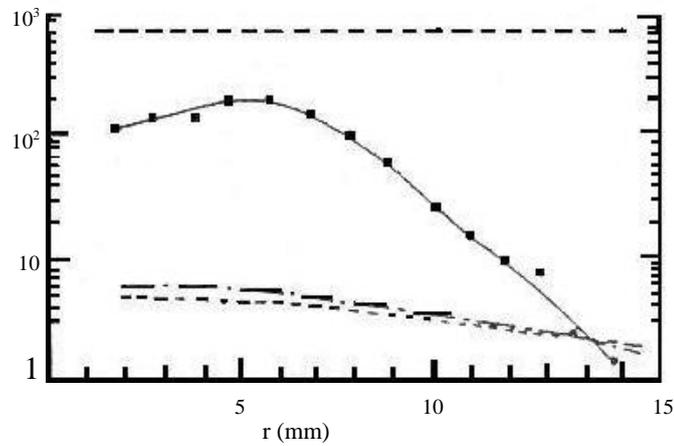


Fig. (4): comparison of diffusion coefficients versus radial position [10,11].

**Modified Bohm Diffusion:**

A modified Bohm diffusion equation given an interpretation of numerical factor of Bohm's formula. This equation suggested that, the 1/16 factor in Bohm's formula could be replaced by instability factor (c-factor) in modified Bohm diffusion equation.

The relation of the diffusion coefficient according to modified Bohm diffusion is equal to [12]:

$$D_{MB} = C \frac{KT}{eB}$$

where C is equal to:

$$C = \left( \frac{\gamma k_y}{2\omega^2} \right) \left| \frac{k_y \phi_1}{B} \right| \left| \frac{\phi_1}{T} \right|$$

It seems from equation (3). The instability factor ( C ) depends on the:

- (a) The fluctuation level of mode  $\left( \frac{\phi_1}{T} \right)$ .
- (b) The ratio of growth rate (  $\gamma$  ) and the frequency (  $\omega$  ).
- (c) The ratio  $v_{1(EXB)} / V_{ph}$  .

From equation (2). It is clear that, the diffusion coefficient is directly connected with instability that causing it.

**Experimental Results:**

In this section, the modified Bohm diffusion equation will apply on experimental results for chu et,al [10,11].

The fluctuation level  $\left( \frac{\phi_1}{T} \right)$  in equation (3) can be taken from figure (2). The phase velocity  $(\omega/k_y)$  of drift wave instability according to maximum

growth rate as predicated by linear theory [10,12] is :

$$V_{ph} = \frac{1}{2} V_{De} = -\frac{1}{2} \frac{KT}{eB} \frac{\nabla n}{n} \quad (4)$$

where  $V_{De}$  is the diamagnetic electron velocity. Where the experimental data of n and  $\nabla n$  can be taken from figure (1). The results of  $v_{ph}$  as function of radial position are plotted in figure (5). While the fluctuated  $E \times B$  drift velocity in radial direction which represent  $(k_y \phi_1 / B)$  in equation (3) can evaluated as:

$$V_{1(EXB)r} = \frac{\Gamma_1}{n_I} \quad (5)$$

where  $\Gamma_1$  is the fluctuated flux which can be taken from figure (3) and  $n_I$  is the fluctuated plasma density which calculated from Figures (1) and (2). Figure (6) shows  $v_{1(EXB)r}$  as unction of radial positions.

In addition to the above parameters, we need the magnitudes of angular frequency (  $\omega$  ) and growth rate (  $\gamma$  ), to apply equation (3).

So, the angular frequency can be determine via use the relation [9,11]:

$$\omega = k_\theta V_{ph} \quad (6)$$

where  $k_\theta$  is the azimuthal wave number, which equal to:

$$k_\theta = \frac{V_{(EXB)r} \cdot B}{\phi_1} \quad (7)$$

in consequence to the above two equation, and by taking the values of  $V_{ph}$  and  $v_{1(EXB)r}$  the variation of w with radial position is shown in figure (7).

For the grown the rate, it is possible to suppose that, the plasma Under study

is in strong turbulence case, which is estimated as[13]:

$$\gamma = \frac{\Gamma k_{\perp}^2}{\nabla n}$$

where  $k_{\perp} = k_r + k_{\theta}$  is the wave number in normal direction to B. The value of  $k_r$  can be taken as:

$$K_r = \frac{w}{v_{1(EXB)r}} \tag{9}$$

By substituting of the values of  $k_r$  and  $k_{\theta}$  in equation (8), the variation of growth rate versus radial position can be noted in figure (8).

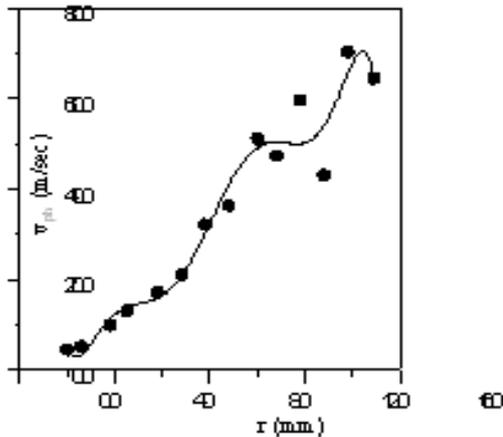


Fig (5): The variation of phase velocity with radial position.

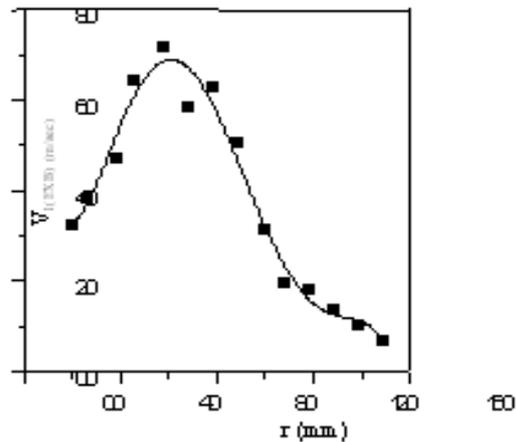


Fig (6): Theradial EXB drift velocity versus radial position.

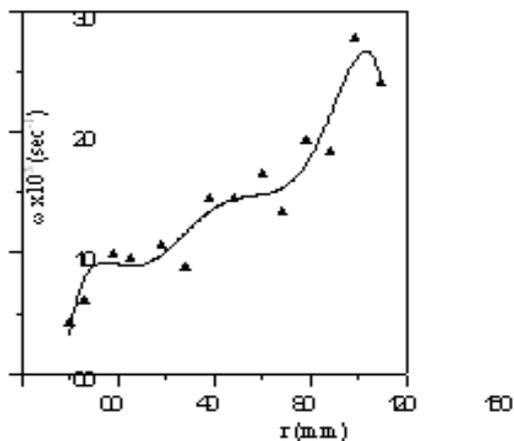


Fig. (7): The variation of angular frequency with radial position.

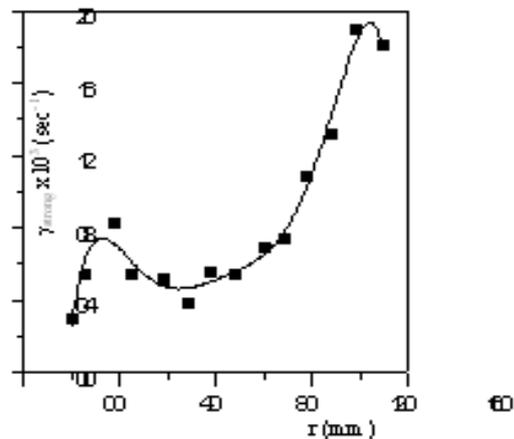


Fig (8): The growth rate versus radial position.

Eventually,if we substitute all parameters which indicated from figure (2),(5),(6),(7),and(8) in equation (4),the instability factor (C) can be calculated .In figure(9) this factor is compared with values of  $C_{exp}$  and  $C_{Bohm}$ .It should be pointed out from figure(9),the instability factor has indirect proportional with plasma

density (because of the instability factor has similar behavior to plasma density).

The modified Bohm diffusion coefficient ( $D_{MB}$ ) can be determined from apply equation (3) after substituting the data of C-factor from figure (9). In figure (10), this diffusion coefficient is compared with  $D_{exe}$  and

$D_{Bohm}$ . One should be observed from figure (10), the behavior of  $D_{MB}$  with C-factor. For this reason, we can say

that, the behavior of instability factor was responsible for the behavior of plasma diffusion coefficient.

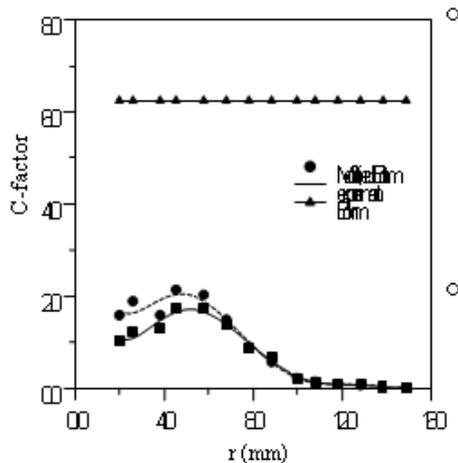


Fig. (9): The comparison of instability factor.

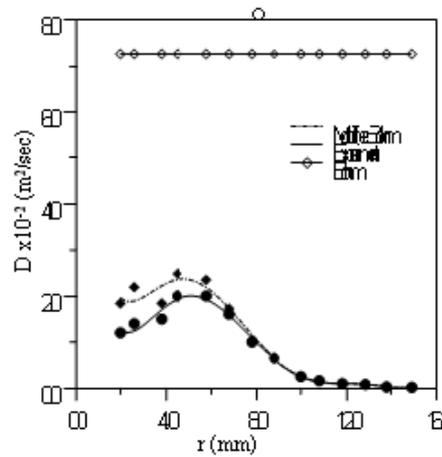


Fig. (10): The comparison of radial diffusion coefficient

### Turbulent plasma:

If we apply the concept of modified Bohm diffusion equation on two turbulence cases, strong turbulence case, where  $\gamma \cong \omega$ , and weak turbulence case, where  $\gamma \ll \omega$ . Let us treat with this subject in slab geometry, where B in Z -direction,  $\nabla n$  in X-direction (corresponding to r-direction), and E in Y-direction (corresponding to  $\theta$ -direction).

The particale flux associated with drift ware can be estimated as:

$$\Gamma = \frac{\gamma}{2} \frac{|k_y \phi|^2}{\omega B} \frac{dn}{dx} \quad (10)$$

This equation can be studied in two cases:

i. Strong turbulence:

Kadomtsev [13], gave a relation of  $e\Phi_1/KT$  as:

$$\frac{e\phi_1}{KT} = \frac{\omega}{\omega_{De}} \frac{1}{k_{\perp} \Lambda} \quad (11)$$

where  $\omega_{De}$  and  $\Lambda$  are diamagnetic drift frequency of electron ( $\omega_{De} = k_y v_{De}$ ) and density scale length ( $\Lambda = -n/\nabla n$ ), respectively. By using the equations (10) and (11) the corresponding particle flux becomes:

$$\Gamma_{st} = -\frac{\gamma}{k_{\perp}^2} \frac{dn}{dx} \quad (12)$$

Based on fick's law, the diffusion coefficient ( $D_{st}$ ) for the flux associated with drift ware in strong turbulence can be calculated as:

$$D_{st} = \frac{\gamma}{k_{\perp}^2} \quad (13)$$

where C-factor is:

$$C_{st} = \frac{\gamma}{k_{\perp}^2} \frac{B[T]}{T[eV]} \quad (14)$$

ii. Weak turbulence:

Kadomtsev [12], gave another relationship of  $e\Phi/KT$  which is equal:

$$\left(\frac{e\phi_1}{KT}\right)^2 \cong \frac{\gamma}{\omega} \left(\frac{\omega}{\omega_{De}} \frac{1}{k_{\perp} \Lambda}\right)^2 \quad (15)$$

For using the same technique which is used in strong turbulence case, the particle flux ( $\Gamma_{wt}$ ) becomes:

$$\Gamma_{wt} = -\frac{\gamma^2}{\omega k_{\perp}^2} \frac{dn}{dx} \quad (16)$$

where

$$D_{wt} = \frac{\gamma^2}{\omega k_{\perp}^2} \quad (17)$$

and

$$C_{wt} = \frac{\gamma^2}{\omega k_{\perp}^2} \frac{B[T]}{T[ev]} \quad (18)$$

Measured quantities of particle flux, diffusion coefficient, and instability factor for two turbulence cases are shown in figure (11),(12),and(13). In figure (11) the particle flux for strong turbulence, and weak turbulence, cases are compared with the flux associated with drift wave. While Figure (12) shows, the diffusion coefficient for both turbulent cases are compared with  $D_{exp}$ . For flux associated with drift wave. The instability factor  $C_{st}$  and  $C_{wt}$  are compared with  $C_{exp}$ . In figure (13).

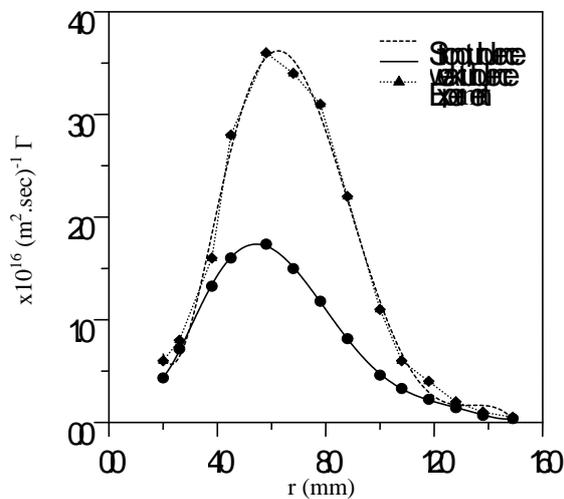


Fig. (11): The version of particle flux with position.

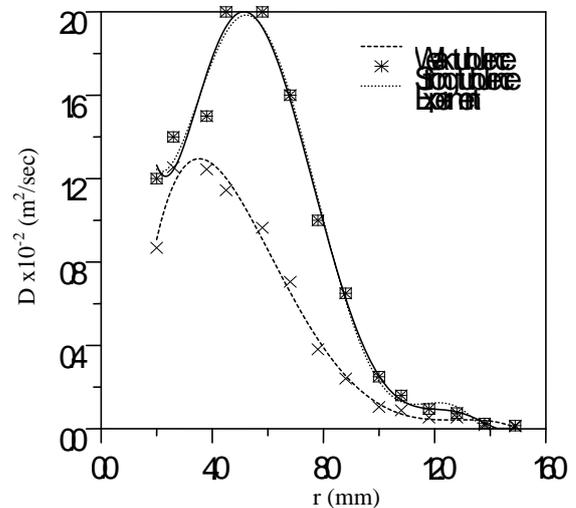


Fig. (12): The differences between diffusion coefficient

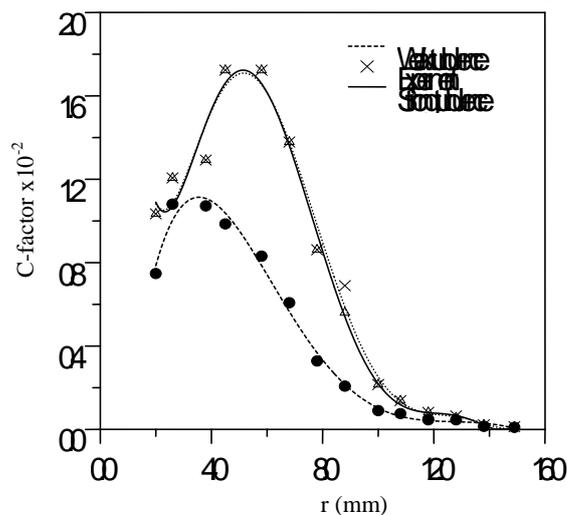


Fig. (13): The version of C-factor with radial position.

These figures (11), (12), and (13) shows there are fitting between the results of strong turbulence case with experimental, and there are approximate between the results of weak turbulence case with experiment. This reason caused by, we assume the plasma under study is in strong turbulence cases, and the turbulent will be weak in center and edge of plasma column.

### Discussion and conclusion:

It is clear from figures (9), and (10) that, the Bohm diffusion equation results has a behavior are for from that for experiment. This arises from the fact that, the normal form of  $D_B$  did not take the fluctuation properties of plasma parameters in to account. For this reason, the Bohm diffusion equation is not suitable for the explanation the experimented behavior for all experimented quantities.

While the figures (11), (12), and (13) shows, the turbulent is weak in the center and edge of plasma because of the anomalous diffusion is weak in that region.

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## الانتشار الشاذ في البلازما المضطربة

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## الخلاصة:

ان العلاقة الشبه وضعية قد درست في المنطقه الداخليه الى عمود البلازما في منظومة Q-Mchine. ان عامل اللاستقرارية (C-factor) ومعادلة بوم المحورة قد حسبت في هذه المنطقه. ان النتائج في هذه الدراسة قد قورنت مع نتائج معادلة بوم والنتائج العملية. ان نتائج هذه المقارنة قد بينت ان النتائج العملية تتفق مع معادلة بوم المحورة التي قد اخذت بنظر الاعتبار عامل اللاستقرارية.

ان معامل الانتشار وعامل الاستقرارية قد حسبت للاضطراب القوي والضعيف. ان نتائج كلتا الحالتين قد فحصت مع نتائج معادلة بوم للانتشار بالاضافة الى النتائج العملية. حيث قد وجدت ان افضل انطباق يمكن الحصول عليها في نتائج الاضطراب القوي مع النتائج العملية. بينما تم الحصول على اتفاق تقريبي بين نتائج الاضطراب الضعيف مع النتائج العملية في المنطقه القريبه من مركز وحافة البلازما.