مقارنة طرق تقدير معالم نموذج الانحدار
في حالة ظهور مشكلة التعدد الخطي والقيم الشاذة

أ.م.د. نزار مصطفى جواد   م. غفران اسماعيل كمال

كلية الادارة والاقتصاد
جامعة بغداد

المستخلص

Ridge   RLAV   Ridge   LSE
   MM   Ridge   MM   WRID
.   MM   Ridge   RMM .RMM
                        .   RMM

ABSTRACT

A simulation study is used to examine the robustness of some estimators on a multiple linear regression model with problems of multicollinearity and non-normal errors, the Ordinary least Squares (LS) ,Ridge Regression, Ridge Least Absolute Value (RLAV), Weighted Ridge (WRID), MM and a robust ridge regression estimator MM estimator, which denoted as RMM this is the modification of the Ridge regression by incorporating robust MM estimator . finialy, we show that RMM is the best among the other estimators.

1- المقدمة وهدف البحث

$$\beta$$

$$\sum_{i=1}^{n} (y_i - x_i'\beta)^2 = \sum_{i=1}^{n} (r_i)^2$$

$$\beta$$

$$\hat{\beta}_{LS} = (X'X)^{-1} X'Y \text{------------------------}(1)$$

.

.

.

.

2- Ridge

$(X'X)$

β

β β

LS   Ridge   (1970)   Kennard   Horel

β .

Conditional number (CN)

X $(X'X)$. CN :

$$CN = \frac{\lambda_{max}}{\lambda_{min}} \geq 1 \quad \text{------------------------------(2)}$$

λ $(X'X)$.

Belsley   (1980)

CN 5   10

CN 30   100 .

Horel and Kennard   (1970) (a,b)

$(K > 0)$

$X'X$:

$$\beta^{\hat{}}(k) = (X'X + kI)^{-1}X'Y \quad \text{------------------------(3)}$$

:

I $(p*p)$

k>0

k   (b

(1970)   Kennard ,Horel   k   :

$$k = \frac{pS_{LS}^2}{\beta'^{\hat{}}_{LS}\beta^{\hat{}}_{LS}} \quad \text{------------------------------(4)}$$

$$S_{LS}^2 = \frac{(Y - X\beta^{\hat{}}_{LS})'(Y - X\beta^{\hat{}}_{LS})}{n - p} \quad \text{------------------------(5)}$$

p

k=0 $\beta^{\hat{}}_{LS} = \beta^{\hat{}}_{RID}$

k>0 $\beta^{\hat{}}_{RID}$

$k \to \infty$ $\beta^{\hat{}}_{RID} \to 0$.

Kennard & Horel (1970 a)

, k>0 $MSE_{\beta^{\hat{}}_{RID}} < MSE_{\beta^{\hat{}}_{LS}}$

3-

.

Edgeworth (1887)

M   **Huber (1973)**   (**LAV**)

Rousseeuw (1987)   .

(LMS)   **50%**

Yohai (1987)   **(LTS)**

:   **MM**.

-          T_0 θ_0 **50%** .

-

$$r_i(T_0) = y_i - T_0' x_i \text{-----------------------}(6)$$

$$1 \le i \le n$$

M-

$$S_n = S(r(T_0)) \text{--------------------} (7)$$

ρ_0 **M –**   **Huber (1973)**

:

$$\frac{c}{d} = 0.5 \text{----------------------------------}(8)$$

$$d = \max \rho_0(u)$$

-          **M –**

ρ_1 **M –**   :

$$\rho_1(u) \le \rho_0(u) \text{---------------------------}(9)$$

$$\sup \rho_1(u) = \sup \rho_0(u) = d \text{------------}(10)$$

$\psi(t)$ ρ

:

$$\psi(t) = \rho'(t)$$

$$\rho(t), \psi(t)$$

T_1 MM –   .

$$\sum_{i=i}^{n}\psi_i(\frac{r_i}{S_n})x_i = 0 \text{ -----------------------(11)}$$

$$S(T_1) \leq S(T_0) \text{ ---------------------------(12)}$$

$$S(\theta) = \sum_{i=1}^{n}\rho(\frac{r_i}{S_n}) \text{-----------------------(13)}$$

Ridge   **-4**

$\beta_{RID}^{\hat{}}$

Ridge

.   **Ridge**

Ridge

.

Ridge

Ridge   Montgomery   Askin (1980)   **Ullah ,Vinod (1981)**
Ridge   **pfaffenberger, Dielman (1984)**   **(WRID)**
.**(RLAV)**

Ridge   **-4-1**

(1980) Askin ,Montgomery

:

$$\beta_{wls}^{\hat{}} = (X'WX)^{-1} X'WY \text{ -------------------(14)}$$

W

.   **OLS** $\beta_{WLS}^{\hat{}}$

$x_i \rightarrow \sqrt{w_{ii}}\, x_i$
$y_i \rightarrow \sqrt{w_{ii}}\, y_i$

$$w_i = \frac{\psi(y_i - x_i'\beta)}{(y_i - x_i\beta)} \text{ -------------------------(15)}$$

WRID

$$\hat{\beta}_{WRID} = (x'wx + kI)^{-1} x'wy \text{ --------------(16)}$$

Ridge

Lawrence (1984),   **Ridge**

(RLAV)   **Ridge**

$$\hat{\beta}_{RLAV} (x'x + k^*I)^{-1} x'y \text{ ------------------------(17)}$$

k **(19)**   **(18)** k^*

               : k^*

$$k^* = \frac{\rho S_{LAV}^2}{\hat{\beta}'_{LAV} \hat{\beta}_{LAV}} \text{ ------------------------(18)}$$

$$\hat{S}_{LAV} = \frac{(y - x\hat{\beta}_{LAV})'(y - x\hat{\beta}_{LAV})}{n - p} \text{ ------------ (19)}$$

$$\hat{\beta}_{LAV}$$

$$\min_{\beta} \sum_{i=1}^{N} \left| y_i - x_i'\beta \right|$$

5-

RMM

.

(25,50)

            :   **0.5**   **0.8**   **0.9**

-                        .

-                   .

-   - t **3** .

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i$$

$$\beta_0 = \beta_1 = \beta_2 = 1$$

$$x_{ij} = (1 - \rho^2)z_{ij} + \rho z_{ij} \qquad\qquad \text{i=1,2,...n}$$
$$\text{j=1,2}$$

z_{ij}

(1000)

:　　　　　　　　　　　　　　　　(SE)　　　　　　(RMSE)

$$Bias = E(\beta^\wedge) - \beta$$

$$= \overline{\beta}_j - \beta_j$$

$$\overline{\beta}_j = \frac{\sum_{j=1}^{L} \beta_j}{L} \qquad\qquad \text{L=1000}$$

$$MSE(\beta_j^\wedge) = E(\beta^\wedge - \beta)^2 = (\overline{\beta}_j - \beta_j)^2 + \frac{1}{L}\sum_{j=1}^{n}(\beta_j^\wedge - \overline{\beta}_j)^2$$

$$[Var(\beta_j^\wedge) + (Bias)^2]^{\frac{1}{2}} \qquad (MSE(\beta_j^\wedge))^{\frac{1}{2}}$$

.

.　　　　　　　　　　　β_1, β_2　　　　　　　β_0

(5,3,1)
.　　　　(6,4,2)

تحليل نتائج تجربة المحاكاة

(1)　　　　　　　　　　　　　　　　　LS

.

(2)　　　　　LS

OLS　　RMM

LS.

RMM
LS　WRID　MM　　　　　　　Ridge　RLAV
LS　　　　　　　　　　　　　.

(2)　　　　　　　　　(1)
RMM　　　　　　　　　　　　　　　　　　　RMM

.

(2)　　　　RMM　　　　　　　　Ridge　RLAV
WRID , MM , LS

.

MM   (3,4)

.

.   RMM   RMM, RLAV , WRID

(3)   t –

RMSE   SE   (5)   .   (5,6)

RMSE   RMM   .   RMM

. (6)

Ridge   MM   RMM

. ρ^2

الاستنتاجات

RMM .

RMM   WRID ,RLAN ,RMM   Ridge

RLAV ,WRID   .

المصادر

1-   (1990)

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جدول (1) التحيز وجذور متوسطات الخطأ والخطأ المعياري لطريقة المربعات الصغرى وطرق التقدير الاخرى للتوزيع الطبيعي

$$\hat{\beta_1}$$

Values of ρ^2	0.0			0.5			0.95		
Method	Bias	RMSE	S.E	Bias	RMSE	S.E	Bias	RMSE	S.E
LS	-0.154 ..0022	.223 .1445	.2189 .1657	-.0089 -.0041	.3321 .2276	.3365 .2341	.0443 -.0522	3.0011 2.0453	3.0006 2.0564
RIDGE	-.0645 -.0231	.2221 .1453	.2341 .1456	-.0432 -.0166	.3213 .2210	.3211 .2321	-.0122 -.0221	1.5664 1.0543	1.5674 1.0785
RLAV	-.0124 -.0200	.2223 .1465	.2144 .1442	-.0432 -.0199	.2540 .2231	.3215 .2119	-.0421 -.0265	1.5668 1.0774	1.5764 1.0764
WRID	-.0151 -.0023	.2744 .1987	.2344 .1654	-.0122 -.0088	.4211 .3421	.4321 .3422	.0665 -.0643	3.5647 2.4567	3.7845 2.3490
MM	-.0154 -.0030	.2344 .1786	.3443 .1339	-.0112 -.0010	.4332 .2887	.4876 .2887	.0498 -.0453	3.7856 2.8976	3.7856 2.8976
RMM	-.0611 -.0198	.1766 .1433	.2346 .1445	-.0432 -.0223	.3765 .2123	.3422 .2987	.0210 -.0101	1.5678 1.0762	1.534 1.0563

$$\hat{\beta_2}$$

Values of ρ^2	0.0			0.5			0.95		
Method	Bias	RMSE	S.E	Bias	RMSE	S.E	Bias	RMSE	S.E
LS	-.0053 .0056	.2234 .1438	.2256 .1456	.0096 .0095	..3456 .2376	.3452 .2346	-.0576 .0577	3.0654 2.0765	3.0111 2.0786
RIDGE	-.0453 -.0453	.2245 .1477	.2216 .1478	-.0234 -.0076	.3178 .2140	.3454 .2145	.0034 .0056	1.5764 1.0664	1.5342 1.0649
RLAV	-.0453 -.0123	.2267 .1456	.2217 .1453	-.0256 -.0087	.3096 .2185	.3459 .2134	.0134 .0245	1.5683 1.0885	1.5332 1.0756
WRID	-.0123 .0034	.2645 .1876	.2675 .1876	.0123 .0134	.4321 .2765	.4387 .2687	-.0978 .0569	3.5664 2.4533	3.5764 2.3442
MM	-.0056 .0023	.2678 .1756	.2687 .1711	.0099 .0123	.4123 .2756	.4765 .2548	-.0443 .00645	3.7694 2.4769	3.7890 2.4768
RMM	-.0543 -.0231	.2267 .1456	.2210 .1456	-.0452 -.0078	.3078 .2344	.3656 ,2534	-.0476 .0065	1.5340 1.0756	1.5764 1.0701

جدول (2) الكفاءة النسبية للمعلمات التقديرية للتوزيع الطبيعي

Estimator 1vs	Estimator2	$\hat{\beta_1}$		Values	OF	ρ^2	$\hat{\beta_2}$
		0.0	0.5	0.95	0.0	0.5	0.95
RMM	LS	1.04	0.97	0.26	0.99	0.78	0.29
		1.00	0.80	0.27	0.95	0.88	0.27
	RID	1.01	1.01	1.01	1.01	0.98	0.99
		1.00	1.00	1.00	1.02	1.00	0.95
	WRID	0.66	0.55	0.16	0.68	0.55	0.18
		0.63	0.63	0.23	0.75	0.63	0.22
	RLAV	1.01	1.01	0.98	0.98	1.01	0.99
		1.00	1.00	1.00	0.99	1.00	0.95
	MM	0.68	0.58	0.16	0.76	0.66	0.17
		0.71	0.64	0.19	0.63	0.63	0.19
MM	LS	1.54	1.42	1.52	1.38	1.54	1.54
		1.33	1.44	1.44	1.44	1.33	1.48
	RID	1.52	1.87	5.52	1.32	1.87	5.62
		1.44	1.63	5.44	1.36	1.63	5.74
	WRID	0.97	0.97	1.11	0.98	0.97	1.01
		0.80.	0.96	1.10	0.88	0.96	1.00
	RLAV	1.54	1.91	5.62	1.34	1.83	5.62
		1.43	1.65	5.44	1.36	1.54	5.44
RLAV	LS	1.01	0.97	0.26	1.02	0.78	0.29
		1.00	0.81	0.28	0.99	0.80.	0.27
	RID	1.03	1.01	1.01	1.01	1.01	1.01
		1.00	1.00	1.00	1.00	1.00	1.0
	WRID	0.66	0.56	0.16	0.76	0.56	0.29
		0.63	0.53	0.20	0.63	0.53	0. 27
WRID	LS	1.55	1.54	1.32	1.54	1.54	1.54
		1.65	1.43	1.24	1.48	1.43	1.43
	RID	1.52	1.92	0.29	1.44	1.87	5.62
		1.74	1.73	0.,27	1.53	1.63	5.44
RID	LS	1.04	0.78	0.29	1.02	0.76	0.29
		1.00	0.80.	0.,27	0.99	0.83	0. 27

جدول (3) التحيز وجذور متوسطات الخطأ والخطأ المعياري لطريقة المربعات الصغرى وطرق التقدير الاخرى لنوزيع كوشي

$$\hat{\beta_1}$$

Values of ρ^2		0.0			0.5			0.95	
Method	Bias	RMSE	S.E	Bias	RMSE	S.E	Bias	RMSE	S.E
LS	2.786	122.564	125.908	2.564	60.564	60.664	6.278	263.786	263.786
	.0123	18.564	18.786	.345	32.896	32.453	5.786	340.785	340.765
RIDGE	.089	44.765	44.786	.756	27.785	27.454	-1.076	50.764	50.734
	-.432	7.895	7.786	-.231	9.765	8.765	.456	73.342	73.987
RLAV	-.534	.756	.576	-.534	.786	.665	-.234	1.856	1.987
	-.578	.786	.534	.567	.776	.576	-.345	1.056	1.007
WRID	-.290	.576	.456	-.124	.655	.576	.189	3.376	3.564
	-.276	,423	.389	-.189	,433	.456	-.134	1.459	1.456
MM	.043	,489	.576	.006	.789	.634	,576	6.342	6.785
	-.002	.256	.342	-.027	.453	.476	-.256	3.997	3.897
RMM	-.564	.789	.534	-.455	.756	.556	-.126	1.534	1.564
	-.755	.786	.568	-.546	.790	.589	-.345	.956	.908

$$\hat{\beta_2}$$

Values of ρ^2		0.0			0.5			0.95	
Method	Bias	RMSE	S.E	Bias	RMSE	S.E	Bias	RMSE	S.E
LS	3.976	144.432	143.897	1.234	78.897	78.876	-5.342	245.897	245.908
	-.238	17.432	17.564	-.645	36.896	36.984	-5.564	344.765	345.876
RIDGE	.756	41.876	42.776	-.123	21.563	21.876	1.786	46.765	45.907
	-.345	6.985	6.954	-.365	9.563	9.564	-.876	74.908	74.786
RLAV	-.576	.7543	.553	.566	.776	.554	-.232	1.908	1.987
	-.534	.776	.566	-.544	.779	.564	-.334	1.098	1.094
WRID	-.234	.556	.433	-.234	.554	.559	-.267	3.443	3.432
	-.299	.455	.432	-.231	.590	.522	-.121	1.432	1.987
MM	-.012	.487	.445	-.022	.633	.645	-.564	6.786	6.321
	.023	.254	.231	.032	.443	.498	-.238	3.908	3.943
RMM	-.534	.732	.654	-.543	.765	.543	-.349	1.786	1.654
	-.588	.775	.687	-.587	.786	.556	-.228	1.897	.998

جدول (4) الكفاءة النسبية للمعلمات التقديرية لتوزيع كوشي

		$\hat{\beta_1}$					$\hat{\beta_2}$
Estimator 1 vs	Estimator2	0.0	0.5	Values 0.95	OF 0.0	ρ^2 0.5	0.95
RMM	LS	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
	RID	0.00 0.001	0.01 0.07	0.01 0.00	0.00 0.014	0.01 0.06	0.01 0.00
	RLAV	1.00 1.00	0.94 1.00	0.67 0.90	1.00 1.00	0.95 0.98	0.67 0.87
	WRID	2.25 2.99	1.55 2.44	0.21 .47	2.28 2.39	1.26 2.44	0.21 0.07
	MM	3.18 7.71	1.56 2.32	0.06 0.09	3.06 7.23	1.26 2.83	0.07 0.09
MM	LS	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
	RID	0.00 0.01	0.001 0.002	0.012 0.003	0.00 0.002	0.001 0.002	0.018 0.003
	RLAV	0.32 0.13.	0.73 0.34	10.87 13.77	0.33 0.14	0.76 0.34	10.77 13.66
	WRID	0.54 0.43	1.21 0.85	3.62 7.44	0.74 0.36	1.33 0.84	3.32 7.44
RLAV	LS	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
	RID	0.00 1.00	0.001 0.007	0.001 0.000	0.00 0.014	.001 .006	.001 0.00
	WRID	2.66 2.63	1.56 2.53	0.36 0.50	2.76 2.63	1.56 2.53	0.29 0. 57
WRID	LS	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
	RID	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.005	1.87 0.002	0.005 0.00
RID	LS	0.14 0.18	0.28 0.07.	0.05 0.06	0.08 0.17	0.08 0.05	0.04 0. 05

جدول (5) التحيز وجذور متوسطات الخطأ والخطأ المعياري لطريقة المربعات الصغرى وطرق التقدير الاخرى لتوزيع t- بدرجة حرية 3

$$\hat{\beta_1}$$

Values of ρ^2		0.0			0.5			0.95	
Method	Bias	RMSE	S.E	Bias	RMSE	S.E	Bias	RMSE	S.E
LS	-.0012	.3765	.3452	.0123	.5764	.5566	.2331	5.6543	5.1324
	-.0056	.2355	.2785	-.0453	.3452	.3224	-.1342	3.564	3.7856
RIDGE	-.0987	.3247	.3490	-.0342	.4432	.4332	.0987	2.7645	2.5433
	-.0564	.2332	.2543	-.0554	.3498	.3432	-.0453	1.8958	1.7653
RLAV	-.1234	.3421	.3298	-.0121	.4231	.4338	.1234	1.7856	1.7654
	-0972	.2112	.2112	-.0543	.3897	.3555	-.2314	1.3490	1.1233
WRID	-.0022	.3442	.2998	-.0543	.4879	.4432	.1453	3.8754	3.8976
	-.01223	.2217	.2113	-.0004	.3221	.3324	-.0231	2.6545	2.7623
MM	.0034	.3445	.3421	-.0054	.5341	.5443	.2341	4.7856	4.2312
	-.0765	.2765	.2218	-.0088	.3987	.3321	-.0875	2.8976	2.9987
RMM	-.1342	.3765	.3329	-.0983	.4875	.3998	.1121	1.8976	1.7654
	-.0543	.2786	.2221	-.0453	.3287	.3221	-.0564	1.8977	1.2212

$$\hat{\beta_2}$$

Values of ρ^2		0.0			0.5			0.95	
Method	Bias	RMSE	S.E	Bias	RMSE	S.E	Bias	RMSE	S.E
LS	-.0007	.3455	.3456	.0087	.5764	.5764	-.2341	5.1434	5.1433
	-.0013	.2345	.2987	.0123	.3762	.3789	.1564	3.5644	3.4566
RIDGE	-.0934	.3667	.3564	-.0645	.4563	.4576	-.1456	2.3454	2.5433
	-.0542	.2667	.2134	-.0345	.3256	.3121	.0234	1.2987	1.6754
RLAV	-.0176	.3978	.3342	-.0723	.4571	.4567	-.1356	1.7543	1.7756
	-.0056	.2134	.2221	-.0398	.3121	.3111	-.0045	1.3324	1.1543
WRID	-.0453	.3775	.3987	-.0023	.4765	.5762	-.1234	3.6554	3.4589
	-.0056	.2456	.2887	.0003	.3112	.3452	.0934	2.5996	.24766
MM	.0033	.3998	.3122	.0034	.5763	.4378	-.2234	4.5633	4.2223
	.0043	.2111	.2776	.0156	.3121	.3211	.0945	2.9876	2.8343
RMM	.-.0123	.3448	.3514	-.0765	.4235	.4456	-.1234	1.7123	1.7433
	-.0234	.2658	.2113	-.0633	.3116	.3214	-.0015	1.1124	1.1133

جدول (6) الكفاءة النسبية للمعلمات التقديرية لتوزيع t

Estimator 1 vs	Estimator2	$\hat{\beta_1}$ 0.0	0.5	Values 0.95	OF 0.0	ρ^2 0.5	$\hat{\beta_2}$ 0.95
RMM	LS	0.82 0.99	0.49 0.69	0.11 0.10	0.87 0.94	0.53 0.67	0.11 0.10
	RID	0.92 0.92	0.84 0.92	0.47 0.44	0.97 0.94	0.87 0.94	0.22 0.23
	RLAV	1.00 1.00	0.94 1.00	0.97 0.90	1.00 1.00	0.95 0.98	0.97 0.87
	WRID	1.25 1.39	0.66 1.07	0.21 0.22	1.28 1.39	0.66 0.94	0.25 0.26
	MM	1.18 1.71	0.56 1.02	0.15 0.16	1.16 1.23	0.66 1.00	0.15 0.15
MM	LS	0.61 0.74	0.80 0.67	0.74 0.70	0.78 0.60	0.79 0.67	0.74 0.70
	RID	0.69 0.72	1.38 0.90	3.04 2.97	0.84 0.67	1.27 0.92	3.07 2.99
	RLAV	0.73 0.74	1.73 0.99	6.87 6.13	0.83 0.64	1.55 0.94	6.12 6.66
	WRID	0.94 1.03	1.21 1.05	1.32 1.44	1.04 0.96	1.13 0.94	132 1.44
RLAV	LS	0.83 0.99	0.50 0.69	0.12 0.11	0.88 0.94	0.55 0.67	0.12 0.11
	RID	0.93 0.97	0.50 0.96	0.49 0.48	0.95 0.96	.88 .93	.94 0.94
	WRID	1.36 1.33	0.76 1.03	0.22 0.24	1.26 1.43	0.77 0.98	0.29 0. 27
WRID	LS	0.63 0.69	0.72 0.64	0.55 0.47	0.73 0.65	0.71 0.70	0.56 0.47
	RID	0.71 0.68	1.23 0.85	2.23 1.87	0.77 0.76	1.13 0.99	2.30 2.00
RID	LS	0.89 1.08	0.58 0.76	0.24 0.23	0.92 0.98	0.62 0.73	0.24 0. 25