



## Prediction of Equilibrium Mixing Index and Optimum Mixing Time for Three solid materials in Fluidized Column

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### Abstract

Equilibrium and rate of mixing of free flowing solid materials are found using gas fluidized bed. The solid materials were sand (size 0.7 mm), sugar (size 0.7 mm) and 15% cast iron used as a tracer. The fluidizing gas was air with velocity ranged from 0.45-0.65 m/s while the mixing time was up to 10 minutes. The mixing index for each experiment was calculated by averaging the results of 10 samples taken from different radial and axial positions in fluidized QVF column 150 mm ID and 900 mm height.

The experimental results were used in solving a mathematical model of mixing rate and mixing index at an equilibrium proposed by Rose. The results show that mixing index increases with increasing air velocity and mixing time until it reaches an optimum value then decrease to an equilibrium value. The results also show the dependency of the mixing index on the particle size of the tracer component.

The statistical analysis of the obtained theoretical mixing index versus the experimental mixing index shows that the standard error is 0.92 % (about 1 %) with the correlation coefficient of 0.9857 and the total residual of 0.002 for the ninth observation of the mixing index (theoretical versus experimental) over 95% confidence level.

**Keywords:** Rate of mixing, equilibrium mixing index, fluidized bed.

### 1. Introduction

Mixing is the treatment of two or more components in such way that the individual particles of different components in the mixture are evenly distributed and lie adjacent to each other within the highest possible probability (Vaizoglu, 1999).

Solid mixing is a freely bubbling fluidized bed is caused not only by the vertical movement of bubbles and bursting at the bed surface, but also by the lateral motion of bubbles as a result of the interaction and coalescence of neighboring bubbles. Mixing of solids occurs in both axial and radial directions. However, vertical solid mixing is generally many times faster than that due to lateral motion (Kunii and Levenspiel, 1991).

Solid mixing is required in daily life, for instance in cooking foods, as well as in industrial, agricultural and pharmaceutical processes. However, unlike a liquid mixture, the

homogeneity cannot be realized spontaneously (Nozaki and Behaviormetrika, 1980). Thus, although fluidized beds are known for their good mixing characteristics, the solid mixing is often incomplete when particles of different properties are present in the bed (Hoffmann et al., 1993).

Statistical analysis has been a major tool of solids mixing investigations because of the random nature of mixing process (Fan and Wing, 1975). The statistical degrees of mixing are essentially different forms of the mean deviation, standard deviation or variance. However, only an index based on the variance can be adapted to be theoretically independent of sample size, and thus only this type of index will give consistently accurate values over the whole course of solids mixing (Roseman and Donald, 1962).

Not only gas velocity, but also particle properties like shape, size, density, stickiness, and size distribution influence the mechanisms of axial and radial transport of particles within the

bed. The competitive mechanism of segregation and mixing occur simultaneously producing equilibrium through the bed (Hoffmann et al., 1993).

Talmor and Beneati (cited in Rasha, 2007) measured the mixing rate of particles in a fluidized bed using a tracer technique. They conclude that the mixing of the particles is due to bulk circulation and that the rate of mixing rate depends on the excess gas flow rate over that required for minimum fluidization. Talmor and Beneati state that the solids circulation is proportional to the excess flow rate which is dependent on the mean particle size.

The behavior of systems of non- identical solid particles is a matter of practical concern in industrial operations dealing with the mixing of free-flowing materials. In such systems, both mixing and demixing occur. One of the major goals of recent efforts has been the formulation of rate-expressions to describe the progress of mixing. Knowledge of the rate is essential in understanding the mechanisms of mixing as well as for the improvement of mixer design (Fan et al., 1975).

In any mixer there is an interaction of mixing; demixing and equilibrium stage is reached when the rate of mixing is equal to rate of demixing. The position of this equilibrium and hence the quality of final mix depends on the mechanisms involved in the mixer. The mixing degree can be applied only when the mixing value scale has been specified. Each mixture may be quite well characterized by the variance or standard deviation calculated on the basis analysis of samples taken at random. A scale thus specified is proposed and presented in Table 1 (Boss and Poland, 1986).

**Table 1,**  
**Proposed Scale of Mixture Quality (Boss and Poland, 1986).**

Quality of mixture	Boundary value of M
Bad	0.7
Un-satisfactory	0.7 – 0.8
Fairly good	0.8 – 0.9
Good	0.9 – 0.94
Very good	0.94 – 0.96
Excellent	> 0.96

Rose (1959) proposed an expression to evaluate the degree of mixing:

$$M = 1 - \frac{\delta}{\delta_b} \quad \dots(1)$$

The mixing index can be determined as follows (Spiegel, 1972):

Mean value can be estimated from the analysis of the sample by the formula:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{i=n} X_n \quad \dots(2)$$

Standard deviation of large set of data is given by:

$$\delta = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}} \quad \dots(3)$$

Standard deviation of totally segregated mixture (before mixing) can be estimated from:

$$\delta_b = \sqrt{X(1-X)} \quad \dots(4)$$

## 2. Rose Model for Mixing Rate (Cited in Clyde, 1966)

Since mixing and unmixing undoubtedly occur simultaneously, a single relationship which was proposed by Rose should account for both phenomena. For mixing, the differential form of equation is:

$$\frac{dM}{dt} = A(1-M) \quad \dots(5)$$

The constant A depends upon the geometrical and physical parameter of the mixer and the powder.

For unmixing, the analogous expression is:

$$\frac{dM}{dt} = -B\phi \quad \dots(6)$$

Where:  $\phi$  depends upon the distribution of the two components in the mixer, and B is a constant depends on densities, size distribution and shape of particles... etc.

The entire process is thus described by the combined differential equations:

$$\frac{dM}{dt} = A(1-M) - B\phi = a_0 + a_1t \quad \dots(7)$$

The first term in equation (7) represents the mixing while the second term represents an unmixing.

A complication is introduced because the unmixing potential conceivably can be either positive or negative.

Considering this tendency to be due to difference in particle density. So if a dense component is placed on top of less one, it is easy to see that there is at first a positive tendency to mix, then the tendency will shift to an unmixing one as the heavier particles collect in the lower portion of the mixer.

Thus  $\phi$  passes from positive one for fully segregated condition, through zero for the completely mixed condition and becomes negative for the other completely segregated state.

This situation is summarized by:

Fully segregated condition,  $M=0 \quad \phi = +1$  ... (8)

Complete mixing condition,  $M=1 \quad \phi = 0$  ... (9)

Complete segregated condition,  $M=0 \quad \phi = -1$  ... (10)

A schematic representation of the mixing, which is expressed mathematically by:

$$M = 1 - \phi^2 \quad \dots(11)$$

Differentiating this expression with respect to time (t) gives:

$$-2\phi \frac{d\phi}{dt} = \frac{dM}{dt} \quad \dots(12)$$

Substituting equation (12) into equation (7), integrating and rearranging results in:

$$A\phi - B = C \exp\left(-\frac{At}{2}\right) \quad \dots(13)$$

Since  $\phi$  is expressed by  $\pm\sqrt{1-M}$ , then equation (13) can be rearranged to:

$$\pm\sqrt{1-M} = \frac{C}{A \exp\left(\frac{At}{2}\right)} + \frac{B}{A} \quad \dots(14)$$

Using the boundary conditions (equations 8-10), the value of constant C was determined, the equation becomes:

For  $\phi$  +ve:

$$M = \eta \left[ 1 - \left\{ \left( 1 - \frac{B}{A} \right) \exp\left(-\frac{At}{2}\right) + \frac{B}{A} \right\}^2 \right] \quad \dots(15)$$

For  $\phi$  -ve:

$$M = \eta \left[ 1 - \left\{ \frac{B}{A} - \left( 1 + \frac{B}{A} \right) \exp\left(-\frac{At}{2}\right) \right\}^2 \right] \quad \dots(16)$$

where  $\eta$  is the efficiency for actual mixer.

Curves corresponding to these equations show the main features exhibited by the results of test upon practical mixing machines i.e., for prolonged mixing the mixture will according to circumstances attain a definite degree of mixing either progressive improvement in the homogeneity of the mixing or by passing through a maximum in the perfection of mixing (Rose, 1959).

Finally it will be noticed that when the mixing process is carried out for an infinite time, the degree of mixing attains an equilibrium value  $M_e$  given by:

$$M_e = \eta \left[ 1 - \left( \frac{B}{A} \right)^2 \right] \quad \dots(17)$$

Actual mixer efficiency can be calculated as shown below:

$$\eta = \frac{\left[ 1 - \left( \frac{B}{A} \right)^2 \right]}{M_e} \quad \dots(18)$$

The aim of this work is to study the rate of mixing and mixing index at equilibrium by using an air fluidized bed, and solving a mathematical model proposed by Rose (cited in Clyde, 1966).

### 3. Materials and Experimental Setup

Sand and sugar with particle size of 0.7 mm were used and cast iron as a trace component with particle sizes 0.4, 0.5, 0.7 mm was used.

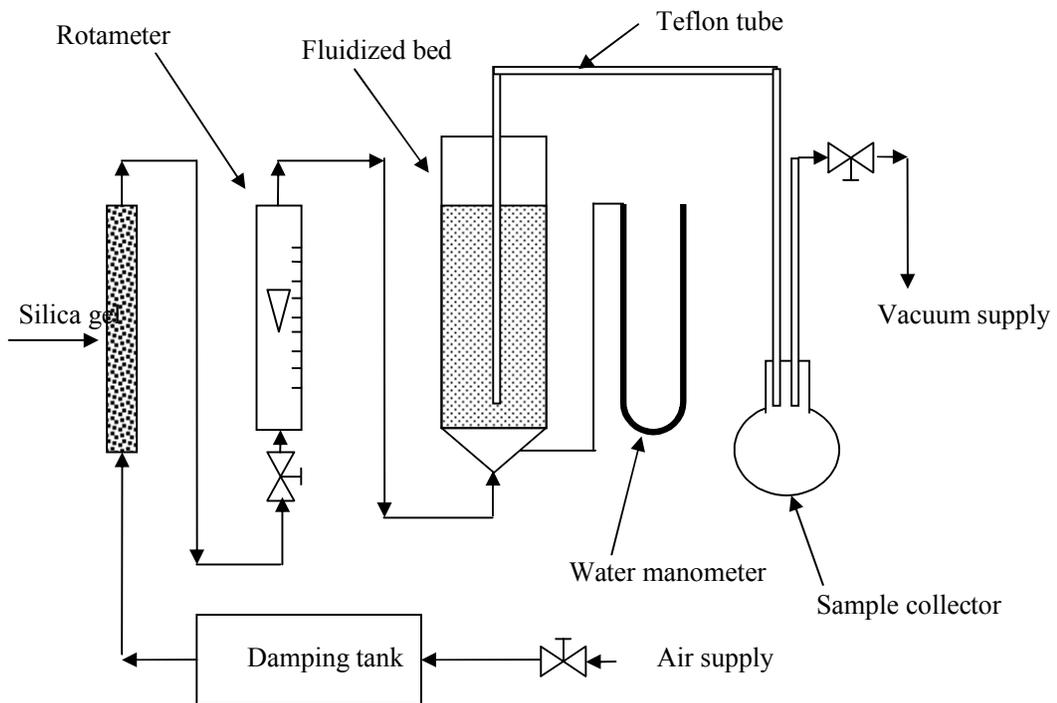
Each component was close-sieved to give a narrow size distribution. The physical properties of the solid materials are tabulated in Table 2.

The choice of using the materials was strongly based on the utilized available and easy analyzable materials. A mixture of sand, salt and cast iron could be analyzed gravimetrically by simply weighing, removing cast iron by magnetic bar from sand and sugar and re-weighing.

A schematic diagram of the experimental setup is shown in Figure 1.

**Table 2,**  
**Physical Properties of the Solid Materials (Perry, 1984)**

Materials	Real density, kg/m <sup>3</sup>	Shape factor	Surface properties
Sand	2530	0.6-0.861	Non cohesive, free flowing
Sugar	2000	0.65	Same
Cast iron	7030	0.578	Same besides magnetic



**Fig.1. Schematic Diagram of Experimental Setup.**

The experimental apparatus consisted of the following parts:

1. Fluidization column: The experiments were conducted in a 150 mm ID Q.V.F glass column, 900 mm height and open at the top to the atmosphere.
2. Air distributor plate: A perforated plate was used to support the bed material and to distribute the air uniformly through the bed; it is covered with wire screen to prevent particles leakage.
3. Air flow rate measurement: Air was supplied by means of a compressor (type LMF) used as a fluidizing gas. The air was fed to a coil then to a damping tank to minimize plus fluctuations in flow rate. The air was metered by means of calibrated feeding rotameter located before entering the fluidizing bed system, which was regulated by means of a glob valve for each rotameter.
4. The pressure drop across the bed was measured by using a U-tube manometer filled with water.

5. Samples from the fluidized bed were drawn by means of a teflon pipe connected to the main vacuum line available in the laboratory.
6. Auxiliary equipments were: Drying oven (funditor Ltd., London and Wembley), Test sieve shaker, (type Endecott) with timer and balance with maximum weight 1500 g (type Satorius), accuracy 2 orders.

#### 4. Procedure

The total weight of the charge was 3000 g; the weight of sand and sugar was equal and the concentration of cast iron was 15% by weight.

The procedure of all experiments was basically the same:

1. After weighing each component, they were introduced into the column carefully to obtain a horizontal interface between them.
2. Air at chosen flow rate was set for a desired time to ensure that mixing was achieved.
3. After the desired time, the air flow rate was shut off, and when the bed settled down ten samples were taken from ten different axial and radial positions. These were positioned 1cm above the air distributor and spaced 2 cm axially between them and 1.5 cm radially.
4. For each sample the sugar was dissolved using water, the sand was filtered and dried. The sand was re-weighed and the weight of sugar was obtained by subtracting the sand weight from total weight.

### 5. Results and Discussion

#### 5.1. Estimation of the Minimum Fluidization Velocity

Figure 2 represent a plot of pressure drop across the bed versus air velocity for a mixture of the three components to find their minimum fluidized velocity. The weight of sand and sugar was equal and the concentration of cast iron was 15% by weight. This figure indicates that the pressure drop across the bed increase with air velocity increasing, and then it decrease when the air velocity reaches an optimum value, these observations agree with Davidson and Harrison (1971).

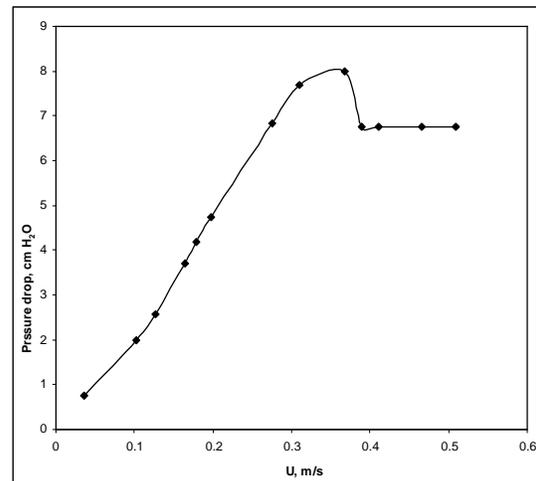


Fig.2. Estimation of Minimum Fluidization Velocity.

#### 5.2. Effect of Air Velocity and Particle Size on Rate Mixing

Figures 3 to 5 show the interaction effect between air velocity and mixing time on rate of mixing and mixing index while keeping the particle size constant. It can be noticed that the mixing index increases with increasing mixing time until the mixing index reaches an optimum value then it decreases however mixing time increases until it reaches an equilibrium value, also mixing index increases with increasing air velocity and the optimum value of air velocity that gives the maximum mixing index is 0.55 m/s, after this value however the air velocity is increased, the mixing index is decreased and rate of segregation increases until the rate of mixing reaches an equilibrium value. According to Scarlett (1964), The mixing index depends on the rate of bubbling but this does not necessarily increases with increasing gas flow rate so any further increasing in air velocity may lead to decrease in mixing index, it is the same for mixing time, mixing index increases with increasing mixing time until it reaches an optimum value then the rate of segregation increase until it reaches an equilibrium value and this agrees with Boss et al. (1986).

Figures 6 to 8 show the interaction effect between particle sizes of components and mixing time on rate of mixing and mixing index while keeping the air velocity constant. The maximum mixing index and equilibrium state of mixing have been reached by using the same particle sizes of each component (0.7 mm). The variation

between the components sizes can be considered the major parameter influencing the segregation intensity and as a consequence the equilibrium degree of mixing. When the difference between particles sizes of components increases the maximum rate of mixing and equilibrium state of mixing decrease. It has been mentioned previously that the segregation problem appears as the difference in particle sizes and density increases and according to Fan et al. (1975), the differences in size and/or density on rate appears to follow the path of rising very rapidly initially, passing through a maximum, and diminishing to a point of equilibrium.

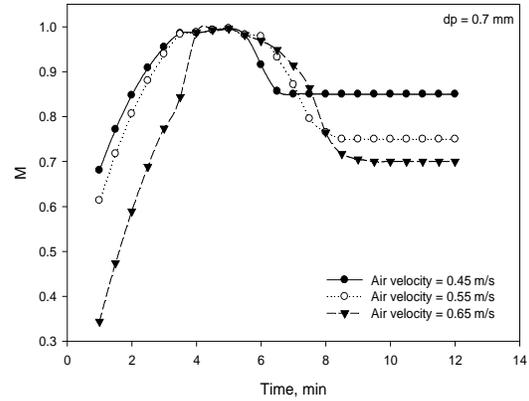


Fig.5. Mixing Index vs. Mixing Time at Various Air Velocities and Tracer Particle Diameter 0.7mm.

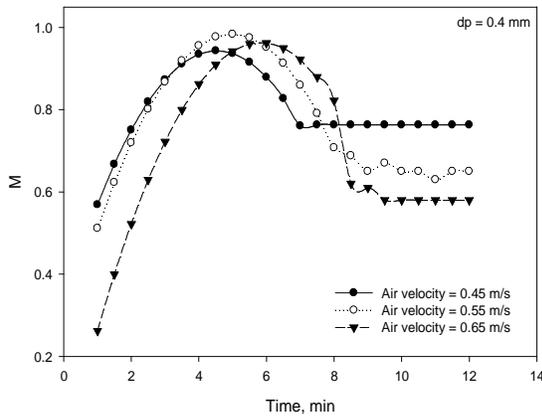


Fig.3. Mixing Index vs. Mixing Time at Various Air Velocities and Tracer Particle Diameter 0.4mm.

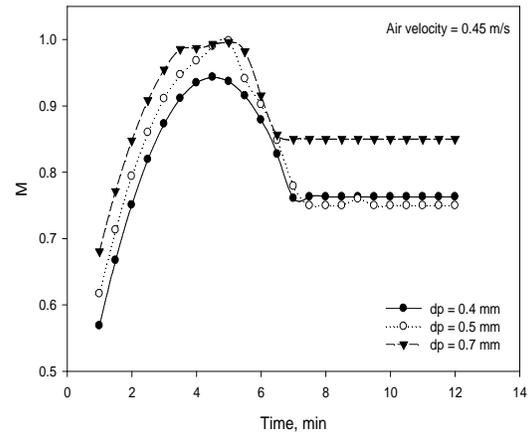


Fig.6. Mixing Index vs. Mixing Time at Various Particle Diameter of Tracer at Air Velocity 0.45m/s.

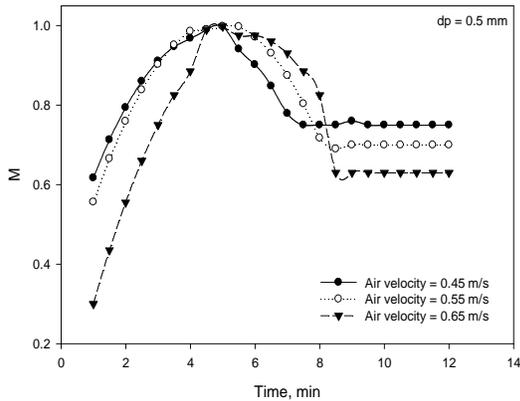


Fig.4. Mixing Index vs. Mixing Time at Various Air Velocities and Tracer Particle Diameter 0.5mm.

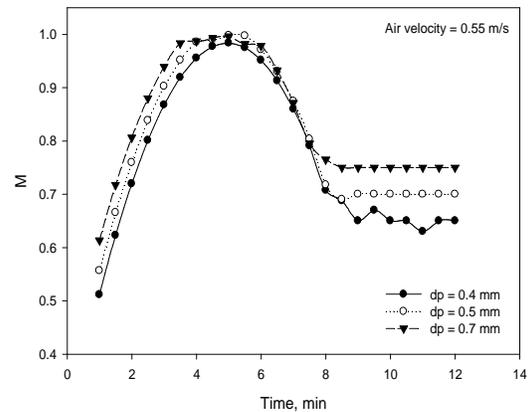


Fig.7. Mixing index vs. Mixing Time at Various Particle Diameter of Tracer at Air Velocity 0.55m/s.

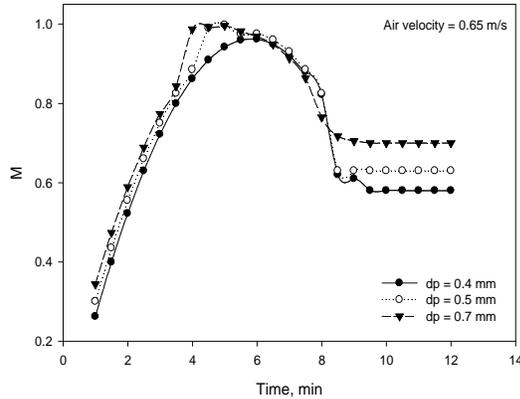


Fig.8. Mixing Index vs. Mixing Time at Various Particle Diameter of Tracer at Air Velocity 0.65 m/s.

### 5.3. Prediction of Equilibrium Mixing Index and Optimum Mixing Time

A mathematical model for rate of mixing proposed by Rose (1959) (cited in Clyde, 1966) was used to predict optimum time of mixing.

Experimental data of rate of mixing versus time were fitted with second order polynomial equation according to time as:

$$M = a_0 + a_1t + a_2t^2 \quad \dots(19)$$

Then the first derivative of the fitted equation ( $\frac{dM}{dt}$ ) is obtained and equal to the equation 4, this gives:

$$\frac{dM}{dt} = A(1 - M) - B\phi = a_1 + 2a_2t \quad \dots(20)$$

Constants (A and B) are easily calculated when substitute  $\phi = \pm\sqrt{1 - M}$  (re-arranging of Eq. 11) in equation 20.

The values of A and B were calculated for negative value of  $\phi$ , because of heavier component (dense component) was placed below (or above) the lighter component in the fluidized bed, and then actual mixer efficiency was calculated from equation 16. The results of actual mixer efficiency are summarized in Table 3.

The higher values of actual mixer efficiency obtained at air velocity 0.55 m/s. This observation supported the present results which mentioned that the mixing index increases with increasing mixing time and air velocity until the air velocity

reaches an optimum value 0.55 m/s at which the maximum mixing index is reached.

Table 3, Actual Mixer Efficiency Results.

Air velocity (u), m/s	Particle size (dp), mm	Actual mixer efficiency, -
0.45	0.4	0.887
0.45	0.5	0.821
0.45	0.7	0.897
0.55	0.4	0.929
0.55	0.5	0.899
0.55	0.7	0.915
0.65	0.4	0.728
0.65	0.5	0.775
0.65	0.7	0.897

After that, an average value of actual efficiency of mixer was used to predict equilibrium rate of mixing using equation 17.

The results show that optimum time of mixing for sand-sugar system was about 4.5 to 5.0 minutes for all experiments conditions.

Figures 9 to 11 show maximum, experimental and theoretical (obtained from Rose model) mixing index (M) versus different air velocity at constant components particle size. Theoretical mixing index was always greater slightly than experimental mixing index; this may be due to friction between the particles during the experiments.

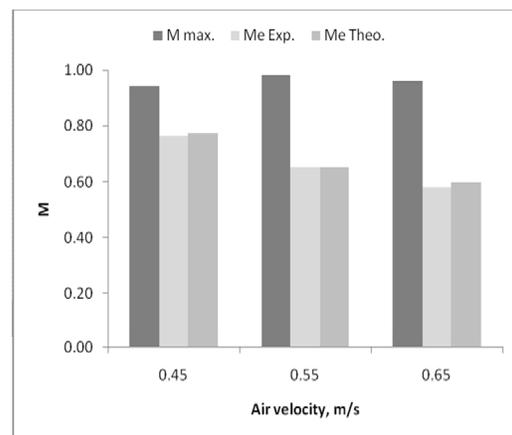


Fig.9. Comparison between Maximum and Equilibrium Experimental Mixing Index and Theoretical Mixing Index at Various Air Velocity and dp Of Tracer = 0.4 mm.

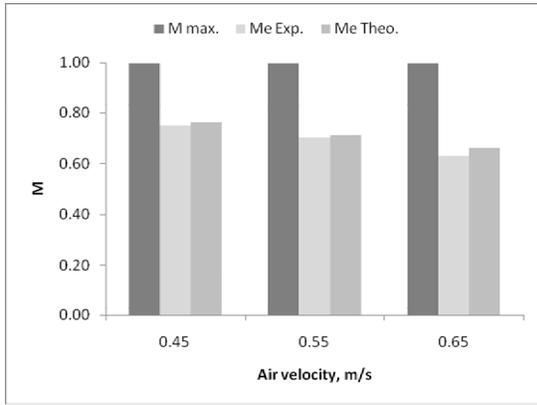


Fig.10. Comparison between Maximum and Equilibrium Experimental Mixing Index and Theoretical Mixing Index at Various Air Velocity and dp Of Tracer = 0.5 mm.

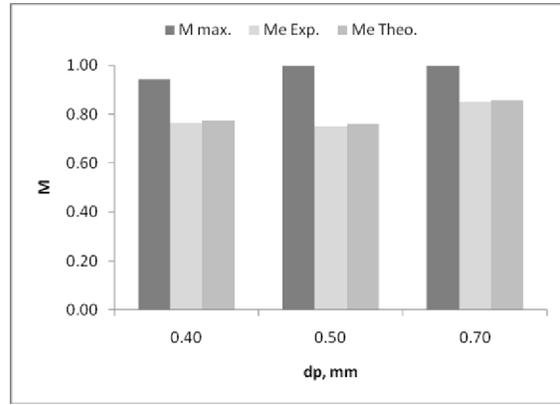


Fig.12. Comparison between Maximum and Equilibrium Experimental Mixing Index and Theoretical Mixing Index at Various Particle Diameter of Tracer and Air Velocity is 0.45 m/s.

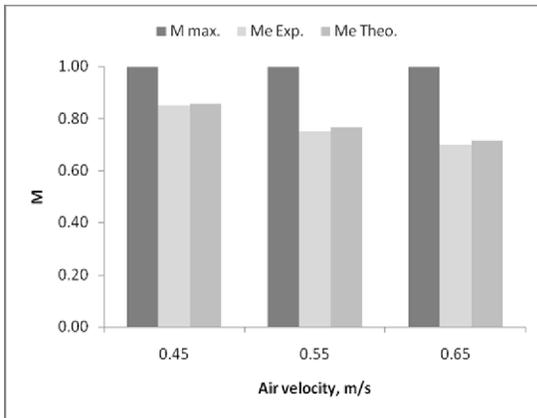


Fig.11. Comparison between Maximum and Equilibrium Experimental Mixing Index and Theoretical Mixing Index at Various Air Velocity and dp Of Tracer = 0.7 mm.

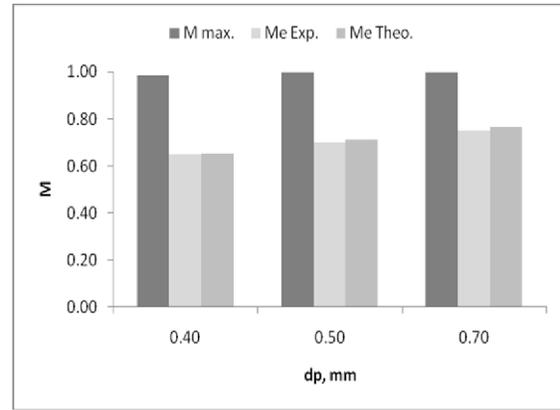


Fig.13. Comparison between Maximum and Equilibrium Experimental Mixing Index and Theoretical Mixing Index at Various Particle Diameter of Tracer and Air Velocity is 0.55 m/s.

Maximum mixing index (Figures 8 to 10) was found to increase with the air velocity, while the effect of particle size on maximum mixing index was limited therefore no significant difference in maximum mixing index as particle size change as shown in Figures 12 to 14.

The statistical analysis of the obtained theoretical mixing index versus experimental mixing index shows that the standard error is 0.92 % (about 1 %) with correlation coefficient of 0.9857 and total residual of 0.002 for the ninth observation of the mixing index (theoretical versus experimental) over 95% confidence level.

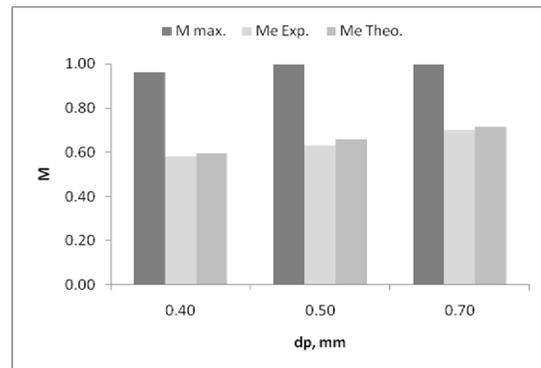


Fig.14. Comparison between Maximum and Equilibrium Experimental Mixing Index and Theoretical Mixing Index at Various Particle Diameter of Tracer and Air Velocity is 0.65 m/s.

## 6. Conclusions

1. The rate of mixing increases with air velocity and time of mixing until the air velocity reaches 0.55 m/s, then the rate of mixing decreased and rate of segregation increases until reaches an equilibrium value of mixing.
2. The maximum rate of mixing and equilibrium state of mixing have been reached when using the same particle size of the main and trace components (0.7 mm)
3. The results show that optimum time of mixing for sand-sugar system was about 4.5 to 5.0 minutes for all experiments conditions.
4. Theoretical mixing index were always greater slightly than experimental mixing index.
5. A mathematical model proposed by Rose is sufficient to describe mixing behavior for sand- sugar -cast iron system of particle size 0.4 to 0.7 mm, air velocity 0.45 to 0.65 m/s over mixing time up to 12 min.

## Nomenclature

A	constant
$a_0$	constant
$a_i$	constant
B	constant
C	constant
$d_p$	particle diameter (mm)
M	mixing index
$M_e$	mixing index at equilibrium
n	number of samples
t	time (min)
$U_{mf}$	minimum fluidized velocity (m/s)
X	tracer percentage
$X_i$	tracer percentage in each sample
$\delta$	standard deviation
$\delta_0$	standard deviation of totally segregated mixture
$\eta$	efficiency of actual mixer
$\Phi$	constant

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## التنبؤ بمعامل الخلط عند التوازن وزمن الخلط الامثل لثلاثة مواد صلبة في عمود التميع

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### الخلاصة

تم دراسة معامل الخلط عند الاتزان ومعدل الخلط لمواد حرة الجريان بأستخدام عمود هواء متميع. المواد التي تم خلطها هي الرمل ( قطر الجسيمات 0.7 ملم) والسكر (قطر الجسيمات 0.7 ملم) وبرادة الحديد وهي تمثل المادة الكاشفة بتركيز 10% وزنا. معدل سرعة الهواء كان بين 0.45 و 0.65 م/ثا بينما زمن الخلط كان لحد 10 دقائق. معامل الخلط لكل تجربة كان يحسب بأخذ عشرة نماذج ذات مواقع مختلفة قطريا وخطيا من عمود التميع ذو القطر الداخلي 150 ملم وأرتفاعه 900 ملم.

تم أستخدام النتائج العملية في حل موديل رياضي لمعدل الخلط ومعامل الخلط عند الأتزان مقترح من قبل Rose. النتائج التجريبية أستعملت لحل موديل رياضي لمعدل الجريان و معامل الخلط عند الاتزان أقترح من قبل Rose. النتائج بينت ان معامل الخلط يزداد بزيادة سرعة الهواء وزمن الخلط حتى يصل للقيمة المثلى ثم ينخفض الى أن يصل للاتزان. نتائج معدل الخلط بينت أعتماها على مقدار الفرق بين حجم جسيمات المادة الكاشفة.

أشار التحليل الإحصائي لكل من معامل الخلط النظري و العملي إلى أن الخطأ المعياري كان 0.92 % (حوالي 1 %) و معامل ارتباط 0.9807. ومجموع المتبقيات لتسع مشاهدات كان 0.002 لمستوى ثقة 95%.