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# Transient Thermal Stress Prediction Due To Flow of Coolant Through Hot Pipe

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**Abstract**

Transient thermal stresses in pipe wall due to coolant turbulent flow through pipe are investigated in this study with transient isothermal hot boundary in the external pipe radius. Three Different values of Reynolds are tested (3000, 5000 and 8000). Finite volume method was used to calculate the velocities and temperatures fields in the working fluid (air). Thermal resistance method was used to calculate the transient temperature distribution in the pipe wall and then the transient thermal stresses were calculated. Minimum thermal stress was located nearly in the mid plane of the pipe.

## الاجهادات الحرارية العابرة بسبب جريان مائع مبرد خلال انبوب ساخن

**الخلاصة**

في هذه الدراسة, تم حساب الاجهادات الحرارية الزمنية المتولدة في جدار انبوب نتيجة مرور جريان اضطرابي مبرد مع تسليط درجات حرارة متغيرة مع الزمن على السطح الخارجي للانبوب. اختبرت ثلاث قيم لاعداد رينولدز. استخدمت طريقة الاحجام المحددة لحساب السرع ودرجات الحرارة في الهواء المار خلال الانبوب. استخدمت طريقة المقاومات الحرارية العددية لحساب التوزيع الحراري في جدار الانبوب ثم حسبت الاجهادات الحرارية في الجدار. لقد وجد ان ادنى قيمة للاجهادات الحرارية تقع في منتصف نصف قطر الانبوب تقريبا.

**List of Symbols**

L = length of the pipe (m)
t = thickness of the pipe (m)
$\alpha$ = thermal expansion coefficient of the pipe material ($°C^{-1}$)
E = modulus of elasticity of the pipe material (Gpa)
$\sigma_\theta$ = tangential stress (Mpa)
$\sigma_r$ = radial stresses (Mpa)
$\sigma_x$ = axial stress (Mpa)
$\varepsilon$ = turbulent dissipation variable
$\nu$ = Poisson's ratio
r = radial coordinate ( m)
r* = dimensionless radius of a grid point $= (r-r_i)/(r_o-r_i)$
$r_o$ = pipe outer radius ( m)
$r_i$ = pipe inner radius ( m)
x = axial coordinate ( m)
u = fluid axial velocity (m/s)
v = radial velocity (m/s)
P = pressure (Kpa)
Pr = laminar Prantdl number
$Pr_t$ = turbulent Prantdl number
Re = laminar Reynolds number
$Re_t$ = turbulent Reynolds number
$\mu$ = fluid dynamic laminar viscosity ( Kg/m.s)
$\mu_t$ = fluid dynamic turbulent viscosity (Kg/m.s)
$\rho$ = fluid density( $Kg.m^{-3}$)
t = thickness of the pipe(m)

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T = temperature (°C)

T* = dimensionless temperature at a grid point
= (T–Tinlet)/(Tmean–Tinlet)

k = turbulent kinetic energy generation variable (J)

$k_f$ = thermal conductivity of the fluid( $W.m^{-1}.K^{-1}$)

$k_s$ = thermal conductivity of the solid ( W.$m^{-1}$ .$K^{-1}$)

R = thermal resistance ( °C/w)

c = specific heat ( J/KJ.K)

V = volume ( $m^3$)

n = time step ( sec)

## Introduction

Flow through pipes with heat transfer finds wide applications in industry. The transient thermal stresses, which develop in the pipe, limit the heat transfer rate in pipe flow. Thermal stresses are taken into consideration as important phenomena in many manufacturing processes and design applications.

Ootao and Tanigawa studied three-dimensional solution for transient thermal stresses of functionally graded rectangular plate due to non-uniform heat supply, Their paper is concerned with the theoretical treatment of transient thermo-elastic problem involving a functionally graded rectangular plate due to non-uniform heat supply. They obtain thethree-dimensional solution for the simple supported rectangular plate. Furthermore, the influence of the non-homogeneity of the material is investigated. Al-Zaharnah, Yilbas and Hashmi, studied steady thermal stresses due to turbulent flow in pipes. A turbulent flow in thick pipe with external heating is considered. The flow and temperature fields in a pipe and in the fluid are predicted using a numerical scheme; which employs a control volume approach. A k-ε model is introduced to account for the turbulence. The thermal stresses developed in the pipe due to heat transfer are predicted. The simulations are repeated for different pipe materials and fluids. It is found that the temperature gradient in the pipe changes rapidly in the vicinity of the solid-fluid interface. The effective stress developed at mid-plane of the pipe is independent of the Reynolds number; however, the pipe material affects the effective stress considerably. Shahani and Nabavi studied transient thermal stress intensity factors for an internal longitudinal semi-elliptical crack in a thick-walled cylinder. The stress intensity factors are derived for an internal semi-elliptical crack in a thick-walled cylinder subjected to transient thermal stresses. The problem of transient thermal stresses in a thick-walled cylinder is solved analytically. Rahimi, Owen and Mistry studied thermal stresses in boiler tubes arising from high-speed cleaning jets. Their study is concerned with a possible scenario where a soot-blower jet is placed closer to the heated surface than would normally be the case, possibly because of stubborn deposits or as a result of a more compact plant design. Experimental data are presented for the surface heat transfer distribution when an under-expanded jet from a 12:7 mm convergent nozzle with supply pressure ratio of 5.08 impinges onto a cylindrical surface for nozzle-to-surface spacing of 3, 6 and 10 nozzlediameters. The heat transfer rate is seen to be very high, and to have steep gradients close to the stagnation zone. Calculations have been carried out to determine the thermal stresses generated when such a jet, with a stagnation temperature of 300 K, impinges onto a 50 mm diameter steel tube of wall thickness 5 mm and inside temperature 803K.Yapıcı and Ba¸stürk studied numerical solutions of transient temperature and thermally induced stress distributions in a solid disk heated with radially periodic expanding and contracting ring heat flux. The applied heat transfer rate, *Q*, regularly increases from 3.14 to 311W and then decreases to 3.14 W in one period depending on the area of heated ring. The temperature and thermal stress distributions were obtained as per the moving ring heat flux per unit area. The maximum thermal stress ratio profiles also vary quasi-periodically by peaking at that rings. Furthermore, the thermal stress ratio profiles at the processed surface peak at the heated ring except the first ring and they are repeated with a time interval of one period by almost not changing the peaked shapes of themselves. Irfan and Chapman studied thermal stresses in radiant tubes due to axial, circumferential and radial temperature distributions. It was found that axial temperature gradients are not a source of thermal stresses as long as the temperature distribution is linear. Spikes in the axial temperature gradient are a source of high thermal stresses. Symmetric circumferential gradients generate thermal stresses. Radial temperature gradients create bi-axial stresses and can be a major source of thermal stress in radiant tubes.

Transient thermal stresses in pipes due to flow of coolant are seemed to have shortage in the literature. In this paper, the induced transient thermal stress due to turbulent coolant flow of air through heated pipe will be investigate based on temperature distribution of the pipe wall. The temperature distribution will be calculated using thermal resistance numerical method while the temperature and velocities in the fluid will be calculated by finite volume method. Three values of Reynolds number are tested, 3000, 5000 and 8000.

## 2. Mathematical Model

### 2.1 Flow Through Pipe

The governing equations that describe the flow and heat through the pipe are continuity, momentum and energy equations. These equations describe steady, axisymmetric flow two-dimensional, turbulent and incompressible flow takes the following forms [7]:-

(i) Continuity Equation

$$\frac{\partial}{\partial z}(\rho u)+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v)=0 \quad ....(1)$$

(ii) Momentum equations

u-momentum (z-direction)

$$\frac{1}{r}\left[\frac{\partial}{\partial z}(\rho r u u)+\frac{\partial}{\partial r}(\rho r u v)\right]=-\frac{\partial p}{\partial z}+\frac{1}{r}\left[\frac{\partial}{\partial z}\left(r\mu_{eff}\frac{\partial u}{\partial z}\right)+\frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial u}{\partial z}\right)\right]+Su \quad ....(2)$$

v-momentum (r-direction)

$$\frac{1}{r}\left[\frac{\partial}{\partial z}(\rho r u v)+\frac{\partial}{\partial r}(\rho r v v)\right]=-\frac{\partial p}{\partial z}+\frac{1}{r}\left[\frac{\partial}{\partial z}\left(r\mu_{eff}\frac{\partial v}{\partial z}\right)+\frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial v}{\partial r}\right)-\mu_{eff}\frac{v}{r^2}\right]+Sv \quad ...(3)$$

(iii) Energy Equation

$$\frac{1}{r}\left[\frac{\partial}{\partial z}(\rho r u T)+\frac{\partial}{\partial r}(\rho r v T)\right]=\frac{1}{r}\left[\frac{\partial}{\partial z}\left(r\Gamma_{eff}\frac{\partial T}{\partial z}\right)+\frac{\partial}{\partial r}\left(r\Gamma_{eff}\frac{\partial T}{\partial r}\right)\right]+S_T \quad ...(4)$$

Where, Su and Sv are given by

$$Su=\frac{\partial}{\partial z}\left(\mu_{eff}\frac{\partial u}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial v}{\partial z}\right) \quad ...(5)$$

$$Sv=\frac{\partial}{\partial z}\left(\mu_{eff}\frac{\partial u}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial v}{\partial r}\right)-\mu_{eff}\frac{v}{r^2} \quad ...(6)$$

This effective viscosity coefficient $\mu_{eff}$ is:

$$\mu_{eff}=\mu+\mu_t \quad ...(7)$$

This effective diffusion coefficient $\Gamma_{eff}$ is:

$$\Gamma_{eff} = \Gamma + \Gamma_t = \frac{\mu}{\sigma} + \frac{\mu_t}{\sigma_t}$$ or

$$\Gamma_{eff} = \frac{\mu_{eff}}{\sigma_{eff}} = \frac{\mu + \mu_t}{\sigma_{eff}} \quad ...(8)$$

Where $\sigma_{eff}$ is the effective Prandtl number including the turbulent dynamic viscosity and turbulentdiffusion coefficients. To solve the governing equations (1) to (4), a mathematical expression for effective kinematics viscosity ($\nu_{eff}$) and effective diffusion coefficient $\Gamma_{eff}$ will be required through the use of a turbulence model.

**2.2 Turbulence Model**
**(Standard k-$\varepsilon$)**

One of the most widely used turbulence models is the two-equation model of kinetic energy (k) and its dissipation rate ($\varepsilon$). This model is used in the present work. The turbulence according to Launder and Spalding [8] is assumed to be characterized by its kinetic energy and dissipation rate ($\varepsilon$). This model relates the turbulent viscosity to the local values of $\rho$, k and $\varepsilon$ by the expression.

$$\mu_t = \rho c_\mu k^2 / \varepsilon \quad ...(9)$$

Where $c_\mu$ is an empirical constant. The distribution of k and $\varepsilon$ over the flow field is calculated from the following semi-empirical transport equations for k and $\varepsilon$ [8].

**(i) Turbulence Energy, k**

$$\frac{1}{r}[\frac{\partial}{\partial z}(\rho r u k) + \frac{\partial}{\partial r}(\rho r v k)] =$$
$$\frac{1}{r}[\frac{\partial}{\partial z}(r\Gamma_k \frac{\partial k}{\partial z}) + \frac{\partial}{\partial r}(r\Gamma_k \frac{\partial k}{\partial r})] + G - C_D \rho \varepsilon \quad ...(10)$$

**(ii) Energy Dissipation Rate, $\varepsilon$**

$$\frac{1}{r}[\frac{\partial}{\partial z}(\rho r u \varepsilon) + \frac{\partial}{\partial r}(\rho r v \varepsilon)] =$$
$$\frac{1}{r}[\frac{\partial}{\partial z}(r\Gamma_\varepsilon \frac{\partial \varepsilon}{\partial z}) + \frac{\partial}{\partial r}(r\Gamma_\varepsilon \frac{\partial \varepsilon}{\partial r})] + c_1 \frac{\varepsilon}{k} G - c_2 \rho \varepsilon^2 / k \quad ....(11)$$

Where

$$G = \mu_t \{2[(\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial r})^2 + (\frac{v}{r})^2] + (\frac{\partial u}{\partial r} + \frac{\partial v}{\partial r})^2\} + S_G \quad ...(12)$$

$S_G$ Given by

$$S_G = -\frac{2}{3}\mu_t [\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial x}]^2$$

The values of the empirical constant used here are given in Table (1). These values are recommended by Launder and Spalding [8].

The governing equations of flow are discretized according to a finite volume scheme [7]. Corrections of the pressure gradient and axial velocity profile at each axial location, satisfying the overall continuity, is achieved using the SIMPLE algorithm, i.e. velocity components and pressure are successively predicted and corrected according to the initial guess for the pressure at each location is assigned.

**2.3 The Boundary Conditions**

The relevant boundary conditions for the conservative equations of flow and solid are:

1) At r = 0:

$$\frac{\partial u}{\partial r} = 0, \quad v = 0 \quad and \quad \frac{\partial T}{\partial r} = 0 \quad ...(13)$$

2) At r = $r_i$:

No-slip conditions are considered, i.e.:

$$u = v = 0 \quad ....(14)$$

3) At x = 0:

Uniform flow and uniform temperature were assumed.

4) At x = L (outlet):

All the gradients of the variables were set to zero, i.e.:

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{....(15)}$$

Where $\phi$ is the general property.

5) At $r = r_o$:

Transient temperature is assumed, i.e.:

$$T_{r=ro} = T_{in} + (T_{max} - T_{in}) \sin(\pi * \frac{\tau}{5}) \quad \text{...(16)}$$

Where, $\tau$ is the time (from 1 to 5) in minute, $T_{in}$ = 20 °C and $T_{max}$ = 40 °C, i.e. the temperature of the surface (at $r = r_o$) changes from 20 °C at time = 0 to 40 °C at time = 5 minutes.

6) At solid-fluid interface, i.e.:

$$k_s \frac{\partial T_s}{\partial r} = k_f \frac{\partial T_f}{\partial r} \quad \text{....(17)}$$

And

$$T_s = T_f \quad \text{....(18)}$$

## 2.4 Conduction in Pipe Wall

Transient thermal resistance analogue with explicit scheme was used to calculate temperatures in pipe wall due to conduction [9],

$$T_i^{n+1} = \frac{\sum_j \left(T_j^{n+1} / R_{ij}\right) + \left(C_i / \Delta\tau\right) T_i^n}{\sum_j \left(1/R_{ij}\right) + C_i / \Delta\tau} \quad \text{...(19)}$$

Where the resistances in the r and z directions are:

$$R_r = \frac{\Delta r}{(r_i + \Delta r/2) 2\pi \Delta x\, k}$$

$$R_z = \frac{\Delta x}{(r_i) 2\pi \Delta r\, k}$$

$$C_i = \rho_i \, c_i \, \Delta V_i$$

For stability,

$$\Delta\tau = \left[\frac{C_i}{\sum_j \left(1/R_{ij}\right)}\right]_{min} = 0.0024$$

second

Table(2) shows the values of inlet velocity, outer and inner radii associated with Reynolds number.

## 2.5 Thermal Stress [10, 11]

$$\sigma_\theta = \frac{E\alpha}{(1-\nu) r^2}\left[\frac{r^2 + r_i^2}{r_o^2 - r_i^2}\int_{r_i}^{r_o} T.\, rdr + \int_{r_i}^{r} T.\, rdr - T.r^2\right] \text{....(20)}$$

$$\sigma_r = \frac{E\alpha}{(1-\nu) r^2}\left[\frac{r^2 - r_i^2}{r_o^2 - r_i^2}\int_{r_i}^{r_o} T.\, rdr + \int_{r_i}^{r} T.\, rdr\right] \text{...(21)}$$

$$\sigma_z = \frac{E\alpha}{(1-\nu)}\left[\frac{2}{r_o^2 - r_i^2}\int_{r_i}^{r_o} T.\, rdr - T\right] \text{.....(22)}$$

$$\sigma = \left[\sigma_\theta^2 + \sigma_r^2 + \sigma_z^2 - (\sigma_\theta\sigma_r + \sigma_\theta\sigma_z + \sigma_r\sigma_z)\right]^{\frac{1}{2}} \text{...(23)}$$

Table (3) shows the physical properties of pipe wall which is necessary for calculation of isothermal field and thermal stress in the wall pipe. Because of the numerical values of temperatures in each node of the pipe wall, numerical integration was performed to calculate the thermal stress in equations[20] to[22].

## 2.6 Procedure of The Solution

The time step for transient conduction problem in pipe wall is 0.0024 second. 2500 time step iterations were performed to reach 6 second. Each 6 second, the finite volume subroutine was called to calculate fluid velocities and temperature fields and thermal stress subroutine to calculate thermal stresses in pipe wall. 40 is the number of callings to reach 5 minute. 26*16 are number of nodes were used for finite volume method and same number of nodes were used for conduction pipe wall calculations.

## 3. Results and Discussion

The turbulent flow through a pipe with external transient heating was considered for air, at three Reynolds numbers (3000, 5000 and 8000). The transient resulting thermal stresses across the pipe thickness were computed numerically. The k-ε model was used to account for the turbulence.

Figure 1 shows the boundary layer development for three different values of Reynolds number 3000, 5000 and 8000. The velocities changes from constant values at inlet to fully-developed.

Figure 2 shows the isothermal contours in fluid and pipe walls for three values of Reynolds number. The maximum temperature is at external radius and decreases toward internal radius. Because of the high thermal conductivity of the metal, the pipe wall gains nearly the maximum temperature, between 35 and 40 °C. The high gradient in temperatures appears in interface between metal and fluid because of the cooling of the coolant. The dimensionless axes were used to show a whole domain of calculation. The isotherm contours varied from inlet fluid temperature (20 °C) to outer surface of the pipe (40 °C).

Figure 3 shows the thermal stress distribution in pipe wall for three Reynolds numbers (3000, 5000 and 8000). Higher values appear at internal and external radius. Minimum value appears nearly at mid radius for Reynolds number equals to 3000. For higher Reynolds number, the minimum value location shifts to inner radius. The values of thermal stresses vary from 2.96 to 4.25 GPa.

Figure 4 shows the variation of thermal stress with time at three positions points ($r_o$, mid radius $(r_o+r_i)/2$ and $r_i$) for three Reynolds numbers at half length of the pipe. The thermal stress increases with the time because of temperature increases with time until it reaches maximum value. At Re = 3000, thermal stresses seems equal at inner and outer radii. At Re = 5000, outer radius location had higher thermal stress than inner radius. Similar behavior was observed for Re = 8000.

## 4. Conclusions

1. Higher gradient of temperature located at the solid-fluid interface.
2. The thermal stresses increases with the time as long as the temperature of the surface increases with the time.
3. Minimum thermal stresses located at nearly half radius between internal and external radii.

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**Table (1) Values of constants in the (k-$\varepsilon$ ) model**

| $c_\mu$ | $c_D$ | $c_1$ | $c_2$ | $\sigma_k$ | $\sigma_\varepsilon$ |
|---|---|---|---|---|---|
| 0.09 | 1.0 | 1.44 | 1.92 | 1.0 | 1.3 |

**Table (2) Values of inlet velocities and radii associated with Reynold's number**

| Reynold no. | Inlet velocity (m/s) | Inner radius (m) | Outer radius (m) |
|---|---|---|---|
| **3000** | **1.245** | **0.021** | **0.025** |
| **5000** | **1.440** | **0.030** | **0.040** |
| **8000** | **2.300** | **0.030** | **0.040** |

**Table 3 Physical properties of steel pipe**

| Thermal expansion coefficient(1/K) | Modulus of elasticity(GPa) | Poisson' ratio | Density ($kg/m^3$) | Specific heat (J/kg K) | Conductivity ($W/m^2$ K) |
|---|---|---|---|---|---|
| 0.37E - 03 | 210 | 0.3 | 7850 | 460 | 43 |

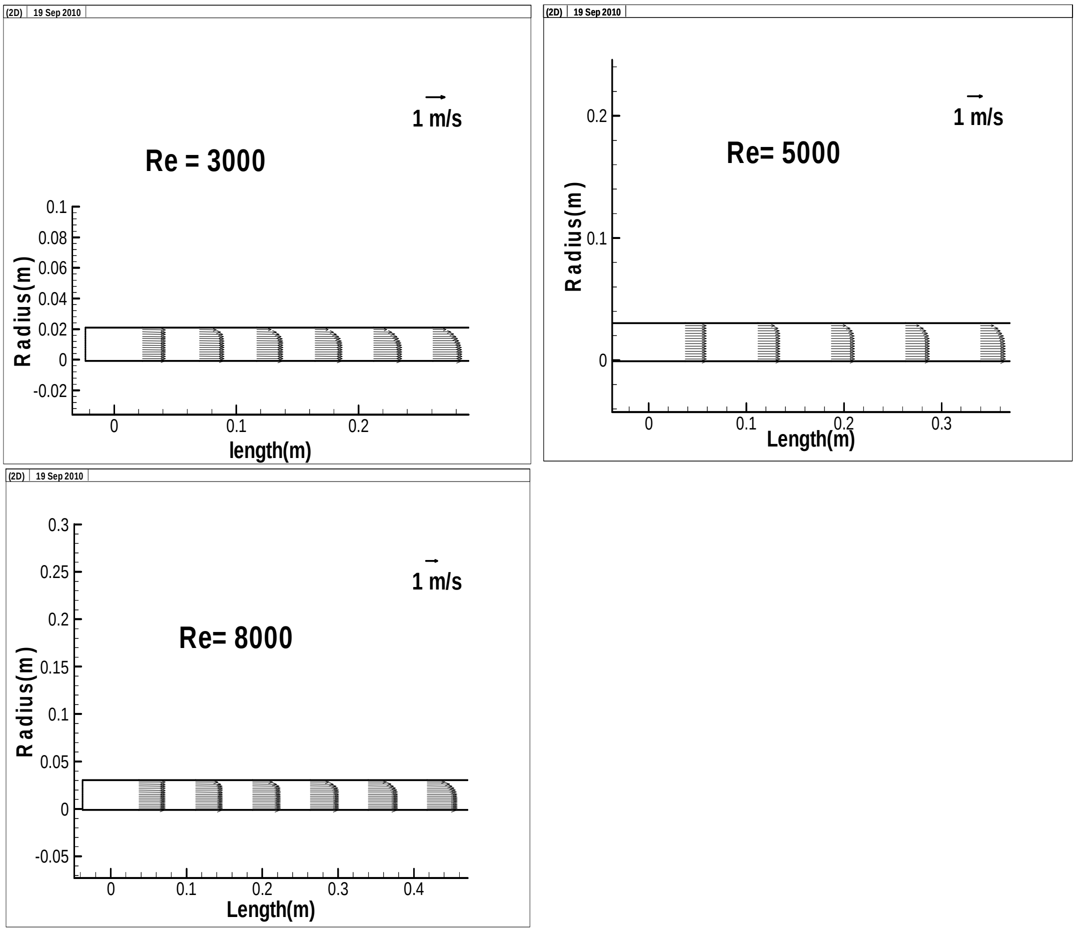
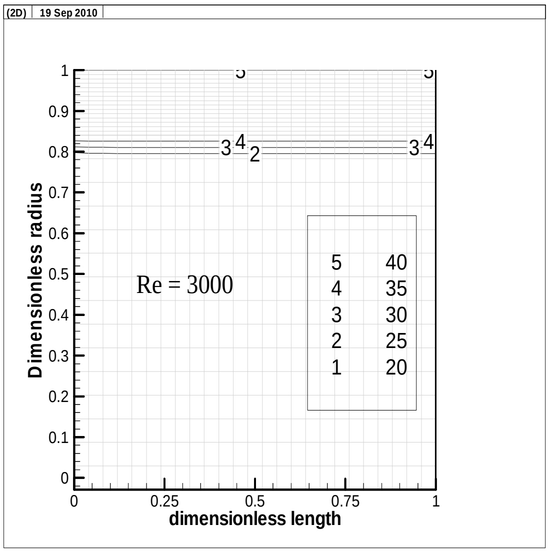
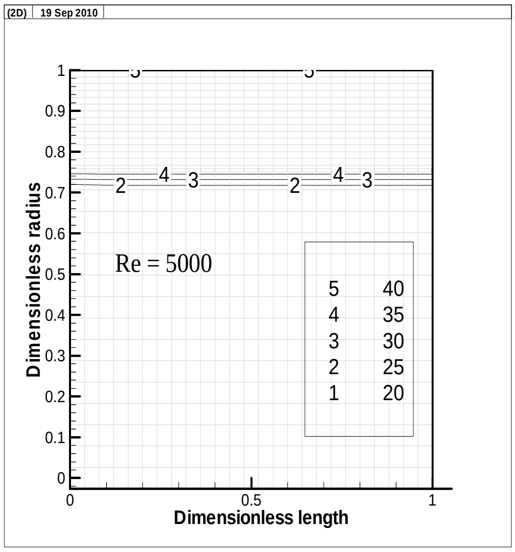



**Figure (1) Boundary layer development**



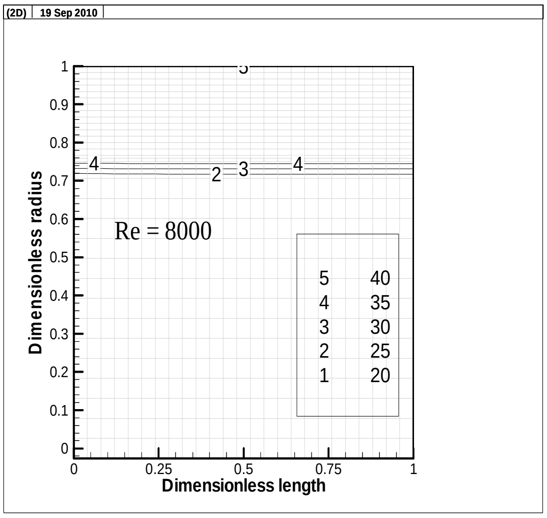




**Figure (2) Isothermal contours in pipe wall and fluid for Re = 3000, 5000 and 8000**

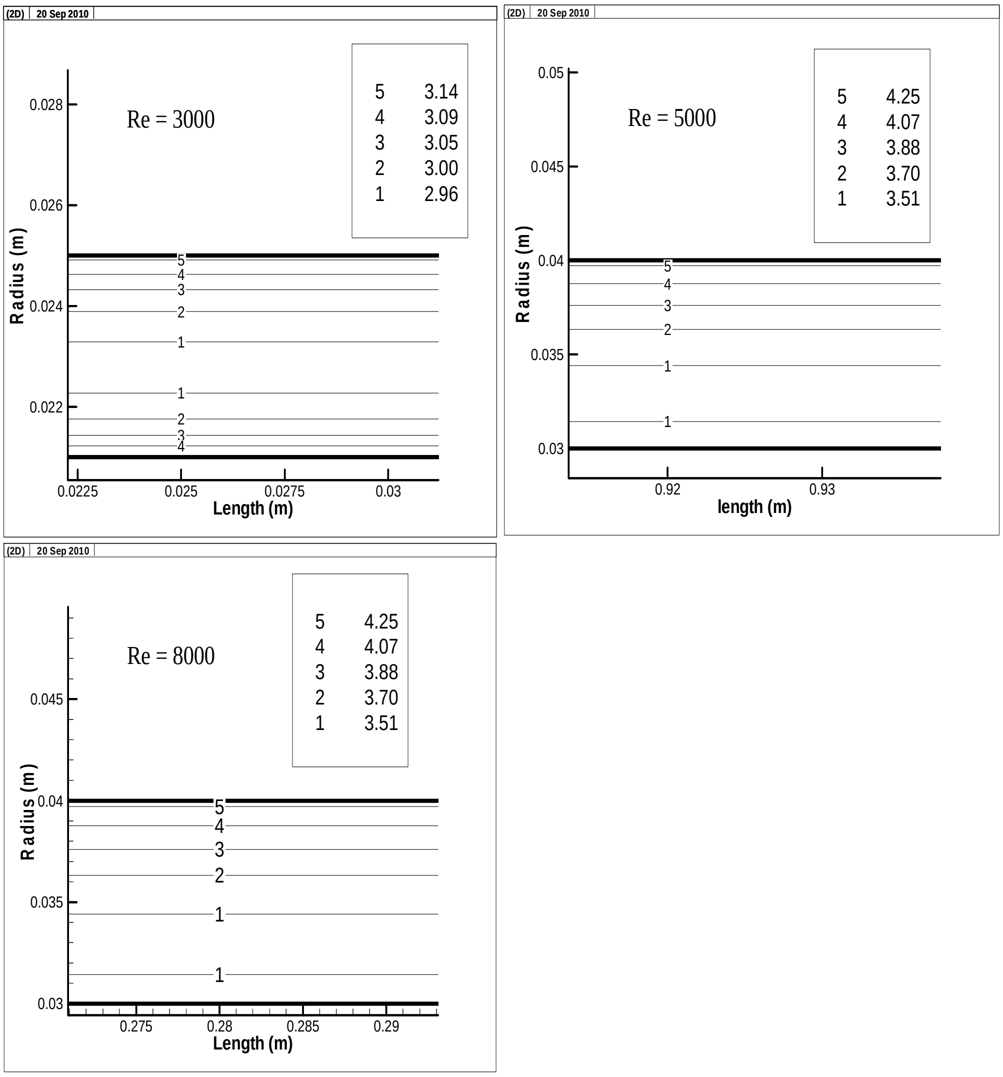



**Figure (3) Stress contours (MPa) in pipe wall for three Re = 3000, 5000 and 8000**

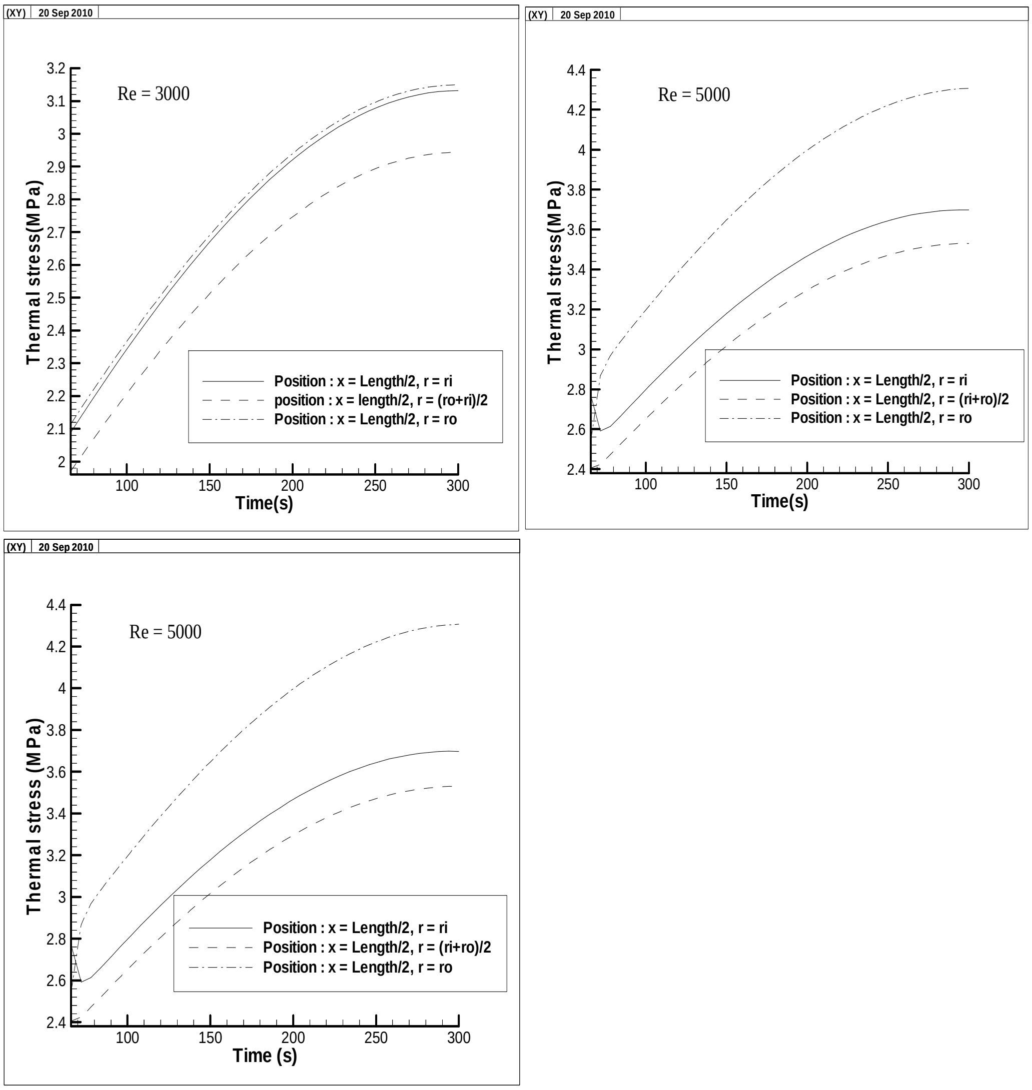



**Figure (4) Transient thermal stress in pipe wall at three positions for three Reynolds number (3000, 5000 and 8000)**