Nonlinear Dynamics of Thick Composite Laminated Plates Including the Effect of Transverse Shear and Rotary Inertia

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Abstract

In this work, a suggested analytical solution for nonlinear dynamic analysis of (fiber-reinforced) composite laminated thick plate is developed by using first-order shear deformation theory (FSDT).

A computer program was built for this purpose for anti-symmetric cross-ply and angle-ply, simply supported thick laminated plate and the developed equations are solved by using (MATLAB V.7) program. The finite-element solution is also adopted using (ANSYS V.8) package, to confirm the analytical results.

The results presented show the effect of plate thickness-to-side ratio (h/a), aspect ratio (a/b), number of layers (N), the degree of orthotropic ratio (E_1/E_2), fiber orientation, boundary conditions, lamination scheme, and the effect of shear deformation and rotary inertia on the thick laminated plate.

Keywords: *Composite, Plate, Shear, Inertia*

1. Introduction

Composites are not single materials but a family of materials whose stiffness, strength, density, thermal and electrical properties can be tailored. The matrix, the reinforcement material, the volume and shape of the reinforcement, the location of the reinforcement, and the fabrication method etc. can all be varied to achieve required properties. The equations which control the status of thick composite plate (isotropic or orthotropic) material do not take the effect of shear deformations or rotary inertia together. These motivated the need to derive equations that control the status of plates which was made of thin or thick materials to end with a cumulative study.

Reddy and Phan [1] used a higher-order shear deformation theory to determine the natural frequencies and buckling loads of elastic plates. They obtained exact solution of simply supported plates and the results are compared with the exact solution of three dimensional elasticity theories and compared between first-order shear deformation theory and the classical plate theory. Then, Debal, Bagchi and Kennedy, [2] presented a method for solving the linear dynamic problem of thick plate using the finite element method, the minimum number of degrees of freedom for an element was employed in order to reduce the computer solution time.

Game San and Sivadas. [3] Analyzed the free vibration characteristics of orthotropic circular cylinder shells by using Love's first approximation shell theory, Semi-analytical finite element method was used as a method of solution, for plates with clamped and simply supported boundary condition and with thickness varying along the axial direction.

The influence of thickness distribution on natural frequencies, especially on lowest natural frequency, the effect of degree of orthotropic was investigated on natural frequencies of shell.

Then, Zaghloul and Kennedy [4] presented the nonlinear solution of unsymmetrical filamentary-composite laminates with angle-ply and cross-ply configurations, the solution was achieved by means of a finite difference iterative technique and the theoretical results were compared with experimental findings. Konaka, Venkat eswara and Raju [5] studied the effect of geometric nonlinearity on the free flexural vibrations of moderately thick rectangular plates and used the finite element formulation to obtain the non-linear to linear period ratios of some rectangular plates, the effect of shear deformation was included and used for the analysis. The result presented was for both simply supported and clamped boundary conditions.

Iyengar [6] presented a higher order linearization method for analyzing nonlinear random vibration problems. The non-linear terms of the given equation were replaced by unknown linear terms. These were described by extra nonlinear differential equations. The numerical results on steady state variance and function were obtained. These were found to be better than the simple linearization results.

2. ANALYSIS

Nonlinear dynamics of thick composite laminated plates including the effect of shear and rotary inertia: The (FSDT) is based on the displacement field; [7].

$$
\begin{aligned}
u_1(x,y,z,t) &= u(x,y,t) + Z\psi_x(x,y,t) \\
u_2(x,y,z,t) &= v(x,y,t) + Z\psi_y(x,y,t) \\
u_3(x,y,z,t) &= w(x,y,t)
\end{aligned}
\qquad (1)
$$

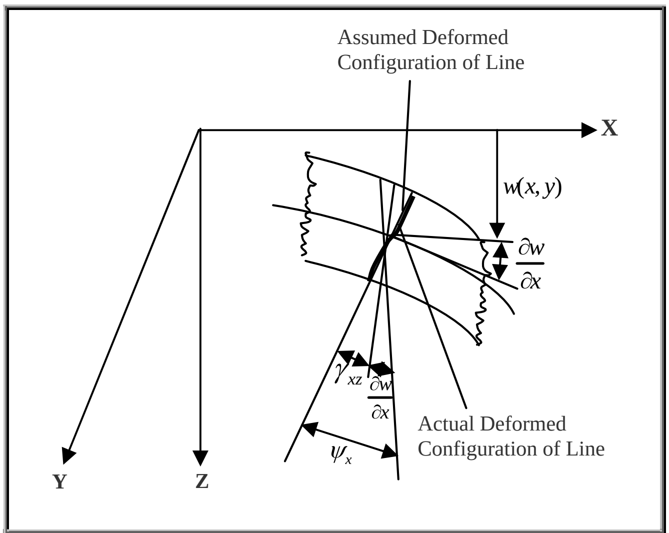

Figure (1) Definition of assumed deformation state for section x = constant

2.1 Stress–Strain Variation in a Laminate

According to the (FSDT) theory, the strain field will be:

$$\varepsilon_x = \frac{\partial u_1}{\partial x}\ ,\ \varepsilon_y = \frac{\partial u_2}{\partial y}\ ,\ \varepsilon_z = \frac{\partial u_3}{\partial z} = 0$$

$$\gamma_{xy} = \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\ ,\ \gamma_{yz} = \frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial y}\ ,\ \gamma_{xz} = \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} \qquad (2)$$

According to [8] and by substituting equation (1) into equation (2), the strain field will be;

$$\varepsilon_x = \frac{\partial u}{\partial x} + Z\frac{\partial \psi_x}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial \chi}\right)\wedge 2,$$
$$\varepsilon_y = \frac{\partial v}{\partial y} + Z\frac{\partial \psi_y}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)\wedge 2,$$
$$\varepsilon_z = \psi_x\wedge_2 + \psi_y\wedge_2\ ,\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + Z\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y},$$
$$\gamma_{yz} = \frac{\partial w}{\partial y} + \psi_y\ ,\ \gamma_{xz} = \frac{\partial w}{\partial x} + \psi_x \qquad (3)$$

The general equations of stress- strain relation are:-

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & \bar{Q}_{26} \\ 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 \\ 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & 0 & 0 & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} \qquad (4)$$

By substitution equ. (3) in equ. (4):

2.2 Resultant Laminate Forces and Moments

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & \bar{Q}_{26} \\ 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 \\ 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & 0 & 0 & \bar{Q}_{66} \end{bmatrix} \left(\begin{bmatrix} u,x \\ v,y \\ \psi y + w,y \\ \psi x + w,x \\ u,y + v,x \end{bmatrix} + z \begin{bmatrix} \psi x,x \\ \psi y,y \\ 0 \\ 0 \\ \psi x,y + \psi y,x \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(w,x)^2 \\ \frac{1}{2}(w,y)^2 \\ 0 \\ 0 \\ w,x * w,y \end{bmatrix} \right) \qquad (5)$$

$$\begin{bmatrix} N_x \\ N_y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau \end{bmatrix} dz = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau \end{bmatrix} dz$$
$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k Z\ dz = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k Z\, dz \qquad (6)$$

Substituting equation (5) into equation (6), gives:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} * \begin{bmatrix} u,x \\ v,y \\ u,y+v,x \\ \psi x,x \\ \psi y,y \\ \psi x,y+\psi y,x \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(w,x)^2 \\ \frac{1}{2}(w,y)^2 \\ w,x * w,y \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (7)$$

And the effect of shear; [8]

$$\begin{bmatrix} Q_y \\ Q_x \end{bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{bmatrix} \psi y + w, y \\ \psi x, y + w, x \end{bmatrix} \quad (8)$$

where: Qx and Qy are forces per unit length due to shear effect

$$A_{ij} = \sum_{k=1}^{n} (Z_k - Z_{k-1}) \ \overline{Q}_{ij}^{(k)} \quad \text{for } i, j = 4, 5. \quad (9)$$

2.3 General Equations of Motion

The equations of motion for (FSDT) obtained in the same method are applied to obtain the general equations of motion for (CLPT); [9].

For (FSDT) the following equations of motion for nonlinear laminate plates are:

$$\begin{aligned}
&\frac{\partial N_x}{\partial_x} + \frac{\partial N_{xy}}{\partial_y} = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \psi_x}{\partial t^2} \\
&\frac{\partial N_{xy}}{\partial_x} + \frac{\partial N_y}{\partial_y} = I_1 \frac{\partial^2 v}{\partial t^2} + I_2 \frac{\partial^2 \psi_y}{\partial t^2} \\
&\frac{\partial Q_x}{\partial_x} + \frac{\partial Q_y}{\partial_y} + N(w, n,) = I_1 \frac{\partial^2 w}{\partial t^2} + q(x, \ y, \ t) \\
&\frac{\partial M_x}{\partial_x} + \frac{\partial M_{xy}}{\partial_y} - Q_x = I_2 \frac{\partial^2 u}{\partial t^2} + I_3 \frac{\partial^2 \psi_x}{\partial t^2} \\
&\frac{\partial M_{xy}}{\partial_x} + \frac{\partial M_y}{\partial_y} - Q_y = I_2 \frac{\partial^2 v}{\partial t^2} + I_3 \frac{\partial^2 \psi_y}{\partial t^2}
\end{aligned} \quad (10)$$

Where

$$N(w,n,) = \frac{\partial\left(Nx \frac{\partial w}{\partial x}\right)}{\partial x} + \frac{\partial\left(Nxy \frac{\partial w}{\partial x}\right)}{\partial y} + \frac{\partial\left(Nxy \frac{\partial w}{\partial y}\right)}{\partial x} + \frac{\partial\left(Nxy \frac{\partial w}{\partial y}\right)}{\partial y} . + \frac{\partial\left(Ny \frac{\partial w}{\partial y}\right)}{\partial y} \quad (11)$$

Substituting equations (7) into equation (11) the general equations of motion are.

$$
\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \end{bmatrix} \left| \begin{bmatrix} u_{,x}w_{,xx} \\ v_{,y}w_{,xx} \\ (u_{,y}+v_{,x})w_{,xx} \\ \psi_{x,x}w_{,xx} \\ \psi_{y,y}w_{,xx} \\ (\psi_{x,y}+\psi_{y,x})\psi_{xx} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(w_{,x})^{2}w_{,xx} \\ \frac{1}{2}(w_{,y})^{2}w_{,x} \\ w_{,x}w_{,y}w_{,xx} \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| + \begin{bmatrix} A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \end{bmatrix} \left| \begin{bmatrix} u_{,xx}w_{,x} \\ v_{,xy}w_{,x} \\ (u_{,xy}+v_{,xx})w_{,x} \\ \psi_{x,xx}w_{,x} \\ \psi_{y,xy}w_{,x} \\ (\psi_{x,xy}+\psi_{y,xx})w_{,x} \end{bmatrix} \right.
$$

$$
\left. + \begin{bmatrix} (w_{,x})^{2}w_{,xx} \\ w_{,y}w_{,xy}w_{,y} \\ (w_{,x}*w_{,xy}+w_{,y}*w_{,xx})w_{,x} \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| + 2\begin{bmatrix} A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \end{bmatrix} \left| \begin{bmatrix} u_{,x}*w_{,xy} \\ v_{,y}*w_{,xy} \\ (u_{,y}+v_{,x})w_{,xy} \\ \psi_{x,xx}*w_{,xy} \\ \psi_{y,y}*w_{,xy} \\ (\psi_{x,y}+\psi_{y,x})*w_{,xy} \end{bmatrix} \right.
$$

$$
\left. + \begin{bmatrix} \frac{1}{2}(w_{,x})^{2}*w_{,,xy} \\ \frac{1}{2}(w_{,y})^{2}*w_{,xy} \\ w_{,x}*w_{,y}*w_{,xy} \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| + \begin{pmatrix} A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \end{pmatrix} \left| \begin{bmatrix} u_{,xy}*w_{,x} \\ v_{,yy}*w_{,x} \\ (u_{,yy}+v_{,xy})w_{,x} \\ \psi_{x,xy}*w_{,x} \\ \psi_{y,yy}*w_{,x} \\ (\psi_{x,yy}+\psi_{y,xy})*w_{,x} \end{bmatrix} \right.
$$

$$
\left. \begin{bmatrix} (w_{,x})^{2}*w_{,xy} \\ w_{,y}w_{,x}w_{,yy} \\ (w_{,x}w_{,yy}+w_{,y}*w_{,xy})w_{,x} \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| + \begin{bmatrix} A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \end{bmatrix} \left| \begin{bmatrix} u_{,xx}*w_{,y} \\ v_{,xy}*w_{,y} \\ (u_{,xy}+v_{,xx})w_{,y} \\ \psi_{x,xx}*w_{,y} \\ \psi_{y,xy}*w_{,y} \\ (\psi_{x,xy}+\psi_{y,xx})*w_{,y} \end{bmatrix} + \begin{bmatrix} w_{,x}w_{,xx}w_{,y} \\ (w_{,y})^{2}w_{,xy} \\ (w_{,x}w_{,xy}+w_{,y}w_{,xx})w_{,xy} \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| \Bigg|
$$

$$
+ \begin{bmatrix} A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \end{bmatrix} \left| \begin{bmatrix} u_{,x}*w_{,yy} \\ v_{,y}*w_{,yy} \\ (u_{,y}+v_{,x})w_{,yy} \\ \psi_{x,x}*w_{,yy} \\ \psi_{y,y}*w_{,yy} \\ (\psi_{x,y}+\psi_{y,x})*w_{,yy} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(w_{,x})\wedge 2w_{,yy} \\ \frac{1}{2}(w_{,y})^{2}w_{,yy} \\ w_{,x}w_{,y}w_{,yy} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \right|
$$

$$
+ \begin{bmatrix} A_{[illegible]} & A_{[illegible]} & A_{[illegible]} & B_{[illegible]} & B_{[illegible]} & B_{[illegible]} \end{bmatrix} \left| \begin{bmatrix} u_{,xy}*w_{,y} \\ v_{,yy}*w_{,y} \\ (u_{,yy}+v_{,xy})w_{,y} \\ \psi_{x,xy}*w_{,y} \\ \psi_{y,yy}*w_{,y} \\ (\psi_{x,yy}+\psi_{y,xy})*w_{,y} \end{bmatrix} + \begin{bmatrix} w_{,x}w_{,xy}w_{,y} \\ (w_{,y})^{2}w_{,yy} \\ (w_{,x}w_{,yy}+w_{,y}w_{,xy})w_{,y} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \right| \qquad (12)
$$

Then substitution equ. (8) And (12) into (10), we get:-

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \end{bmatrix} \left| \begin{bmatrix} u,_{xx} \\ v,_{xy} \\ (u,_{xy}+v,_{xx}) \\ \psi x,_{xx} \\ \psi y,_{xy} \\ (\psi x,_{xy}+\psi y,_{xx}) \end{bmatrix} + \begin{bmatrix} w,_{x}w,_{xx} \\ (w,_{y})w,_{xy} \\ (w,_{x}w,_{xy}+w,_{y}w,_{xx}) \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| + \begin{bmatrix} A_{16} & A_{12} & A_{26} & B_{16} & B_{16} & B_{66} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{16} & B_{26} \end{bmatrix}$$

$$\left| \begin{bmatrix} u,_{xy} \\ v,_{yy} \\ (u,_{yy}+v,_{xy}) \\ \psi x,_{xy} \\ \psi y,_{yy} \\ (\psi x,_{yy}+\psi y,_{xy}) \end{bmatrix} + \begin{bmatrix} w,_{x}w,_{xy} \\ (w,_{y})w,_{yy} \\ (w,_{x}w,_{yy}+w,_{y}w,_{xy}) \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| = \begin{bmatrix} I_1 & 0 & I_2 & 0 \\ 0 & I_1 & 0 & I_2 \end{bmatrix} \begin{bmatrix} u,_{tt} \\ v,_{tt} \\ \psi x,_{tt} \\ \psi y,_{tt} \end{bmatrix}$$

$$\begin{bmatrix} A_{45} & A_{55} \end{bmatrix} \begin{bmatrix} \psi_{y,x} + w_{,xy} \\ \psi_{x,x} + w_{,xx} \end{bmatrix} + \begin{bmatrix} A_{44} & A_{45} \end{bmatrix} \begin{bmatrix} \psi_{y,y} + w_{,yy} \\ \psi_{x,y} + w_{,xy} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \end{bmatrix} \left| \begin{bmatrix} u,_{x} * w,_{xx} \\ v,_{y} * w,_{xx} \\ (u,_{y}+v,_{x})w,_{xx} \\ \psi x,_{x} * w,_{xx} \\ \psi y,_{y} * w,_{xx} \\ (\psi x,_{y}+\psi y,_{x}) * w,_{xx} \end{bmatrix} \right.$$
$$\begin{bmatrix} u,_{xx} * w,_{x} \\ v,_{xy} * w,_{x} \\ (u,_{xy}+v,_{xx})w,_{x} \\ \psi x,_{xx} * w,_{x} \\ \psi y,_{xy} * w,_{x} \\ (\psi x,_{xy}+\psi y,_{xx}) * w,_{x} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(w,_{x})^{2} * w,_{yy} \\ \frac{1}{2}(w,_{y})^{2} * w,_{yy} \\ w,_{x}w,_{y}w,_{yy} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} (w,_{x})^{2} * w,_{xx} \\ w,_{y}w,_{x}w,_{x} \\ (w,_{x}w,_{xy}+w,_{y}w,_{xx})w,_{x} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \end{bmatrix} \left| \begin{bmatrix} u,_{x} * w,_{yy} \\ v,_{y} * w,_{yy} \\ (u,_{y}+v,_{x})w,_{yy} \\ \psi x,_{x} * w,_{yy} \\ \psi y,_{y} * w,_{yy} \\ (\psi x,_{y}+\psi y,_{x}) * w,_{yy} \end{bmatrix} \right. + \begin{bmatrix} u,_{xy} * w,_{y} \\ v,_{yy} * w,_{y} \\ (u,_{yy}+v,_{xy})w,_{y} \\ \psi x,_{xy} * w,_{y} \\ \psi y,_{yy} * w,_{y} \\ (\psi x,_{yy}+\psi y,_{xy}) * w,_{y} \end{bmatrix} +$$
$$\begin{bmatrix} \frac{1}{2}(w,_{x})^{2} * w,_{yy} \\ \frac{1}{2}(w,_{y})^{2} * w,_{yy} \\ w,_{x}w,_{y}w,_{yy} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} w,_{x}w,_{xx}w,_{y} \\ (w,_{y})^{2} * w,_{yy} \\ (w,_{x}w,_{yy}+w,_{y}w,_{xy})w,_{y} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \rho(x,y,t) + I_1 w,_{tt} \qquad (13)$$

$$
\begin{bmatrix} B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix}
\begin{bmatrix} u_{,xx} \\ v_{,xy} \\ u_{,xy} + v_{,xx} \\ \psi_{x,xx} \\ \psi_{y,xy} \\ \psi_{x,xy} + \psi_{y,xx} \end{bmatrix}
+ \begin{bmatrix} w_{,x} * w_{,xx} \\ w_{,y} * w_{,xy} \\ (w_{,x} w_{,xy} + w_{,y} w_{,xx}) \\ 0 \\ 0 \\ 0 \end{bmatrix} +
$$

$$
\begin{bmatrix} B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \end{bmatrix}
\begin{bmatrix} u_{,xy} \\ v_{,yy} \\ u_{,yy} + v_{,xy} \\ \psi_{x,xy} \\ \psi_{y,yy} \\ \psi_{x,yy} + \psi_{y,xy} \end{bmatrix}
\begin{bmatrix} w_{,x} * w_{,xy} \\ w_{,y} * w_{,yy} \\ (w_{,x} w_{,yy} + w_{,y} w_{,xy}) \\ 0 \\ 0 \\ 0 \end{bmatrix}
- \begin{bmatrix} A_{45} & A_{55} \\ A_{44} & A_{45} \end{bmatrix}
\begin{bmatrix} [\psi_y + w_{,y}] \\ [\psi_x + w_{,x}] \end{bmatrix}
$$

$$
= \begin{bmatrix} I_2 & 0 & I_3 & 0 \\ 0 & I_2 & 0 & I_3 \end{bmatrix}
\begin{bmatrix} u_{,tt} \\ v_{,tt} \\ \psi_{x,tt} \\ \psi_{y,tt} \end{bmatrix}
$$

The general equations of motion can be re-written, as:

$$
-[M][\ddot{\Delta}] + [K]_{linear}[\Delta] + \begin{bmatrix} K_{13} \\ K_{23} \\ K_{33} \\ K_{43} \\ K_{53} \end{bmatrix}_{nonlinear} + \begin{bmatrix} K_{31} & K_{32} & 0 & K_{34} & K_{35} \end{bmatrix}_{nonlinear} \begin{bmatrix} U \\ V \\ W \\ \Psi \\ \Psi \end{bmatrix} = -\vec{F}(x,y,t) \qquad (14)
$$

Where:

$$
[\Delta] = [u(x,y,t) \quad v(xy,t) \quad w(xy,t) \quad \psi_x(x,y,t) \quad \psi_y(x,y,t)]^T
$$
$$
[f] = [0 \quad 0 \quad q(x,y,t) \quad 0 \quad 0]^T \qquad (15)
$$

[M] and [k]linear and [k]nonlinear are as in [10]

3. General Solution for Equations of Motion

The general equations of motion, for different states without damping, are:

$$
[m][\ddot{\Delta}] + [k][\Delta] = [f]
$$

To solve the above equations, the actual displacement equations shown in [10] (depending on the theory used) for simply supported laminated plates are substituted into general equations of motion. Then, by pre-multiplying the result by $[\overline{\Delta}(x,y)]^T$ and integration of xy, a new form of equation of motions is obtained ;

$$
[M][\ddot{\Delta}(t)] + [K][\Delta(t)] = [F] \qquad (16)
$$

$$
[\overline{\Delta}(x,y)] = [\overline{u}(x,y) \quad \overline{v}(x,y) \quad .....^T \qquad (17)
$$

Respectively, where these matrices are shown in [10]

A program is built for solving by using first order shear deformation theory (FSDT), for anti-symmetric (cross-play) and (angle-ply) simply supported laminated plates subjected to uniformly, sinusoidal distribution for pulse and ramp dynamic loading. The results are the dynamic response and stresses in each layer.

New program was conducted to evaluate the mechanical properties and tested in different situations (ssss and cccc). The tested model varies between even schemes of (six layers) and odd one (nine layers), two different plates dimensions were selected for the entire tests (0.125*0.125 and 0.25*0.25)

4. The Results and Discussions

4.1 Frequency Results

4.1.1 Effect of Number of Layers , boundary condition

Table (1) shows the effect of number of layer for first five resonant frequencies (Hz) of the plate. The frequencies increase with resonance and with increasing the number of layers and for clamped plates the frequencies increase more than for the simply supported plates.

Table (1) Experimental reading of the first five natural frequencies (Hz) of the manufactured composite plates for type one and two end condition

	mode	4 -layer		6-layer		9- layer	
		SSSS	CCCC	SSSS	CCCC	SSSSS	CCCC
125*125 mm	1 st	141.0	173.9	289.1	350.7	487.0	539.3
	2 nd	202.2	244.5	344.7	409.8	545.1	620.9
	3rd	206.4	271.5	380.4	445.2	717.0	796.3
	4th	319.6	354.1	688.6	762.1	798.7	863.4
	5th	542.1	577.1	1446.2	1620.6	1468.0	1661.8
250*250 mm	1 st	129.9	160.9	252.7	308.1	407.7	473.7
	2 nd	138.4	166.0	295.7	348.7	486.6	553.1
	3rd	182.9	211.2	347.4	418.2	500.0	554.8
	4th	258.9	287.1	536.4	768.4	708.9	879.4
	5th	476.9	517.3	1196.4	1562.2	1386.0	1595.6

4.2Deflection Results

4.2.1 Effect of Loading Condition

Figs. (3) and (4) represent the variation of central transverse deflection with time for anti-symmetric cross-ply (0/90/0/…) simply supported thick laminated plates under different bending loading (sinusoidal loading q(x,y)=qo sin(π x/a)sin(π y/b)) and (uniform loading q(x,y)=qo) respectively, (plus loading q(x,y,t)=P(x,y)) and (ramp loading q(x,y,t)=P(x,y) t/to) for (qo=10 N/cm2 ,to=0.0005 sec). The magnitude of deflection due to pulse loading is higher than ramp loading by 110%, because it is subjected suddenly with a constant value of the time.

Figs. (5) and (6) represent the variation of central transverse deflection with time for anti-symmetric cross-ply (0/90/0/…) simply supported thick laminated plates under different dynamic loading (pulse and ramp loading) respectively (sinusoidal q(x,y)=qosin(π x/a)sin(π y/b)) and (uniform q(x,y)=qo) loading. The deflection's magnitude due to uniform loading is higher than the deflection due to sinusoidal loading by 60% because the value of the uniform loading is higher than the sinusoidal loading.

4.2.2 Effect of Number of Layers and Fiber Orientation

Figs. (7) and (8) represent the variation of central transverse deflection with time for anti-symmetric cross-ply and angle-ply simply supported thick laminated plates under sinusoidal (pulse and ramp) loading respectively.

The figures show that the deflection's magnitude of the anti-symmetric cross-ply (0/90/…) laminated is higher than the anti-symmetric angle-ply (45/-45/…) laminated with 15% because at (θ=45/-45/…) .the extension and bending stiffness A16, A26, D16 and D26 appear to have a significant effect compared with (θ=0/90/…).

Figs. (9) and (10) show comparison of the central transverse deflection for four- layer symmetric thick laminated plate about the middle plane (0/90/90/0) and (90/0/0/90) with four-layer anti-symmetric cross-ply (0/90/0/90/) thick laminated plates of simply supported edges, subjected to sinusoidal (ramp and pulse) loading respectively. The central deflection for symmetric (0/90/90/0) plate is equal to symmetric (90/0/0/90) plate and less than that for anti-symmetric cross-ply laminated plate (0/90/0/90) because symmetric conditions where there is no coupling between bending and extension and stiffnesses (Bij) are equal to zero.

4.2.3 Effect of Modulus Ratio, Thickness, Aspect ratio and boundary condition

Fig. (11) represents the effect of the degree of orthotropy (E1/E2) of (E2=10.6Gpa) on the central transverse deflection of simply supported anti-symmetric cross-ply thick laminated plates subjected to sinusoidal pulse loading condition. The figure shows, that the central deflection decreases with increase in the material's degree of orthotropy ratio (E1/E2). The decreases in deflections are 40% and 33% for increasing (E1/E2) (10 to 20) and (20 to 30) respectively, because increasing of the (E1/E2) ratio means increasing of (E1), so the stiffness of the plate or deflection resistance is increased.

Fig. (12) shows the effect of the thickness-to-length ratio (h/a), on the transverse central deflection of the simply supported anti-symmetric cross-ply thick laminated plates (a=1m) subjected to sinusoidal ramp loading condition. The results shows,that increasing (h/a) ratio decreases the deflection of plates. The deflections of plates decrease by 95%, 85% for increasing (h/a) from (0.02 to 0.05) and (0.05 to 0.1) respectively.

4.2.4 Effect of the Layer Position and Fiber Orientation

Figs. (13) and (14) show the stress (σx) at the middle plane of the layers in each layer for two-layer anti-symmetric cross-ply (0/90) thick laminated plates under (sinusoidal and uniformly) ramp loading respectively. The maximum value of (σx) is at (t=±h/2) and the stress (σx) at the middle plane of plate is zero, i.e., at (Z=0), the stress (σx) at the middle plane of layer-1 (7.6 MPa) and (12.5 MPa) for sinusoidal and uniformly loading respectively and the stress (σx) is symmetric about the middle plane.

Figs. (15) and (16) show the shear stress (τxy) in the middle plane of each layer for four- layer anti-symmetric cross-ply (0/90/0/…) and angle-ply (45/-45/45/…) laminated plates under uniform ramp loading. The maximum value of (τxy) is at ($t=\pm h/2$), and the τxy are symmetric about the middle plane for cross-ply and anti-symmetric for angle-ply thick laminated plates. Fig. (17) shows the stress (σx) with time at the middle plane of layer-1 for four-layer anti-symmetric cross-ply (0/90/0/…), angle-ply (45/-45/..) and different fiber orientation (0/45/90/-45) laminated plates, under uniformly ramp loading condition. From the results, the stress (σx) for angle-ply (45/-45/…) is less than that for cross-ply (0/90/…) with 50%.

4.2.5 The Effect of the Shear Deformation and Rotary Inertia

In this section, the effect of the transverse shear deformation and rotary inertia is discussed to test the validity of the present work (analytical solution) and finite-element technique. therefore, a case of square, simply-supported, four-layer anti-symmetric cross-ply (0/90/0/…) thick laminated plate was studied for the purpose of comparing the present work (analytical solution) with shear deformation (S.D.), i.e., built using (F.S.D.T) with finite-element technique with shear deformation and without shear deformation, i.e., using (C.L.P.T).

Figs. (17) and (18) show results for the variation of the maximum non-dimensional deflection parameter (coefficient α=WmaxE22h3/qoa4) with plate thickness to length ratio (h/a) for above laminated plates having orthotropy ratio (E1/E2=10) and (E1/E2=40) with (E2=10.6 GPa) respectively, and are subjected to a sinusoidal loading ($P(x,y)=qo\sin(\pi x/a)\sin(\pi y/b)$).

From the figures, good agreement is obtained between the present work with (S.D.) and the finite-element analysis with (S.D.). The figures show that for a laminate in which the modulus ratio (E1/E2=10) the increase in deflection due to transverse shear deformation effect is 20% at thickness ratio (h/a=0.1) and for a laminate of the modulus ratio (E1/E2=40) the increase in deflection is 30% at thickness ratio (h/a=0.1), the effects of transverse shear deformation on the deflection behavior of cross-ply laminate considerably increase with the degree of orthotropy of individual layers. Thus, the criterion for applying classical thin-plate theory (C.P.T) to laminated composite plates is not as simply defined as it is for thick homogeneous isotropic plates.

Figs. (18 and 19)show results of two finite-element analysis for the distributions of transverse shear stresses (τxz) and (τyz) across the thickness respectively, for above laminated plate and which had a thickness ratio (h/a=0.02), (a=1m) and subjected to uniformly ramp loading. Results of the finite-element analysis which does not include shear deformation are less than those which include shear deformation.

Considerable difference can be noted in the results of the two analyses, particularly in the distribution of transverse shear stress parallel and neer to the edges. This is also true for thick isotropic plates but the differences are not as pronounced because of the influence of material properties and transverse heterogeneity.

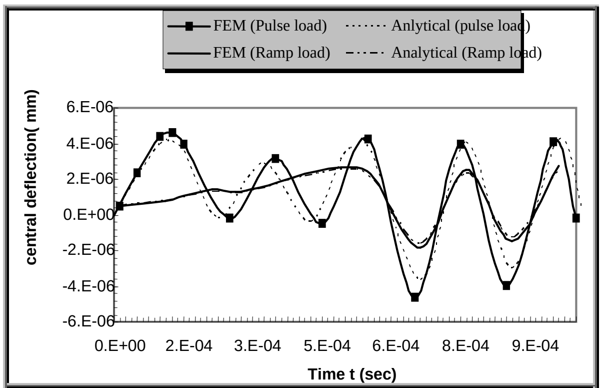

Figure (2) Central deflection of (cross-ply) thick laminated plate for different sinusoidal dynamic load for (N=4

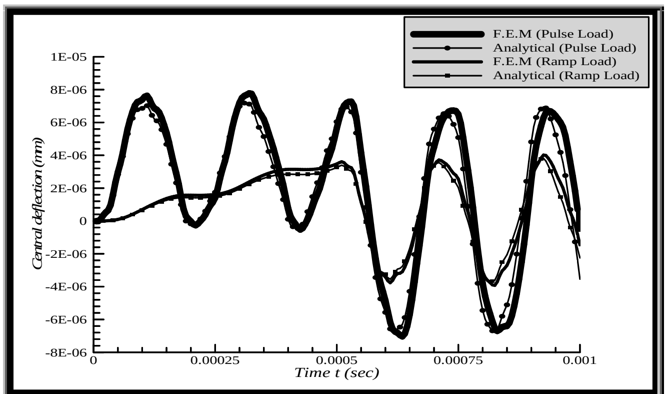

Figure (3) Central Deflection of (Cross-Ply) Thick Laminated Plate for Different Uniform Dynamic Loads for (N=4).

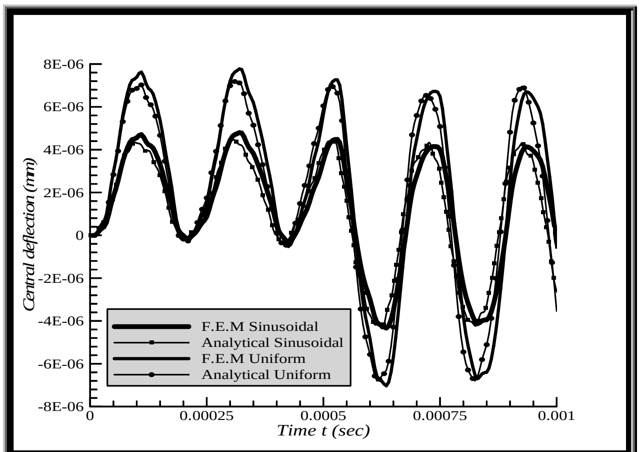

Figure (4) Central Deflection of (Cross-Ply) Thick Laminated Plate for Different Pulse Dynamic Loads for (N=4)

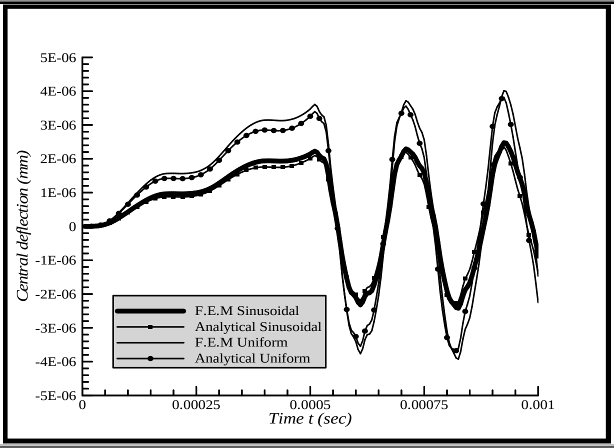

Figure (5) Central Deflection of (Cross-Ply) Thick Laminated Plate for Different Ramp Dynamic Loads for (N=4).

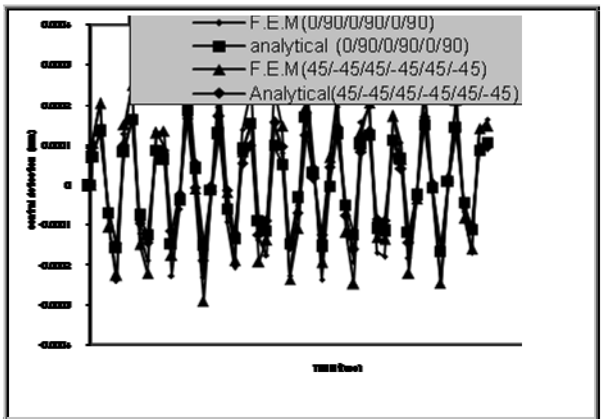

Figure (6) Central Deflection Due to Sinusoidal Pulse for Cross-Ply (0o/ 90o/ ….) and Angle–Ply (45o/ -45o/ ….) Thick Laminated Plate For (N=4).

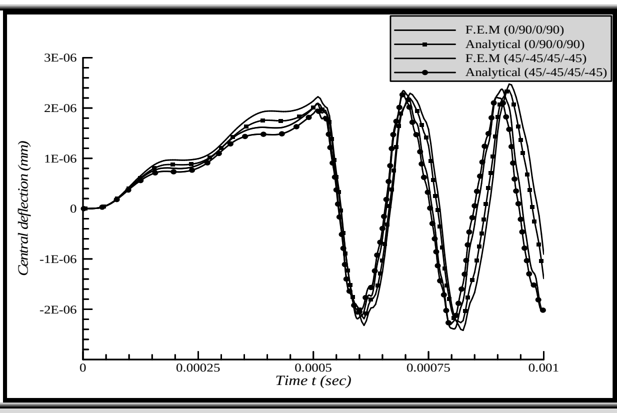

Figure (7) Central Deflection Due to Sinusoidal Ramp for Cross-Ply (0o/ 90o/ ….) and Angle–Ply (45o/ 45o/….) Thick Laminated Plate For (N=4).

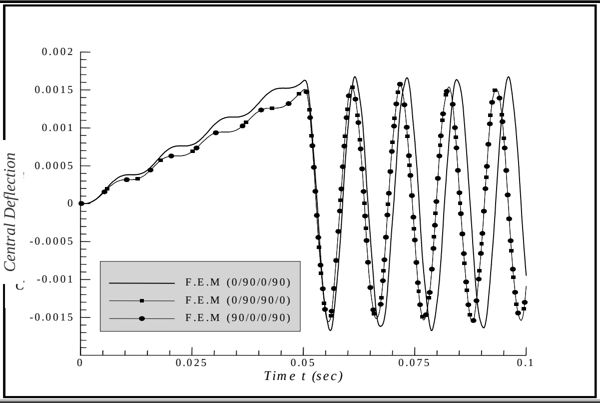

Figure (8) Central Deflection due to Sinusoidal Ramp Loading for Different Fiber Orientations of Thick Laminated Plate (N=4).

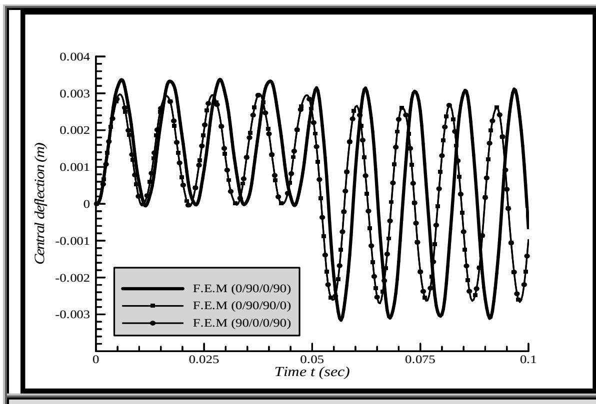

Figure (9) Central Deflection due to Sinusoidal Pulse Loading for Different Fiber Orientations of Thick Laminated Plate (N=4).

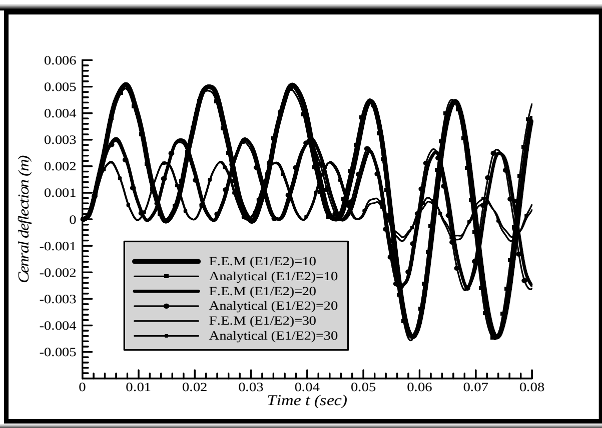

Figure (10) Effect of Orthotropy (Modulus Ratio) (E1/E2) on the Central Deflection of Cross-Ply (0o/90o/….), 4- Layer, Thick Laminated Plate Under Sinusoidal Pulse Loading.

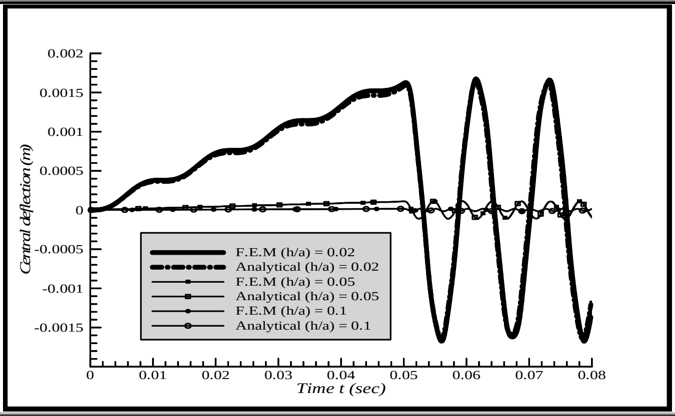

Figure (11) Effect of Thickness (h) on the Central Deflection of Cross-Ply,4- Layer, Thick Laminated Plate Under Sinusoidal ramp Loading.

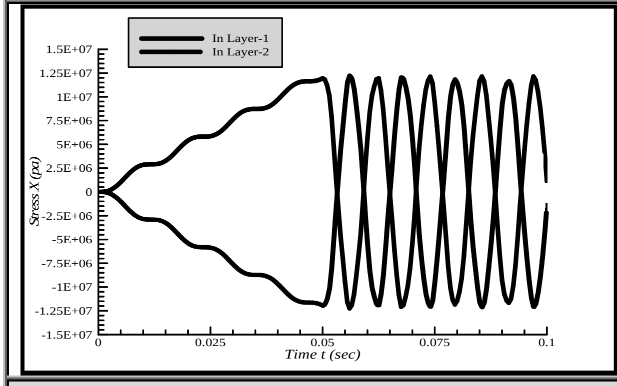

Figure (13) σx) In Middle Plane of Each Layer Due to Uniform Ramp Loading For Tow Layer Cross–Ply ($0^o/90^o$) Thick Laminated Plates.

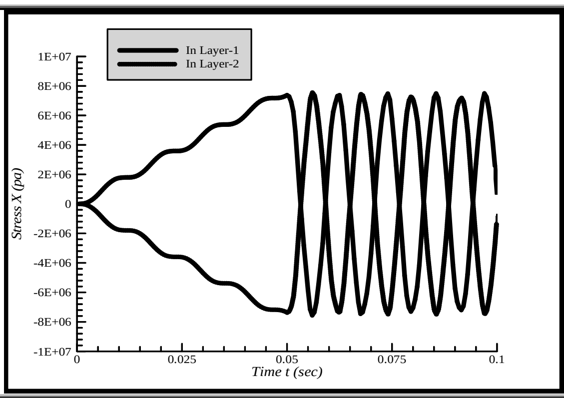

Figure (12) σx) In Middle Plane of Each Layer Due to Sinusoidal Ramp Loading For Tow Layer Cross–Ply (0o/90o) Thick Laminated Plates.

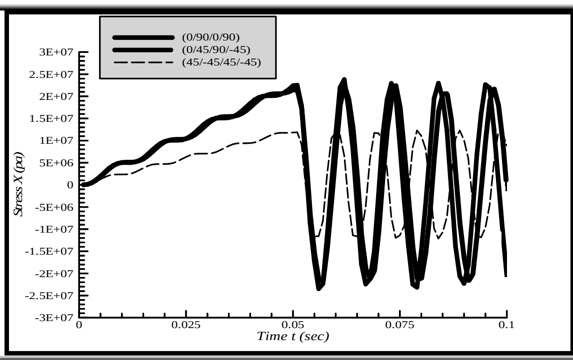

Figure (16) σx) In Layer-1 due to Uniform Ramp Loading For Different Fiber Orientations of Thick Laminated Plates (N=4).

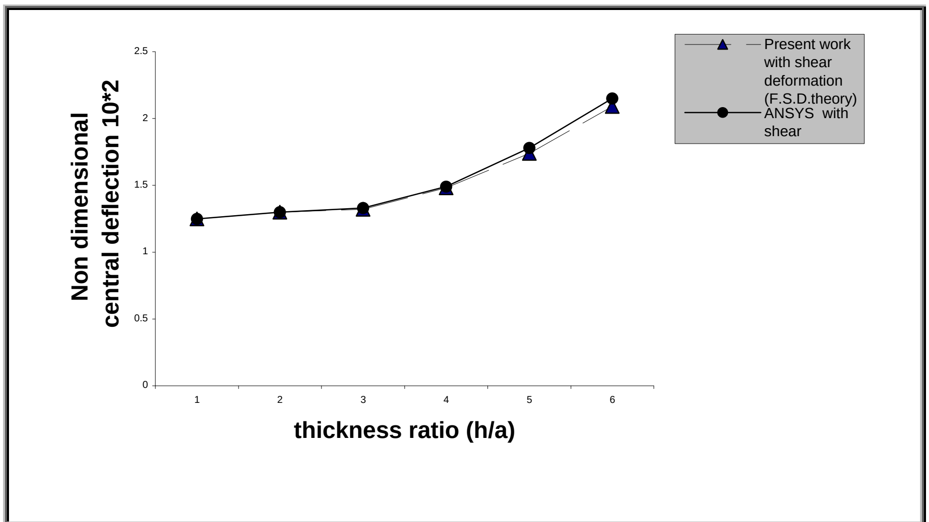

Figure (17) The Effect of Shear Deformation on The Deflection of Square,4-Layers, Anti-Symmetric Cross–Ply ($0^o/90^o/….$) Laminated Plates, Under Sinusoidal Ramp Loading

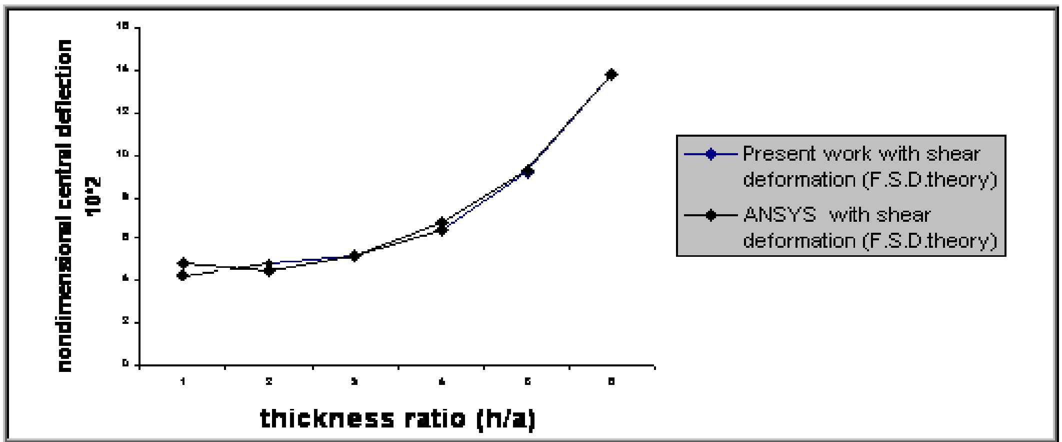

Figure (18) The Effect of Shear Deformation on The Deflection of Square, 4-Layers, Anti-Symmetric Cross–Ply (0o/90o/….) Laminated Plates, Under Sinusoidal Ramp Loading

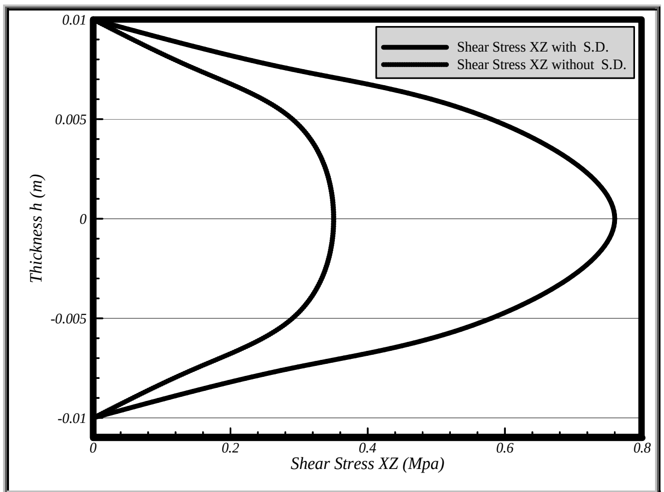

Figure (19) The Effect of Shear Deformation on the Distribution of Transverse Shear Stress, τ_{xz}) Cross the Thickness of 4-Layers, Anti-Symmetric Cross–Ply ($0^o/90^o$/….) Laminated Plates, Subjected to Uniform Ramp Loading.

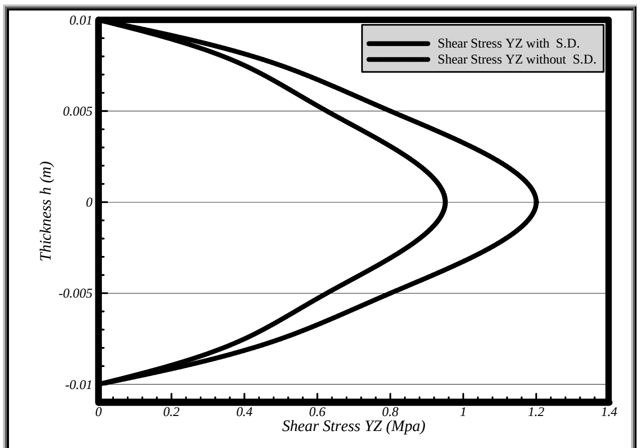

Figure (20) The Effect of Shear Deformation on the Distribution of Transverse Shear Stress, τyz) Cross the Thickness of 4-Layers,Anti-Symmetric Cross–Ply (0o/90o/….) Laminated Plates, Subjected to Uniform Ramp Loading.

5. Conclusions

1. The suggested analytical solution is a powerful tool for static and dynamic analyses of thick composite laminated plates.
2. The central deflection decreases with increase in the material's degree of orthotropy ratio (E1/E2). The decreases in deflections are 40% and 33% for increasing (E1/E2) (from 10 to 20) and (from 20 to 30) respectively because increasing of the (E1/E2) ratio means increasing of (E1), so the stiffness of the plate or deflection resistance is increased.
3. The transverse shear stresses (τxz) and (τyz) in case of using anti-symmetric angle-ply (45/-45/-45…) lamination is higher than using anti-symmetric cross-ply (0/90/0/…) lamination.
4. There is no effect of the number of layers on the transverse shear stress (τxz) and (τyz) at the middle plane and the values of stress will remain constant in spite of the increase in the numbers of layers.
5. The shear stress (τxz) at middle plane decreases with increasing the aspect ratio (a/b) and the orthotropy ratio of plates (E1/E2).

6. References

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ملخص

في هذه البحث تم اقتراح تحليل رياضي للتصرف اللاخطي الحركي (الديناميكي) للصفائح السميكة المركبة المقواة بالليف والمصنوعة على شكل طبقات باستخدام نظريات الطبقة المفردة **"Single Layer Theories" (FSDT)**

تم بناء برنامج صمم لهذا الغرض، للصفائح غير المتناظرة والمثبتة **(Simply Supported)** تحل معادلاته باستخدام برنامج **(MATLAB V.7)** ولدعم وإسناد النتائج التحليلية تم وضع الحل العددي باستخدام طريقة العناصر المحددة**(FEM)** وباستخدام برنامج**(ANSYS V.8)** .

تمت دراسة تأثير نسبة الطول إلى السمك **((h/a)**، نسبة الباع **((a/b)**، نسبة درجة الارثوتروبية **E1/E2**، زاوية التركيب ، وحالة التثبيت للصفيحة وكذلك تأثير تشوه القص والقصور الذاتي الدوار للصفائح السميكة **.(Lamination Angle)**