

## A Mathematical Development of Gordon, Mills and Welch Generator Using Galois Field and Trace Polynomials

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### Abstract

The paper presents a newly developed method depending on trace polynomial with new algorithm which has been written in Matlab language to modify the Gordon, Mills and Welch generator by increasing the complexity of the output sequence to increase the security of this generator concerning the designed feature that limits the ability of anti-jammer. Good results are obtained compared with Gordon, Mills and Welch generator using Berlekamp-Massey method for determining the complexity of the output sequences. Moreover, this paper has the useful properties of the trace function. Some examples are given of show conciliated the results of this proposed method.

**Keywords :** Gordon, Mills and Welch Generator, Trace polynomial, Complexity and Sequence.

التطوير الرياضي لمولد جوردن - ملز - ويلش باستخدام  
حقل غالوا و متعددات حدود الأثر

الخلاصة

يقدم البحث طريقة مطورة مع خوارزمية جديدة تعتمد على حقل غالوا و متعددات حدود الأثر (Trace polynomials) لتطوير مولد جوردن - ملز - ويلش (GMW) اللاخطي عن طريق زيادة تعقيد المتتابعة الناتجة من هذا المولد لزيادة أمنيته و قدرته على مواجهة التداخل و التشويش في أنظمة الاتصالات وقد حصلنا على تعقيد جيد بالمقارنة مع مولد الـ (GMW) من خلال إعطاء بعض الأمثلة التي تبين كفاءة المولد الجديد. استخدمت لغة (Matlab) لبرمجة هذه الطريقة كما تم ذكر الخصائص المهمة لدالة الأثر.

### 1. Introduction

A pseudo-noise (PN) generator is a mechanism for generating a PN-sequence of binary or real digits. Pseudo-Noise generators generate (PN) sequences which are used as spectrum-spreading modulations for direct sequence spread spectrum design for digital communication system and as a key in cipher systems [1,2]. The resulting sequence is called pseudo-noise

sequence since it is periodic and there is no algorithm using a finite state machine which can produce a truly random sequence [2]. PN-sequences are characterized by three properties; namely: period, complexity and randomness. These properties define the measure of security for the sequences [3,4]. The complexity of the sequence is one of important properties for security of the

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information from unauthorized person.

Gordon, Mills and Welch generator is one of these PN generators. It produces a binary sequence with good period and randomness properties but with relatively small complexity. The idea of this paper is to increase the complexity of the produced sequence but with remaining the same period. The new generator has a sequence with high complexity and good randomness properties. Hence, it has enough ability for the security of the information from interceptor and jammer.

## 2. The Complexity of The Periodic Sequence :

The complexity of the sequence (or generator) in the cipher and communication systems is the length of the minimum linear feedback shift register (LFSR) that can generate the sequence [5,6]. We can characterize the LFSR of length (n) by the characteristic polynomial  $f(x)$  as:

$$f(x) = c_0 + c_1x + c_2x^2 + \dots + c_{n-1}x^{n-1} + x^n$$

where  $c_0, c_1, \dots, c_{n-1}$  are 0 or 1.

For a sequence with a known complexity (L), we need (2L) consecutive bits to deduce the entire sequence since if (2L) consecutive bits are given, then we can write a system of L-equations in the L unknown variables and find its unique solution. This gives the characteristic polynomial of the minimal LFSR that can generate the given sequence. Hence for acceptable security, we need to have a sequence or generator with high complexity [2,7]. There are several methods to determine the complexity of the sequences like

Berlekamp-Massey method which is given in the following section, Z-transform and matrices techniques [7].

## 3. The Berlekamp-Massey Method

Berlekamp-Massey (BM) technique [3,5,6] uses the description based on the synthesis of a shift register where it is used to determine the complexity and the minimal characteristic polynomial that can generate the given sequence .

Berlekamp-Massey method gives [6,7]:

1. The polynomial  $C(D)$  .
2. The linear equivalence or the complexity (L) of the sequence .

BM technique is explained in the following algorithm :

### *Berlekamp-Massey (BM)*

#### *Algorithm [5,6,7]:*

#### **Step 1:**

Input :

- (1) The period (n) of the sequence (S).
- (2) The digits  $S_i, i=0,1,\dots, n-1$  of the sequence (S).

#### **Step 2:**

Put  $C(D) = 1, B(D) = 1, L=0, b=1, x=1$  and  $N=0$

#### **Step 3:**

If  $N = n$ , then stop. Otherwise compute (d) :

$$d = S_N + \sum_{i=1}^L c_i S_{N-i}$$

#### **Step 4:**

If  $d = 0$ , then  $x = x + 1$ , and go to (step 7)

#### **Step 5:**

If  $d \neq 0$  and  $2L > N$ , then

- $C(D) = C(D) - db^{-1} D^x B(D)$
- $x=x+1$  and go to (step 7)

**Step 6:**

If  $d \neq 0$  and  $2L \leq N$ , then

- $T(D) = C(D)$
- $C(D) = C(D) - db^{-1}D^x B(D)$
- $L = N+1-L$
- $B(D) = T(D)$
- $b = d$
- $x = 1$

**Step 7:**

$N = N+1$   
and go to (step 3).

**4. Galois Field Arithmetic [1,7] :**

**Definition(1):**

A finite field  $F$  is called a Galois field denoted by  $GF(q^m)$  if the number of elements of  $F$  is  $q^m$  (i.e. the order of  $F$  is  $q^m$ ), where  $q$  is a prime number and  $m$  is a natural number.  $q$  is called the characteristic of the field  $GF(q^m)$ .

**Definition(2):**

It is known that every nonzero element  $\alpha$  of  $GF(q^m)$  which satisfies the equation :

$$\alpha^{q^m-1} = 1$$

is said to be a primitive element of  $GF(q^m)$  if all the powers of  $\alpha$  less than  $(q^m-1)$  are different. Thus, if  $\alpha$  is a primitive element then  $\alpha^i \neq 1$ , for  $0 < i < q^m-1$ .

**Example:**

Consider the field  $GF(2^4)$  obtained by taking the polynomial  $m_\alpha(z) = z^4+z+1$  over  $GF(2)$  as the modulo polynomial.

The powers of  $\alpha$  were reduced to polynomials of degree (3) or less in  $\alpha$ . Table (1) shows the representation elements of  $GF(16)$ .

**5. Trace polynomial :**

A fundamental mathematical tool used in investigation of PN generator is a linear mapping from a finite field onto a subfield. This mapping is called the ‘‘Trace polynomial’’ or the ‘‘Trace function’’ denoted by  $Tr_q^{q^n}(a)$  where  $a \in GF(q^n)$ .

The Trace polynomial from  $GF(q^n)$  to  $GF(q)$  where  $(q>1)$  and  $(n \geq 2)$  is defined as [1]:

$$Tr_q^{q^n}(a) = \sum_{i=0}^{n-1} (a)^{q^i} \dots (1)$$

where  $a \in GF(q^n)$ .

**Example :**

In  $GF(16)$  represented in Table (1), the trace values of  $\alpha^3$  and  $\alpha^5$  are :

$$\begin{aligned} Tr_2^{16}(\alpha^3) &= (\alpha^3) + (\alpha^3)^2 + (\alpha^3)^4 + (\alpha^3)^8 \\ &= \alpha^3 + \alpha^6 + \alpha^{12} + \alpha^9 \\ &= \alpha^3 + (\alpha^2 + \alpha^3) + (1 + \alpha + \alpha^2 + \alpha^3) \\ &+ \end{aligned}$$

**Table (1) The representation elements of  $GF(2^4)$**

Representations			
Power of $\alpha$	Polynomial in $\alpha$	Power of $\alpha$	Polynomial in $\alpha$
$\alpha^0$	1	$\alpha^8$	$1 + \alpha^2$
$\alpha^1$	$\alpha$	$\alpha^9$	$\alpha + \alpha^3$
$\alpha^2$	$\alpha^2$	$\alpha^{10}$	$1 + \alpha + \alpha^2$
$\alpha^3$	$\alpha^3$	$\alpha^{11}$	$\alpha + \alpha^2 + \alpha^3$
$\alpha^4$	$1 + \alpha$	$\alpha^{12}$	$1 + \alpha + \alpha^2 + \alpha^3$
$\alpha^5$	$\alpha + \alpha^2$	$\alpha^{13}$	$1 + \alpha^2 + \alpha^3$
$\alpha^6$	$\alpha^2 + \alpha^3$	$\alpha^{14}$	$1 + \alpha^3$
$\alpha^7$	$1 + \alpha + \alpha^3$		

$(\alpha + \alpha^3) = 1$ . (See table(1)).

$$\begin{aligned} Tr_2^{16}(\alpha^5) &= (\alpha^5) + (\alpha^5)^2 + (\alpha^5)^4 + (\alpha^5)^8 \\ &= \alpha^5 + \alpha^{10} + \alpha^5 + \alpha^{10} = 0, \end{aligned}$$

where  $\alpha^3$  and  $\alpha^5 \in GF(16)$  and the arithmetic operations over  $GF(2)$ .

The useful properties of the trace function are :

**Property 1:** When  $\alpha$  is in  $GF(q^n)$ ,  $Tr_q^{q^n}(a)$  has values in  $GF(q)$ .

**Proof :**

$$[Tr_q^{q^n}(a)]^q = \left( \sum_{j=0}^{n-1} a^{q^j} \right)^q \text{ (Definition)}$$

$$= \sum_{j=0}^{n-1} a^{q^{j+1}} \text{ (Since the arithmetic operations over } GF(q))$$

$$= \sum_{j=1}^{n-1} a^{q^j} + a$$

Since  $(a \in GF(q^n) \Rightarrow a^{q^n} = a)$

$$= Tr_q^{q^n}(a) \text{ (Definition)}$$

Therefore,  $Tr_q^{q^n}(a)$  is a field element whose q-th power equals itself. Only elements of  $GF(q)$  have this property.

**Property 2 :**

Conjugate field elements which have the same trace value.

The proof of this property follows from the definition of Trace polynomial and the previous property.

**Property 3 :** The Trace is linear : for  $a, b \in GF(q)$  and  $a, b \in GF(q^n)$ ,

$$Tr_q^{q^n}(aa + bb) = aTr_q^{q^n}(a) + bTr_q^{q^n}(b)$$

**Proof :-**

$$Tr(aa + bb) = \sum_{j=0}^{n-1} (aa + bb)^{q^j}$$

(Definition)

$$= \sum_{j=0}^{n-1} (a^{q^j} a^{q^j} + b^{q^j} b^{q^j})$$

(Since the arithmetic operations over  $GF(2)$ )

$$= \sum_{j=0}^{n-1} aa^{q^j} + \sum_{j=0}^{n-1} bb^{q^j}$$

Since  $(a \in GF(q) \Rightarrow a^q = a)$

$$= aTr_q^{q^n}(a) + bTr_q^{q^n}(b)$$

(Definition).

### 6. Gordon, Mills and Welch Generator :

Gordon, Mills and Welch (GMW) [1,8,9] suggested the creation of a binary generator depending on "Trace polynomial". When the integer m is a composite, i.e.

$$m = j \times k \quad \dots (2)$$

We define a GMW sequence generator of period  $(2^m - 1)$  as :

$$b_n = Tr_2^{2^j} \left( [Tr_2^{2^m}(a^n)]^r \right) \dots (3)$$

where  $n=0,1,2,\dots,2^m-2$ ,  $\alpha$  is a primitive element of  $GF(2^m)$ ,  $Tr_q^{q^d}(a^n)$  is the "Trace polynomial" from  $GF(q^d)$  to  $GF(q)$  and r is any integer relatively prime to  $2^j-1$  (i.e.  $\gcd(r, 2^j-1) = 1$ ), r in the range  $0 < r < (2^j-1)$ . Figure (1) shows a GMW generator of period  $(2^m-1)$  [10].

The exponent r in Eq.(3) can be written as [1,10]:

$$r = \sum_{i=1}^w 2^{t_i} \quad \dots (4)$$

where the  $t_i$ 's are distinct integers in the range  $0 \leq t_i < j$  for all i and w is the number of ones in the base-2 representation of r.

Hence,

$$b_n = \text{Tr}_2^{2^j} \left( \prod_{i=1}^w \left[ \text{Tr}_2^{2^m} (a^n) \right]^{2^{i_i}} \right) \dots (5)$$

Since the inner trace is a sum of elements from a field of characteristic (2) (i.e. the operations over GF(2) ), all cross-product terms disappear when the trace is squared and Eq.(5) reduces to:

$$b_n = \text{Tr}_2^{2^j} \left( \prod_{i=1}^w \sum_{h=0}^{k-1} a^{n 2^{j h + t_i}} \right) = \text{Tr}_2^{2^j} \left( \sum_{h_1=0}^{k-1} \mathbf{K} \sum_{h_w=0}^{k-1} a^{nC(k,r)} \right) \dots (6)$$

where k is defined in Eq.(2) and

$$C(k,r) = \sum_{i=1}^w 2^{j h_i + t_i} \dots (7)$$

When expanding the outer Trace polynomial in Eq.(6) , cross-product terms again must disappear, and the GMW generator becomes :

$$b_n = \sum_{u=0}^{j-1} \sum_{h_1=0}^{k-1} \mathbf{K} \sum_{h_w=0}^{k-1} a^{nC(k,r)2^u} \dots (8)$$

The GMW generator producing a binary sequence of period  $(2^m-1)$  is computed by the following algorithm:

**GMW Algorithm:**

**Step 1 :**

Input:-

- (1) The minimum polynomial  $m_\alpha(z)$  of the primitive element  $\alpha$  of GF( $2^m$ ) over GF(2).
- (2) The values of j, k and r .

**Step 2 :**

Find the powers-of- $\alpha$  representation for GF( $2^m$ ) elements (i.e.  $a^0, a^1, \dots, a^{2^m-2}$ ) from  $m_\alpha(z)$  over GF(2) .

**Step 3 :**

For all  $n=0,1,\dots,2^m-2$ , evaluate  $b_n$ :

$$b_n = \sum_{u=0}^{j-1} \sum_{h_1=0}^{k-1} \mathbf{K} \sum_{h_w=0}^{k-1} a^{nC(k,r)2^u}$$

where  $C(k,r) = \sum_{i=1}^w 2^{j h_i + t_i}$

and use (step 2 ) to find the sequence of GMW generator .

**Example :**

A GMW sequence of period (63) is:

$$b_n = \text{Tr}_2^{2^3} \left( \left[ \text{Tr}_2^{2^6} (a^n) \right]^3 \right), \quad n=0,1,\dots,62 \dots(9)$$

where  $m_\alpha(z) = z^6 + z^5 + z^2 + z + 1$  over GF(2),  $m=6=3 \times 2 = j \times k$  and  $r=3$  is chosen .

Applying the GMW algorithm yields the following sequence :

$b_n = 000001010010011101011101001011100011001111110010010111001110100.$

**7. Proposed Method for Developing The Gordon, Mills and Welch (GMW) Generator Using Galois Field and Trace polynomial :**

The GMW in previous section is a nonlinear generator which produces a pseudo-noise sequence of period  $(2^m - 1)$  over GF(2), where m is a composite integer as well as the degree of the minimum polynomial

$m_\alpha(z)$  of a primitive element  $\alpha$  of  $GF(2^m)$  over  $GF(2)$ .

We are developing the GMW generator by using “Trace polynomial” from  $GF(q^n)$  to  $GF(q)$  to produce a sequence which has the same period but with complexity higher than GMW generator and it is denoted by (DGMW).

Hence, the **DGMW generator** is defined as :

$$b_n = Tr_2^{2^j} ([Tr_2^{2^m} (a^n)]^r) \oplus_2 S_n \oplus_2 Rec(S_n), \quad n=0,1,\dots,2^m-2 \quad \dots (10)$$

where

$$S_n = Tr_2^{2^m} (a^{n+k}) | Tr_2^{2^m} (a^{n+j+k}),$$

which has period  $(2^m - 1)$ ,  $Rec(S_n)$

is the reciprocal of the sequence

$$(S_n = Tr_2^{2^m} (a^{n+k}) | Tr_2^{2^m} (a^{n+j+k}))$$

,  $n=0,1,\dots,2^m-2$  and it is defined as :

$$\begin{aligned} Rec(S_n) &= Rec(Tr_2^{2^m} (a^{n+k}) | Tr_2^{2^m} (a^{n+j+k})) \\ &= Rec(S_0, S_1, S_2, \dots, S_{2^m-3}, S_{2^m-2}) \\ &= (S_{2^m-2}, S_{2^m-3}, S_{2^m-4}, \dots, S_1, S_0) \end{aligned}$$

,  $\alpha^n \in GF(2^m)$ ,  $n = 0, 1, 2, \dots, 2^m-2$ ,

$m = j \times k$  is a composite integer,  $r$  is an integer in the range  $(0 < r < 2^j-1)$

relatively prime to  $2^j-1$  (i.e.

$\gcd(r, 2^j-1) = 1$ ),  $\oplus_2$  is a modulo

2 addition,  $(|)$  is “OR” function which is defined as  $(0|0=0,$

$0|1=1|0=1|1=1)$  and  $Tr_q^{q^d} (a^n)$  is the

“Trace polynomial” in eq.(1) from  $GF(q^d)$  to  $GF(q)$  which is defined as:

$$Tr_q^{q^d} (a^n) = \sum_{i=0}^{d-1} (a^n)^{q^i}.$$

The following algorithm summarizes the steps for finding the binary sequence of period  $(2^m-1)$  of DGMW generator.

**DGMW Algorithm :**

**Step 1:**

Input:

- (1) The minimum polynomial  $m_\alpha(z)$  of the primitive element  $\alpha$  of  $GF(2^m)$  over  $GF(2)$ .
- (2) The values of  $j, k$  and  $r$ .

**Step 2 :**

Find the power- $\alpha$  representation for  $GF(2^m)$  elements (i.e.  $a^0, a^1, \dots, a^{2^m-2}$ ) using  $m_\alpha(z)$  over  $GF(2)$  as shown in section(4).

**Step 3 :**

For all  $n=0,1, \dots, 2^m-2$  compute:

$$Tr_2^{2^j} ([Tr_2^{2^m} (a^n)]^r) = Tr_2^{2^j} \left( \sum_{h_1=0}^{k-1} \mathbf{K} \sum_{h_w=0}^{k-1} a^{nC(k,r)} \right)$$

where  $C(k, r) = \sum_{i=1}^w 2^{jh_i+t_i}$

(See Eq.(4) and Eq.(7)).

**Step 4 :**

For all  $n=0,1,\dots,2^m-2$  compute:

$$\begin{aligned} S_n &= Tr_2^{2^m} (a^{n+k}) | Tr_2^{2^m} (a^{n+j+k}), \\ Rec(S_n) &= Rec(Tr_2^{2^m} (a^{n+k}) | Tr_2^{2^m} (a^{n+j+k})) \\ &= (S_{2^m-2}, S_{2^m-3}, S_{2^m-4}, \dots, S_1, S_0) \end{aligned}$$

where

$$Tr_2^{2^m} (a^{n+k}) = \sum_{i=0}^{m-1} (a^{n+k})^{2^i}$$

and

$$Tr_2^{2^m} (a^{n+j+k}) = \sum_{d=0}^{m-1} (a^{n+j+k})^{2^d}$$

**Step 5 :**

For all  $n=0,1,\dots,2^m-2$ , evaluate  $b_n$  in Eq.(10) by using (step 3) and (step 4) as follows:

$$b_0 = \text{Tr}_2^{2^j} ([\text{Tr}_2^{2^m} (a^0)]^r) \oplus_2 S_0 \oplus_2 S_{2^m-2}$$

$$b_1 = \text{Tr}_2^{2^j} ([\text{Tr}_2^{2^m} (a)]^r) \oplus_2 S_1 \oplus_2 S_{2^m-3}$$

**M**

$$b_{2^m-2} = \text{Tr}_2^{2^j} ([\text{Tr}_2^{2^m} (a^{2^m-2})]^r) \oplus_2 S_{2^m-2} \oplus_2 S_0$$

where

$$\text{Tr}_2^{2^j} ([\text{Tr}_2^{2^m} (a^n)]^r) = \sum_{v=0}^{j-1} \left( \sum_{i=0}^{k-1} (a^n)^{2^{ji}} \right)^{2^v r}$$

**8. Illustrative Examples :**

**Example (1) :**

Consider the following GMW generator :

$$b_n = \text{Tr}_2^{2^4} ([\text{Tr}_2^{2^8} (a^n)]^{11}) , \dots (11)$$

where  $n = 0, 1, \dots, 254$ ,  $\alpha \in \text{GF}(2^8)$ ,  $m_\alpha(z) = z^8 + z^6 + z^5 + z + 1$  over  $\text{GF}(2)$ ,  $m=8=4 \times 2 = j \times k$  and  $r=11$ . The GMW generator is developed using Eq.(10) as :

$$b_n = \text{Tr}_2^{2^4} ([\text{Tr}_2^{2^8} (a^n)]^{11}) \oplus_2 S_n \oplus_2 \text{Rec}(S_n) \dots (12)$$

where :

$$S_n = \text{Tr}_2^{2^8} (a^{n+2}) | \text{Tr}_2^{2^8} (a^{n+6}) ,$$

$$\text{Rec}(S_n) = \text{Rec}(\text{Tr}_2^{2^8} (a^{n+2}) | \text{Tr}_2^{2^8} (a^{n+6})) = (S_{254}, S_{253}, S_{252}, \dots, S_1, S_0)$$

The following table presents the results obtained by applying DGMW algorithm.

**Table (2) The results of DGMW generator for Ex.(1).**

Generators	Output Sequences	Period
$\text{Tr}_2^{2^4} ([\text{Tr}_2^{2^8} (a^n)]^{11})$	000101100111110100 111110...11111	255
$\text{Tr}_2^{2^8} (a^{n+2})   \text{Tr}_2^{2^8} (a^{n+6})$	110110111011111111 111011...11101	255
$\text{Rec}(\text{Tr}_2^{2^8} (a^{n+2})   \text{Tr}_2^{2^8} (a^{n+6}))$	101111111101111101 110100...11011	255
<b>DGMW generator in Eq.(12)</b>	011100100001110110 110001...11001	255

The complexity of GMW and DGMW sequences is computed in

Table (3) by applying Berlekamp-Massey (BM) algorithm.

**Table (3) The complexity of GMW and DGMW sequences.**

<i>The Generators</i>	<i>BM Method</i>
	<i>The Complexity (L)</i>
GMW in Eq.(11) $Tr_2^{2^4} ([Tr_2^{2^8} (a^n)]^{11})$	16
DGMW in Eq.(12) $Tr_2^{2^4} ([Tr_2^{2^8} (a^n)]^{11}) \oplus_2 S_n \oplus_2 Rec(S_n)$	72

It is obvious from Table (3) that the complexity of DGMW sequence (or generator) is higher than GMW sequence. So, the security of DGMW generator is more than GMW, since in DGMW sequence we need (144) consecutive bits to find the entire sequence while in GMW sequence we need only (32) consecutive bits to find the entire sequence (see the definition of the complexity in section(2)).

**Example (2) :**

Consider the following GMW generator :

$$b_n = Tr_2^{2^3} ([Tr_2^{2^6} (a^n)]^3),$$

where  $n = 0, 1, \dots, 62$  ,  $\alpha \in GF(2^6)$  ,  
 $m_\alpha(z) = z^6 + z^5 + z^2 + z + 1$  over  $GF(2)$  ,  
 $m = 6 = 3 \times 2 = j \times k$  and  $r = 3$ . The GMW

generator is developed using Eq.(10) as:

$$b_n = Tr_2^{2^3} ([Tr_2^{2^6} (a^n)]^3) \oplus_2 S_n \oplus_2 Rec(S_n) \dots(13)$$

where:

$$S_n = Tr_2^{2^6} (a^{n+2}) | Tr_2^{2^6} (a^{n+5}),$$

$$Rec(S_n) = Rec(Tr_2^{2^6} (a^{n+2}) | Tr_2^{2^6} (a^{n+5})) = (S_{62}, S_{61}, S_{60}, \dots, S_1, S_0)$$

Table (4) presents the results by applying DGMW algorithm.

**Table (4) The results of DGMW generator for Ex.(2).**

<b>Generators</b>	<b>Output Sequences</b>	<b>Period</b>
$Tr_2^{2^3} ([Tr_2^{2^6} (a^n)]^3)$	00000101001001110101110100101110 0011001111110010010111001110100	63
$Tr_2^{2^6} (a^{n+2})   Tr_2^{2^6} (a^{n+5})$	1111111111110111111101100111111 1001011110110110101001101101111	63
$Rec(Tr_2^{2^6} (a^{n+2})   Tr_2^{2^6} (a^{n+5}))$	11110110110010101101101111010011 1111100110111111011111111111111	63
<b>DGMW generator in Eq.(13)</b>	00001100000101100111110111000010 0101110111111011010001011100100	63

The complexity of GMW and DGMW sequences is computed in

table (5) by applying Berlekamp-Massey (BM) algorithm.



**Table (5) The complexity of GMW and DGMW sequences.**

<i>The Generators</i>	<i>The Complexity (L)</i>
GMW in Eq.(9) $\text{Tr}_2^{2^3}([\text{Tr}_2^{2^6}(a^n)]^3)$	12
DGMW in Eq.(13) $\text{Tr}_2^{2^3}([\text{Tr}_2^{2^6}(a^n)]^3) \oplus_2 S_n \oplus_2 \text{Rec}(S_n)$	42

It is obvious from table (5) that the complexity of DGMW sequence (or generator) is higher than GMW sequence. So, the security of DGMW generator is more than GMW.

**Example (3) :**

Consider the following GMW generator :

$$b_n = \text{Tr}_2^{2^2}([\text{Tr}_2^{2^4}(a^n)]^2), \dots(14)$$

where  $n = 0, 1, \dots, 14$ ,  $\alpha \in \text{GF}(2^4)$ ,  $m_\alpha(z) = z^4 + z + 1$  over  $\text{GF}(2)$ ,  $m = 4 = 2 \times 2 = j \times k$  and  $r = 2$ . The GMW generator was developed using Eq.(10) as :

$$b_n = \text{Tr}_2^{2^2}([\text{Tr}_2^{2^4}(a^n)]^2) \oplus_2 S_n \oplus_2 \text{Rec}(S_n) \dots (15)$$

where

$$S_n = \text{Tr}_2^{2^4}(a^{n+2}) | \text{Tr}_2^{2^4}(a^{n+4}),$$

$$\text{Rec}(S_n) = \text{Rec}(\text{Tr}_2^{2^4}(a^{n+2}) | \text{Tr}_2^{2^4}(a^{n+4}))$$

$$= (S_{14}, S_{13}, S_{12}, \dots, S_1, S_0)$$

Table (6) presents the sequences of GMW and DGMW generators by applying their algorithms respectively and shows their complexities by using BM algorithm.

**Table (6) The sequences of GMW and DGMW generators with their complexities.**

	<i>Generators</i>	<i>Output Sequences</i>	<i>Period</i>	<i>Complexity (L)</i>
1	GMW in Eq.(14) $b_n = \text{Tr}_2^{2^2}([\text{Tr}_2^{2^4}(a^n)]^2)$	00010011010 1111	15	4
2	DGMW in Eq.(15) $b_n = \text{Tr}_2^{2^2}([\text{Tr}_2^{2^4}(a^n)]^2) \oplus_2 S_n \oplus_2 \text{Rec}(S_n)$	11010001110 1100	15	12

It is obvious from Table (6) that the complexity of DGMW sequence (or generator) is higher than GMW sequence. So, the security of DGMW generator is more than GMW.

Table (7) presents a comparison between GMW and

DGMW generators depending on the complexity (L) of their sequences by applying BM algorithm when  $n=0,1,\dots,4094$ ,  $\alpha \in GF(2^{12})$ ,  
 $m_a(z) = z^{12} + z^6 + z^4 + z + 1$ ,  
 $m = 12, j = 4, k = 3$  and  $r = 13$ .

**Table (7) The complexity of GMW and DGMW generators of period 4095**

<i>Generators</i>	<i>Period</i>	<i>Complexity (L)</i>
GMW with $m_a(z) = z^{12} + z^6 + z^4 + z + 1$ , $m = 12, j = 4, k = 3$ and $r = 13$	4095	36
DGMW with $m_a(z) = z^{12} + z^6 + z^4 + z + 1$ , $m = 12, j = 4, k = 3$ and $r = 13$	4095	156

**9. Conclusion:**

The cipher and communication systems depend on the degree of the security of the key generators to give an acceptable protection to the confidential information from the unauthorized person. So, the paper presents a proposed method to increase the security of the Gordon, Mills and Welch (GMW) generator using Trace polynomial and from the Tables (3), (5), (6) and (7) the following results are listed :

- The sequence of DGMW generator has the same period of GMW generator with good random properties.
- The generator DGMW is more secure than GMW generator

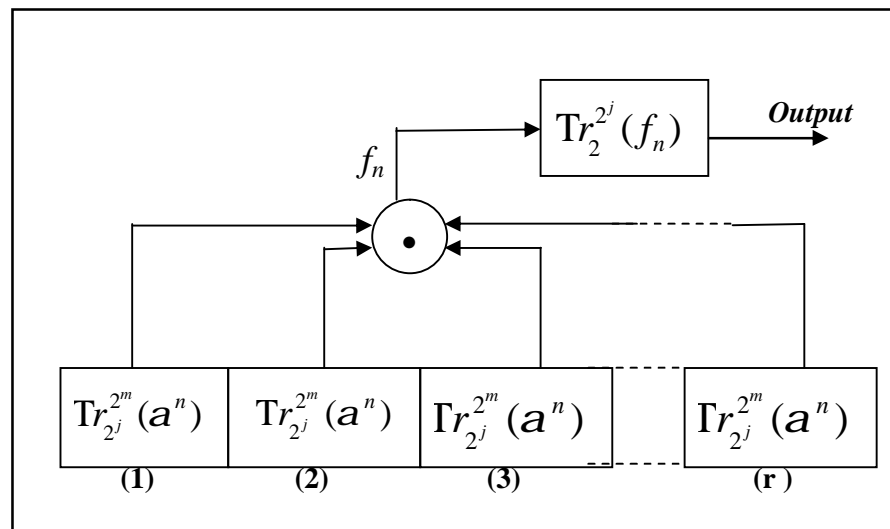
since it has higher complexity than GMW generator.

- DGMW generator gives a better accuracy and is consistent with the output sequence than GMW generator.

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**Figure (1) The GMW generator.**