

# تقدير دالة الانحدار اللامعلمية باستخدام دوال لب قانونية

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-

المستخلص

.2010

2007

Gaussian

## Abstract

This research aims to review the importance of estimating the nonparametric regression function using so-called Canonical Kernel which depends on re-scale the smoothing parameter, which has a large and important role in Kernel and give the sound amount of smoothing .

We has been shown the importance of this method through the application of these concepts on real data refer to international exchange rates to the U.S. dollar against the Japanese yen for the period from January 2007 to March 2010. The results demonstrated preference the nonparametric estimator with Gaussian on the other nonparametric and parametric regression estimators (Simple and Multiple linear regressions).



## 1-1 المقدمة

Kernel

1942

Jacob Wolfowitz

[8] 2004 Randles, Hettmansperger and Cansella

Y

X

## 1-2 هدف البحث

## 2- الجانب النظري

2-1 :

 $K(x)$ 

$$K(x) \propto I(|x| \leq 1)$$

:

$$Y_i = m(X_i) + \varepsilon_i, \quad i = 1, 2, \dots, n$$

... (1)

m(x)

h

$$\int K(x) dx = 1$$

[7][6]:

Beta Kernel

$$K(x) = \frac{1}{B(0.5, \gamma + 1)} (1 - x^2)^\gamma I(|x| \leq 1), \quad \gamma = 0, 1, 2, \dots \quad \dots (2)$$



$\gamma = 1, 2, 3$   
 $\gamma$

Uniform  
Triweight Biweight Epanchnikov  
: Gaussian Kernel Beta Kernel

$\gamma = 0$

$$K(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \quad \dots (3)$$

$$\sigma^2 = \frac{1}{(2\gamma + 3)}$$

: (Cosinus )

$$K(x) = \frac{1}{2} \exp\{-|x|/\sqrt{2}\} \text{Sin}(|x|/\sqrt{2} - \frac{\pi}{4}) \quad \dots (4)$$

2-2

Watson (1964) Nadaraya  
:

(1964)  
[9][5][3]

$$\hat{m}(x) = \frac{\sum_{i=1}^n K_h(X_i - x)}{\sum_{i=1}^n K_h(X_i - x)} \quad \dots (5)$$

p

:

$$m(z) \approx \sum_{j=0}^p \beta_j (z - x)^j$$

:

$$\beta_j = \frac{m^{(j)}(x)}{j!}$$

$$\sum_{i=1}^n (Y_i - \sum_{j=0}^p \beta_j (X_j - x)^j)^2 K_h(X - x)$$



$p=1$

$\beta$   
:

$$m(y) \approx m(x) + m'(x)(y - x) \equiv \beta_0 + \beta_1(y - x)$$

:  $\beta_1$  و  $\beta_0$

$$\hat{m}(x) = \hat{\beta}_0 = n^{-1} \sum_{i=1}^n W_h(x) Y_i \quad \dots (6)$$

:

$$W_h(x) = W_i / n^{-1} \sum_{i=1}^n W_i$$

$$W_i \equiv K_h(X_i - x) [S_{n,2} - (x - X_i) S_{n,1}] \quad \dots (7)$$

$$S_{n,l} = \sum_{i=1}^n K_h(X_i - x) (x - X_i)^l, \quad l = 1, 2 \quad \dots (8)$$

.  $h$

: [7][6][5]

2-3

:

$$MSE(\hat{m}(x)) \approx \frac{h^4}{4} (m''(x))^2 d_{ks}^2 + (nh)^{-1} C_{ks}^2 f^{-1}(x) \sigma^2(x) \quad \dots (9)$$



:

$$K_s(u) = s^{-1}K(u/s), \quad u = \frac{X_i - x}{h} \quad \dots (10)$$

rescaled

s

:

$$(nhf(x))^{-1} \sigma^2(x)(s^{-1} C_k) + \frac{h^4}{4} m''(x)(s^2 d_k)^2 \quad \dots (11)$$

$$C_k = \lim_{n \rightarrow \infty} Var(K) = \int_{-\infty}^{\infty} K^2(u) du \quad \dots (12)$$

$$d_k = \lim_{n \rightarrow \infty} bias(K) = \int_{-\infty}^{\infty} u^2 K(u) du$$

h

: s

$$S^{-1} C_k = (S^2 d_k)^2$$

$$S = S^* = (C_k / d_k^2)^{0.2} \quad \dots (13)$$

$$.S = S^* \quad (10) \quad K^*$$

:

$$\begin{aligned} (\int u^2 K^*(u) du)^2 &= \int (K^*(u))^2 du \\ \Rightarrow (S^*)^{-1} C_k &= (d_k^{2/5} / C_k^{1/5}) C_k = d_k^{2/5} / C_k^{4/5} \quad \dots (14) \end{aligned}$$

:

$$MSE(\hat{m}(x)) \approx (d_k^{0.5} C_k)^{4/5} [(nhf(x))^{-1} \sigma^4(x) + \frac{h^4}{4} (m''(x))^2] \quad \dots (15)$$

 $S^*$ 

:

 $h_j$ 

$$h_j^* = h_j / S_j^* \quad \dots (16)$$

 $h_2, h_1$ 

$$\frac{h_2}{S_2^*} = \frac{h_1}{S_1^*} \Rightarrow h_2 = h_1 \left( \frac{S_2^*}{S_1^*} \right) \quad \dots (17)$$

:

$$h_j = h_i \left( \frac{S_j^*}{S_i^*} \right) \quad \dots (18)$$

(10)

 $K_j$  $S^*$ 

(18)

 $h_j$  $h_i$ [6].  $K_i$ 

(1)

(18)

 $S_j^*/S_i^*$ 

$S_j^*/S_i^*$	Uniform	Triangle	Epanchikov	Quartic	Triweight	Gaussian	Cosinus
Uniform	1	0.715	0.786	0.663	0.584	1.74	0.761
Triangle	1.398	1	1.099	0.927	0.817	2.432	1.063
Epanchnikov	1.272	0.91	1	0.844	0.743	2.214	0.968
Quartic	1.507	1.078	1.185	1	0.881	2.623	1.146
Triweight	1.711	1.225	1.345	1.136	1	2.978	1.302
Gaussian	0.575	0.411	0.452	0.381	0.336	1	0.437
Cosinus	1.315	0.941	1.033	0.872	0.768	2.288	1



Epanchnikov Kernel

Gaussian Kernel

: (18)

Gaussian Kernel

$$h_{Epan} = 2.214 * h_{Gauss}$$

: Gaussian Kernel

$$h_{Gauss} = h_{Epan} (1/2.214)$$

Confidence Intervals:[7][5] فترات الثقة -3

Piecewise

confidence interval

:

: 3-1

:

-1

$$E(|Y|^{2+k} / X = x) \quad \sigma^2(x)$$

 $x_1, x_2, \dots, x_n$  -2

 $. j = 1, 2, \dots, n \quad f(x_j) > 0$ 

$$3 - \int |K(u)|^{2+k} du < \infty, \text{ for some } k > 0.$$

$$4 - h = cn^{-0.2}.$$



$$(nh)^{0.5} \left\{ \frac{\hat{m}(x_j) - m(x_j)}{\{\sigma^2(x_j)C_k / f(x_j)\}^{0.5}} \right\}_{j=1}^n \xrightarrow{L} N(B, I) \quad \dots (19)$$

$$B = \left[ \frac{d_k}{2} m''(X_j) h^2 \right]_{j=1}^n \quad \dots (20)$$

$x_n, \dots, x_2, x_1$

:

$(Y / X = x)$

3-2

.1

.2

$$\hat{\sigma}^2(x) = n^{-1} \sum_{i=1}^n W_h(x) (Y_i - \hat{m}(x))^2 \quad \dots (21)$$

$W_h(x)$

:

.3

$$CLO = \hat{m}(x) - \frac{c_\alpha C_K^{0.5} \hat{\sigma}(x)}{(nh \hat{f}(x))^{0.5}}$$

... (22)

$$CUP = \hat{m}(x) + \frac{c_\alpha C_K^{0.5} \hat{\sigma}(x)}{(nh \hat{f}(x))^{0.5}}$$

. x

$\hat{m}(x)$

$[CLO, CUP]^{c_\alpha}$  .4





## 4- الجانب العملي

Matlab

39

. 2010

2007

2002

%80

%50

( )

3

.[2]

Gaussian Kernel

Epanchnikov Kernel

(18)

[9]



(2)

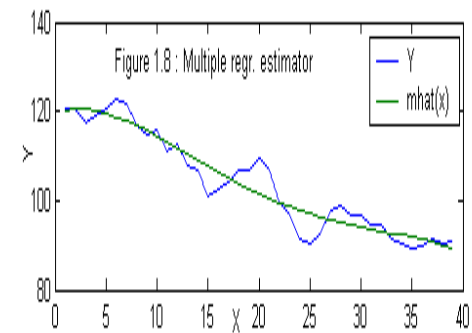
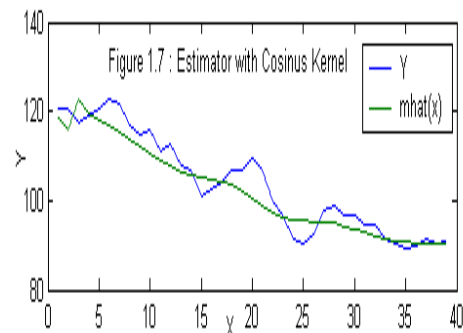
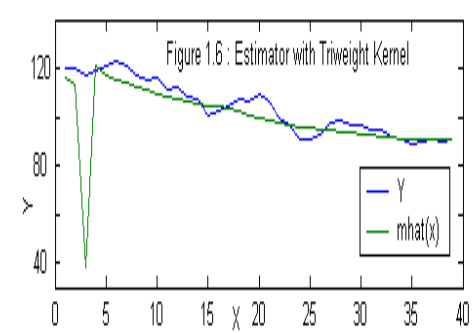
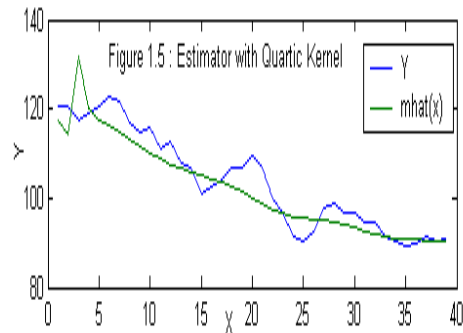
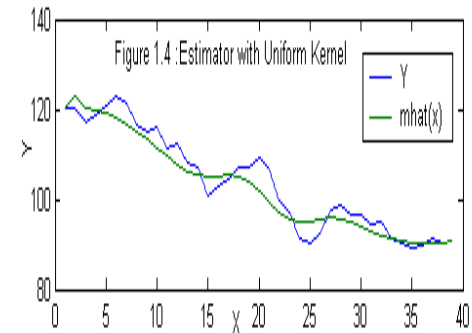
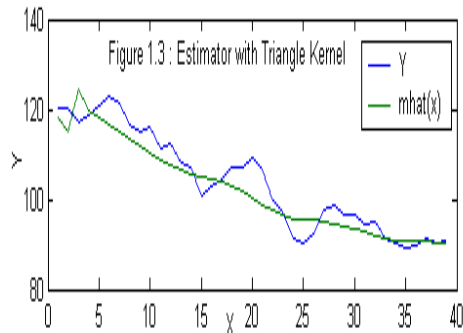
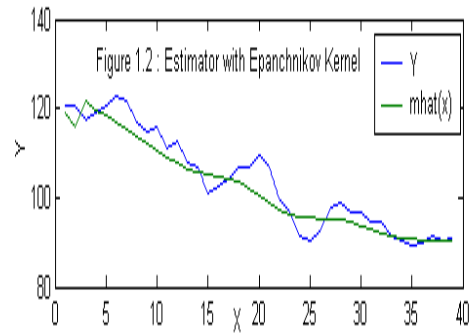
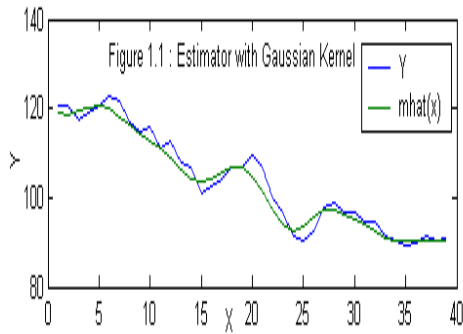
Kernel	Gaussian	Epanchnikov	Triangle	Uniform	Quartic	Triweight	Cosinus	Linear Regression	Multiple Regression
h	3.5573	4.6301	4.0781	2.7053	3.7812	3.4422	1.5548	-	-
MASE	4.4743	12.1645	14.1532	9.1461	18.9494	174.8244	12.7255	13.2066	10.3155
Efficiency	1	0.3678	0.3161	0.4892	0.2361	0.0256	0.3516	0.3388	0.4337



(1)

2010

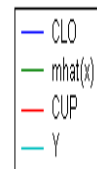
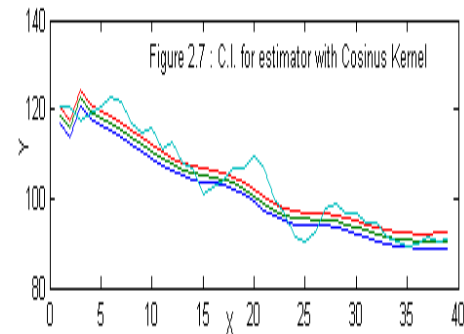
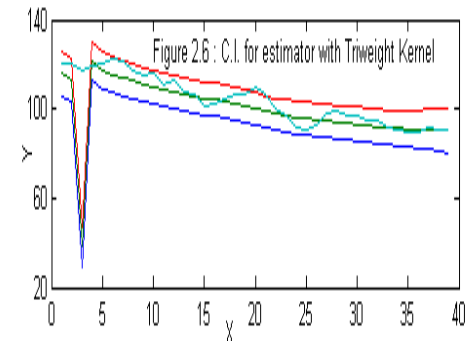
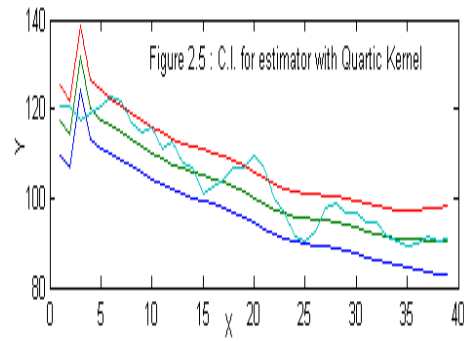
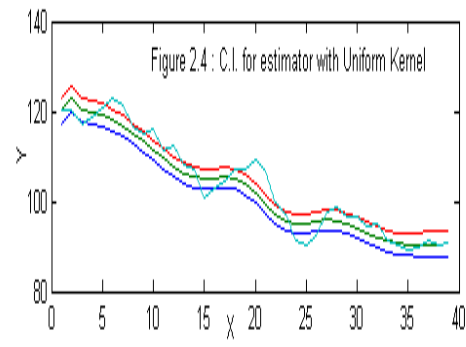
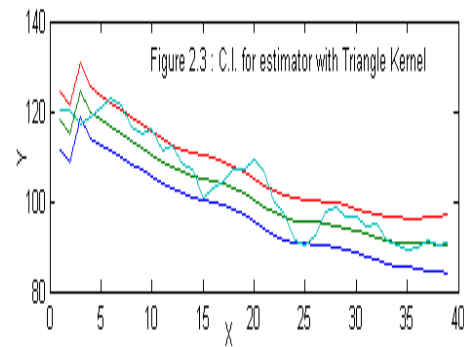
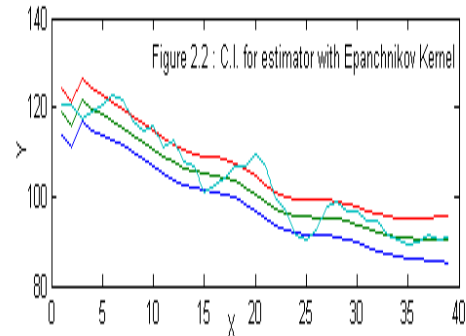
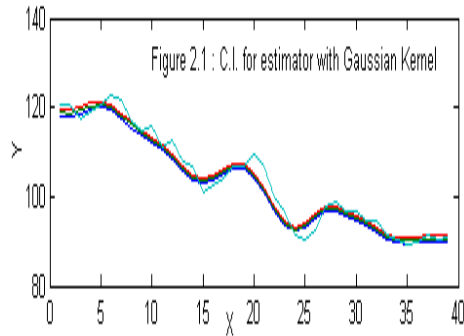
2007





(2)

CLO ) 2010 mhat(x) 2007 CUP





## 5-1 تفسير النتائج والاستنتاجات

Gaussian Kernel (2) .1  
Uniform Kernel

$$\hat{m}(x) = -0.0001x^4 + 0.0062x^3 - 0.175x^2 + 0.6473x + 119.8731$$

(1)

Triweight .2

Gaussian Kernel

(2) (2) .3  
Gaussian Kernel

) Cosinus Kernel

Uniform Kernel

.(

Gaussian



## References

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